Problem Set IV

1. **Autoencoder (30%).** Train an autoencoder (AE) network (provided in Matlab) with aligned faces obtained from the PS3 question 1. Reconstruct the training data with different sizes of latent (hidden) layers to answer the following questions.

   No regularity in AE:

   1) Can you recover the original aligned faces with a full size of hidden layers? Plot the results.

   2) Set $1\%, 3\%, 10\%$ of hidden layer size (compare with original data dimension) to plot out the reconstruct face and report the reconstruction error.

   With regularity in AE:

   3) Increase the weight w of L_2 regularity term in AE, plot out the reconstruction errors with different weights $w = \{0.1, 0.2, 0.3, 0.4, 0.5\}$.

2. **Regression (70%).** Given data (X, Y) with $X \in \mathbb{R}^d$ and $Y \in \{0, 1\}$, our goal is to train a classifier that will predict an unknown class label $\tilde{y}$ from a new data point $\tilde{x}$. Consider the following model:

$$Y \sim \text{Ber}\left(\frac{1}{1 + e^{-X^T\beta}}\right),$$
$$\beta \sim \text{N}(0, \sigma^2 I).$$

   This is a **Bayesian logistic regression** model. Your goal is to derive and implement a MAP (maximum a posterior) Bayesian inference on β.

   (a) Write down the formula for the unormalized posterior of $\beta \,|\, Y$, i.e.,

$$p(\beta \,|\, y; x, \sigma) \propto \prod_{i=1}^{n} p(y_i \,|\, \beta; x_i) p(\beta; \sigma)$$

   (b) Show that this posterior is proportional to $\exp(-U(\beta))$, where

$$U(\beta) = \sum_{i=1}^{n} (1 - y_i) x_i^T \beta + \log(1 + e^{-x_i^T \beta}) + \frac{1}{2\sigma^2} \|\beta\|^2.$$

   (c) Implement MAP to infer β.

   (d) Use your code to analyze the `iris` data (provided in txt file), looking only at two species, *versicolor* and *virginica*. The species labels are your Y data, and the four features, petal length and width, sepal length and width, are your X data. Also, add a constant term, i.e., a column of 1's to your X matrix. Use the first 30 rows for each species as training data and leave out the last 20 rows for each species as test data (for a total of 60 training and 40 testing). Use the estimated β to get a prediction, $\tilde{y}$, of the class labels for the test data.

   (e) Compare this to the true class labels, y, and see how well you did by estimating the average error rate, $\text{E}[|y - \tilde{y}|]$ (a.k.a. the zero-one loss). What values of σ, ϵ, and L did you use?