

# Green Ammonia Production

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## Sets

$T$  set of timesteps  $t$  that the model will optimize for  
 $I^{EY}$  set of available EY models to build

## Parameters

$d$  demand for end fuel in total simulation period

$\theta_t^W$  capacity factor of wind energy source at timestep  $t$

$\theta_t^S$  capacity factor of solar energy source at timestep  $t$

$\theta^{HSstore}$  efficiency of storing hydrogen

$\theta^{BSstore}$  efficiency of storing energy in battery

$\theta^{HSdeploy}$  efficiency of deploying hydrogen from storage

$\theta^{BSdeploy}$  efficiency of deploying energy from battery

$\theta^{FCdeploy}$  efficiency of converting hydrogen into electricity (in fuel cell)

$\theta^{HSavail}$  vaporization rate of hydrogen storage (often .1 percent/day or .0041 percent per hour)

$\theta^{HBhydrogen}$  efficiency of hydrogen input into HB process

$\theta^{HBnitrogen}$  efficiency of nitrogen input into HB process

$c^W$  CAPEX cost for wind technology generation

$c^S$  CAPEX cost for solar technology generation

$c_i^{EY}$  CAPEX cost for electrolysis model  $i \in I^{EY}$

$c^{HS}$  CAPEX cost for hydrogen storage

$c^{BS}$  CAPEX cost for battery storage

$c^{FC}$  CAPEX cost of fuel cell

$c^{ASU}$  CAPEX cost for ASU

$c^{HB}$  CAPEX cost of haber-bosch process (ammonia plant)

$o^{Wfixed}$  Fixed OPEX cost for wind technology generation

$o^{Sfixed}$  Fixed OPEX cost for solar technology generation

$o_i^{EYfixed}$  Fixed OPEX cost for EY operations in model  $i \in I^{EY}$

$o^{HSfixed}$  Fixed OPEX cost for HS operations

$o^{BSfixed}$  Fixed OPEX cost for BS operations

$o^{FCfixed}$  Fixed OPEXC cost of fuel cell

$o^{ASUfixed}$  Fixed OPEX cost for ASU operations

$o^{HBfixed}$  Fixed OPEX cost for HB operations

$o_i^{EYvariable}$  Variable OPEX cost for EY operations in model  $i \in I^{EY}$

$e_i^{EY}$  energy usage per unit (MWh)output of H2 in EY in model  $i \in I^{EY}$  (kg)

$e^{FC}$  energy density of hydrogen(33.6 KWh of usable energy per kg of hydrogen)

$e^{ASU}$  energy usage (MWh) per unit output of N2 in ASU (kg)

$e^{HB}$  energy usage (MWh) per unit output of NH3 in HB (kg)

$m^{ASU}$  minimum operation of nameplate capacity for ASU

$m^{HB}$  minimum operation of nameplate capacity for HB

$s_i^{EY}$  stack size capacity for electroyzer model  $i \in I^{EY}$  (in MW)

$n$  plant lifetime (number of years) that the plants will be in operation (often 20-30 years)

$n^{FC}$  fuel cell lifetime (often taken as 5 years)

$\delta^{ASU}$  ramping rate for ASU based on decimal percentage

$\delta^{HB}$  ramping rate for HB based on decimal percentage

## Decision Variables

$W^{Cap}$  total nameplate wind capacity to build (MW)

$S^{Cap}$  total nameplate solar capacity to build (MW)

$EY_i^{Cap}$  total number of model stacks  $i \in I^{EY}$  to build (integer)

$BS^{Cap}$  total battery storage capacity to build (MW)

$HS^{Cap}$  total hydrogen storage capacity to build (kg H2 output)

$FC^{Cap}$  fuel cell nameplate capacity (MW)

$ASU^{Cap}$  total ASU capacity to build (kg N2 output)

$HB^{Cap}$  total Haber-Bosch plant capacity to build (kg NH3 output)

$EY_{i,t}^{Gen}$  amount of H2 to produce from stacks of EY model type  $i \in I^{EY}$  at timestep  $t$

$ASU_t^{Gen}$  amount of nitrogen to generate at timestep  $t$

$HB_t^{Gen}$  amount of ammonia to generate at timestep  $t$

$HS_t^{Store}$  amount of hydrogen to store at timestep  $t$

$BS_t^{Store}$  amount of energy to store at timestep  $t$

$HS_t^{Avail}$  amount of hydrogen in storage at end of timestep  $t$

$BS_t^{Avail}$  amount of energy available in battery at end of timestep  $t$

$HS_t^{Deploy}$  amount of hydrogen to deploy to HB at timestep  $t$

$FC_t^{Deploy}$  amount of energy to deploy onto islanded grid from fuel cell at timestep  $t$

$BS_t^{Deploy}$  amount of energy to release into islanded grid at timestep  $t$

## Optimization Model

Objective

$$\min \quad W^{cost} + S^{cost} + EY^{cost} + HS^{cost} + BS^{cost} + FC^{cost} + ASU^{cost} + HB^{cost} \quad (1)$$

where

$$W^{cost} = W^{Cap}(c^W + \sum_{t=0}^n \frac{o^{Wfixed}}{(1+r)^t}) \quad (2)$$

$$S^{cost} = S^{Cap}(c^S + \sum_{t=0}^n \frac{o^{Sfixed}}{(1+r)^t}) \quad (3)$$

$$EY^{cost} = \sum_{i \in I^{EY}} (s_i^{EY} EY_i^{Cap}(c_i^{EY} + \sum_{t=0}^n \frac{o_i^{EYfixed}}{(1+r)^t})) + \sum_{t=0}^n \frac{(\sum_{i \in I^{EY}} o_i^{EYvariable} EY_{i,t}^{Gen})}{(1+r)^t} \quad (4)$$

$$HS^{cost} = HS^{Cap}(c^{HS} + \sum_{t=0}^n \frac{o^{HSfixed}}{(1+r)^t}) \quad (5)$$

$$BS^{cost} = BS^{Cap}(c^{BS} + \sum_{t=0}^n \frac{o^{BSfixed}}{(1+r)^t}) \quad (6)$$

$$FC^{cost} = FC^{Cap}(\frac{n}{n^{FC}} c^{FC} + \sum_{t=0}^n \frac{o^{FCfixed}}{(1+r)^t}) \quad (7)$$

$$ASU^{cost} = e^{ASU} ASU^{Cap}(c^{ASU} + \sum_{t=0}^n \frac{o^{ASUfixed}}{(1+r)^t}) \quad (8)$$

$$HB^{cost} = e^{HB} HB^{Cap}(c^{HB} + \sum_{t=0}^n \frac{o^{HBfixed}}{(1+r)^t}) \quad (9)$$

S.t.

$$\sum_{t \in T} HB_t = d \quad (10)$$

$$E_t^{Demand} \leq E_t^{Gen} \quad \forall t \in T \quad (11)$$

$$E_t^{Demand} = \sum_{i \in I^{EY}} (e_i^{EY} EY_{i,t}^{Gen}) + e^{HS} HS_t^{Avail} +$$

$$\frac{BS_t^{Store}}{\theta_{BSstore}} + e^{ASU} ASU_t^{Gen} + e^{HB} HB_t^{Gen} \quad \forall t \in T \quad (12)$$

$$E_t^{Gen} = \theta_t^{Wind} W_t^{cap} + \theta_t^{Solar} S_t^{cap} + BS_t^{Deploy} + FC_t^{Deploy} \quad \forall t \in T \quad (13)$$

$$BS_t^{Avail} = (1 - \theta^{HS_{avail}})BS_{t-1}^{Avail} + BS_t^{Store} - \frac{BS_t^{Deploy}}{\theta^{BS_{deploy}}} \quad \forall t \in T, BS_0^{Avail} = 0 \quad (14)$$

$$BS_t^{Avail} \leq BS^{Cap} \quad \forall t \in T \quad (15)$$

$$BS_t^{Store} \leq BS^{Cap} - BS_{t-1}^{Avail} \quad \forall t \in T, BS_0^{Store} \leq BS^{Cap} \quad (16)$$

$$\frac{BS_t^{Deploy}}{\theta^{BS_{deploy}}} \leq BS_{t-1}^{Avail} \quad \forall t \in T, BS_0^{Deploy} = 0 \quad (17)$$

$$FC_t^{Deploy} \leq FC^{Capacity} \quad \forall t \in T \quad (18)$$

$$FC_t^{Deploy} \leq \theta^{FC_{deploy}} e^{FC} HS_{t-1}^{Avail} \quad \forall t \in T, FC_0^{Deploy} = 0 \quad (19)$$

$$HS_t^{Avail} = HS_{t-1}^{Avail} + HS_t^{Store} - \frac{HS_t^{Deploy}}{\theta^{HS_{deploy}}} - \frac{FC_t^{Deploy}}{\theta^{FC_{deploy}} e^{FC}} \quad \forall t \in T, HS_0^{Avail} = 0 \quad (20)$$

$$HS_t^{Avail} \leq HS^{Cap} \quad \forall t \in T \quad (21)$$

$$HS_t^{Store} \leq HS^{Cap} - HS_{t-1}^{Avail} \quad \forall t \in T, HS_0^{Store} \leq HS^{Cap} \quad (22)$$

$$\frac{HS_t^{Deploy}}{\theta^{HS_{deploy}}} \leq HS_{t-1}^{Avail} \quad \forall t \in T, BS_0^{Deploy} = 0 \quad (23)$$

$$\frac{HS_t^{Store}}{\theta^{HS_{store}}} \leq \sum_{i \in I^{EY}} EY_{i,t}^{Gen} \quad \forall t \in T \quad (24)$$

$$EY_{i,t}^{Gen} \leq \frac{s_i^{EY}}{e_i^{EY}} EY_i^{Cap} \quad \forall i, t \in I^{EY}, T \quad (25)$$

$$ASU_t^{Gen} \leq ASU^{Cap} \quad \forall t \in T \quad (26)$$

$$HB_t^{Gen} \leq HB^{Cap} \quad \forall t \in T \quad (27)$$

$$|ASU_t^{Gen} - ASU_{t-1}^{Gen}| \leq \delta^{ASU} ASU^{Cap} \quad \forall t \in T, ASU_0^{Gen} \leq ASU^{Cap} \quad (28)$$

$$|HB_t^{Gen} - HB_{t-1}^{Gen}| \leq \delta^{HB} HB^{Cap} \quad \forall t \in T, HB_0^{Gen} \leq HB^{Cap} \quad (29)$$

$$ASU_t^{Gen} \geq m^{ASU} ASU^{Cap} \forall t \in T \quad (30)$$

$$HB_t^{Gen} \geq m^{HB} HB^{Cap} \forall t \in T \quad (31)$$

$$\frac{1}{\theta^{HBhydrogen}} \left( \sum_{i \in I^{EY}} EY_{i,t}^{Gen} - \frac{HS_t^{Store}}{\theta^{HSstore}} + HS_t^{Deploy} \right) = HB_t^{Gen} \forall t \in T \quad (32)$$

$$\frac{1}{\theta^{HBnitrogen}} (ASU_t^{Gen}) = HB_t^{Gen} \forall t \in T \quad (33)$$

$$EY_i^{Cap} \in Z^+ \forall i \in I^{EY} \quad (34)$$

All other decision variables are non-negative reals unless explicitly stated

## Objective and Constraint Explanations

1. Objective: minimize total system operating costs of wind, solar, electrolyzer, hydrogen storage, battery storage, fuel cell, air separation unit, and ammonia plant (Haber-Bosch process)
2. Wind Costs: CAPEX + fixed operation costs multiplied by total number of time periods including the time value of money
3. Solar Costs: CAPEX + fixed operation costs multiplied by total number of time periods including the time value of money
4. Electrolyzer Costs: size of specific electrolyzer stack multiplied by number of stacks to build of that model CAPEX + fixed OPEX + variable electrolyzer generation OPEX costs (capex and opex for EY aligns with USD/MW while variable OPEX is in dollars/kg H<sub>2</sub> )

5. Hydrogen Storage Costs: Hydrogen storage CAPEX + fixed OPEX including time value of money (costs are in USD/kg)
6. Battery Storage Costs: Battery storage CAPEX + fixed OPEX including TVOM
7. Fuel Cell Costs: Fuel cell CAPEX + fixed OPEX including TVOM. For CAPEX need to factor in that lifetime of fuel cell is shorter than lifetime of other plants, so will need to keep building new fuel cells every  $n^{FC}$  years in total  $n$  time period. This leaves us with a total number of  $\frac{n}{n^{FC}}$  to build.
8. Air Separation Unit Costs: CAPEX for ASU + fixed OPEX + variable OPEX (need to multiply by energy usage for ASU as ASU capacity is in kg and the respective CAPEX and OPEX costs are in USD/MW)
9. Ammonia Plant Costs: CAPEX for Ammonia plant + fixed OPEX (need to multiply by energy usage for HB as well as HB capacity is in kg and the respective CAPEX and OPEX costs are in USD/MW)

#### Constraints

10. Total generation of ammonia over time period must meet demand targets
11. Energy consumed by total process operations must be less than or equal to total energy available at each timestep for simulation
12. Energy demand (sum over all power consuming operations): required energy for total generation of electrolyzers + energy required to store  $x$  kg of H<sub>2</sub> + required energy to store and save in battery storage + energy required for ASU to obtain N<sub>2</sub> + energy required for HB process to produce NH<sub>3</sub> (assumed respective process efficiencies are encapsulated by their energy usage parameter)
13. Energy available (sum over all power suppliers at each timestep): respective wind and solar generation + battery storage deployment + fuel cell energy deployment
14. Available power in battery: equal to previous charge + how much you decided to store - how much you deploy and the requirement to deploy for each timestep. Start off with zero charge

15. Battery availability upper bound: Charge of battery must be no more than total capacity of battery
16. Battery storage definition: can only add new charge to battery up to maximum capacity minus what space you already take up.
17. Battery deploy definition: can only deploy enough energy that you have available
18. Fuel cell operation: Fuel cell deployment must be less than or equal to capacity to build.
19. Fuel cell energy deployment definition: must be less than hydrogen available in storage (converting HS (in kg) to MWh and then accounting for efficiency of fuel cell)
20. Hydrogen available definition: equal to previous timestep hydrogen in storage + new hydrogen added to storage - hydrogen deployed (including losses from inefficiencies) - hydrogen deployed through fuel cell. Start off with nothing in storage
21. Hydrogen availability upper bound: can not store more than what the tank can hold
22. Hydrogen storage definition: can only store up to max capacity minus what space you already take up.
23. Hydrogen deploy definition: must have enough hydrogen in storage to deploy how much you want including efficiency
24. Hydrogen storage source: You can only store up to how much hydrogen you produce from EY process
25. Total hydrogen upper bound: you can only generate up to max capacity in kg for each module type
26. Nitrogen generation upper bound: can only produce up to Nitrogen capacity
27. Ammonia generation upper bound: can only generate up to max capacity output for Haber-Bosch process



28. ASU ramping constraint (whether you increase or decrease) must be below a certain percentage of nameplate capacity. Can start off at any production quantity though.
29. Ammonia plant ramping constraint (whether you increase or decrease) must be below a certain percentage of nameplate capacity. Can start off at any production quantity though.
30. Nitrogen generation lower bound: can't fall below a certain production output for nitrogen production
31. Ammonia generation lower bound: can't fall below a certain production output for Ammonia production
32. Hydrogen input for ammonia production: must have stoichmetric balance for hydrogen input for 1 output of ammonia. Hydrogen input is the hydrogen sent directly from electroyzer to plant + any released from storage)
33. Nitrogen input for ammonia production: nitrogen input for ammonia production must meet stoichmetric balance with hydrogen
34. How much EY you build for each model is limited to all real positive integers for each model (all other decision variables are continuous non-negative)