# Green Hydrogen Production

Julian Florez

July 28, 2022

## Sets

$T$ set of timesteps $t$ that the model will optimize for
$I^{EY}$ set of available EY models to build

## Parameters

$d$ demand for end fuel (hydrogen in this case) in total simulation period

$\theta_t^W$ capacity factor of wind energy source at timestep $t$
$\theta_t^S$ capacity factor of solar energy source at timestep $t$
$\theta^{HSstore}$ efficiency of storing hydrogen
$\theta^{BSstore}$ efficiency of storing energy in battery
$\theta^{HSdeploy}$ efficiency of deploying hydrogen from storage
$\theta^{BSdeploy}$ efficiency of deploying energy from battery
$\theta^{HSavail}$ vaporization rate of hydrogen storage (often .1 percent/day or .0041 percent per hour

$c^W$ CAPEX cost for wind technology generation
$c^S$ CAPEX cost for solar technology generation
$c_i^{EY}$ CAPEX cost for electrolysis model $i \in I^{EY}$
$c^{HS}$ CAPEX cost for hydrogen storage
$c^{BS}$ CAPEX cost for battery storage

$o^{Wfixed}$ Fixed OPEX cost for wind technology generation
$o^{Sfixed}$ Fixed OPEX cost for solar technology generation
$o_i^{EYfixed}$ Fixed OPEX cost for EY operations in model $i \in I^{EY}$
$o^{HSfixed}$ Fixed OPEX cost for HS operations
$o^{BSfixed}$ Fixed OPEX cost for BS operations

$o_i^{EYvariable}$ Variable OPEX cost for EY operations in model $i \in I^{EY}$

$e_i^{EY}$ energy usage per unit (MWh)output of H2 in EY in model $i \in I^{EY}$ (kg)
$e^{FC}$ energy density of hydrogen(33.6 KWh of usable energy per kg of hydrogen)

$s_i^{EY}$ stack size capacity for electroyzer model $i \in I^{EY}$ (in MW)

$n$ plant lifetime (number of years) that the plants will be in operation (often 20-30 years)
$n^{FC}$ fuel cell lifetime (often taken as 5 years)

$r$ discount rate on annual basis

## Decision Variables

$W^{Cap}$ total nameplate wind capacity to build (MW)
$S^{Cap}$ total nameplate solar capacity to build (MW)
$EY_i^{Cap}$ total number of model stacks $i \in I^{EY}$ to build (integer)
$BS^{Cap}$ total battery storage capacity to build (MW)
$HS^{Cap}$ total hydrogen storage capacity to build (kg H2 output)
$EY_{i,t}^{Gen}$ amount of H2 to produce from stacks of EY model type $i \in I^{EY}$ at timestep $t$

$HS_t^{Store}$ amount of hydrogen to store at timestep $t$
$BS_t^{Store}$ amount of energy to store at timestep $t$

$HS_t^{Avail}$ amount of hydrogen in storage at end of timestep $t$
$BS_t^{Avail}$ amount of energy available in battery at end of timestep $t$

$HS_t^{Deploy}$ amount of hydrogen to deploy to HB at timestep $t$
$BS_t^{Deploy}$ amount of energy to release into islanded grid at timestep $t$

## Optimization Model

Objective

$$\min \quad W^{cost} + S^{cost} + EY^{cost} + HS^{cost} + BS^{cost} \tag{1}$$

where

$$W^{cost} = W^{Cap}(c^{W} + \sum_{t=0}^{n} \frac{o^{Wfixed}}{(1+r)^t}) \tag{2}$$

$$S^{cost} = S^{Cap}(c^{S} + \sum_{t=0}^{n} \frac{o^{Sfixed}}{(1+r)^t}) \tag{3}$$

$$EY^{cost} = \sum_{i \in I^{EY}} (s_i^{EY} EY_i^{Cap}(c_i^{EY} + \sum_{t=0}^{n} \frac{o_i^{EYfixed}}{(1+r)^t})) + \sum_{t=0}^{n} \frac{(\sum_{i \in I^{EY}} o_i^{EYvariable} EY_{i,t}^{Gen})}{(1+r)^t} \tag{4}$$

$$HS^{cost} = HS^{Cap}(c^{HS} + \sum_{t=0}^{n} \frac{o^{HSfixed}}{(1+r)^t})) \tag{5}$$

$$BS^{cost} = BS^{Cap}(c^{BS} + \sum_{t=0}^{n} \frac{o^{BSfixed}}{(1+r)^t})) \tag{6}$$

S.t.

$$E_t^{Demand} \leq E_t^{Gen} \qquad \forall t \in T \tag{7}$$

$$E_t^{Demand} = \sum_{i \in I^{EY}} (e_i^{EY} EY_{i,t}^{Gen}) + e^{HS} HS_t^{Avail} + \frac{BS_t^{Store}}{\theta^{BSstore}} \qquad \forall t \in T \tag{8}$$

$$E_t^{Gen} = \theta_t^{Wind} W_t^{cap} + \theta_t^{Solar} S_t^{cap} + BS_t^{Deploy} \qquad \forall t \in T \tag{9}$$

$$BS_t^{Avail} = BS_{t-1}^{Avail} + BS_{t-1}^{Store} - \frac{BS_{t-1}^{Deploy}}{\theta^{BSdeploy}} \quad \forall t \in T, BS_0^{Avail} = .3BS^{Cap} \tag{10}$$

$$BS_t^{Avail} \leq BS^{Cap} \qquad \forall t \in T \tag{11}$$

$$BS_t^{Store} \leq BS^{Cap} - BS_t^{Avail} \qquad \forall t \in T \tag{12}$$

$$\frac{BS_t^{Deploy}}{\theta^{BSdeploy}} \leq BS_t^{Avail} \qquad \forall t \in T \tag{13}$$

$$HS_t^{Avail} = (1-\theta^{HSavail})HS_{t-1}^{Avail} + HS_{t-1}^{Store} - \frac{HS_{t-1}^{Deploy}}{\theta^{HSdeploy}} \quad \forall t \in T, HS_0^{Avail} = 0 \tag{14}$$

$$HS_t^{Avail} \leq HS^{Cap} \qquad \forall t \in T \tag{15}$$

$$HS_t^{Store} \leq HS^{Cap} - HS_t^{Avail} \qquad \forall t \in T \tag{16}$$

$$\frac{HS_t^{Deploy}}{\theta^{HSdeploy}} \leq HS_t^{Avail} \qquad \forall t \in T \tag{17}$$

$$\frac{HS_t^{Store}}{\theta^{HSstore}} <= \sum_{i \in I^{EY}} EY_{i,t}^{Gen} \qquad \forall t \in T \tag{18}$$

$$EY_{i,t}^{Gen} \leq \frac{s_i^{EY}}{e_i^{EY}} EY_i^{Cap} \qquad \forall i,t \in I^{EY}, T \tag{19}$$

$$\sum_{t \in T} \sum_{i \in I} ((EY_{i,t}^{Gen}) - \frac{HS_t^{Store}}{\theta^{HSstore}} + HS_t^{Deploy}) = d \tag{20}$$

$$EY_i^{Cap} \in Z^+ \qquad \forall i \in I^{EY} \tag{21}$$

All other decision variables are non-negative reals unless explicitly stated

# Objective and Constraint Explanations

1. Objective: minimize total system operating costs of wind, solar, electroyzer, hydrogen storage, and battery storage

2. Wind Costs: CAPEX + fixed operation costs multiplied by total number of time periods including the time value of money

3. Solar Costs: CAPEX + fixed operation costs multiplied by total number of time periods including the time value of money

4. Electroyzer Costs: size of specific electroyzer stack multiplied by number of stacks to build of that model CAPEX + fixed OPEX + variable electroyzer generation OPEX costs (capex and opex for EY aligns with USD/MW while variable OPEX is in dollars/kg H2 )

5. Hydrogen Storage Costs: Hydrogen storage CAPEX + fixed OPEX including time value of money (costs are in USD/kg)

6. Battery Storage Costs: Battery storage CAPEX + fixed OPEX including TVOM

   Constraints

7. Energy consumed by total process operations must be less than or equal to total energy available at each timestep for simulation

8. Energy demand (sum over all power consuming operations): required energy for total generation of electroyzers + energy required to store x kg of H2 + required energy to store and save in battery storage

9. Energy available (sum over all power suppliers at each timestep): respective wind and solar generation + battery storage deployment

10. Available power in battery: equal to previous charge + how much you decided to store - how much you deploy and the requirement to deploy for each timestep. Start off with zero charge

11. Battery availability upper bound: Charge of battery must be no more than total capacity of battery

12. Battery storage definition: can only add new charge to battery up to maximum capacity minus what space you already take up.

13. Battery deploy definition: can only deploy enough energy that you have available

14. Hydrogen available definition: equal to previous timestep hydrogen in storage (taking into account vaporization rate) + new hydrogen added to storage - hydrogen deployed (including losses from inefficiencies) - hydrogen deployed through fuel cell. Start off with nothing in storage

15. Hydrogen availability upper bound: can not store more than what the tank can hold
16. Hydrogen storage definition: can only store up to max capacity minus what space you already take up.
17. Hydrogen deploy definition: must have enough hydrogen in storage to deploy how much you want including efficiency
18. Hydrogen storage source: You can only store up to how much hydrogen you produce from EY process
19. Total hydrogen generation upper bound: you can only generate up to max capacity in kg for each module type
20. Meet demand rule: total hydrogen production + hydrogen deployed must meet the required demand targets
21. How much EY you build for each model is limited to all real positive integers for each model (all other decision variables are continuous non-negative)