Hw_2: Ch 6: Ex 32, 34, Ch 7: Ex 2, 6, 16, and 20

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Chp 6: 32, 34

32a) Let $X_1, ...X_n$ be a random sample from a uniform distribution on $[0, \theta]$. Then the mle of θ is $\hat{\theta} = Y = \max(X_i)$. Use the fact that $Y \leq y$ iff each $X_i \leq y$ to the derivative the cdf of Y. Then show that the pdf of $Y = \max(X_i)$ is:

$$f_y(y) = \begin{cases} \frac{ny^{n-1}}{\theta^n} & 0 \leq y \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

cdf:

$$F_Y(y) = P(Y \leq y) = P(max(X_i) \leq y$$

all the X_i are small, if max is small:

$$= P(X_1 \leq y * ....X_n \leq y)$$

they are independent:

$$= P(X_1 \leq y) * ....(X_n \leq y)$$

cdf of uniform distribution on $[0, \theta]$:

$$= (\frac{y}{0})^n, 0 \leq y \leq \theta$$

Now taking the derivative:

$$\frac{d}{df_Y} = \begin{cases} \frac{ny^{n-1}}{\theta^n} & 0 \leq y \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

32b) Use the result of part (a) to show that mle is biased but that (n + 1)max(X_i)/n is unbiased.

For the estimator ($E(y) = \theta$) to be unbiased, then:

$$E(Y) = \int_0^\theta y * \frac{ny^n - 1}{\theta^n} dy$$

$$= \frac{n}{\theta^n} \frac{y^n + 1}{n+1} |_0^\theta$$

$$= \frac{n}{n+1} \theta$$

As you can see that $E(Y) \neq \theta$, and $\therefore$ the estimator is biased.

To show that (n + 1)max(X_i)/n is unbiased:

Let $\tilde{Y} = \frac{n+1}{n}$, then

$$E(\tilde{Y}) = E\frac{(n+1)}{n}$$

Using result from (a):

$$= E(Y)\frac{n+1}{n}$$

$$= \theta \frac{n}{n+1} \frac{n+1}{n}$$

Cancel:

$$= \theta$$

This shows that $\tilde{Y}$ is unbiased.