Math 338 Write up_2

Julia Andrus

Note: Hong Le and I discussed and worked on this hw together. We compared answers and asked each other questions.

2.2a) Suppose A=C. What number must be assigned as the measure of "angle" ∠ ABC in order to be consistent with the axioms of angle measure? Explain why assigning any other number would result in a contradiction to one or more of the axioms of angle measure.

We are given that A=C, then we need to assign a number that is between 0 and 180. We need to assign a number between) and 180, because it would be consistent to the Axioms of Angle Measure since the axioms states that assigning a positive number between 0 and 180 would give us an angle measure for $\angle ABC$. Any other number that is not in the interval of 0 and 180 would contradict the properties of the Axioms of angle Measure.

2.2b) Suppose B is between A and C. What number must be assigned as the measure of "angle" ∠ABC in order to be consistent with the axioms of angle measure? Explain why assigning any other number would result in a contradiction to one or more of the axioms of angle measure.

For a number to be assigned as a measure of the angle, that number has to be a positive number between 0 and 180. Having assigning a positive number between 0 and 180 is called to the degree ($° = \epsilon$) of measure and is consistent to the axiom. Any number that is greater or equal 180 and less or equal to 0, then it would be a contradiction of the Axioms of Angle Measure because it would not hold all the properties. If we to assign a number anywhere between 0-180 (ϵ) to $\angle ABC$, then the properties:

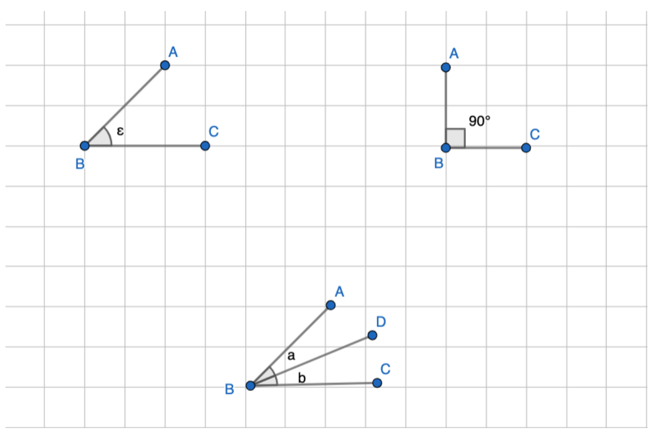
Reference: Class notes

-The degree of measure of right angle is 90°:
-right angle: angle with measure exactly half of a straight angle.

-Measure angle ABC=measure of CBA

-If point D is in the interior of ∠ABC, then the measure of ∠ABC is equal to the sum of the measures of ∠ABD and ∠DBC.

-There exists a unique ray that is the angle bisector of ∠ABC.



As you can see in the drawings, these properties hold with an assigned number between 0-180. Again, if any number that is greater or equal 180 and less or equal to 0, then these properties would not hold which result that $\angle ABC$ would not be consistent to the **Axioms of Angle Measure**.

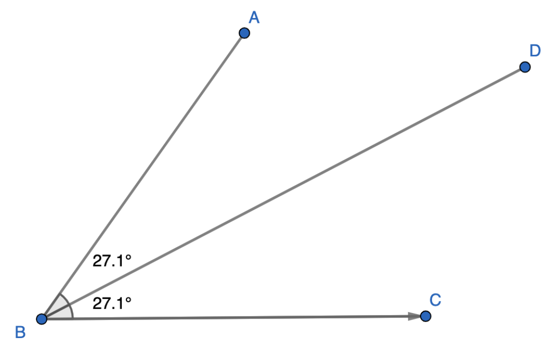
2.4) Congruence and Length Theorem: Two angles are congruent iff they have the same measures.

For iff arguments we have to prove this in two ways:

1) If two angles are congruent, then they have the same measures.

Let ∠ABD and ∠CBD be congruent angles (drawing 1). By definition of ray bisecting an angle, it creates the idea that two angles are adjacent and thus making them the same measures: a ray bisecting an angle ABC resulting adjacent angles ABD and CBD have the same measures. By definition of congruence, there exists a basic rigid motion mapping angle ABD to CBD. This superimposes one figure on to the other figure. So in result, $\angle ABD$ and $\angle CBD$ must have the same measures.

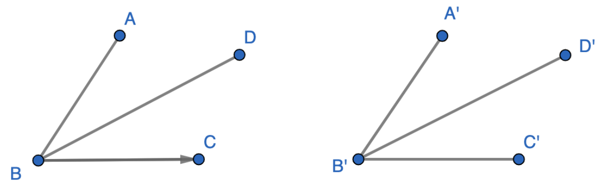
1)



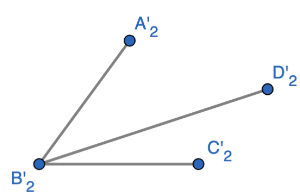
2) If two angles have the same measures, then they are congruent.

Let $\angle ABD$ and $\angle CBD$ be same measures. To prove they are congruent we have to talk about the transformations, which can be explained as being the Axioms of the Isometries. First translate along the $\overrightarrow{BC}$. After that we have $B^{'} = C$ and $\overline{AB} = \overline{A'B^{'}}$. Next, let there be a ray $\overrightarrow{BD}$. If the ray $\overrightarrow{BD}$ = ray$\overrightarrow{B^{'}D'}$, then we don't need to do any more transformations because both original and new drawings have the same angles. So, $\angle A^{'}B^{'}D^{'}$ would be congruent with $\angle C^{'}B^{'}D^{'}$, because of the rays being the same: ray $\overrightarrow{BD}$=ray $\overrightarrow{B^{'}D'}$ (drawing 1).

1)



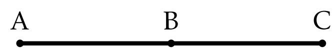
2)



There is a possibility that ray $\overrightarrow{BD} \neq$ ray $\overrightarrow{B^{'}D'}$ (drawing 2), then comes our next transformation. The second transformation is rotation. A rotation centered at B of $\angle ABC$ moving the whole thing counterclockwise or clockwise direction will map $\overrightarrow{B^{'}D'}$to $\overrightarrow{BD}$. Since rotation is an isometry and preserves angle measure, then it would make $\overrightarrow{BD}$ = ray $\overrightarrow{B^{'}D'}$ true. The translation, followed by the rotation makes this statement true.

2.5) Suppose that B is between A and C. Prove $\overline{AB} \cong \overline{AC}$.

We are given B is between A and C, lets draw a picture of this:



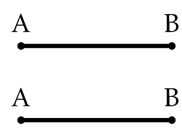
Now that we have a drawing, I want to prove this by using the Axiom of Equivalence:

1.) Reflexive: X≅X (Geometric figures must be congruent to itself

2.) Symmetric: If X≅Y, then Y≅X

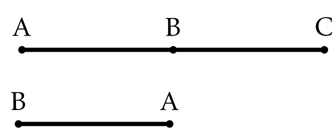
3. Transitive: X≅Y and Y≅Z, then X≅Z

Starting with 1): I want to let X=$\overline{AB}$, then



For this being congruence by itself, there needs to be the idea of sameness; they need to have the same length. The drawing shows AB≅AB. We can use the idea of translation(where the image is translated with the assigned vector).

Now with 2): I want to let X=$\overline{AB}$ and $Y = \overline{BA}$, then X≅Y and Y≅ X ($AB \cong BA$ and $BA \cong AB$):



This is true, because of the Axioms of Length notes that $\overline{AB}$ and $\overline{BA}$. I used the idea of reflexive, which was one of the transformations.

For 3): let $X = \overline{AC}$, $Y = \overline{AB} + \overline{BC}$, and $Z = \overline{BC} + \overline{AB}$.

Assume

$$X \cong Y and Y \cong Z$$

then by $X \cong Y$ (got this idea from the Axioms of Length: AC=AB+BC)

$$\overline{AC} = \overline{AB} + \overline{BC}$$

and by $Y \cong Z$ (AB≅AB) (got this idea from one the basic rigid motions: reflexive)

$$\overline{AB} + \overline{BC} = \overline{BC} + \overline{AB}$$

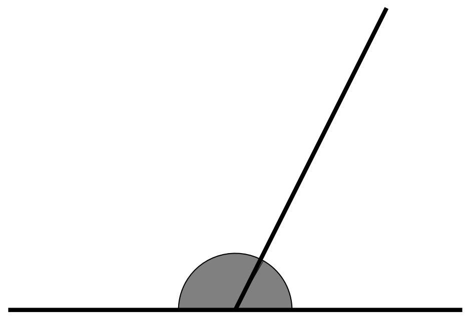
Since both of those $X \cong Y$ and $Y \cong Z$ are true, then $X \cong Z$ must be true

$$\overline{AC} = \overline{BC} + \overline{AB}$$

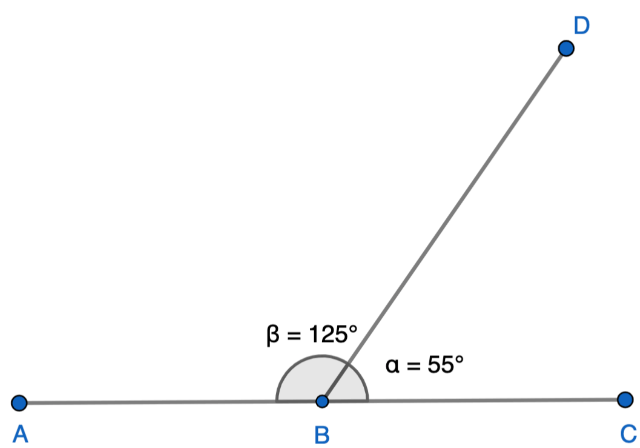
This is true because of the Axioms of the Length (AC= AB+BC). With all these being true, then passes these steps in the Axiom of Equivalence.

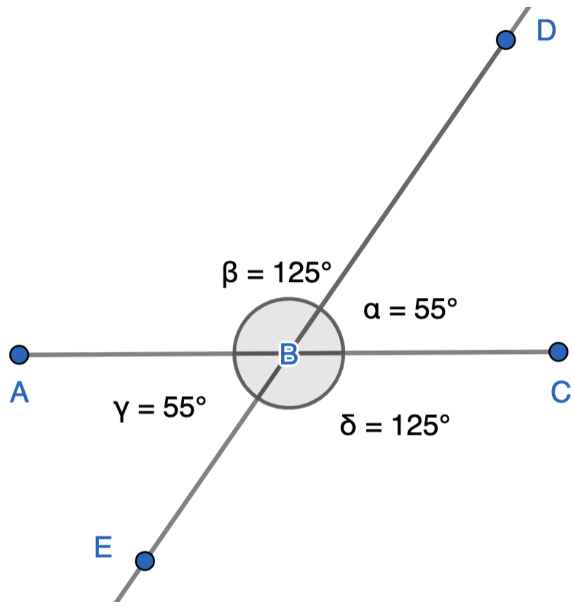
2.6) Suppose ∠ABC and ∠CBD are supplementary and ∠ABC≅∠EBD. Prove ∠ABE≅∠CBD

First, the definition of supplementary is where the two angle's measure is the sum of a straight angle's measure (180):



Now, I want draw what is given: ∠ABC and ∠CBD are supplementary.





I used GeoGebra to draw these.

Let there be segment EB, then we have all four angles equaling to 360° : $\beta + Y + \alpha = 360°$. We are given that ∠ABC and ∠CBD are supplementary and ∠ABC≅∠EBD, then it must be that $\angle ABC$ and $\angle EBD$ have the same angles.

By definition congruence of ∠ABE≅∠CBD, then the idea isometry or isometries hold. So there exists a basic rigid motion mapping angle ABD to CBD. This superimposes one figure onto the other figure, so $\angle ABD$ and $\angle CBD$ must have the same measures.

1) Translation: A rigid motion that slides the plane along a fixed non-zero vector.

If we have a vector A, going to the right, then we would have A beside a translation by vector A is a point A' such that AA'=vector A.

2) Rotation: Rotation about the center 0 of angle θ

If we did a rotation of 360 or 180 degrees, then we still get the two same measures of ∠ABE≅∠CBD.

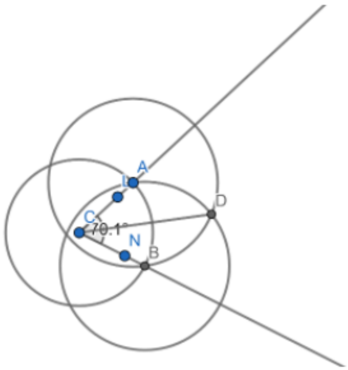
3) Reflection: Reflection of the figures

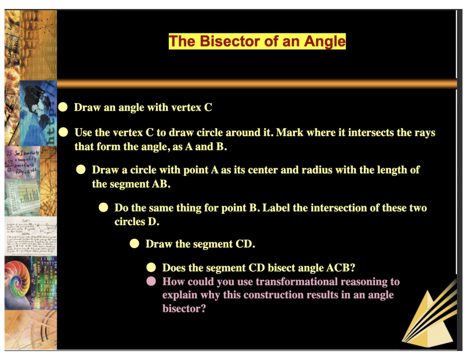
Let there be a perpendicular bisector GH through $\angle ABD$ and $\angle EBC$, then there will be a reflection of $\angle ABE$ and ∠CBD. With the reflection, we can see that $\angle ABE$ and ∠CBD have the same measures. Since the perpendicular bisector intersect the lines, then they form congruent adjacent angles.

At last, ∠ABE≅ ∠CBD because it surpasses the fundamental idea of the axioms of isometries.

5) The construction for the angle bisector in the slides from class has at least two flaws. Identify flaws in the instructions and suggest how to fix them.

Here is our groups' drawing from the slide instructions and the slide with instructions:

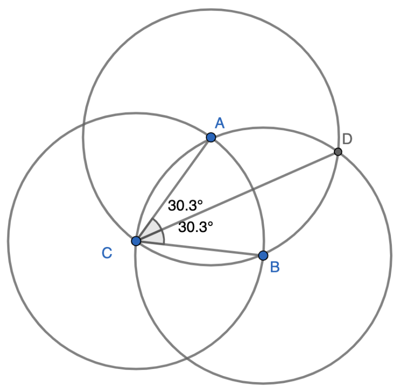




Now that we have the instructions and the drawing together, we can look for the flaws:

1) C has two be intersection too. The instruction says that to use vertex C to draw a circle around it, but that is a mistake because if there is an intersection D then there needs to be an intersection at C to make segment bisect angle ACB.

2) Instead of the third bullet point, it should say draw a circle with point A as its center and having radius with length of AC. This goes same for B with radius length of BC. With this, the drawing would show the idea of equality.



A
D
30.3°
30.3°
C
B