

HW_2: Ch 6: Ex 32, 34, Ch 7: Ex 2, 6, 16, and 20

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Chp 6: 32, 34

32a) Let X_1, \dots, X_n be a random sample from a uniform distribution on $[0, \theta]$. Then the mle of θ is $\hat{\theta} = Y = \max(X_i)$. Use the fact that $Y \leq y$ iff each $X_i \leq y$ to the derivative the cdf of Y . Then show that the pdf of $Y = \max(X_i)$ is:

$$f_Y(y) = \begin{cases} \frac{ny^{n-1}}{\theta^n} & 0 \leq y \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

cdf:

$$F_Y(y) = P(Y \leq y) = P(\max(X_i) \leq y)$$

all the X_i are small, if max is small:

$$= P(X_1 \leq y * \dots * X_n \leq y)$$

they are independent:

$$= P(X_1 \leq y) * \dots * (X_n \leq y)$$

cdf of uniform distribution on $[0, \theta]$:

$$= \left(\frac{y}{\theta}\right)^n, 0 \leq y \leq \theta$$

Now taking the derivative:

$$\frac{d}{df_Y} = \begin{cases} \frac{ny^{n-1}}{\theta^n} & 0 \leq y \leq \theta \\ 0 & \text{otherwise} \end{cases}$$

32b) Use the result of part (a) to show that mle is biased but that $(n + 1)\max(X_i)/n$ is unbiased.

For the estimator $(E(y) = \theta)$ to be unbiased, then:

$$\begin{aligned} E(Y) &= \int_0^\theta y * \frac{ny^n - 1}{\theta^n} dy \\ &= \frac{n}{\theta^n} \frac{y^n + 1}{n + 1} \Big|_0^\theta \\ &= \frac{n}{n + 1} \theta \end{aligned}$$

As you can see that $E(Y) \neq \theta$, and \therefore the estimator is biased.

To show that $(n + 1)\max(X_i)/n$ is unbiased:

Let $\tilde{Y} = \frac{n+1}{n}$, then

$$E(\tilde{Y}) = E \frac{(n + 1)}{n}$$

Using result from (a):

$$\begin{aligned} &= E(Y) \frac{n + 1}{n} \\ &= \theta \frac{n}{n + 1} \frac{n + 1}{n} \end{aligned}$$

Cancel:

$$= \theta$$

This shows that \tilde{Y} is unbiased.

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