

MATH 321: 10-22 IN-CLASS WORK

Exercise 1. Use induction to prove for all natural numbers n ,

$$2^0 + 2^1 + \cdots + 2^n = 2^{n+1} - 1.$$

Exercise 2. Explain what's wrong with the following "proof".

Theorem. Every integer $x \in \mathbb{Z}$ satisfies $x^3 > -1$.

Proof. We prove this by induction. The base case $x = 0$ is easy. Clearly, $0^3 = 0 > -1$. Next is the induction step. Suppose $x^3 > -1$. Then $(x + 1)^3 > x^3$, because the cube function is an increasing function, and so $(x + 1)^3 > -1$. So we have seen that $x^3 > -1$ for all integers x . \square

Exercise 3. Prove that every logical formula is equivalent to one without any \exists s or \forall s.