

MATH 013: 2/12 WORKSHEET

FRACTIONS 1

What is a fraction?

$$\frac{\text{numerator}}{\text{denominator}} \quad \text{or} \quad \text{numerator/denominator}$$

You can think of a fraction as counting pieces of a whole. The *denominator*, written on the bottom of the fraction, is how many pieces the whole was broken into. The *numerator*, written on the top, is how many pieces you have.

- If you eat $3/4$ of a pizza you can think of this as meaning, the pizza was divided into 4 pieces and you got 3 of them.
- If you eat $6/8$ of a pizza you can think of this as meaning, the pizza was divided into 8 pieces and you got 6 of them.
- These are the same quantity of pizza, just cut up into a different number of slices. So these two fractions represent the same quantity.

What does a fraction do?

Another way you can think of a fraction is as shorthand for multiplying and dividing. The numerator represents the multiplication. The denominator represents the division.

- The expression $\frac{3}{4}x$ means multiply the unknown quantity x and divide by 4.
- The expression $\frac{3}{4} \cdot \frac{2}{5}$ can be thought of in two ways: multiply the quantity $\frac{2}{5}$ by 3 and divide by 4, or multiply the quantity $\frac{3}{4}$ by 2 and divide by 5.
- The expression $\frac{5}{7}$ means multiply the quantity 1 by 5 and divide by 7.
- The expression $\frac{9}{0}$ doesn't make sense because division by 0 is undefined.

How to multiply fractions.

This way of thinking tells you how to multiply two fractions.

$$\frac{n}{d} \cdot \frac{m}{c} = \frac{nm}{dc}.$$

You multiply the numerators to get the new numerator, since that represents the total multiplication. And you multiply the denominators to get the new denominator, since that represents the total division.

- $\frac{5}{2} \cdot \frac{3}{4} = \frac{15}{8}$ since multiplying by 5 then 3 is the same as multiplying by 15 and dividing by 2 then by 4 is the same as dividing by 8.
- $\frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12}$ because multiplying by 2 then 3 is the same as multiplying by 6 and dividing by 3 then 4 is the same as dividing by 12.
- $\frac{1}{2} \cdot \frac{x}{y} = \frac{x}{2y}$ because multiplying by 1 then x is the same as multiplying by x and dividing by 2 then y is the same as dividing by $2y$.

Simplifying fractions.

This way of thinking also explains why you can simplify fractions. If you have the same term in both the numerator and denominator then they cancel out.

- $\frac{2 \cdot 3}{2 \cdot 4} = \frac{3}{4}$ because multiplying by 2 then dividing by 2 doesn't change the value.
- $\frac{3xz}{3xy} = \frac{z}{y}$ because multiplying by $3x$ then dividing by $3x$ doesn't change the value.
- $\frac{4}{6} = \frac{2}{3}$ because $4 = 2 \cdot 2$ and $6 = 2 \cdot 3$, and dividing then multiplying by 2 doesn't change the value.
- $\frac{2}{3} \cdot \frac{3}{5} = \frac{2}{5}$ because multiplying by 3 then dividing by 3 doesn't change the value.

Dividing a fraction.

This way of thinking also explains how to divide by a fraction. Division is the opposite of multiplication, so when you divide by a fraction you swap the role of the numerator and denominator. In other words, dividing by a fraction is the same as multiplying by its *reciprocal*—the fraction obtained by swapping the numerator and denominator.

$$\frac{n/d}{m/c} = \frac{n}{d} \cdot \frac{c}{m} = \frac{nc}{dm}$$

- $\frac{3}{1/2} = 3 \cdot 2 = 6$ because dividing by $1/2$ means swapping the roles. Instead of multiplying by 1 you divide by 1 and instead of dividing by 2 you multiply by 2.
- $\frac{3/4}{2/5} = \frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8}$ because you swap the roles of the fraction being divided. The numerator 2 becomes the new denominator, representing division, while the denominator 5 becomes the new numerator, representing multiplication.
- $\frac{x/3}{2a/3} = \frac{x}{3} \cdot \frac{3}{2a} = \frac{x}{2a}$ because we multiply by the reciprocal and can then cancel out the multiplication and division by 3.

PRACTICE PROBLEMS

(1) Multiply the following fractions, simplifying your answer.

(a) $\frac{2}{5} \cdot \frac{7}{3}$

(b) $\frac{2}{4} \cdot \frac{3}{11}$

(c) $\frac{4a}{3} \cdot \frac{6}{7}$

(d) $\frac{2x^2}{3} \cdot \frac{6}{x}$

(e) $\frac{2}{3} \cdot \frac{3}{5} \cdot \frac{4}{5}$

(f) $\frac{a}{2} \cdot \frac{3}{ab} \cdot \frac{x^2}{2y}$

(2) Perform the following divisions, simplifying your answer.

(a) $\frac{2/5}{2/5}$

(b) $\frac{3/4}{4/3}$

(c) $\frac{4/15}{8/3}$

(d) $\frac{2x/3}{3/4}$

(e) $\frac{ax/3}{(6a)/x}$

(f) $\frac{(2ax^2)/15}{(3a^2)/(4x)}$