MATH 243: SECTION 14.5 GROUPWORK

Recall the following form of the chain rule: Suppose $z = z(x, y)$ is a differentiable function of x and y where $x = x(s, t)$ and $y = y(s, t)$ are differentiable functions of s and t. Then,

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}, \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}.$$

There is also a version if z is a function of ≥ 3 variables or if $x, y, \ldots$ are functions of a different number of variables. If z is a function of more variables you need a summand for each variable, and if $x, y, \ldots$ are functions of more variables $u, \ldots$ then you can compute $\frac{\partial z}{\partial u}$ in the same way, replacing all s's or t's in the formula with u's.

(1) Suppose $w = \ln(x^2 + y^2 + z^2)$ and $x = e^t$, $y = e^{-t}$, $z = t$. Calculate $\frac{\partial w}{\partial t}$.

(2) Suppose $z = \frac{x}{y}$ and $x = t\sin(s)$ and $y = s\cos(t)$. Use the chain rule to compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at the point $(s, t) = (1, 0)$.

(3) Suppose wheat production W in a year is a function of the average temperature T and annual rainfall R. Our best science says that average temperature is increasing at a rate of 0.15°C per year and rainfall is decreasing at a rate of 0.1cm per year. Moreover, our best science estimates that, given current productive capacities, that $\partial W/\partial T = -2$ and $\partial W/\partial R = 8$.
- Explain what these two partial derivatives and two derivatives mean.
- Using this information, estimate the current rate of change of wheat production $\mathrm{d}W/\mathrm{d}t$.