MATH 321: 9-8 GROUPWORK

(1) Consider the following logical formula.

$$\forall \varepsilon > 0 \; \exists \delta > 0 \; \forall x \; (|x - a| < \delta \to |f(x) - f(a)| < \varepsilon)$$

Identify the variables in this formula. Which variables are free and which are bound? Translate this formula to an English sentence. Do you recognize the property this formula is a definition of?

(2) Consider the following logical formula.

$$\forall x \forall y \; (y \neq 0 \to \exists z \; (y \cdot z = x))$$

Identify the variables in this formula. Which are free and which are bound? Translate this formula into an English sentence.

Interpret this formula as quantifying over $\mathbb{Z}$ as the universe of discourse. Is the formula true or false? Explain why. Now interpet this formula as quantifying over $\mathbb{Q}$ as the universe of discourse? Is the formula true or false? Explain why. What if you use $\mathbb{R}$ as the universe of discourse?

(3) Consider the four universes of discourse $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$. Come up with logical formulae (with no free variables) which are true in one universe of discourse but not the other three universes of discourse. For an extra challenge, can you use $<$ as your only non-logical symbol in the formulae?