Math 113 Quiz 1

Friday, March 7

Name: _____Answer Key_____

This is the first quiz. There are four parts, for a total of 100 points.
No electronic devices are permitted. You should have with you one page of notes. Turn in your notes with your quiz when finished. Carefully read each question and understand what is being asked before you start. Note that some parts give you a choice of which questions to answer.

1. (10 points) **Core concepts.** Do both questions.

   a. (5 points) Is the following argument valid or invalid? Write a sentence explaining why.

   - If Mr. Kinbote is the king of Zembla, then he should have poetry written about him.
   - Mr. Kinbote is not the king of Zembla.
   - ∴ He should not have poetry written about him.

   Invalid. There could be other reasons to write poetry about him.

   The argument has the form $K \rightarrow P$
   $\neg K$
   $\neg P$

   This is a well-known fallacy called "denying the antecedent".

   b. (5 points) Give an example of an argument which is valid but not sound. Write a sentence explaining why your argument is valid but not sound.

   Ex 1

   - Trump is a good president.
   - ∴ Trump is a good president

   The conclusion is a premise so it is valid, but the premise is not true.

   Ex 2

   - The sky is green.
   - The ocean is pink.
   - ∴ The sky is green and the ocean is pink.

   Valid, but the premises are not true.

2. (15 points) **Translating.** Do both questions, using the following symbolization key.

L	Dr. Humbert is a professor of French literature.
M	Dr. Humbert murdered Ms. Haze
G	Dr. Humbert is of good moral character.
P	Dr. Humbert should be in prison.

a. (5 points) Translate the following English sentences into truth functional logic.

- If Dr. Humbert murdered Ms. Haze then he is not of good moral character and should be in prison.

$$M \rightarrow (\neg G \wedge P)$$

- Dr. Humbert didn't murder Ms. Haze but he nonetheless is not of good moral character.

$$\neg M \wedge \neg G$$

b. (10 points) Translate the following sentences of truth functional logic into English.

- $L \leftrightarrow (G \wedge \neg P)$

Dr. Humbert is a professor of French literature if and only if he is of good moral character & shouldn't be in prison

- $G \vee (L \wedge P)$

Either Dr. Humbert is of good moral character or he is a professor of French literature who should be in prison.

3. (40 points) **Truth tables.** Do 2 of the 3 questions, labeling the work for each problem. If you attempt all 3, cross out the work you do not want to be graded.

a. (20 points) Your friend insists that the argument $A \wedge (A \to B), \therefore A \vee B$ is invalid. Use a truth table to check their work. Say whether they are correct, and write a sentence explaining why.

b. (20 points) Your friend insists that the sentence $(A \to B) \vee (B \to C) \vee (C \to A)$ is a tautology. Use a truth table to check their work. Say whether they are correct, and write a sentence explaining why.

c. (20 points) Your friend insists that the sentences $\neg(\neg A \wedge \neg B)$ and $A \vee B$ are equivalent. Use a truth table to check their work. Say whether they are correct, and write a sentence explaining why.

a)

A	B	A∧(A→B)	A∨B
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

In every row where the premise is true the conclusion is true. Therefore it is valid.

b)

A	B	C	(A→B)∨(B→C)∨(C→A)
T	T	T	T T T
T	F	F	T T F
T	F	T	F T T
T	F	F	F T T
F	T	T	T T T
F	T	F	T T F
F	F	T	T T T
F	F	F	T T T

Tautology, because every row is true.

c)

A	B	¬(¬A ∧ ¬B)	A ∨ B
T	T	T F F F	T
T	F	T F F T	T
F	T	T T F F	T
F	F	F T T T	F

Equivalent, because the columns are the same.

4. (35 points) **Formal proofs**. Do 3 of the 4 problems. If you attempt all 4, cross out the work you do not want to be graded. (Note that c. and d. are each worth 5 points more than a. and b., so if you do both c. and d. you can earn 5 points extra credit.)

a. (10 points) Your friend tells you that they have produced a formal proof showing that the argument $A \lor B, A \to C, \therefore C \lor B$ is valid. You are doubtful. Their proof is below. Identify every error, writing a sentence for each to explain.

1	$A \lor B$	Pr	
2	$A \to C$	Pr	
3	A	$\lor$E, 1	← 1
4	B	$\lor$E, 1	← 1
5	C	$\to$I, 2, 3	← 2
6	$C \lor B$	$\lor$I, 4, 5	← 3

The friend has confused ∨ for ∧. At (1), they applied the rule for ∧E not ∨E. At (3), they applied the rule for ∧I, not ∨I. (2) is →E, not →I

b. (10 points) Your friend asks for help with their logic homework. They have produced a proof, but need to fill in the annotations. Fill in the annotations for them, and say what argument the proof is for.

1	$A \land (A \to B)$	Pr	
2	A	∧E, 1	
3	$A \to B$	∧E, 1	
4	B	→E, 3, 2	
5	$\neg B$	As	
6	$\bot$	¬E, 4, 5	
7	$\neg\neg B$	¬I, 5-6	

The proof is for:
$A \land (A \to B)$
$\therefore \neg\neg B$

c. (15 points) Write a formal proof to verify that the argument $A \lor B$, $A \to C$, $\therefore B \lor C$ is valid. Your proof must include annotations for every line. Note that you probably don't need the full number of lines given.

1	$A \lor B$ Pr
2	$A \to C$ Pr
3	A As
4	C $\to$E, 2,3
5	$B \lor C$ $\lor$I, 4
6	✗
7	B As
8	$B \lor C$ $\lor$I, 7
9	$B \lor C$ $\lor$E, 1, 3–5, 7–9
10	
11	
12	
13	
14	
15	
16	

line 6 got skipped

d. (15 points) Your friend says that the sentences $A \to B$ and $\neg B \land A$ are jointly consistent. Prove them wrong by producing a formal proof demonstrating the sentences are jointly inconsistent. Your proof must include annotations for every line. Note that you probably don't need the full number of lines given.

1	$A \to B$	Pr
2	$\neg B \land A$	Pr
3	A	$\land$E, 2
4	$\neg B$	$\land$E, 2
5	B	$\to$E, 1, 3
6	$\bot$	$\neg$E, 4, 5
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