MATH454 HOMEWORK 12
DUE THURSDAY, NOVEMBER 21

In class we discussed club and stationary subsets on ω_1. This homework generalizes these concecepts to any regular uncountable cardinal κ. To be clear, here are the definitions, which are the definitions for ω_1 but with ω_1 replaced by κ: A set $C \subseteq \kappa$ is *closed* if for all $\alpha < \kappa$ we have that $\sup(C \cap \alpha) \in C$ whenever $C \cap \alpha \neq \emptyset$. And $C \subseteq \kappa$ is *unbounded* if $\sup C = \kappa$. Then $C \subseteq \kappa$ is *club* if C is closed and unbounded. And $S \subseteq \kappa$ is *stationary* if $S \cap C \neq \emptyset$ for all club $C \subseteq \kappa$.

Throughout the remainder fix κ a regular uncountable cardinal.

Exercise 1. Let C be the collection of limit ordinals $< \kappa$. Show that C is club.

Exercise 2. Do Exercise 7.7 from the textbook. (page 155)

Exercise 3. Do Exercise 7.8 from the textbook.

Exercise 4. Do Exercise 7.9 from the textbook.

For $\lambda < \kappa$ a regular cardinal let $E_\lambda = \{\alpha < \kappa : \mathrm{cf}(\alpha) = \lambda\}$.

Exercise 5. Show that if $\lambda < \kappa$ is regular then E_λ is stationary.

Exercise 6. Use the previous exercise to show that if $\kappa > \omega_1$ then the club filter on κ—that is, the collection of subsets of κ which contain a club as a subset—is not an ultrafilter. This is easier than the argument we did for ω_1!