Math 1420: Study Guide for Midterm 1

General comments:

- The exam is about 10 to 15 questions, broken up by the four learning objectives. Each learning objective is scored out of 100, and I will report your grades separately by learning objective.
- Calculators and notes are not allowed for the exam. The questions are written with the fact that these are not allowed in mind. In particular, you should expect that any numbers involved in calculations will be relatively small and manageable.
- Show your work! For one, understanding the process and how to communicate your logic to others is more important than being able to produce a correct answer with no explanation. For another, I cannot give partial credit if you show no work.

Here's what you should know for each learning objective.

1. Conceptual Understanding
   - How to determine limits given the graph of a function.
   - How to find and classify discontinuities given the graph of a function.
   - How the derivative of a function relates to the shape of its graph.
2. Formal Understanding
   - The definition of continuity and how to classify discontinuities based on limits at a point.
   - What the intermediate value theorem says, and how to determine whether it applies in a particular situation.
   - How to use the definition of the derivative to compute the derivative of a function, both at a point and for an arbitrary input x.
3. Rules for Calculations
   - Rules for calculating limits, including: how to calculate a limit for a continuous function, the composite function theorem, the squeeze theorem.
   - How to determine limits as $x \to \pm\infty$.
   - How to calculate a limit for an expression which gives an indeterminate form.
4. Approximations and Applications
   - Given values of a function close to $x = a$, approximate the limit of the function as $x \to a$, or explain why the limit does not exist.
   - Given values of a function close to $x = a$, determine whether the function is continuous at $x = a$.
   - Use difference quotients to approximate the derivative of a function.

Here's some helpful tricks and bits of knowledge you should have for the exam.

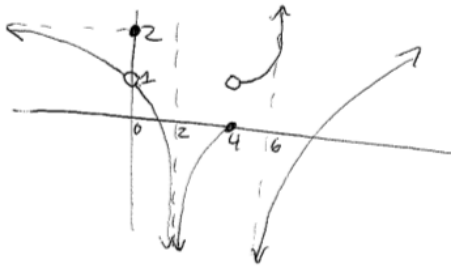
- Know the graphs for basic functions: x^n, $1/x^n$, $\sqrt[n]{x}$, exponential functions, logarithms, and trig functions. Know how to use these to determine limits involving asymptotes.

- The multiplying by the conjugate trick for rewriting an expression.

Here are some sample questions similar to what you should expect to see on the exam.

1. Use the graph of the function $f(x)$ below to determine the limits

$$\begin{array}{lll} \lim_{x\to-\infty} f(x), & \lim_{x\to 0} f(x), & \lim_{x\to 2} f(x), \\ \lim_{x\to 4^-} f(x), & \lim_{x\to 4^+} f(x), & \lim_{x\to 4} f(x), \\ \lim_{x\to 6^-} f(x), & \lim_{x\to 6^+} f(x), & \lim_{x\to 6} f(x). \end{array}$$



2. Consider the function

$$f(x) = \frac{2x(x+1)}{(x+1)(x-2)^2}.$$

Calculate the limits

$$\lim_{x\to-\infty} f(x), \quad \lim_{x\to-1} f(x), \quad \lim_{x\to 0} f(x), \quad \lim_{x\to 2} f(x), \quad \lim_{x\to\infty} f(x)$$

3. Calculate the limit

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x-3}-1}.$$

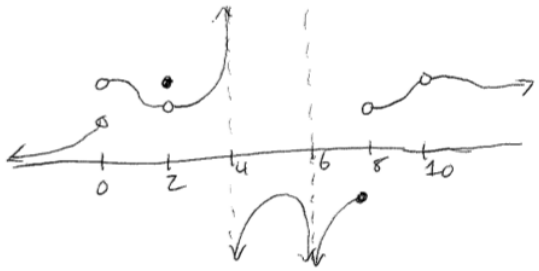
4. Use the squeeze theorem to compute the limit

$$\lim_{x\to 0^+} x^2 \sin(\ln x).$$

5. Using the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, compute the limit

$$\lim_{x\to 0} 3\ln\left(\frac{\sin x}{x}\cdot(x+1)\right).$$

6. Use the graph below of the function $f(x)$ to list all the values of x where $f(x)$ has a discontinuity. For each, state what kind of discontinuity (removable, jump, or infinite) it is.



7. Consider the piecewise-defined function

$$f(x) = \begin{cases} x^2 + 4x & \text{if } x < 0 \\ 2^x - 1 & \text{if } 0 < x < 4 \\ 30 - x^2 & \text{if } 4 < x \end{cases}$$

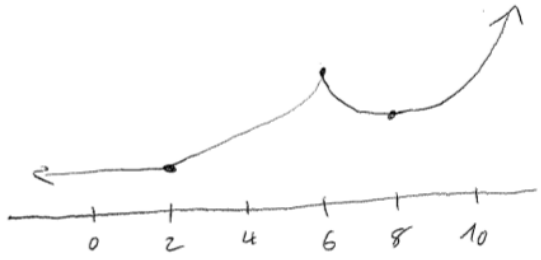
This function is undefined and thus discontinuous at $x = 0$ and $x = 4$. For each discontinuity, determine whether it is a removable discontinuity or a jump discontinuity. Justify your answers with limit calculations.

8. You know that $f(x)$ is continuous on the interval $[-3, 3]$, that $f(-3) = 4$, and $f(3) = 1$. Can you conclude that $f(x)$ has no zeroes? Justify your answer with a sentence.

9. You know that $f(x)$ is continuous everywhere, that $f(0) = 1$, and $f(10) = 20$. Can you conclude that there is an input x for which $f(x) = 10$? Justify your answer with a sentence.

10. Use the graph below of the function $f(x)$ to say whether $f'(x)$ is positive, negative, zero, or undefined at the points:

$$x = 0, \qquad x = 2, \qquad x = 4, \qquad x = 6, \qquad x = 8, \qquad x = 10$$



11. Use the definition of the derivative to calculate $f'(4)$ if $f(x) = \sqrt{x} + 2x$.

12. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = x^3 + 3$.