MATH 210: STUDY GUIDE FOR MIDTERM 1

Here are the big things you should know for the exam.

- How to do implicit differentation.
- How to do logarithmic differentiation.
- How to use derivatives to understand how rates of change relates. (Related rates.)
- How to apply L'Hôpital's rule.
- How to use derivatives to find the absolute maximum or minimum of a function on a closed, bounded interval. How to optimize a quantity to find a maximum or a minimum.
- How to use the first and second derivative to get information about the shape of the graph of a function: where it is increasing versus decreasing, concave up versus concave down, where the local maximums and minimums are, where the inflection points are.
- How to set up Riemann sums to approximate an integral, especially left and right Riemann sums.
- How to calculate indefinite and definite integrals by using antiderivatives.

For the exam you are allowed to create and bring a note card. It must be a 3" $\times$ 5" note card. (If you don't have any I can give you one.) You may print or handwrite it, and can use both the front and back. You will write your name on it and turn it in with your exam.

Here's some sample problems to practice for the exam.

(1) What is the slope of the curve $e^{xy} - e^2 y = 0$ at the point $(2, 1)$?
(2) What is the slope of the ellipse $16x^2 + 9y^2 = 25$ at the points $(\pm 1, \pm 1)$?
(3) Differentiate $a(x) = x^{\ln x}$.
(4) Use logarithmic differentiation to differentiate $b(x) = x^3 e^{2x} \cos x$.
(5) Use logarithmic differentiation, the rule for the derivative of $\ln x$, and the chain rule to derive the quotient rule.
(6) Use implicit differentation and the rule for the derivative of $\tan x$ to derive the rule for $\arctan x$.
(7) The *kinetic energy* of an object in motion is given by the equation $E = \frac{1}{2}mv^2$ where m is the mass of an object and v is its velocity. Suppose a meteor has a mass of 240 kilograms. If its acceleration is a constant 10 meters per square second, what is the rate of change in its kinetic energy when its velocity is 40 meters per second?
(8) A beehive is constructing a honeycomb which is approximately a circle. If the area of the circle is changing at a rate of 15 square inches per day, then how fast is the radius of the circle changing when the honeycomb has a radius of 10 inches?
(9) Evaluate the limit
$$\lim_{x \to 1} \frac{\ln(x)}{x - 1}.$$
(10) Evaluate the limit
$$\lim_{x \to -\infty} x^3 e^x.$$

(11) Evaluate the limit
$$\lim_{x\to\infty} \frac{\arctan x}{x}.$$
(12) Consider the function $f(x) = \frac{x-1}{x^2+3}$. Determine where all local maximums, local minimums, and inflection points of $f(x)$ are. Where is $f(x)$ increasing? Decreasing? Concave up? Concave down?
(13) Find the absolute maximum and absolute minimum of $g(x) = x^2 - 4\sqrt{x}$ on the interval $[0, 4]$.
(14) Find the nonnegative numbers x and y which maximize the quantity $x\sqrt{y}+y$, subject to the constant that the numbers sum to 100.
(15) You are building a fenced in yard for your chickens. You decide it will be a rectangle. You want to keep the roosters separe from the hens, so you need to divide the rectangle into two areas with a fence in between them. For simplicity, you decide to make the two areas equal sized rectangles with the fence in the middle parallel to one of the sides of the outer rectangle of fence. If you want the yard to have a total area of 800 square feet, then what should the dimensions of the yard be in order to minimize the length of fencing you need?
(16) Consider the integral
$$\int_1^3 x^3 - \frac{1}{x}\,\mathrm{d}x.$$
   (a) Set up the left Riemann sum with $N = 2$ pieces which approximates this integral. Evaluate it.
   (b) Set up the right Riemann sum with $N = 120$ pieces which approximates this integral. Don't evaluate it, but your summand should only depend on the index variable.
   (c) Set up the middle Riemann sum with $N = 10$ pieces which approximates this integral. Don't evaluate it.
(17) Evaluate
$$\int e^x - \sin x + \frac{4}{3x^3}\,\mathrm{d}x.$$
(18) Evaluate
$$\int_1^3 \sqrt{3x^3}\,\mathrm{d}x.$$
(19) Evaluate
$$\int \sin x + \sec^2 x - \frac{1}{x}\,\mathrm{d}x.$$
(20) Evaluate
$$\int_{-1}^1 5x^3 + 3\sin x\,\mathrm{d}x.$$