MATH 1420 WRITTEN HOMEWORK 3
DUE FRIDAY, APRIL 7

Remember that for all problems you should write up a complete explanation. That is, don't just show me calculations. You also need to explain why the steps you follow are valid.

Problem 1. *Consider the function $f(x) = \sin(2x)$, defined only on the interval $[0, \pi]$. Determine on what interval(s) this function is increasing/decreasing and where any local maximums and minimums of the function are.*

Do the same thing for the function $g(x) = e^{-2x}\sin(2x)$, again on the interval $[0, \pi]$. Compare your answers. If they're the same, why do you think that is? If they're different, what explains that?

Please give exact answers. I set up the problem so that you could give exact answers in terms of fractions of π. [Hint: As part of this, you'll have to solve a couple trigonometric equations.]

A little bit of background to Problem 1 (you don't have to do respond to any of this, I just wanted to give some context for where this sort of problem shows up):

As you should've learned when you first learned trig, functions like $f(x)$ model *harmonic motion*—periodic motion like a pendulum or a spring, where an object oscillates back and forth, repeating forever. This is good model for lots of physical phenomenons. But a problem with this model is it doesn't take into account friction. If you've ever watched a spring bounce up and down, you'll have noticed that over time it slows down and it goes up and down less and less over time. So a more accurate model for this is what's called *damped harmonic motion*, where the oscillations get shorter over time. This is modeled with a function like $g(x)$, with both a trig term and an exponential term. The trig part is about the harmonic motion, while the exponential part is about the amplitude going down over time. So you can think of this problem as asking, if you compare undamped versus damped harmonic motion, does that change when the spring (or whatever you're modeling) reaches its maximum and minimum? Does that change when it's going up versus down?

Problem 2. *You are an engineer at a factory that produces aluminum cans. You are tasked with figuring out how to minimize the costs for materials to produce the cans. The cans being produced are cylinders with a circular base, and need to have a volume of 25 cubic inches. The material for the top and bottom of the cans costs 0.3 cents per square inch, while the material for the sides must be thicker and costs 0.7 cents per square inch. What dimensions should the can be to minimize the cost of materials while having that mcuh volume, and what is the cost per can? For this, please round your dimensions to the nearest hundredth of an inch, and the cost to the nearest tenth of a cent.*

[Hint: the formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height. For its surface area: the top and bottom are circles, so you can use the usual formula for the area of a circle, and the area of the side is the circumference times the height.]