MATH 321: 9-10 GROUPWORK

(1) Consider the following two logical formulae.

$$\neg\exists x\ \forall y\ (x = y \vee x < y) \quad \text{and} \quad \neg\forall x \forall y\ \exists z\ \forall w\ (w \in z \leftrightarrow (w = x \vee w = y))$$

Use rules for equivalences of formulae to translate each formula into an equivalent form which does not begin with a $\neg$.

(2) Imagine a universe of discourse $U = \{a, b\}$ with two objects a and b. Explain why, over this universe of discourse, $\forall x\ P(x)$ is equivalent to $P(a) \wedge P(b)$. Explain why $\exists x\ P(x)$ is equivalent to $P(a) \vee P(b)$. Can you generalize this to an arbitrary finite universe of discourse $U = \{a_1, a_2, \ldots, a_n\}$? With this connection in mind, compare the quantifier negation laws on page 65 of the textbook to DeMorgan's laws for propositional logic.

(3) Recall that the "there exists a unique object" quantifier $\exists! x$ can be expressed in terms of the quantifiers $\exists$ and $\forall$, namely $\exists! x\ P(x)$ is expressed as $\exists x\ (P(x) \wedge \forall y\ P(y) \to x = y)$. Can you express "there exist exactly two objects so that P holds" using $\exists$ and $\forall$? What about "there exist exactly three objects so that P holds"? What about "there exist exactly n objects so that P holds" for an arbitrary natural number n?