MATH 321: HOMEWORK 7
DUE THURSDAY, NOV 5 BY 11:59PM

For your proofs, you should submit fully written up formal proofs, i.e. not scratchwork.

Problem 1. Do problem #1 from page 295 of the textbook.

Take the following recursive definition of exponentiation with natural number exponents:

- $x^0 = 1$ for all x;
- $x^{n+1} = x^n \cdot x$ for all x and all $n \in \mathbb{N}$.

Problem 2. Use induction and this definition of exponentiation to prove the following rules for exponentiation:

(1) $x^n \cdot y^n = (xy)^n$ for all x, y and all $n \in \mathbb{N}$.
(2) $x^n \cdot x^m = x^{n+m}$ for all x and all $n, m \in \mathbb{N}$.
(3) $(x^n)^m = x^{nm}$ for all x and all $n, m \in \mathbb{N}$.

[Hint: for 2 and 3 there are two natural number variables n and m in the statement of the fact. Which one do you want to do induction on?

Recall the recursive definition of the Fibonacci sequence F_n:

- $F_0 = 1$;
- $F_1 = 1$;
- $F_{n+1} = F_n + F_{n+1}$.

Problem 3. Prove that the Fibonacci sequence follows the pattern odd, odd, even. That is, prove that for all natural numbers m that F_{3m} and F_{3m+1} are odd while F_{3m+2} is even.