

MATH 190: 1/22 WORKSHEET

KNIGHTS AND KNAVES PUZZLES

All of these puzzles involve Knights and Knaves. Knights will only tell the truth, whereas Knaves will only tell lies. However the two are otherwise indistinguishable; you cannot tell which is which just by looking at them. (These puzzles are inspired by the logician Raymond Smullyan, who introduced Knights and Knaves.)

A Fork in the Road.

You are navigating a labyrinth when you come to a fork. Standing at fork are two guards. Your guide, whom you know to be a Knight, tells you that one guard is a Knight and the other is a Knave, but she doesn't know which is which, and that the guards know where the two paths lead. One path leads to certain death while the other leads to fortune. Can you with a single question addressed to a single guard determine which path to take?

The same scenario, except there is a single guard, and you don't know whether they are a Knight or a Knave. Can you ask them a single question to determine which path to take?

The Island of Knights and Knaves.

You are on an island where everyone is either a Knight or a Knave, and you are trying to determine the status of different inhabitants of the island.

- (1) You meet two inhabitants, Alice and Bob. Alice volunteers, "We are both knaves." Determine the status of both Alice and Bob.
- (2) Imagine if instead Alice had said, "We are both knights." Can you determine the status of both Alice and Bob?
- (3) If Alice says, "We are both knights," and Bob says, "Alice is a Knave" can you determine their status?
- (4) Alice says, "We are the same," and Bob replies, "No we are different". Can you determine their status?
- (5) Alice says, "Bob is a Knave," then Bob says, "Both of us are Knights". Can you determine their status?
- (6) You meet Alice, Bob, and Carol. Alice says, "I am a Knight." Bob says, "She's lying." Carol says, "Either Bob is a Knight or else I am a Knave." Can you determine their status?
- (7) Would an islander ever say, "I am a Knave"? Explain.
- (8) If you asked an islander their status, what would they answer? Does that answer give you any information?

Spies on the Island.

You have discovered you are not the only visitor to the island of Knights and Knaves. There are Spies present, who can freely lie or tell the truth as they will.

You are dealing with Alice, Bob, and Carol, and you know that precisely one is a Spy, one is a Knight, and one is a Knave. You inquire as to their identities and this is what they say. Alice says, “Carol is a Knave.” Bob says, “Alice is a Knight.” Carol says, “I am the Spy.” Can you determine their identities?

If-then statements.

Logicians love studying if-then statements, and this madness has gripped the island of Knights and Knaves.

Alice declares, “If Bob is a Knave then we are both Knaves.” Determine whether Alice and Bob are Knights or Knaves.