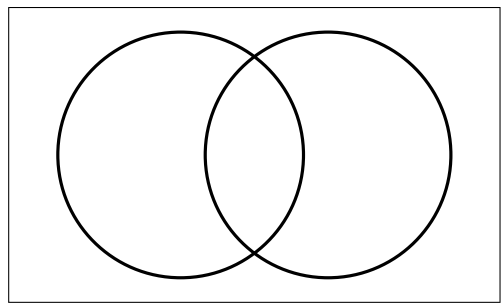
MATH 113: 2/10 WORKSHEET

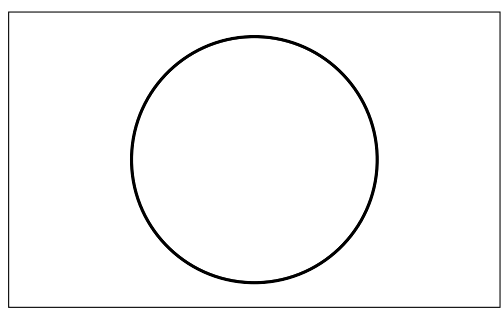
Semantics means the meaning of words or symbols. To describe the semantics of logical connectives we will use *truth tables*. These tables describe connectives as functions, hence the name "truth-functional logic". The left side of the table gives the possible truth value combinations for the input, and the right side gives the output truth value.

As a visual aid, we will also describe the semantics using *Venn diagrams*. In a Venn diagram the interior of the circles represent when each input is true, and we shade in the regions corresponding to which outputs are true.

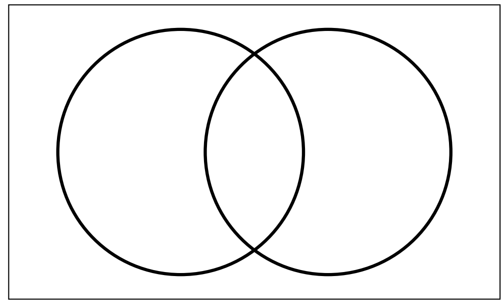
P	Q	$P \wedge Q$
1	1	
1	0	
0	1	
0	0	



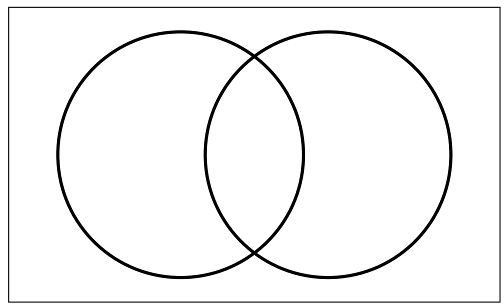
P	$\neg P$
1	
0	



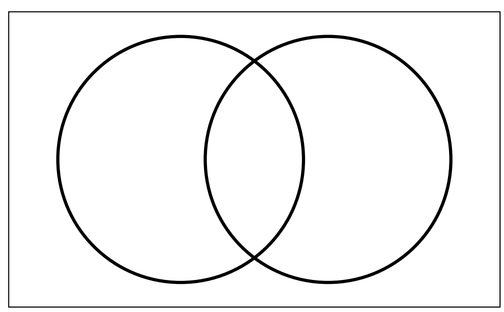
P	Q	$P \vee Q$
1	1	
1	0	
0	1	
0	0	



P	Q	$P \leftrightarrow Q$
1	1	
1	0	
0	1	
0	0	



P	Q	$P \to Q$
1	1	
1	0	
0	1	
0	0	



We can also use truth tables and Venn diagrams to describe the semantics of more complicated sentences in truth-functional logic. When doing this, you determine the behavior of each piece of the sentence, building upward to describe the overall behavior. Compare to how when determining the value of an arithmetic expression you have to follow order of operations in combing the intermediate values.

P	Q	$\neg P \vee Q$
1	1	
1	0	
0	1	
0	0	

One use of truth tables is to show that different sentences are *equivalent*—the same inputs give the same outputs. To check that two sentences are equivalent we compute their truth tables and see that they have the same values.

P	Q	R	$(P \wedge Q) \wedge R$	$P \wedge (Q \wedge R)$

P	Q	R	$(P \vee Q) \vee R$	$P \vee (Q \vee R)$