MATH 321: 9-29 IN-CLASS WORK

We continue to look at how the logical structure of a mathematical statement informs how you can use it in proofs. Today we focus on $\neg$.

If your goal is of the form $\neg P$, how might you try to prove it? One way to try to prove $\neg P$ is known as *proof by contradiction* or, for those who like Latin, *reductio ad absurdum*. For this, you assume P and you try to derive a contradiction—usually something of the form $Q \wedge \neg Q$.

Also, note that you can use proof by contradiction to prove P. This is because P is equivalent to $\neg\neg P$ and so using the template from the previous paragraph you assume $\neg P$ and try to derive a contradiction.

How do you use $\neg P$ as a known? Usually you want to translate it into a different form.

Exercise 1. *Recall from last time that proving B from the assumption of A is the same as proving $A \to B$. With that in mind, check that the formula $P \to$ `false` is equivalent to $\neg P$, and use this to explain why proof by contradiction is a valid method of proof.*

Exercise 2. *Prove the following statement about an integer n: if n^3+3 is odd then n is even. First transform this goal of an if-then statement into a known plus a simpler goal, then use proof by contradiction to prove your new goal.*

[Hint: "n is not even" is equivalent to "n is odd" for integers n.]

The next technique combines working with negations and conditionals. Recall that $P \to Q$ is equivalent to $\neg Q \to \neg P$. So to prove $P \to Q$ you can transform it to: $\neg Q$ is known and you want to derive $\neg P$. This rule of proof is known as *modus tollens*.

Exercise 3. *Use* modus tollens *to prove the following statement about real numbers $x > 0$ and y: if $xy \geq 0$ then $y \geq 0$.*

If you finish the earlier three exercises, attempt this one.

Exercise 4. *Prove that $\sqrt{3}$ is irrational.*