MATH 321: 10-8 IN-CLASS WORK

We continue our investigation of proof strategies. Today we focus on the logical connectives $\wedge$ and $\leftrightarrow$.

And is the most straightforward, so straightforward that we've used it already without comment: To prove $P \wedge Q$, prove P and Q separately. And if you know $P \wedge Q$ then you know both P and Q.

Iff is only a little less straightforward. The way to think of it is, $P \leftrightarrow Q$ is equivalent to $P \to Q \wedge Q \to P$. So if you're trying to prove an iff, you need to prove two conditionals. That is, you assume P and derive Q, and you independently assume Q and derive P. And if you have an iff as a known then it means you have two conditionals as knowns.

An alternate and often useful way to think about iffs goes through the contrapositive: $P \leftrightarrow Q$ is equivalent to $P \to Q \wedge \neg P \to \neg Q$.

An important special case of iff statements are statements about the equality of two sets: $A = B$. These are iff statements in disguise because $A = B$ just means $\forall x \ (x \in A \leftrightarrow x \in B)$.

Exercise 1. *Prove that, for any two sets A and B, that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.*

Exercise 2. *Prove that for every integer n we have $10 \mid n$ iff $2 \mid n$ and $5 \mid n$.*

Exercise 3. *Prove or disprove: for every integer n we have $24 \mid n$ iff $4 \mid n$ and $6 \mid n$.*

Attempt this exercise after completing all the previous ones.

Exercise 4. *Can you figure out the pattern here? What should K be to make the following statement, about arbitrary positive natural numbers a, b, true?*

- *For every integer n we have $K \mid n$ iff $a \mid n$ and $b \mid n$.*