MATH 321: 9-24 IN-CLASS WORK

The logical structure of a mathematical statement/definition/etc. informs its role in proofs. Let's look at the material conditional $\to$. There are two aspects to this, based on whether it's a statement you know already or a statement you are trying to prove. Suppose you know that $P \to Q$. Then, if you know P you can conclude Q. This often appears in the guise $P(x) \to Q(x)$, where you are making a statement about an object x. Then if you know $P(a)$ holds for an appropriate object a you can conclude $Q(a)$.

For the other case, suppose you are trying to prove $P \to Q$. Then the way to do that is to assume P and now try to prove Q. You can think of this as transforming your knowns/goals. If you start with:

knowns	goals
$\vdots$	$P \to Q$

then you can transform it into:

knowns	goals
P	Q
$\vdots$	

In summary: if-then statements are great! They give you extra information which you can use to make progress on a proof.

Exercise 1. *Prove the following statement about a natural number n: If $6 \mid n$ then $2 \mid n$. After you figure out how to prove it, write a paragraph in ordinary mathematical English which gives your proof.*

[Hint: $k \mid n$ means there exists a so that $ka = n$. You are given $6a = n$ for some a. Can you find b so that $2b = n$?]

Exercise 2. *A common and easy mistake in reasoning is the following: You know $P \to Q$ and Q, so you conclude P. Show that this reasoning is not valid by considering the following example: P says $6 \mid n$ and Q says $2 \mid n$. Find a value of n for which Q is true but P is false.*

This exercise is harder than the previous two. Work on them before you try this one.

Exercise 3. *Prove the following statement about a natural number n: If $2 \mid n$ and $3 \mid n$ then $6 \mid n$.*