MATH 321: 10-27 IN-CLASS WORK

The Fibonacci sequence is a sequence F_n of natural numbers defined recursively as follows:

$$F_0 = 1$$
$$F_1 = 1$$
$$F_{n+2} = F_n + F_{n+1}$$

Exercise 1. *Prove that for all n,*

$$\sum_{k=0}^{n} F_k = F_{n+2} - 1.$$

[Hint: use strong induction.]

Exercise 2. *Prove that for all n,*

$$\sum_{k=0}^{n} {F_k}^2 = F_n F_{n+1}.$$

Exercise 3. *Explain what's wrong with the following "proof".*

Theorem. *All horses are the same color.*

Proof. We will prove by induction that for every natural number $n > 1$, if you have a group of n horses then all those horses are the same color. Clearly this suffices by taking some large enough n.

The base case $n = 1$ is easy. If you have a group of just one horse, then it is the same color as itself.

Now do the induction step. Assume we have the result for n, let's try to prove it for $n + 1$. Taking our group of $n + 1$ horses, exclude one horse, call it A, from the group. Then we have n horses, so by inductive hypothesis they are all the same color. It remains only to see that A has the same color as those horses. Now exclude a different horse, call it B, from the group. Then we have n horses, so by inductive hypothesis they are all the same color. Now take a horse C which was in both groups of size n. Then we have A is the same color as C which is the same color as B. So A is the same color as the n horses we separated A from, so they are all the same color. $\square$