MATH 321: 9-15 GROUPWORK

(1) Consider the one element set $A = \{a\}$. How many elements does $\mathcal{P}(A)$ have? List them. Next consider the two element set $B = \{a, b\}$. How many elements does $\mathcal{P}(B)$ have? List them. Generalize this to determine how many subsets a set with n elements has.

(2) For each natural number $n > 0$, let $A_n \subseteq \mathbb{N}$ consist of the multiples of n. (For example, $A_1 = \{0, 1, 2, \ldots\}$, $A_n = \{0, 2, 4, \ldots\}$, $A_3 = \{0, 3, 6, \ldots\}$.) Determine the following sets:
- $A_2 \cap A_3$
- $A_4 \cap A_6$
- Can you generalize this and determine $A_k \cap A_\ell$ for any k and ℓ?
- $A_4 \cap A_5 \cap A_6$
- $\cap\{A_n : n > 0\}$
- $\cup\{A_n : n > 0\}$
- $\cup\{A_n : n \in A_2\}$

(3) Show that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$ for any two sets A and B by showing that the two statements $x \in \mathcal{P}(A \cap B)$ and $x \in \mathcal{P}(A) \cap \mathcal{P}(B)$ are equivalent.