# MATH 195: 2/4 WORKSHEET

**One-to-one.**

A function $f(x)$ is *one-to-one* (synonymously, *injective*) if each output comes from exactly one input. Said another way, one-to-one means different inputs go to different outputs. You can write this in symbols:

$$\text{if } f(x_1) = f(x_2) \text{ then } x_1 = x_2$$
$$\text{if } x_1 \neq x_2 \text{ then } f(x_1) \neq f(x_2)$$

Geometrically, being one-to-one means passing the *horizontal line test*: any horizontal line crosses the graph in at most one point.

**Inverses of functions.**

If $f(x)$ is one-to-one then you can define its *inverse function* $f^{-1}(y)$ as $f^{-1}(y) = x$ if and only if $f(x) = y$. The reason $f^{-1}(y)$ is a function—each input goes to exactly one output—is because $f(x)$ passes the horizontal line test.

- To find a formula for $f^{-1}(y)$, solve the equation $f(x) = y$ for $x$, this then gives a formula for $f^{-1}(y)$. (And if you want to call the input $x$ you can then swap out the variables at the end.)
- Graphically, the graph of $f^{-1}(x)$ is obtained by reflecting the graph of $f(x)$ across the diagonal line $y = x$.
- If $f(x)$ is not one-to-one, you might be able to restrict its domain down to make it one-to-one, and then you can take the inverse.

**Radical functions**

If $n$ is a positive integer, then $\sqrt[n]{x}$ is the inverse of $x^n$.

- When $n$ is odd, $x^n$ is one-to-one and so this can be done directly.
- When $n$ is even, $x^n$ is not one-to-one. To make it one-to-one restrict its domain to $[0, \infty)$ and then take the inverse.

A *radical function* is one of the form $f(x) = \sqrt[n]{x}$. We will sometimes use *radical function* to refer more generally to any function which is a geometric transformation of some $\sqrt[n]{x}$.