

Study guide for Math 211 Midterm 1

February 20, 2025

Things to know

The midterm covers all the material from the first five weeks of class. This amounts to chapter 6 of the textbook. Specifically, you should know all of the following.

- How to use integration to find the volume of a solid based on knowing about its cross sections.
- How to use integration to find the volume of a solid formed by rotation. You should know both the disk/washer and the cylindrical shell methods.
- How to find the arc length of a curve $y = f(x)$.
- How to find the surface area of a surface formed by rotation.
- How to find the mean value of a function on an interval, and how to determine where a function obtains its mean value.
- How to compute with densities and calculate means/centers of mass, whether physical density or probability density.
- How to calculate improper integrals.

You should expect that some questions will ask you to set up but not solve an integral, while others will ask you to solve the integral.

The exam has seven questions.

Formula sheet

Here are the formulas I will give you. Other formulas you will have to remember (or, even better, know the concepts that give rise to them).

$$\begin{aligned}\frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \arctan x &= \frac{1}{1+x^2}\end{aligned}$$

$$\begin{aligned}\int_a^b 2\pi x \, ds \\ \int_a^b 2\pi f(x) \, ds\end{aligned}$$

Sample questions

1. A solid is formed by placing isosceles right triangles along the planar region bounded by the x -axis and the curve $y = 4 - x^2$, so that each triangle has its hypotenuse running vertically from the x -axis to the curve $y = 4 - x^2$. Determine the volume of this solid.
2. The triangle with corners $(0, 0)$, $(2, 5)$, and $(2, 0)$ is rotated around the y -axis to form a solid. Determine the volume of this solid.
3. The region bounded by the curve $y = \cos x$ and the x -axis, where $-\pi \leq x \leq \pi$, is rotated around the x -axis. Determine the volume of the resulting solid.
4. Set up an integral which gives the surface area of the solid from the previous question.
5. You are stacking sheets of thin square paper to make a three dimensional sculpture. The precision of your art is impeccable or $0 \leq x \leq 1$.
6. Set up an integral which gives the arc length of the hyperbola $xy = 1$ from the point $(0.1, 10)$ to $(10, 0.1)$.
7. The curve $y = e^{-x}$, where $0 \leq x < \infty$, is rotated around the x -axis to produce a surface. Set up an integral which gives its surface area.
8. Find the mean value of $y = e^{-x}$ along the interval $[0, \ln 3]$.
9. Determine where the curve $y = x^3 - 3$, where $-1 \leq x \leq 2$ obtains its mean value.
10. A rope of length 10 meters has its density given by $\rho(x) = 1 + \cos(\pi x/5)$ kilograms per meter at x meters from the endpoint. Determine the total mass of the rope. Where is its center of mass?
11. Confirm that $\rho(x) = e^{-x}$, where $0 \leq x < \infty$ is a probability density function.
12. The function $\rho(x) = \frac{1}{100}$, where $-43 \leq x \leq 57$ is a probability density function. Determine its mean.
13. Calculate

$$\int_3^{\infty} x^{-3} dx.$$

14. Calculate

$$\int_0^1 \ln x dx.$$

[Hint: differentiate $x \ln x - x$.]