

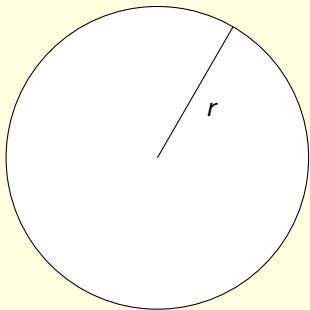
# Math 210: Calculus I

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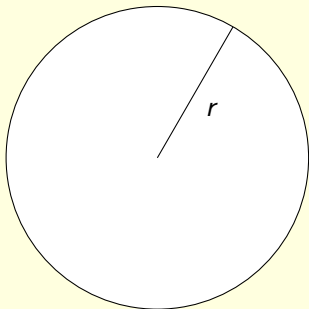
# Circles



$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

# Circles



The circumference formula is the definition of  $\pi$ ; the circumference is a ratio of the radius, and  $2\pi$  is the number that gives that ratio.

But where does the area formula come from?

$$\text{Circumference} = 2\pi r$$

$$\text{Area} = \pi r^2$$

# The area of a circle



Sato Moshun's calculation of the area of the circle, 17th century

# The big idea of calculus

Looking at what happens when a quantity becomes infinitely small or two quantities get infinitely close to each other gives you a lot of information.

We call infinitely small quantities **infinitesimals**.

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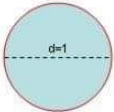
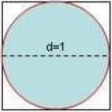
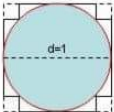
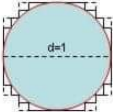
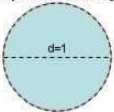

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## Big Concepts

- **Derivative**: Instantaneous rate of change or slope of a function
- **Integral**: Area under a curve

# But we have to be careful!

<p>Draw a circle</p> 	<p>Draw a square around it. Perimeter = 4</p> 
<p>Remove corners. Perimeter is still 4!</p> 	<p>Remove more corners. Perimeter is still 4!</p> 
<p>Repeat to infinity</p> 	<p><b><math>\pi = 4!</math></b></p>  <p>Problem Archimedes?</p>

# A brief history of the calculus

- (Before 17th century) Many different mathematicians around the globe (such as Archimedes of Syracuse and Madhava of Sangamagrama) use infinitesimals to make calculations—volumes, areas, lengths.
- (17th–18th centuries) Gottfried Wilhelm Leibniz and Isaac Newton independently invent a systematic framework for this species of calculation, and prove the **fundamental theorem of calculus** which explains the connection between **derivatives** and **integrals**. Later mathematicians build on their work and the modern discipline of calculus is born.

But this framework faced criticism for its use of infinitesimals.

- (19th century) Karl Weierstrass and other mathematicians show how to redo calculus in a framework that doesn't use infinitesimals
- (1960s) Abraham Robinson gives a mathematically rigorous framework for calculus using infinitesimals.