EXPERIMENT 13

Active Filters

13.1 Application

Filters are used to amplify or attenuate different parts of a signal based on frequency. They are essential for analog signal processing of sensor measurements, wireless communication signals, audio signals, and more. Even if the majority of signal processing and filtering will be done digitally, the analog front end must ensure that only the desired signal at the right amplitude gets to the analog to digital converter. Filters with gain or multiple stages can easily be built using active circuit elements. Active filters allow for isolating multiple stage filters from each other as well as amplifying the signal.

13.2 Active Filters

Active filters is a rather broadly defined term to refer to any filter circuit that use an "active" element to help with filtering. Typically, this means using operational amplifiers (op amps) as either part of the filter or as buffers between filter stages. In order to analyze active filter circuits, we need to derive the transfer function $H(s) = \frac{v_{\text{out}}(s)}{v_{\text{in}}(s)}$ for the circuit. Once we have the transfer function, we can compare it to the standard filter transfer functions to determine the filter type and parameters.

13.2.1 First order filters

First order filters have the following form:

$$H(s) = \frac{A(s)}{\omega_0 + s} \tag{13.1}$$

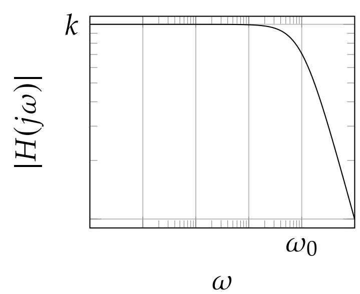

Figure 13.1: First order low pass

Where $A(s)$ is either $k\omega_0$ or ks. When $A(s) = k\omega_0$, the filter is a low pass filter with a single pole at ω_0. This filter has a pass band gain of k and a −3 dB cutoff frequency at its only pole, ω_0. The first order low pass magnitude and phase response are shown in equation (13.2) and equation (13.3), respectively. The magnitude response is plotted in figure 13.1.

$$|H(j\omega)| = \frac{k\omega_0}{\sqrt{\omega_0^2 + \omega^2}} \tag{13.2}$$

$$\angle H(j\omega) = -\operatorname{atan}\left(\frac{\omega}{\omega_0}\right) \tag{13.3}$$

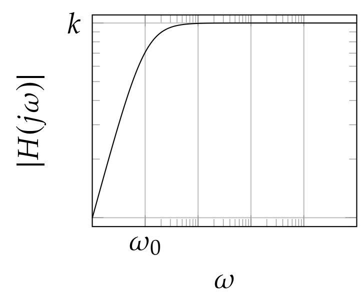

Figure 13.2: First order high pass

When the first order response numerator $A(s) = ks$, the filter is high pass. The transfer function has a zero at $s = 0$ and a pole at ω_0. This filter has a pass band gain of k and a −3 dB cutoff frequency at its only pole, ω_0. The first order high pass magnitude and phase response are shown in equation (13.4) and equation (13.5), respectively. The magnitude response is plotted in figure 13.2.

$$|H(j\omega)| = \frac{k\omega}{\sqrt{\omega_0^2 + \omega^2}} \tag{13.4}$$

$$\angle H(j\omega) = \frac{\pi}{2} - \operatorname{atan}\left(\frac{\omega}{\omega_0}\right) \tag{13.5}$$

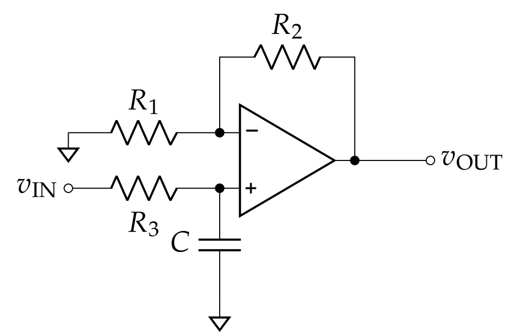

Figure 13.3: Noninverting low pass amplifier

Low pass circuits

Two simple low pass circuits are shown in figure 13.3 and figure 13.4. The transfer function of the noninverting circuit can be found using virtual short circuit analysis. The result is shown in equation (13.8). This transfer function has a passband gain of $k = 1 + \frac{R_2}{R_1}$ and a cutoff frequency of $\omega_0 = \frac{1}{R_3 C}$.

$$H(s) = \frac{\left(1 + \frac{R_2}{R_1}\right)\frac{1}{R_3 C}}{s + \frac{1}{R_3 C}} \tag{13.6}$$

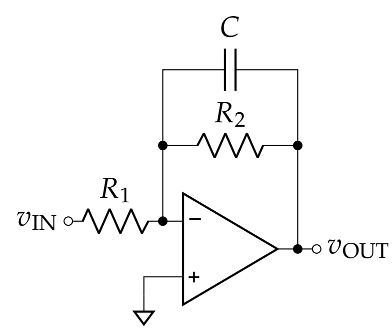

Figure 13.4: Inverting low pass amplifier

The inverting low pass filter can also be solved using virtual short circuit analysis on the laplace equivalent circuit. It has the transfer function given in equation (13.7). It has a passband gain of $k = -\frac{R_2}{R_1}$ and a cutoff frequency of $\omega_0 = \frac{1}{R_2 C}$. Be careful to note that the negative gain adds a phase shift of π, so the phase response of this circuit is actually $\angle H(j\omega) = \pi - \operatorname{atan}(\omega/\omega_0)$.

$$H(s) = \frac{-\frac{R_2}{R_1}\frac{1}{R_2 C}}{s + \frac{1}{R_2 C}} \tag{13.7}$$

High pass circuits

The corresponding high pass circuits are shown in figure 13.5 and figure 13.6. The analysis and results are similar to the low pass circuits. The non inverting high pass transfer function is shown in equation (13.8). This transfer function has a passband gain of $k = 1 + \frac{R_2}{R_1}$ and a cutoff frequency of $\omega_0 = \frac{1}{R_3 C}$.

$$H(s) = \frac{s\left(1 + \frac{R_2}{R_1}\right)}{s + \frac{1}{R_3 C}} \tag{13.8}$$

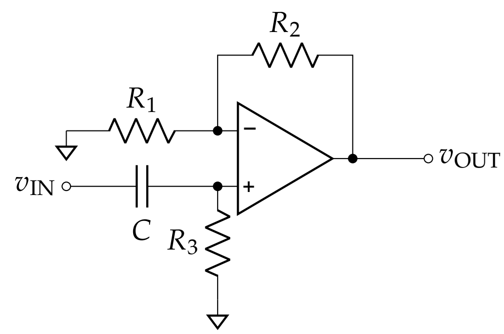

Figure 13.5: Noninverting high pass amplifier

The inverting high pass filter has the transfer function given in equation (13.9). It has a passband gain of $k = -\frac{R_2}{R_1}$ and a cutoff frequency of $\omega_0 = \frac{1}{R_1 C}$. The negative gain adds a phase shift of π to the high pass as well, so the phase response of this circuit is actually $\angle H(j\omega) = \pi/2 - \mathrm{atan}(\omega/\omega_0)$.

$$H(s) = \frac{-\frac{R_2}{R_1}s}{s + \frac{1}{R_1 C}} \tag{13.9}$$

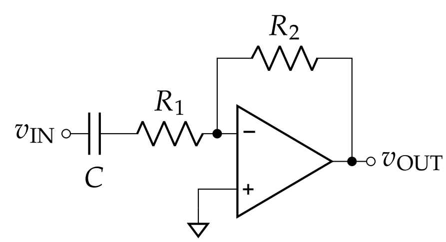

Figure 13.6: Inverting high pass amplifier

The high pass circuits can be used for more than just filters. They both block DC input signals so they can be used to connect multiple stages of circuits together with different DC biases. When used as a DC blocking circuit, the circuit will high pass the input signal, so it is important to set the cutoff frequency low enough that it does not affect the performance of the circuit.

13.2.2 Second order filters

Second order filters have the form

$$H(s) = \frac{A(s)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \tag{13.10}$$

The cutoff or resonance frequency is determined by ω_0, and the response at the cutoff is determined by Q.[1] The zeros of $A(s)$ determine the overall response of the filter. The value k in each response determines the overall gain of the filter in

[1] For low and high pass filters, Q changes the shape of the knee in the filter response. For a band pass or band stop filter, Q changes the width of the pass or stop band, respectively. Filters with high Q and some gain have the risk of becoming unstable and oscillating.

the passband.

$$A(s) = k\omega_0^2 \qquad \text{Low Pass} \qquad (13.11)$$
$$A(s) = ks^2 \qquad \text{High Pass} \qquad (13.12)$$
$$A(s) = k\frac{\omega_0}{Q}s \qquad \text{Band Pass} \qquad (13.13)$$
$$A(s) = k\left(s^2 + \omega_0^2\right) \qquad \text{Band Stop}$$

Low pass circuit

The most ubiquitous active filter circuit is the Sallen-Key(2) filter depicted in figure 13.7. The elements can be easily reconfigured to implement a band pass, low pass, and high pass response. This specific filter implements a second order lowpass filter with an overall passband gain of 1. While possible to add additional gain, we will omit it to simplify analysis.

(2) This topology was invented by R.P. Sallen and E.L. Key in 1955 as a method to eliminate inductors from filter circuits [1].

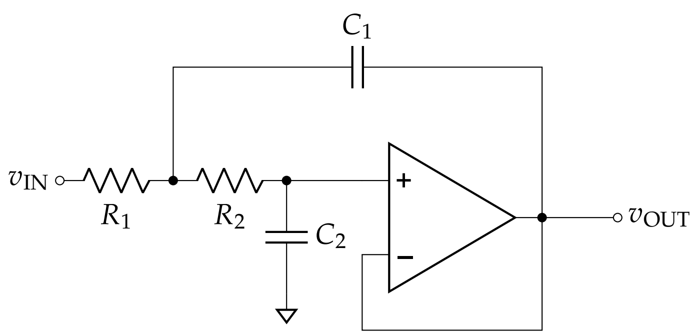

Figure 13.7: Low pass Sallen-Key filter circuit

This circuit implements the transfer function

$$H(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right) + \frac{1}{R_1R_2C_1C_2}} \qquad (13.14)$$

We can compare equation (13.14) with equation (13.11) to determine the cutoff frequency ω_0, Q factor, and gain k of the filter.

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} \qquad (13.15)$$
$$Q = \sqrt{\frac{R_1R_2C_1}{(R_1+R_2)^2C_2}} \qquad (13.16)$$
$$k = 1$$

For a maximally flat, or Butterworth, filter response, use $Q = \frac{1}{\sqrt{2}}$. A maximally flat filter is usually a good place to start, but there are other filter response types that allow for sharper rolloffs at the expense of ripple in the frequency response. There are also limitless solutions to the above design equations. Additional constraints, such as input impedance, need to be used to pick the actual values in your circuit. (3)

(3) Filter circuit design is easier when broken apart into separate stages, but there are nonidealities that add up when too many op amps are used. As with any design, there are many tradeoffs and it is always important to keep track of loading between stages.

High pass circuit

The Sallen-Key circuit used for the low pass only needs to have the capacitors and resistors exchanged to become a high pass circuit. The new circuit is shown in figure 13.8.

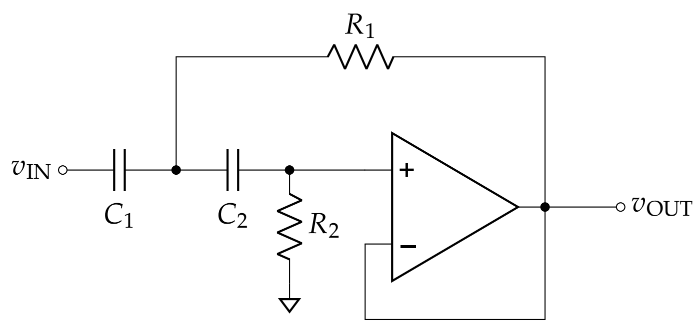

Figure 13.8: High pass Sallen-Key filter circuit

The frequency response of the modified circuit is then

$$H(s) = \frac{s^2}{s^2 + s\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}} \tag{13.17}$$

Once agian, we can compare equation (13.17) with equation (13.12) to determine the cutoff frequency ω_0, Q factor, and gain k of the filter.

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} \tag{13.18}$$

$$Q = \sqrt{\frac{C_1C_2R_2}{(C_1 + C_2)^2R_1}} \tag{13.19}$$

$$k = 1$$

Band pass circuit

The band pass circuits implemented using the Sallen-Key topology have limitations on the relationship between the gain and Q of the circuit, so we will look at a different band pass topology. A multiple feedback band pass circuit is shown in figure 13.9.

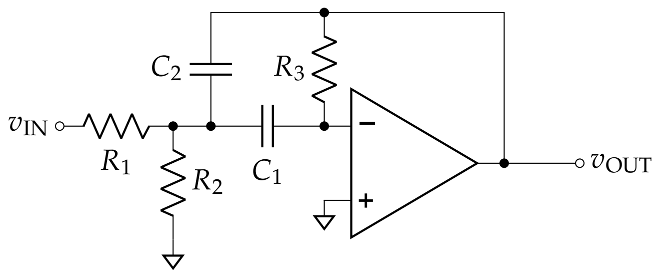

Figure 13.9: Multiple feedback band pass circuit

The frequency response of the multiple feedback bandpass circuit is

$$H(s) = -\frac{s\frac{1}{R_1C_2}}{s^2 + s\frac{C_1+C_2}{C_1C_2R_3} + \frac{1}{R_3C_1C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{13.20}$$

We can then compare equation (13.20) to the standard band-pass response form in equation (13.13) to determine the filter parameters.

$$\omega_0 = \sqrt{\frac{1}{R_3C_1C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{13.21}$$

$$Q = \sqrt{\frac{C_1C_2R_3}{(C_1 + C_2)^2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{13.22}$$

$$k = -\frac{R_3}{R_1}\frac{C_1}{C_1 + C_2} \tag{13.23}$$

13.3 Prelab

Task 13.3.1: Prelab Questions

It is recommended that you use an electronic equation solver for these design problems. Provide the actual values and the closest values available in your kit. If you use an equation solver, provide the input you gave to the solver as your work.

1. Use equations (13.15) and (13.16) to design the low pass filter needed for task 13.4.1.
2. Use equations (13.21) to (13.23) to design a filter with $k = -5$, $Q = 10$, and $f_0 = \frac{\omega_0}{2\pi} = 10\,\mathrm{kHz}$. Start with $C_1 = C_2 = 10\,\mathrm{nF}$. You will use this solution for task 13.4.3.

13.4 Tasks

Task 13.4.1: Design of an active filter

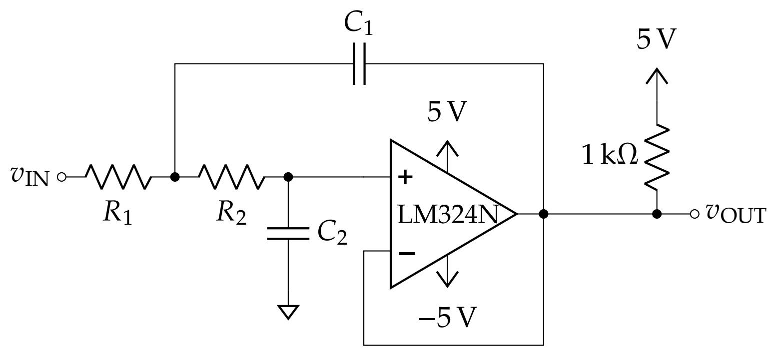

Figure 13.10: Low Pass Filter

1. *Compute* the values of R_1 and R_2 necessary to build a second order Sallen-Key low pass filter with a cutoff frequency of 20 kHz and Q of $\frac{1}{\sqrt{2}}$. Use $C_1 = 10\,\text{nF}$ and $C_2 = 1\,\text{nF}$.
2. **EC:** *Simulate* the response of the filter using AC SPICE analysis. Only use resistor values you can make using resistors in your kit. Calculate the error between the simulated cutoff frequency and the ideal cutoff frequency.
3. *Construct* the circuit using your designed values.[a]
4. *Measure* the gain and phase response of the filter from 100 Hz to 50 kHz.
5. *Capture* an oscilloscope screenshot showing v_{IN} and v_{OUT} at the cutoff frequency.
6. *Calculate* the error between the ideal cutoff frequency and the cutoff frequency of the constructed circuit.

[a] The 1 kΩ resistor is necessary due to limitations of the output stage of the LM324N [2], but is not necessary for all opamp circuits.

Task 13.4.2: Single supply active filters

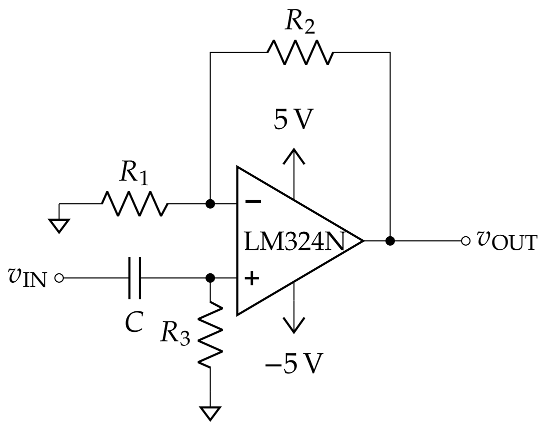

Figure 13.11: High Pass Filter

1. *Calculate* the cutoff frequency and gain of the filter in figure 13.11 when $R_1 = 1\,\text{k}\Omega$, $R_2 = 33\,\text{k}\Omega$, $R_3 = 330\,\text{k}\Omega$, and $C = 1\,\mu\text{F}$.
2. *Construct* the filter in figure 13.11.
3. *Measure* the frequency response of the filter for $0.2\,\text{Hz} < f < 20\,\text{Hz}$. Use a $0.1\,\text{V}_{\text{p-p}}$ input. You will need to do this manually on the ADALM, as the network tool does not work below 1 Hz.
4. **EC:** *Replace* the −5 V supply with ground. *Apply* a $0.1\,\text{V}_{\text{p-p}}$ 2 Hz sine wave to v_{IN} and *capture* an oscilloscope plot of v_{IN} and v_{OUT}. Does the filter still work properly?
5. **EC:** In order to bias the opamp correctly, we need to set the DC value of the inputs to a value inside the common mode input range. *Construct* the revised circuit in figure 13.12. Set the potentiometer so that V_{ref} is in the center of the common mode input range of the op amp.
6. **EC:** *Apply* a $0.1\,\text{V}_{\text{p-p}}$ 5 Hz signal to the input and record the gain. Why is the gain different than the original high pass filter circuit?

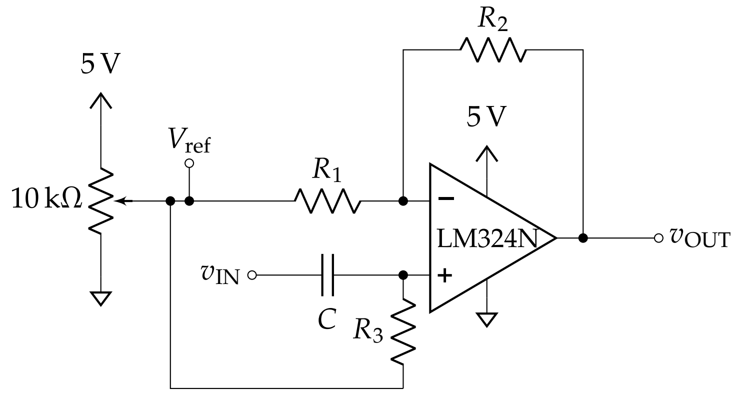

Figure 13.12: Single Supply High Pass Filter

7. **EC:** *Construct* the additional buffer circuit for V_{ref} shown in figure 13.13.

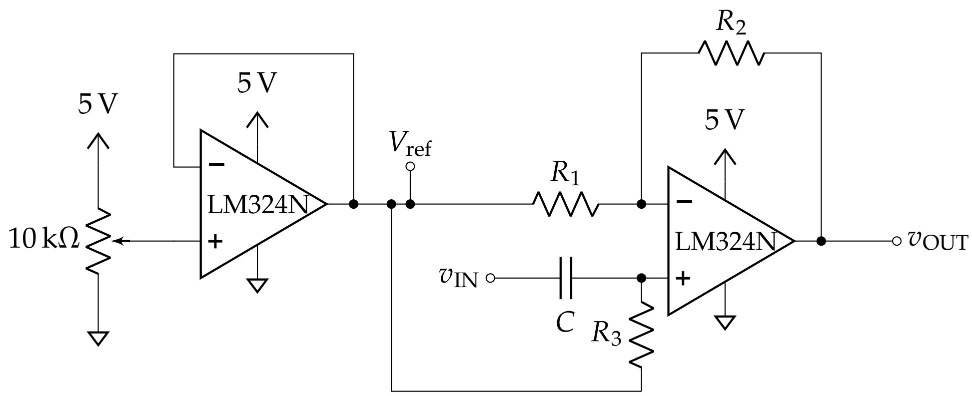

Figure 13.13: Corrected Single Supply High Pass Filter

8. **EC:** *Adjust* the potentiometer so that V_{ref} is still at the desired bias voltage.
9. **EC:** *Measure* the gain of the circuit with the buffer added at 5 Hz. You will need to reduce the amplitude of the input until the output is no longer clipping. *Capture* an oscilloscope screenshot showing v_{IN} and v_{OUT}.

Task 13.4.3: Infrared communication system

Infrared data signals are typically transmitted using a technique called on off keying, where a binary 1 is represented by the light being on and a binary 0 is represented by the light being off. Unfortunately, this system does not allow for multiple transmissions in a single area and allows for interferers to affect the signal reception. In order to combat this, we will design a system that allows us to use an infrared source with the intensity modulated by a specific frequency and a corresponding filter in the receiver that only allows that frequency through.

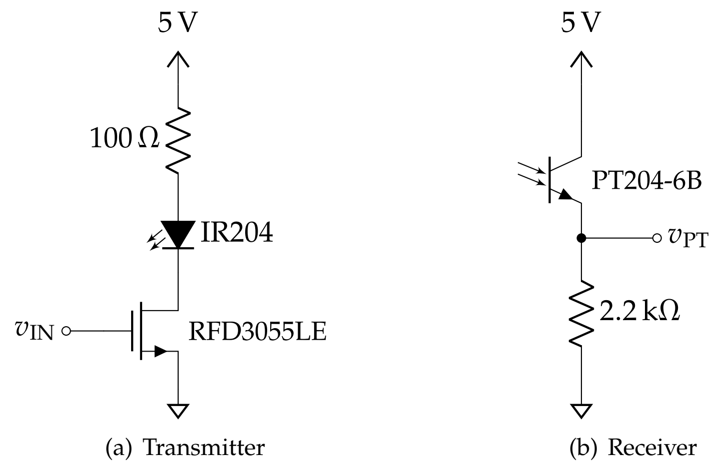

Figure 13.14: Infrared communication building blocks

1. *Construct* the infrared transmitter and receiver circuits in figure 13.14 on opposite sides of your breadboard. Use an infrared LED (IR204) [3] as the transmitter and an infrared phototransistor (PT204-6B) [4] as the receiver.[a]
2. *Apply* a 0 V to 5 V 10 kHz square wave to v_{IN}.
3. *Aim* the transmitter LED at the receiver phototransistor.
4. *Capture* an oscilloscope printout showing v_{IN} and v_{PT}.

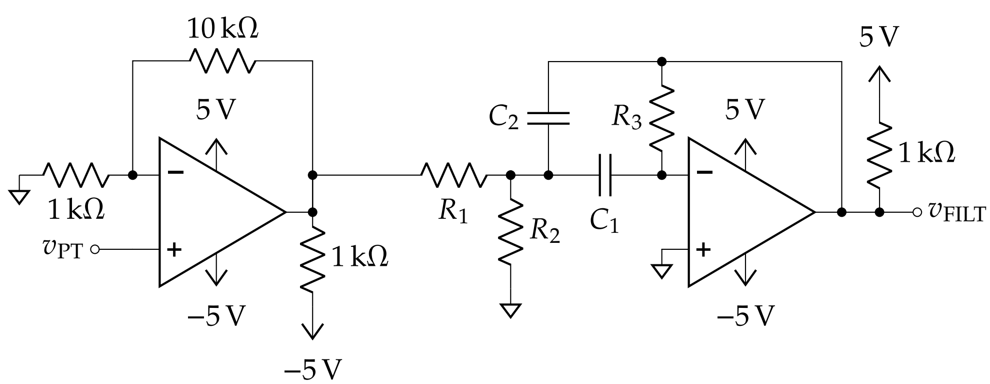

Figure 13.15: Receiver amplifier and filter

5. *Calculate* the values of R_1, R_2, and R_3 necessary in the multiple feedback band pass amplifier to have $k = -5$, $Q = 10$, and $f_0 = 10\,\text{kHz}$ given $C_1 = C_2 = 10\,\text{nF}$.
6. *Construct* the circuit in figure 13.15.
7. *Measure* the magnitude and phase response of the circuit using a $50\,\text{mV}_{\text{p-p}}$ sine wave as v_{PT}.

8. *Measure* the center frequency, gain at the center frequency, bandwidth, and Q of the filter using the magnitude response of the circuit.

9. *Connect* the output of the phototransistor circuit in section 13.4 to the input of the filter circuit in figure 13.15.

10. Once again, *Apply* a 0 V to 5 V square wave to v_{IN} while aiming the LED at the phototransistor. Set the square wave frequency to center frequency you measured in step 8.

11. *Capture* an oscilloscope screenshot of v_{IN} and v_{FILT}.

12. *Record* the peak to peak voltage of v_{FILT} while sweeping the square wave frequency of v_{IN} from 1 kHz to 20 kHz. Record at least one data point per 500 Hz.

[a] A phototransistor is a device that changes the amount of current flowing through it based on the amount of received light.

13.5 References

[1] R. P. Sallen and E. L. Key, "A practical method of designing rc active filters," *IRE Transactions on Circuit Theory*, vol. 2, no. 1, pp. 74–85, 1955.

[2] *LMx24-N, LM2902-N low-power, quad-operational amplifiers*, LM324N, SNOSC16D, Texas Instruments Inc., Jan. 2015. [Online]. Available: `http://www.ti.com/lit/ds/symlink/lm324-n.pdf`.

[3] *3mm infrared LED*, IR204-A, DIR-0000132 Rev. 6, Everlight, Nov. 2016. [Online]. Available: `http://www.everlight.com/file/ProductFile/IR204-A.pdf`.

[4] *3mm phototransistor t-1*, PT204-6B, DPT-0000293 Rev. 2, Everlight, Jun. 2013. [Online]. Available: `http://www.everlight.com/file/ProductFile/PT204-6B.pdf`.