

Derivativa

$$\textcircled{1} f(x; \beta) = (\beta_3 + \beta_4 x) \arctan\left(\frac{x - \beta_1}{\beta_2}\right) + \beta_5 x + 1$$

$$\begin{aligned} \frac{\partial}{\partial x} f(x) &= \beta_4 \arctan\left(\frac{x - \beta_1}{\beta_2}\right) + (\beta_3 + \beta_4 x) \frac{1}{1 + \left(\frac{x - \beta_1}{\beta_2}\right)^2} \left(\frac{1}{\beta_2}\right) + \beta_5 \\ &= \beta_4 \arctan\left(\frac{x - \beta_1}{\beta_2}\right) + \frac{\beta_3 + \beta_4 x}{\frac{\beta_2^2 + (x - \beta_1)^2}{\beta_2^2}} \cdot \frac{1}{\beta_2} + \beta_5 \end{aligned}$$

$$= \boxed{\beta_4 \arctan\left(\frac{x - \beta_1}{\beta_2}\right) + \frac{\beta_2 (\beta_3 + \beta_4 x)}{\beta_2^2 + (x - \beta_1)^2} + \beta_5}$$

$$\textcircled{2} f_1(x; \beta) = (\beta_3 + \beta_4 x) \frac{x - \beta_1}{\sqrt{\beta_2 + (x - \beta_1)^2}} + \beta_5 x + 1$$

$$\frac{\partial}{\partial x} f_1(x; \beta) = \beta_4 \frac{x - \beta_1}{\sqrt{\beta_2 + (x - \beta_1)^2}} + (\beta_3 + \beta_4 x) \left[\frac{\sqrt{\beta_2 + (x - \beta_1)^2} (1) - (x - \beta_1) \left(\frac{1}{2}\right) (\beta_2 + (x - \beta_1)^2)^{-\frac{1}{2}} (2(x - \beta_1))}{\beta_2 + (x - \beta_1)^2} \right] + \beta_5$$

$$= \beta_4 \frac{x - \beta_1}{\sqrt{\beta_2 + (x - \beta_1)^2}} + (\beta_3 + \beta_4 x) \left[\frac{1}{\sqrt{\beta_2 + (x - \beta_1)^2}} - \frac{(x - \beta_1)^2}{(\beta_2 + (x - \beta_1)^2)^{3/2}} \right] + \beta_5$$

$$= \boxed{\beta_4 \frac{x - \beta_1}{\sqrt{\beta_2 + (x - \beta_1)^2}} + \frac{(\beta_3 + \beta_4 x)}{\sqrt{\beta_2 + (x - \beta_1)^2}} \left[1 - \frac{(x - \beta_1)^2}{(\beta_2 + (x - \beta_1)^2)} \right] + \beta_5}$$

$$\textcircled{3} f_2(x; \beta) = -(\beta_3 + \beta_4 x) (1 + e^{(x - \beta_1)/\beta_2})^{-1} + \beta_5 x + 1$$

$$\frac{\partial}{\partial x} f_2(x; \beta) = -\beta_4 (1 + e^{(x - \beta_1)/\beta_2})^{-1} + (\beta_3 + \beta_4 x) (1 + e^{(x - \beta_1)/\beta_2})^{-2} \left(e^{\frac{x - \beta_1}{\beta_2}} \cdot \frac{1}{\beta_2} \right) + \beta_5$$

$$= \boxed{-\beta_4 (1 + e^{(x - \beta_1)/\beta_2})^{-1} + \frac{(\beta_3 + \beta_4 x)}{\beta_2} \frac{e^{\frac{x - \beta_1}{\beta_2}}}{(1 + e^{(x - \beta_1)/\beta_2})^2} + \beta_5}$$