# **PARE Model**

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## Point-to-Area Random Effects

Link to this project's Github Repo (contains all datasets).

This site is a guided tutorial on fitting the Point-to-Area Random Effects model as part of the paper "Spatial-Temporal Extreme Modeling for Point-to-Area Random Effects (PARE)", presented at SDSS 2023 and submitted to the accompanying special issue of the Journal of Data Science.

This paper concerns how to model extreme values when data are available at the point level, but results are desired at the area level.

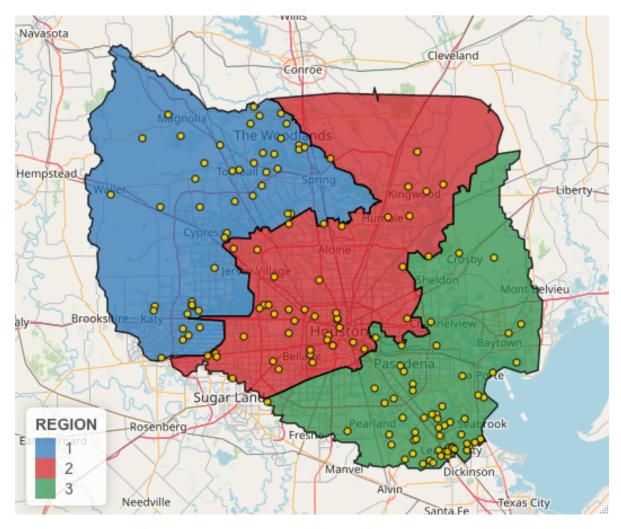


Figure 1: A map of the rainfall gauges (point-level data) and the hydrologic regions (areal data) used in the analysis.

## 1 PARE model

## 1.1 Load data, functions, and packages

```
library(dplyr)
library(sf)
library(spdep)
library(extRemes)
library(ggplot2)
thresh = 253
stations <- read.csv("Data/station_info.csv")</pre>
stat_nos <- stations[,1]</pre>
source("Code/HelperFunctions_PARE/format_data.R")
source("Code/HelperFunctions PARE/jitter n sym.R")
source("Code/HelperFunctions_PARE/get_sym_car_mat.R")
source("Code/HelperFunctions PARE/get par car.R")
source("Code/HelperFunctions_PARE/get_se.R")
source("Code/HelperFunctions_PARE/pars_to_rl.R")
source("Code/HelperFunctions_PARE/rl_with_ci.R")
source("Code/HelperFunctions_PARE/make_varcov_mats_car.R")
source("Code/HelperFunctions_PARE/rl_ci_to_plot_vec.R")
source("Code/HelperFunctions_PARE/create_mat.R")
source("Code/HelperFunctions_PARE/win_rl_to_plot_dat.R")
source("Code/HelperFunctions_PARE/get_latex_table.R")
```

### 1.2 Format time series of GPD fits

```
## load watershed data
ws_regs <- sf::read_sf("Data/watershed_region_updated")</pre>
st_crs(ws_regs) <-sf::st_crs(2278)
## window object generated in Fit_GPD.qmd
## Load rolling GPD fits (takes a bit)
window <- readRDS("Data/window_1day_dclust_updated.rds") # list of lists (82 x 601) of GF
window_CVM <- readRDS("Data/window_CVM_updated.rds")</pre>
window_AD <- readRDS("Data/window_AD_updated.rds")</pre>
## Extract windows (note varying # stations)
dat_win_22 <- format_data(window[[22]], stations, ws_regs, window_CVM[[22]]) # 34</pre>
dat_win_52 <- format_data(window[[52]], stations, ws_regs, window_CVM[[52]]) # 28
dat_win_82 <- format_data(window[[82]], stations, ws_regs, window_CVM[[82]]) # 149</pre>
## save values (faster than reading entire window[[]] object)
save(dat_win_22, file = "Data/dat_win_22")
save(dat_win_52, file = "Data/dat_win_52")
save(dat_win_82, file = "Data/dat_win_82")
```

## 1.3 Construct spatial weight matrices

```
i Generating the Hausdorff matrix
The hausMat function and additional documentation can be found in the hausdorff
Github repository.

# (make sure ws_regs is projected first)
distMat <- hausMat(ws_regs, f1 = 0.5)
distMat/5280

# save output
# saveRDS(distMat, file = "~/Documents/GitHub/Spatial_Extreme_Value_Modeling/Data/hMat_m</pre>
```

```
# read in previously computed Hausdorff matrix and convert units to miles
hMat <- readRDS(file = "Data/hMat_med.rds")</pre>
```

```
hMat_miles <- hMat/5280

## Create weight matrix for each window
## Accounts for different numbers of stations (varying point-to-area structure)
## Jiters the values to give valid weight matrix
set.seed(24)
D_22 <- get_sym_car_mat(dat_win_22, hMat_miles)
D_52 <- get_sym_car_mat(dat_win_52, hMat_miles)
D_82 <- get_sym_car_mat(dat_win_82, hMat_miles)</pre>
```

## 1.4 Fit models to each window for each parameter

To obtain region-level estimates of the GPD parameters, fit a conditional auto-regressive model for each GPD parameter for each window, a total of 9 models.

```
## window 22
## Fit models
car_shape_22 <- spatialreg::spautolm(shape ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                       data = dat_win_22, family="CAR",
                                       listw=mat2listw(1/D_22, style = "C"))
car_ln.scale_22 <- spatialreg::spautolm(log(scale) ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                          data = dat_win_22, family="CAR",
                                          listw=mat2listw(1/D_22,style = "C"))
car_rate_22 <- spatialreg::spautolm(rate ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                      data = dat_win_22, family="CAR",
                                      listw=mat2listw(1/D_22, style = "C"))
## extract parameter estimates
par_22 <- get_par_car(car_ln.scale_22, car_shape_22, car_rate_22)</pre>
se_22 <- get_se(car_ln.scale_22, car_shape_22, car_rate_22)[3,]</pre>
car_varcov_22 <- make_varcov_mats_car(car_ln.scale_22, car_shape_22) # se of scale 8.5, 4</pre>
st_devs_22 \leftarrow data.frame(Reg1 = c(sqrt(diag(car_varcov_22[[1]])), se_22[1]),
                          Reg2 = c(sqrt(diag(car_varcov_22[[2]])), se_22[2]),
                          Reg3 = c(sqrt(diag(car_varcov_22[[3]])), se_22[3]))
row.names(st_devs_22) <- row.names(par_22)</pre>
## window 52
## Fit models
car_shape_52 <- spatialreg::spautolm(shape ~ -1 + Reg1 + Reg2 + Reg3,</pre>
```

```
data = dat_win_52, family="CAR",
                                       listw=mat2listw(1/D_52, style = "C"))
car_ln.scale_52 <- spatialreg::spautolm(log(scale) ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                          data = dat_win_52, family="CAR",
                                          listw=mat2listw(1/D_52, style = "C"))
car_rate_52 <- spatialreg::spautolm(rate ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                      data = dat_win_52, family="CAR",
                                      listw=mat2listw(1/D_52, style = "C"))
## Extract parameter estimates
par_52 <- get_par_car(car_ln.scale_52, car_shape_52, car_rate_52)</pre>
se_52 <- get_se(car_ln.scale_52, car_shape_52, car_rate_52)[3,]</pre>
car_varcov_52 <- make_varcov_mats_car(car_ln.scale_52, car_shape_52) # se of scale 8.5, 4</pre>
st_devs_52 \leftarrow data.frame(Reg1 = c(sqrt(diag(car_varcov_52[[1]])), se_52[1]),
                          Reg2 = c(sqrt(diag(car_varcov_52[[2]])), se_52[2]),
                          Reg3 = c(sqrt(diag(car_varcov_52[[3]])), se_52[3]))
## window 82
## fit models
car_shape_82 <- spatialreg::spautolm(shape ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                       data = dat_win_82, family="CAR",
                                       listw=mat2listw(1/D_82, style = "C"))
car_ln.scale_82 <- spatialreg::spautolm(log(scale) ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                          data = dat_win_82, family="CAR",
                                          listw=mat2listw(1/D_82, style = "C"))
car_rate_82 <- spatialreg::spautolm(rate ~ -1 + Reg1 + Reg2 + Reg3,</pre>
                                      data = dat_win_82, family="CAR",
                                      listw=mat2listw(1/D_82, style = "C"))
## Extract parameter estiamtes
par_82 <- get_par_car(car_ln.scale_82, car_shape_82, car_rate_82)</pre>
se_82 <- get_se(car_ln.scale_82, car_shape_82, car_rate_82)[3,]</pre>
car_varcov_82 <- make_varcov_mats_car(car_ln.scale_82, car_shape_82) # se of scale 8.5, 4</pre>
st_devs_82 \leftarrow data.frame(Reg1 = c(sqrt(diag(car_varcov_82[[1]])), se_82[1]),
                          Reg2 = c(sqrt(diag(car_varcov_82[[2]])), se_82[2]),
                          Reg3 = c(sqrt(diag(car_varcov_82[[3]])), se_82[3]))
```

### 1.4.1 Table of parameter estimates

#### [1] "Parameters"

```
Reg1 Reg2 Reg3
scale 230.85855709 200.15626351 198.45979895
```

shape -0.01912063 0.05329551 0.14923817 rate 0.02814622 0.03138875 0.03426008

#### [1] "Standard Errors"

Reg1 Reg2 Reg3 scale 5.3213140942 3.0221032163 4.5999740248 shape 0.0313947819 0.0204812330 0.0315739471 rate 0.0005872697 0.0003877783 0.0005903755

#### [1] "Parameters"

Reg1 Reg2 Reg3 scale 171.95531347 202.3189569 217.80371166 shape 0.17232481 0.1340430 0.16194601 rate 0.02981064 0.0317399 0.03388214

#### [1] "Standard Errors"

Reg1 Reg2 Reg3 1 4.4827004098 2.7176004289 6.6253934853 2 0.0131459687 0.0067930158 0.0153266094 3 0.0006522969 0.0003391368 0.0007591396

#### [1] "Parameters"

Reg1 Reg2 Reg3 scale 215.07848444 236.68456472 223.98647293 shape 0.20456458 0.16327472 0.18610913 rate 0.05510062 0.04333857 0.05629901

#### [1] "Standard Errors"

Reg1 Reg2 Reg3
1 4.323191261 4.894934168 3.699433095
2 0.016361115 0.016841083 0.013420936
3 0.002272369 0.002339033 0.001864008

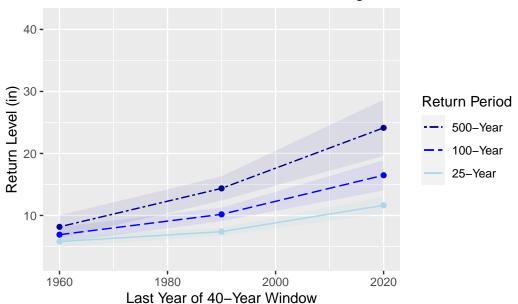
### 1.5 Calculate return levels + CIs

```
### Estimates and CIs for visuals
car_rl_25_22 <- rl_with_ci(par_mat = par_22,</pre>
                             varcov_list = car_varcov_22,
                             return_period = 25,
                             type = "ci", alpha = 0.05)/254
car_rl_100_22 <- rl_with_ci(par_22, car_varcov_22, 100, "ci")/254</pre>
car_rl_500_22 <- rl_with_ci(par_22, car_varcov_22, 500, "ci")/254
car_rl_25_52 <- rl_with_ci(par_52, car_varcov_52, 25, "ci")/254
car_rl_100_52 <- rl_with_ci(par_52, car_varcov_52, 100, "ci")/254</pre>
car_rl_500_52 <- rl_with_ci(par_52, car_varcov_52, 500, "ci")/254</pre>
car_rl_25_82 <- rl_with_ci(par_82, car_varcov_82, 25, "ci")/254</pre>
car_rl_100_82 <- rl_with_ci(par_82, car_varcov_82, 100, "ci")/254
car_rl_500_82 <- rl_with_ci(par_82, car_varcov_82, 500, "ci")/254</pre>
win_1 <- rl_ci_to_plot_vec(car_rl_25_22, car_rl_100_22, car_rl_500_22)
win_2 <- rl_ci_to_plot_vec(car_rl_25_52, car_rl_100_52, car_rl_500_52)
win_3 <- rl_ci_to_plot_vec(car_rl_25_82, car_rl_100_82, car_rl_500_82)
plot_dat <- win_rl_to_plot_dat(win_1, win_2, win_3)</pre>
reg1 <- plot_dat[[1]] # Region 1 data frame for plotting</pre>
reg2 <- plot_dat[[2]]</pre>
reg3 <- plot_dat[[3]]</pre>
```

#### 1.5.1 Visualize return levels

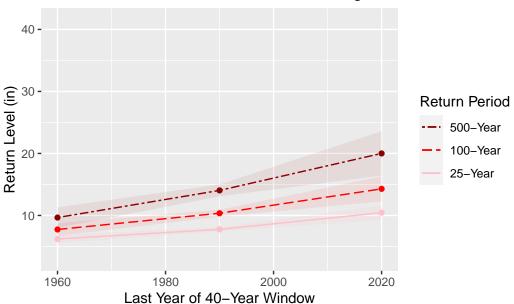
```
ggplot(data=reg1, aes(x=c(1960, 1990, 2020))) +
    coord_cartesian(ylim = c(3, 41.5)) +
    geom_line(aes(y=RL_500, color="500-Year", linetype = "500-Year")) + geom_point(aes(y=R)
    geom_line(aes(y=RL_100, color="100-Year", linetype = "100-Year")) + geom_point(aes(y=R)
    geom_line(aes(y=RL_25, color="25-Year", linetype = "25-Year")) + geom_point(aes(y=RL_2)
    labs(x="Last Year of 40-Year Window", y="Return Level (in)", title="Estimated Return I scale_colour_manual(name = "Return Period", values = c('500-Year' = "darkblue", '100-Year')
    scale_linetype_manual(values = c("500-Year" = "solid"), breaks = c('500-Year', '100-Year')
```

## Estimated Return Levels - Model 1 - Region 1

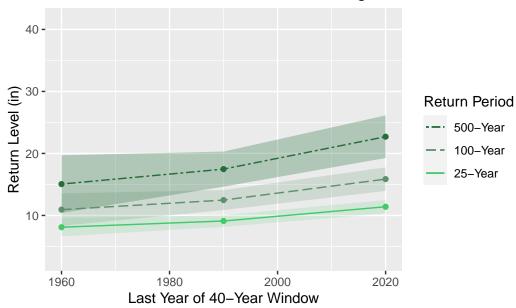


```
ggplot(data=reg2, aes(x=c(1960, 1990, 2020))) +
   coord_cartesian(ylim = c(3, 41.5)) +
   geom_line(aes(y=RL_500, color="500-Year", linetype = "500-Year")) + geom_point(aes(y=RL_geom_line(aes(y=RL_100, color="100-Year", linetype = "100-Year")) + geom_point(aes(y=RL_geom_line(aes(y=RL_25, color="25-Year", linetype = "25-Year")) + geom_point(aes(y=RL_25, labs(x="Last Year of 40-Year Window", y="Return Level (in)", title="Estimated Return Level scale_colour_manual(name = "Return Period", values = c('500-Year' = "darkred", '100-Year breaks = c('500-Year', '100-Year', '25-Year'))+
   scale_linetype_manual(values = c("500-Year" = "solid"), breaks = c('500-Year', '100-Year', '100-Ye
```

## Estimated Return Levels - Model 1 - Region 2



## Estimated Return Levels - Model 1 - Region 3



### 1.5.2 Table of return levels

```
Reg2
               Reg1
                                    Reg3
25-yr RL 5.7812602 6.2000477 8.1017057
          0.4268857 0.3273666 0.7538876
SE
100-yr RL 6.8995253 7.7328867 10.9382561
          0.6450349 0.5268412 1.3344426
SE
500-yr RL 8.1611487 9.6604967 15.0588530
          0.9508076 0.8372663 2.3736256
  ## Table 7
  car_rl_25_52_se <- rl_with_ci(par_52, car_varcov_52, 25, "se")/254
  car_rl_100_52_se <- rl_with_ci(par_52, car_varcov_52, 100, "se")/254
  car_rl_500_52_se <- rl_with_ci(par_52, car_varcov_52, 500, "se")/254</pre>
  # Latex output
  car_rl_52_list <- list(car_rl_25_52_se, car_rl_100_52_se, car_rl_500_52_se)</pre>
  get_latex_table_RL(car_rl_52_list)
&25-yr RL&7.391(0.3201)&7.759(0.1733)&9.104(0.4793)\\
&100-yr RL&10.177(0.5587)&10.354(0.2883)&12.477(0.8223)\\
&500-yr RL&14.367(0.9991)&14.038(0.4891)&17.472(1.4436)\\
  rbind(t(car rl_25_52 se), t(car rl_100_52 se), t(car rl_500_52 se))
                Reg1
                           Reg2
                                      Reg3
25-yr RL
           7.3910220 7.7591772 9.1036876
SE
           0.3201394 0.1732745 0.4792676
100-yr RL 10.1767483 10.3537503 12.4771511
SE
           0.5586972 0.2883097 0.8222935
500-yr RL 14.3667665 14.0375739 17.4724334
SE
           0.9991024 0.4890933 1.4436359
  ## TABLE 3
  car_rl_25_82_se <- rl_with_ci(par_82, car_varcov_82, 25, "se")/254
  car_rl_100_82_se <- rl_with_ci(par_82, car_varcov_82, 100, "se")/254</pre>
  car_rl_500_82_se <- rl_with_ci(par_82, car_varcov_82, 500, "se")/254
  # Latex output
  car_rl_82_list <- list(car_rl_25_82_se, car_rl_100_82_se, car_rl_500_82_se)</pre>
  get_latex_table_RL(car_rl_82_list)
```

```
&25-yr RL&11.634(0.6874)&10.442(0.5857)&11.399(0.5461)\\
&100-yr RL&16.48(1.2435)&14.291(1.0239)&15.856(0.9666)\\
&500-yr RL&24.131(2.3097)&20.002(1.8186)&22.7(1.7504)\\
```

```
rbind(t(car_rl_25_82_se), t(car_rl_100_82_se), t(car_rl_500_82_se))
```

Reg1 Reg2 Reg3
25-yr RL 11.6342489 10.4422599 11.3992557
SE 0.6873749 0.5856544 0.5461398
100-yr RL 16.4795690 14.2913625 15.8560177
SE 1.2435110 1.0239273 0.9666489
500-yr RL 24.1305826 20.0023744 22.7002292
SE 2.3097267 1.8186075 1.7503929

# 2 Block Kriging and Regional Max code

The code in the previous few files has been curated for ease-of-use with a focus on the PARE model, which is the novel part of the paper.

Code for all three models PARE, Block Kriging, and Regional Max methods the full analysis as originally included in [?] is available on Github.

## 3 Fitting GPD to moving windows

The methodology used here is described in [?]. If fitting to more rolling windows than described in the paper this material accompanies, you will need to run this code to generate the object window.

## 3.1 Setup and Declustering

```
# Recall PRCP = Precipitation is measured as "tenths of mm" (254 = 1 inch)
# To get inches: x/254
# To get mm: x/10
library(lubridate)
library(extRemes)
library(stats)
library(dplyr)
library(xts)
library(gnFit)
library(ismev)
# library(tseries)
# library(trend)
# library(astsa)
# library(ggmap)
# Load in the precipitation data
precip <- read.csv("../scrape-NOAA-data/Data/all_stations_precip_UDP_updated_2021.csv")</pre>
### Declustering
precip_dclust = precip
for ( i in seq(from=2, to=length(precip[1,]))){
  station = precip[,i]
  non_na = which(station>=0)
  station[is.na(station)] <- 0</pre>
```

```
dec = extRemes::decluster(station, threshold=0, clusterfun = "max", replace.with = 0)
    stat_clus = as.numeric(dec)

station = stat_clus
    precip_dclust[non_na,i] = station[non_na]
}

### format
precip_dclust[,1] <- lubridate::ymd(as.character(precip_dclust[,1])) #put Date column into
precip_dclust <- as.data.frame(precip_dclust) #make it a data frame

### Set threshold
thresh <- 253  # for 1-day</pre>
```

## 3.2 Fit GPD to entire series by station

```
fulldata_no_na <- data_no_na(precip_dclust, thresh) # saving data corresponding to fullfi

## Goodness of fit measures

CVMp <- ADp <- NULL

for(h in c(1:numstat)){ # for some reason 481 was not working with gof fn, so use gof_fix_
    if(!sum(is.na(fullfits[[h]]))){
        gof_h <- gof_fix_error(fulldata_no_na[[h]], dist="gpd", pr=fullfits[[h]]$results$par,
        CVMp[h] <- gof_h$Wpval
        ADp[h] <- gof_h$Apval
    }else{
        CVMp[h] <- ADp[h] <- NA
    }
}</pre>
```

## 3.3 GPD moving window fits

This code chunk generates the "window" file which is the starting point for the PARE model. It will take some time to run and the resulting file is about 1.6 Gb.

```
# start <- pasteO(startyr, startday)
# endyr <- startyr + 39
# end <- pasteO(endyr, endday)
# labelyr[j] <- lubridate::year(as.Date(end))
#
# start <- which(precip_dclust$Date==start) #finding indexes corresponding to start & end <- which(precip_dclust$Date==end)
# sub <- precip_dclust[start:end, ] #subset data to those 40 years
# window[[j]] <- fitgpdR(sub, thresh) # GPD fit with extRemes package
# startyr <- startyr + 1
# }
#
## Large file-- takes time to save/load
#saveRDS(window, file="Data/window_1day_dclust_updated.rds")
#window <- readRDS(file="Data/window_1day_dclust_updated.rds")
#window <- readRDS(file="Data/window_1day_dclust_updated.rds")
## file too big for quarto to load-- could try some of the solutions listed</pre>
```

### 3.4 Calculate return levels

Using the GPD parameter estimates to calculate the return level estimates.

```
# Saving Return Levels for Easy Plotting ------
# in mm
# this is set up differently for easier plotting
# saving vector of RLs for each station across 79 windows

trend_RL <- list()
for(i in 1:numstat){
   RL <- NULL
   for(j in 1:numcol){
     fit <- window[[j]][[i]]
     if(!sum(is.na(fit)) == TRUE){
        RL[j] <- extRemes::return.level(fit, return.period=100)/10 # divide by 10 to get mm
     }else{
        RL[j] <- NA
     }
}</pre>
```

```
}
  trend_RL[[i]] <- RL</pre>
# trend_RL[[588]] # Hobby
trend_RL_25 <- list()</pre>
trend_RL_100 <- list()</pre>
trend_RL_500 <- list()</pre>
for(i in 1:numstat){
  RL_25 <- RL_100 <- RL_500 <- NULL
  for(j in 1:numcol){
    fit <- window[[j]][[i]]</pre>
    if(!sum(is.na(fit)) == TRUE){
      RL_25[j] <- extRemes::return.level(fit, return.period=25)/10 # divide by 10 to get
      RL_100[j] <- extRemes::return.level(fit, return.period=100)/10 # divide by 10 to ge
      RL_500[j] <- extRemes::return.level(fit, return.period=500)/10 # divide by 10 to ge
      RL_25[j] \leftarrow RL_100[j] \leftarrow RL_500[j] \leftarrow NA
    }
  }
  trend_RL_25[[i]] <- RL_25
  trend_RL_100[[i]] <- RL_100
  trend_RL_500[[i]] <- RL_500
}
#labelyr <- 1939:2017
labelyr <- 1939:2020
```

### 3.5 Goodness of fit measures

This code chunk performs goodness of fit tests to determine for each of the rolling windows which of the stations have achieved a good fit to the GPD distribution. These goodness-of-fit results are used in the extreme value model fitting process as part of the data cleaning. The spatial models are only applied to stations which do not show evidence of a lack of GPD fit for a particular window.

```
# Goodness of Fit ------
# Now going to do GOF for ALL stations, for EACH WINDOW FIT
### Should save RDS to access saved output instead of having to run again
window_CVM <- matrix(nrow = numstat, ncol = numcol)</pre>
window_AD <- matrix(nrow = numstat, ncol = numcol)</pre>
for(j in 1:numcol){
 CVMp <- ADp <- NULL
  for(i in 1:numstat){
    fit <- window[[j]][[i]]</pre>
    if(!sum(is.na(fit)) == TRUE){
      gof_h <- gof_fix_error(fit$x, dist="gpd", pr=fit$results$par, threshold=thresh)</pre>
      CVMp[i] <- gof_h$Wpval</pre>
      ADp[i] <- gof_h$Apval
    }else{
      CVMp[i] <- ADp[i] <- NA
    }
  }
  window_CVM[, j] <- CVMp</pre>
  window_AD[, j] <- ADp</pre>
}
window_CVM <- as.data.frame(window_CVM)</pre>
window_AD <- as.data.frame(window_AD)</pre>
# saveRDS(window_CVM, file="Data/window_CVM_updated.rds")
# saveRDS(window_AD, file="Data/window_AD_updated.rds")
window_CVM <- readRDS("Data/window_CVM_updated.rds")</pre>
window_AD <- readRDS("Data/window_AD_updated.rds")</pre>
# window_CVM[i, j] gives CVM (Cramer Von Mises) p-value for station i, window j (similar
# window[[j]][[i]] gives gpd fit output for station i, window j
```