

# Solutions: Assignment 2 Due 10/7 at Midnight

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NOTE: I forgot to include a relevant detail for Part 2, number 7. The change is **bolded**. Sorry!!! Part 2 number 8 should be easier to answer now.

## Part 1: Math

In class, we have worked with “Signal plus noise Model” (equation 1.5)

$$\text{Model: } x_t = 2 \cos(2\pi \frac{t+15}{50}) + w_t$$

$$\text{Mean function: } \mathbb{E}(x_t) = 2 \cos(2\pi \frac{t+15}{50})$$

1. [5 points] The mean function is derived in Example 2.4. Describe what happens in each step of the computation [3 points], and provide a “math stress” rating (1 = effortless, 100 = nightmare) and 3 emojis[2 points]. This is personal and there is no right answer.
2. [5 points] Is the signal plus noise model stationary in the mean?
3. [5 points] Write down  $\gamma_x(s, t)$ , the autocovariance function of  $x_t$  [3 points]. You may accomplish this in any way, including asking me personally in office hours or asking a classmate. Just make sure you cite the source![2 points]
4. [6 points] Consider the model:

$$y_t = x_t - 2 \cos(2\pi \frac{t+15}{50})$$

Compute the mean function of  $y_t$  [3 points]. Is  $y_t$  stationary in the mean?[1 point] How do you know?[2 points]

## Part 2: Code

Note: I have set the code chunks here to have `eval: false` in the code chunk. Change that to `true` so that I can run your code easily.

0. [5 points] All your code runs without errors (unless that's the point), and if there is a message, explain what it means. (Bonus: to be nice to me, submit a rendered pdf)
1. [5 points] Simulate from an AR(1) process with coefficient 0.7 and 10 data points.

```
library(astsa)

# your code here

w <- rnorm(10)
x_t <- stats::filter(x = w, filter = 0.7, method = "recursive")
```

2. [6 points] Look at the documentation for the `stats::lag` function (run `?lag` in the console). State what package the function is in and what the function does[4 points]. Using `k = 1` compute a `lag(1)` version of `x_t` that you simulated above[2 points].

```
x_t_lag1 <- stats::lag(x_t, k = 1)
```

3. [3 points] Run the following code and compare `x_t` and `x_t_lag1`.

```
cbind(x_t, x_t_lag1)
```

Time Series:

Start = 0

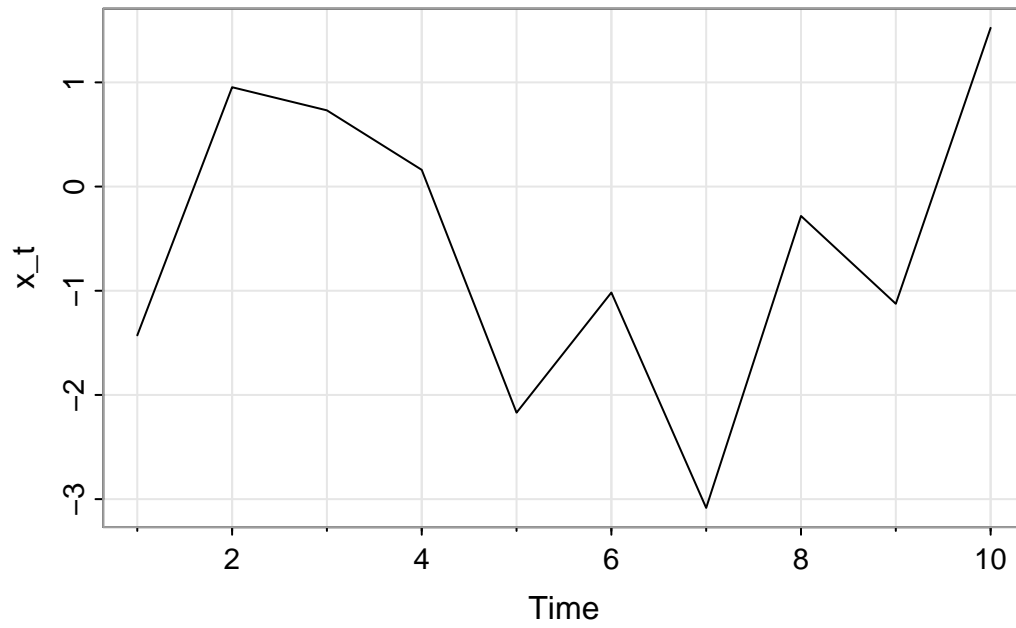
End = 10

Frequency = 1

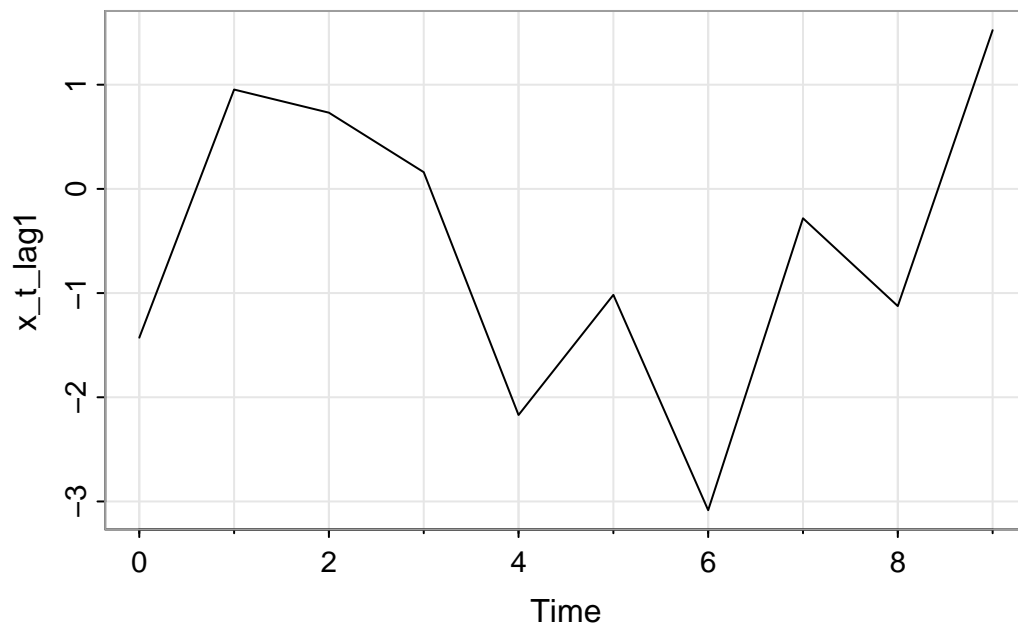
	x_t	x_t_lag1
0	NA	-1.4280561
1	-1.4280561	0.9532596
2	0.9532596	0.7313082
3	0.7313082	0.1602877
4	0.1602877	-2.1702800
5	-2.1702800	-1.0169647
6	-1.0169647	-3.0840613
7	-3.0840613	-0.2821006
8	-0.2821006	-1.1250459
9	-1.1250459	1.5230642
10	1.5230642	NA

4. Make a time series plot of  $x_t$  and  $x_{t-1}$ . Do you notice the same features as when in the previous question?

```
tsplot(x_t)
```



```
tsplot(x_t_lag1)
```

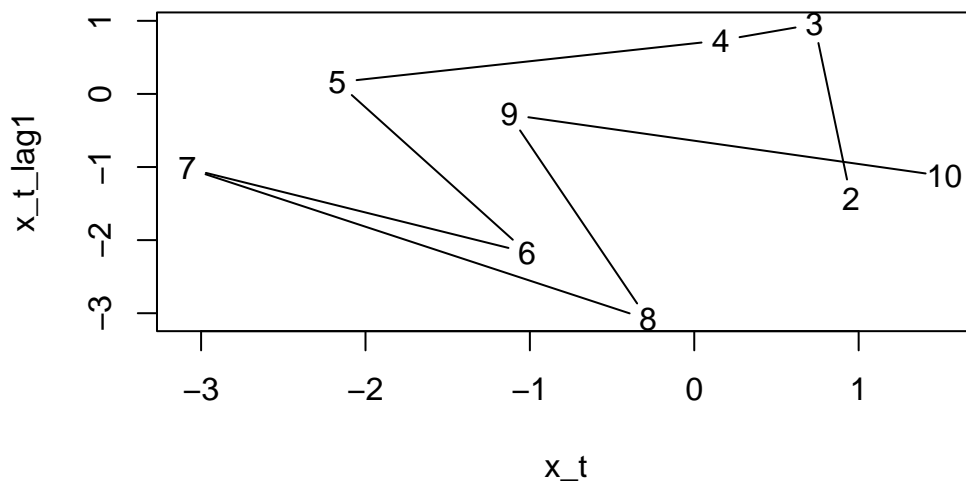


5. Run the below code. Why are the plots different? Are either particularly useful?

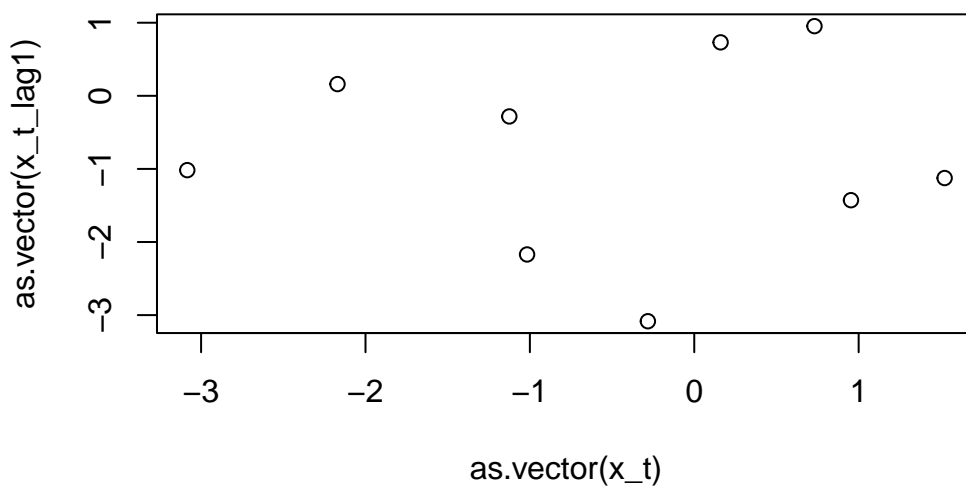
```
plot(x_t, x_t_lag1)
plot(as.vector(x_t), as.vector(x_t_lag1))
```

6. Instead of using `stats::lag`, use `dplyr::lag` to create a new version of `x_t_lag`. Repeat the code from steps 2-5. Describe how the output has changed.

```
x_t_lag1 <- dplyr::lag(as.vector(x_t), n = 1)
plot(x_t, x_t_lag1)
```



```
plot(as.vector(x_t), as.vector(x_t_lag1))
```



7. Re-simulate an AR(1) process as in number 1, but this time with 100 observations. Also recompute  $x_{t-1}$ . Fit an intercept-free regression model to

**predict  $x_t$  from  $x_{t-1}$ .** Provide the value of the slope estimate and interpret the value in the context of this simulation.

```
w <- rnorm(100)
x_t <- stats::filter(x = w, filter = 0.7, method = "recursive")

x_t_lag1 <- dplyr::lag(as.vector(x_t), n = 1)

linear_model <- lm(x_t ~ -1 + x_t_lag1)
summary(linear_model)
```

Call:

```
lm(formula = x_t ~ -1 + x_t_lag1)
```

Residuals:

Min	1Q	Median	3Q	Max
-2.99483	-0.76753	0.03296	0.66949	2.60954

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
x_t_lag1	0.61592	0.07891	7.805	6.59e-12 ***
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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.05 on 98 degrees of freedom

(1 observation deleted due to missingness)

Multiple R-squared: 0.3833, Adjusted R-squared: 0.377

F-statistic: 60.92 on 1 and 98 DF, p-value: 6.594e-12

8. [11 points] Plot the acf of  $x_t$ [2 points] and the acf of the residuals from the regression model[4 points]. Which looks more like white noise?[2 points] What does this tell you about the temporal structure in  $x_t$  and its residuals?[3 points]

```
acf(x_t)
acf(residuals(linear_model))
```

### Part 3: Reading

[9 points] Read sections 2.8 and 2.9 from Forecasting Principles and Practice. Make 3 connections [3 points each] to content from the course textbook (equations or similar examples.).