Circular Motion and Gravity
Orbital Simulation
PHYS 442

Dr. Schultz

October 9, 2015

Date Performed:	September 18, 2015
Partners:	Whole class
Instructor:	Me

1 Objective

Explored the motion of a particle under the influence of a gravitational force. Specifically we look at attractive inverse square distance forces, Hookean forces, escape velocity, circular orbits, kinetic energy, potential energy and elliptical orbits. These are defined in 1.1:

1.1 Definitions

Law of Universal Gravitation The law of universal gravitation states the force of gravity between two point masses is directly proportional to each mass and inversely proportional to the distance between them. This is also true for masses outside of spherically symmetric mass distributions.Homer (2014)

$$F_g = \frac{mMG}{r^2}$$

Hookean Forces Inside a uniformly dense sphere of mass the force is Hookean, with an attractive force proportional to the displacement from equilibrium. The effective spring constant is $K = \frac{mMG}{R^3}$.

$$F_g = \frac{mMG}{R^3}r$$

Gravitational Constant The universal gravitation constant G determines the strength of the gravity force from a given mass. It may also be considered as the force that 1 kg exerts on another 1 kg mass separated by 1 meter.

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Escape Velocity Escape velocity is the initial velocity required to escape gravitational attraction. An object launched at the escape velocity will never come back (escape).

$$v_{escape} = \sqrt{\frac{2MG}{r}}$$

Kinetic Energy Kinetic energy is the energy associated with motion.

$$KE = \frac{mv^2}{2}$$

Potential Energy The potential associated with the universal gravitation force is written as follows.

$$PE = -\frac{mMG}{r}$$

Circular Orbit A circular orbit is an orbit with a constant radius r.

Elliptic Orbit An elliptic orbit is a closed orbit with changing radius r.

2 Simulation

The simulation applies a central acceleration to the orbiting particle. Outside the boundary of the central mass we have the following acceleration.

$$a = \frac{K}{r^2}$$

Inside the boundary of the central mass ($r < R$) we have the following acceleration.

$$a = \frac{K}{R^3} r$$

Here R is the radius of the central mass and K is a constant determined by the user of the simulation. K is MG. For this simulation the radius was set to $R = 6$ and the constant was set to $K = -0.1$ making the acceleration attractive.

The initial position $\overrightarrow{r}_0$ and initial velocity $\overrightarrow{v}_0$ are set by the user.

3 Sample Calculation

3.1 Circular Orbit

Given $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we find the $\overrightarrow{v}_0$ for circular orbit.

$$F_{net} = ma$$

$$\frac{mMG}{r^2} = m\frac{v^2}{r}$$

$$\frac{K}{r^2} = \frac{v^2}{r}$$

$$v = \sqrt{\frac{K}{r}}$$

$$v = \sqrt{\frac{0.1}{10}} = 0.1$$

The velocity must be tangential and therefore $\overrightarrow{v}_0$ must be perpendicular to $\overrightarrow{r}_0$.

$$\overrightarrow{v}_0 = (0, 0.1)$$

3.2 Escape Velocity

Given $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we find the $\overrightarrow{v}_0$ for escape from the central mass' gravitational attraction. Escape is associated with a total mechanical energy of zero.

$$PE + KE = 0$$

$$-\frac{mMG}{r} + \frac{mv^2}{2} = 0$$

$$-\frac{K}{r} + \frac{v^2}{2} = 0$$

$$v_{escape} = \sqrt{\frac{2K}{r}}$$

$$v_{escape} = \sqrt{\frac{2(0.1)}{10}} = 0.14$$

4 Results and Conclusions

4.1 Circular Orbit

For the conditions $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\overrightarrow{v}_0 = (0, 0.1)$ will give a circular orbit. Running the simulation yields the following orbit. We can see it is circular.

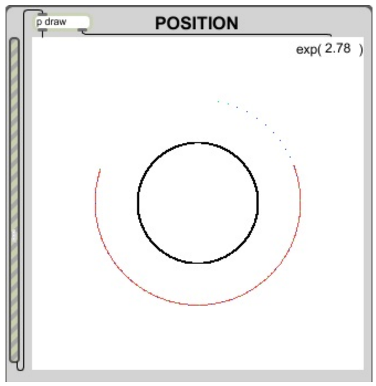

Figure 1: Circular Orbit

4.2 Escape Velocity

For the conditions $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\overrightarrow{v}_0 = (0, 0.14)$ will give an escape from the gravitational attraction. We can observe in the simulation the object indeed escapes and the total energy is zero. See the following figure.

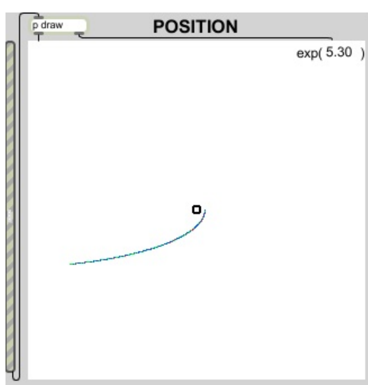

Figure 2: Escape Velocity

4.3 Elliptical Orbit

For the conditions $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we use an initial velocity $\vec{v}_0 = (0, 0.12)$ that is between the one for circular orbit and escape. As the figure shows the resulting orbit is elliptical.

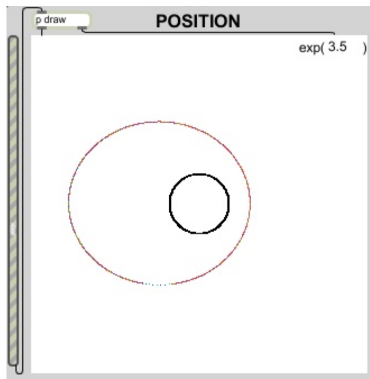

Figure 3: Elliptic Orbit

5 Discussion

With this lab I was able to understand how orbital motion works. With the results we obtained we are now able to distinguish between a circular motion, an elliptical motion and an escape situation. Now we are able to calculate and determine all this three situations. Moreover, we are now possible to see the relationship between the law of universal gravitational and the velocities of both circular and escape motion; we can see the influence of gravity in the motion of a particle. For this we needed to learn the formulas for Kinetic Energy, Potential Energy, Escape Velocity, Centripetal Velocity and the Law of Universal Gravitation. After knowing all this formulas and doing some practice we were then able to establish relationships between the related the formulas. For example, while the force of gravity is:

$$F_g = \frac{mMG}{r^2}$$

the escape velocity formula is:

$$v_{escape} = \sqrt{\frac{2MG}{r}}$$

. It is possible to see how they relate to one another. By being able to relate the formulas, it is easier to determine which orbit an object has and the velocity it acquires. Last but not least, this knowledge is important because nowadays circular orbits are even more present in our life. The space is full of satellites and for them to be efficient they need to be in circular orbit around the Earth. This lab gives us the right knowledge to determine the correct velocity to launch the satellite, which is only one of the examples of orbits in our everyday life. All in all, this lab taught us how to determine the orbits. Because of that, I am now capable to understand and put into practice all that relates to orbits; we can say that we now master the subject and understand the relationship between the formulas and the simulations we made.

References

Homer, J. (2014). *Physics*. Oxford, 3rd edition.