Paper Code: COMP717 Assignment 1

Due Tuesday, 29 April 2025

Name: Julia Calma **ID number: 22167597**

Instructions:

Please attach this sheet to the front of your assignment.

- This assignment contributes 35% towards your final grade. The total mark is 100.
- This assignment has two parts. Part 1 has base problems; Part 2 has a choice of two options A and B. You **can team up with 1-2 other students** to complete this assignment. If you work as a team, all students should sign this page, and provide a brief note on their contributions to the assignment.
- **We encourage you to explore and utilize the capabilities of AI tools like ChatGPT as valuable resources in your learning journey. These tools can be powerful aids when used thoughtfully and ethically. No Full Answer Generation: You must not use AI to generate complete answers or substantial portions of your assignments. Transparency and Attribution: You are required to clearly state in your submission where and how you have used AI tools. Be specific about the prompts you used and the output you received. Critical Evaluation: It is essential to critically evaluate the information provided by AI tools. Remember that AI can sometimes produce inaccurate, biased, or misleading information. Always verify the information with reliable sources. Focus on Learning: The goal of using AI is to enhance your learning, not to replace it.**

  Check Canvas on more details on AI usage policy.
- Submit an electronic copy through Canvas before 11:59 p.m. on Tuesday, 29 April 2025. The submission requirement is specified at the end of each option.

The School of Computer and Mathematical Sciences regards any act of cheating including plagiarism, unau- thorised collaboration and theft of another student's work most seriously. Any such act will result in a mark of zero being given for this part of the assessment and may lead to disciplinary action.

Please sign to signify that you understand what this means, and that the assignment is your own work.

Jula

Signature:

Part 1

Question 1

		Player 2				
		1	**2**	**3**	**4**	**5**
Player 1	**1**	(5,5)	(10,0)	(10,0)	(10,0)	(10,0)
	2	(0,10)	(5,5)	(10,0)	(10,0)	(10,0)
	3	(0,10)	(0,10)	(5,5)	(10,0)	(10,0)
	4	(0,10)	(0,10)	(0,10)	(5,5)	(10,0)
	5	(0,10)	(0,10)	(0,10)	(0,10)	(5,5)

(b) Answer the following questions(5 marks) (6 marks)

- When player 2 chooses 4, what are the best responses for player 1? **1, 2, 3**
- When player 1 chooses 3, what are the best responses for player 2? **1,2**
- When player 2 chooses 2, what are the best responses for player 1? **1**
- When player 1 chooses 1, what are the best responses for player 2? **1**
- For player 1, is the strategy of choosing 4 strictly or very weakly dominated by another strategy? If so, which ones? **Yes it is a very weakly dominated by another strategy, This is when player 1 choose strategy 1.**
- For player 2, is the strategy of choosing 1 strictly or very weakly dominated by another strategy? If so, which ones? **No, player 2 choosing 1 is not dominated by any strategy.**

(c) What is the Nash equilibrium of this game? (4 marks)

- Find this out by applying the concept of dominated strategies to rule out a succession of inferior strategies until only one choice remains.
  - We observe that for player 2, strategy 5 is very weakly dominated by strategy 4 because 4 always gives equal and better payoff than 5 and is strictly better in at least one case.

		Player 2				
		1	**2**	**3**	**4**	**5**
Player 1	**1**	(5,5)	(10,0)	(10,0)	(10,0)	(10,0)
	2	(0,10)	(5,5)	(10,0)	(10,0)	(10,0)
	3	(0,10)	(0,10)	(5,5)	(10,0)	(10,0)
	4	(0,10)	(0,10)	(0,10)	(5,5)	(10,0)
	5	(0,10)	(0,10)	(0,10)	(0,10)	(5,5)

  - We observe that for player 1, strategy 5 is very weakly dominated by strategy 4 because 4 always gives equal and better payoff than 5 and is strictly better in at least one case.

		Player 2				
		1	2	3	4	~~5~~
Player 1	1	(5,5)	(10,0)	(10,0)	(10,0)	~~(10,0)~~
	2	(0,10)	(5,5)	(10,0)	(10,0)	~~(10,0)~~
	3	(0,10)	(0,10)	(5,5)	(10,0)	~~(10,0)~~
	4	(0,10)	(0,10)	(0,10)	(5,5)	~~(10,0)~~
	~~5~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~

- We observe that for player 2, strategy 4 is very weakly dominated by strategy 3 because 3 always gives equal and better payoff than 4 and is strictly better in at least one case.

		Player 2				
		1	2	3	~~4~~	~~5~~
Player 1	1	(5,5)	(10,0)	(10,0)	~~(10,0)~~	~~(10,0)~~
	2	(0,10)	(5,5)	(10,0)	~~(10,0)~~	~~(10,0)~~
	3	(0,10)	(0,10)	(5,5)	~~(10,0)~~	~~(10,0)~~
	4	(0,10)	(0,10)	(0,10)	~~(5,5)~~	~~(10,0)~~
	~~5~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~

- We observe that for player 1, strategy 4 is very weakly dominated by strategy 3 because 3 always gives equal and better payoff than 4 and is strictly better in one case.

		Player 2				
		1	2	3	~~4~~	~~5~~
Player 1	1	(5,5)	(10,0)	(10,0)	~~(10,0)~~	~~(10,0)~~
	2	(0,10)	(5,5)	(10,0)	~~(10,0)~~	~~(10,0)~~
	3	(0,10)	(0,10)	(5,5)	~~(10,0)~~	~~(10,0)~~
	~~4~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~	~~(10,0)~~
	~~5~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~

- We observe that for player 2, strategy 3 is very weakly dominated by strategy 2 because 2 always gives equal and better payoff than 3 and is strictly better in two cases.

		Player 2				
		1	2	~~3~~	~~4~~	~~5~~
Player 1	1	(5,5)	(10,0)	~~(10,0)~~	~~(10,0)~~	~~(10,0)~~
	2	(0,10)	(5,5)	~~(10,0)~~	~~(10,0)~~	~~(10,0)~~
	3	(0,10)	(0,10)	~~(5,5)~~	~~(10,0)~~	~~(10,0)~~
	~~4~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~	~~(10,0)~~
	~~5~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~

- We observe that for player 1, strategy 3 is very weakly dominated by strategy 2 because 2 always gives equal and better payoff than 3 and is strictly better in at least one case.

		Player 2				
		1	2	~~3~~	~~4~~	~~5~~
Player 1	1	(5,5)	(10,0)	~~(10,0)~~	~~(10,0)~~	~~(10,0)~~
	2	(0,10)	(5,5)	~~(10,0)~~	~~(10,0)~~	~~(10,0)~~
	~~3~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~	~~(10,0)~~	~~(10,0)~~
	~~4~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~	~~(10,0)~~
	~~5~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(0,10)~~	~~(5,5)~~

- We observe that for player 2, strategy 2 is very weakly dominated by strategy 1 because 1 always gives equal and better payoff than 2 and is strictly better in two cases.

		Player 2				
		1	2	3	4	5
Player 1	1	(5,5)	(10,0)	(10,0)	(10,0)	(10,0)
	2	(0,10)	(5,5)	(10,0)	(10,0)	(10,0)
	3	(0,10)	(0,10)	(5,5)	(10,0)	(10,0)
	4	(0,10)	(0,10)	(0,10)	(5,5)	(10,0)
	5	(0,10)	(0,10)	(0,10)	(0,10)	(5,5)

- We observe that for player 1, strategy 2 is very weakly dominated by strategy 1 because 1 always gives equal and better payoff than 2 and is strictly better in two cases.

		Player 2				
		1	2	3	4	5
Player 1	1	(5,5)	(10,0)	(10,0)	(10,0)	(10,0)
	2	(0,10)	(5,5)	(10,0)	(10,0)	(10,0)
	3	(0,10)	(0,10)	(5,5)	(10,0)	(10,0)
	4	(0,10)	(0,10)	(0,10)	(5,5)	(10,0)
	5	(0,10)	(0,10)	(0,10)	(0,10)	(5,5)

In conclusion, this leaves only one option: players 1 and 2 should both choose strategy 1.

Question 2. Extensive Form Game (15 marks)

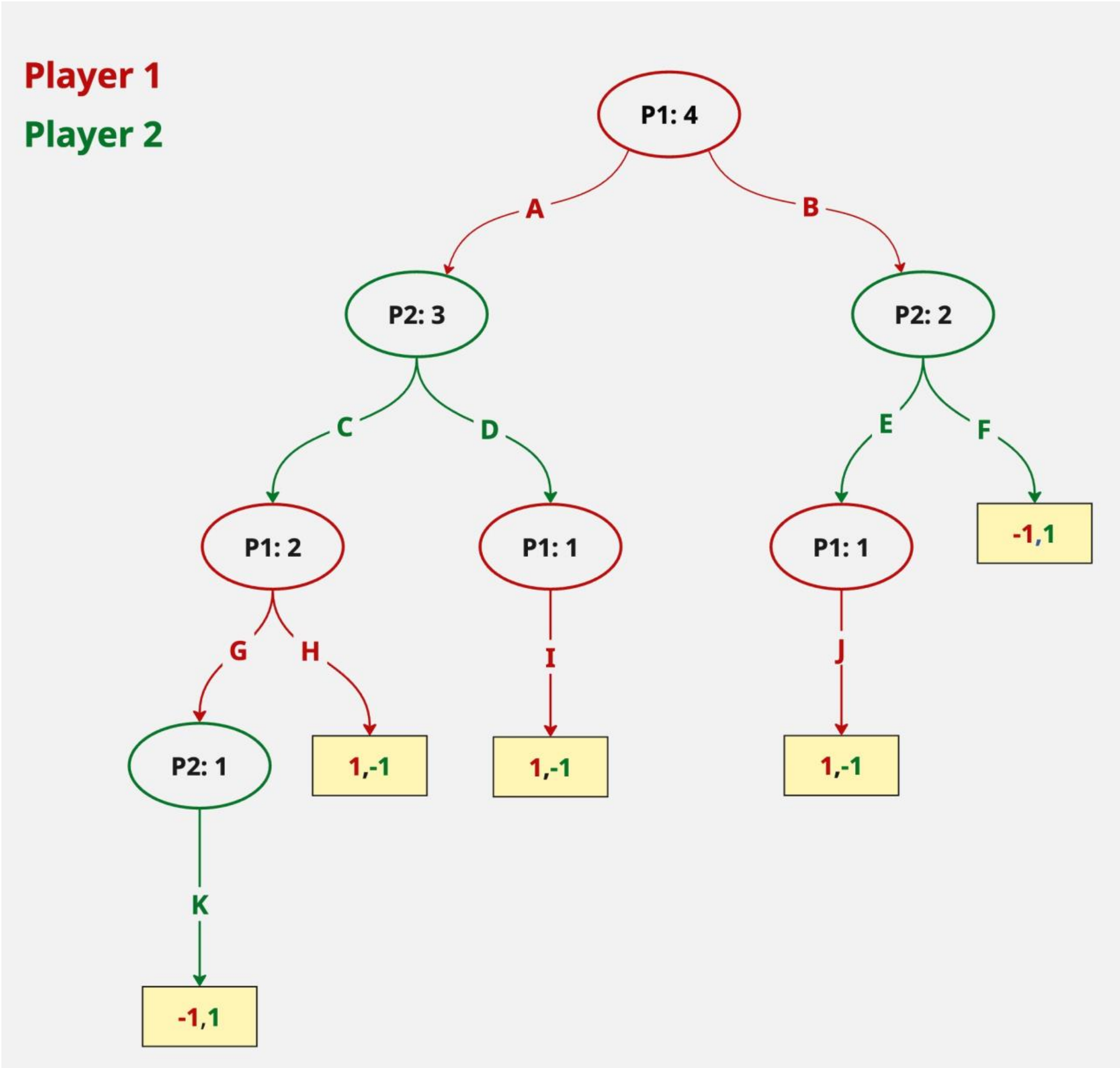
Consider a variant of the Take-away game discussed in the lecture:

• There is a pile of 4 chips on the table.

• Two players take turns to remove 1 or 2 chips from the table, with player 1 starting first.

• The player removing the last chip(s) wins the game, and get a reward of 1; and the opponent gets a reward of -1.

(a) Represent this game in Extensive Form. (Note that players have more than 1 decision node and a strategy is a selection of an action at each decision node. If some node has just 1 action, then that action will be chosen as part of the strategy.) (4 marks)



(b) List all the pure strategies of the two players. (4 marks)

Note that you may first find the number of decision nodes for each player, and label the nodes in an order.

Pure strategies for Player 1: $S_1 = \{(A,G,I,J); (A,H,I,J); (B,G,I,J); (B,H,I,J)\}$

This can be simplified slightly since actions I and J are the only available options at their respective nodes: $S_1 = \{(A,G); (A,H); (B,G); (B,H)\}$

Pure strategies for Player 2: $S_2 = \{(C,E,K); (C,F,K); (D,E,K); (D,F,K)\}$

Since K is the only action available at the bottom node, this can be simplified to: $S_2 = \{(C,E); (C,F); (D,E); (D,F)\}$

(c) Represent this game in normal form.

(Each row or column is a pure strategy for the respective player).

	(C, E)	(C, F)	(D, E)	(D, F)
(A, G)	-1,1	-1,1	1, -1	1, -1
(A, H)	1, -1	1, -1	1, -1	1, -1
(B, G)	1, -1	-1,1	1, -1	-1,1
(B, H)	1, -1	-1,1	1, -1	-1,1

(d) Find all of the pure strategy Nash Equilibria of this game in the normal form. Indicate if an equilibrium is subgame perfect.

The pure strategy Nash Equilibria of this game are:

	(C, E)	(C, F)	(D, E)	(D, F)
(A, G)	-1,1	-1,1	1, -1	1, -1
(A, H)	1,-1	1,-1	1,-1	1,-1
(B, G)	1, -1	-1,1	1,-1	-1,1
(B, H)	1,-1	-1,1	1,-1	-1,1

1. (A,H), (C,E) with payoff (1,-1)
2. (A,H), (C,F) with payoff (1,-1)
3. (A,H), (D,E) with payoff (1,-1)
4. (A,H), (D,F) with payoff (1,-1)

These are the only strategies where neither player has an incentive to deviate.

(3 marks)

From the 4 Nash equilibria, the following are subgame perfect:

- (A, H), (C, F)
- (A, H), (D, F)

These are the only subgame perfect equilibria because they follow the optimal path at every decision node based on backward induction.

Task 1

(a) Rationality and Knowledge (10 marks)

In terms of game theory, rationality can be defined as a player's motivation to maximise their own payoffs, based on the assumption that they will make consistent, logical decisions aimed at achieving the most favourable result from the available options. A player's knowledge about other players' rationality is important. This is because if a player believes that others will act unpredictably or irrationally, they may choose not to opt for the theoretically ideal approach. On the contrary, provided all players believe that everybody else is rational and strategic, their decisions are more inclined in accordance with Nash equilibrium.

V100: Game 1 winners' rationale (from different streams):

P25138	26	Date of birth
P25144	26	just random
P25047	24	Why not
P25086	24	My birthday
P25140	24	my lucky number

The majority of the winners in V100 won by pure luck, with reasoning such as “it’s random” or “lucky number.” Based on the data, there was little evidence of strategic reasoning among the participants in this game. This suggests that success was more coincidental than calculated. In contrast, the winners in V10A and V10B won through rational thinking. Specifically for V10A, the reason is that players had experience with the game and therefore had knowledge about the quantity of the average from the initial game. The data shows that many of the chosen numbers in V10A were strategically positioned, indicating that players were attempting to select values 2/3 closer to the previous average in V100. Hence, their win is best explained by rational thinking based on prior exposure and learning. In V10B, we observe that there were a lot of winners. This is likely because the range of numbers to pick from was smaller (1–10), and players were explicitly given a rule. As seen in the data, most players chose 10 and explained their choice by stating that the final answer would be 1.5 times the average. This shows rational reasoning driven by the rule itself. In this case, players were not merely guessing but rather were strategically responding to the mechanism. To conclude, the data supports that V100 winners succeeded mostly by luck, while V10A winners relied on a combination of experience and rational adaptation, and V10B winners won primarily through clear, rule-based strategic thinking.

V10A: Game 2 winners’ rationale (from different streams):

P25032	23	If people go choose lower i will be closser to the mark
P25014	17	It is 2/3 of the previous answer.
P25002	17	2/3 of previous answer
P25129	17	I think that after people see the most common number, some people would choose the new common answer which would drive the average down
P25116	23	Can feel it in my bones

V10B: Game 3 winners’ rationale (who chose 10):
*Most winners from this game all chose 10 as a number (out of 10).

1st Round Player (ID)	Chosen Number	Overall Average	Target (Multiplier * Avg)	2nd Round Winner(s)
S16	10	8.96	13.44	Hugh Henley McCutcheon
S25	10	8.96	13.44	Lewis Henderson
S23	10	8.96	13.44	Nikita Rawat
S36	8	8.96	13.44	
S20	10	8.96	13.44	Yeoungjun Kim
S22	10	8.96	13.44	Ethan Segvich
S14	10	8.96	13.44	Peter Guilbert
S35	10	8.96	13.44	Trey Baker
S24	10	8.96	13.44	Ranwei Zhang
S34	10	8.96	13.44	Sahil Dhanda
S17	5	8.96	13.44	
S21	10	8.96	13.44	Rui Deng
S29	10	8.96	13.44	Sangmin Lee
S2	10	8.96	13.44	Daniel Eavestaff
S5	10	8.96	13.44	Jeffery Tolmie
S27	10	8.96	13.44	Pattarapong Lertlum
S33	5	8.96	13.44	
S18	8	8.96	13.44	
S13	10	8.96	13.44	Dan Thorpe
S19	10	8.96	13.44	Cameron John
S9	10	8.96	13.44	Campbell Mathew Boulton
S15	5	8.96	13.44	

10	it guarantees either a win or draw. you can work this out manually by eliminating each minimum number one at a time - 1 will only draw with 1 and will lose to every other number so it should be ignored. ignoring 1, 2 should not be played cause it only draws with 2 and loses to everything else. this pattern repeats for every minimum number, so 10 should win
10	the more the merrier I think
10	1.5 times the average, so the number should go up. Therefore the highest number available should be chosen.
10	If their option is greater than 10 then I always win (or tie)
10	if we choose number closer to 10 we have higher chance of winning cause the average will be multiplies by 1.5
10	Because the final answer will need to be times 1.5 for the average number. For example, even if one player choice 1 and another player choice 10. And the final result would be (1+10)/2 x 1.5 = 8.25. And 8.25 is more close to number 10. Therefore, I'll go for number 10.
10	This is the opposite of the other game, which has perfect play as guessing 1 as you can't get any closer than 75 bringing down the average, which keeps happening in this case increasing the number up and up
10	I have chosen 10 as no matter what the other player chooses if it is averaged out and multiplied by 1.5 it will be closer to 10 than their number and if they choose 20 then that wil just result in a tie..
10	1.5 times the average means that the highest number will generally closer

(b) Nash equilibrium

The pure strategy Nash equilibrium in this game can be defined as the best number to choose, given what other players pick, where no one would want to change their choice.

V100 (Game1) was the first game played, meaning players had no prior exposure to the rules or strategic expectations. The instructions stated that the winner would be the one whose number was closest to ⅔ of the group's average. Given the situation, most players were unlikely to apply a deep strategy. They may have relied on randomness and intuition. However, if players were perfectly rational, they would use the repetitive reasoning logic. First reasoning, a player might think that the average is 50, and so 2/3 of that is 33.3 (33 round off). Then a second reasoning, a player might think that if everyone also thinks like this, won't they also pick around 33, so then the average now is 33, in whcih 2/3 of that is 22. Now the average is 22. Third reasoning goes the same, if a player think that everyone will pick 22, then they will choose the 2/3 of that which is 14.6666666667 (15 round off). Repeating this process, the number will get smaller and smaller, ultimately reaching 1. Therefore, **1 is the Nash equilibrium** and the rational number to pick for the game **V100**. However, the winning number in V100 was 26, which is higher than the hypothetical equilibrium. This difference reflects the fact that the majority of players did not reason to the nash equilibrium. Several players chose randomly, relied on intuition, and reasoned out their own logic. This shows that players did not fully use rational thinking and had no prior knowledge about other players.

In the next game, V10A, players were given the previous winning number from V100, which provided them with prior knowledge that they could use to form a more strategic response. For example, if the previous winning number was 26, many players reasoned that the best choice in this round would be ⅔ of 26, which is approximately 17.. However, some players likely anticipated that others would use the same logic, and therefore chose **⅔ of 17**, arriving at around **11**. This iterative reasoning could continue further, eventually reaching towards **1**. Under perfect rationality and common knowledge, this process would lead to the same theoretical outcome as in V100: Nash equilibrium 1. In reality, though, the winning numbers in V10A were 23 and 17, which are still significantly higher than the equilibrium. This suggests that some degree of rational reasoning did occur, but players did not fully iterate the process. Many justifications referred to "⅔ of the previous answer", indicating that players based their choices on past outcomes and were thinking about how others might also adapt their strategies. This behaviour reflects a moderate level of strategic reasoning, where players were not only using logic but were also considering the likely thought processes of others. While the choices didn't meet the Nash equilibrium, the shift from random guessing in V100 to more calculated responses in V10A shows that players were beginning to act more rationally and account for others' knowledge of the game.

P25014	17	It is 2/3 of the previous answer.
P25002	17	2/3 of previous answer

In V10B, the pure strategy Nash equilibrium is 10. The game used a rule where the winning number was calculated as 1.5 times the average of two players' chosen numbers. Given this rule, the most rational and strategic option is to choose 10, since any lower number would reduce the average and decrease the chance of winning. This game also used a smaller number range (1 to 10) compared to the earlier games, which made it easier for players to identify the best choice. With fewer options and a clear rule, players were more likely to reason accurately and choose the equilibrium. In practice, most players chose 10, and 10 was the winning number, showing that their choices were based on logical thinking and shared expectations about what others would do. This demonstrates strong strategic reasoning and alignment with the Nash equilibrium.

(c) Player Classification (15 marks) *Table in separate file: 22167597_Classification Table.pdf or 22167597_Classification Table.xlsx *

Classification Scheme

Class	Label	Criteria	Keywords
Highly Rational	H	Accurate number - Chose 1 in V100/V10A and reasoned it as "2/3 of average" etc. - Chose 10 in V10B and reasond it as "1.5 * average so number should be higher" or maximasation - Showed gradual understanding and good analytical reasoning across the games.	"2/3", "average","1.5*average", "optimal", "maximise"
Moderately Rational	M	- Chose moderately low values in V100/V10A and attempted to rationalise them. - Chose number around 10 in V10B with insufficient justification. - Presented partial strategic logic, but not the best possible option.	"based on previous answer","average", "2/3","middle"

		- Showed attempts at logical thinking, though some inconsistency were presented across games.	
Irrational (Random)	I	- Chose significantly different values compared to the equilibrium with nonsensical reasoning (e.g., "birthday", "random", "luck number"etc.) - Reasons are either missing or insignificant. - Shows no indication of learning and understanding of the game structure across game sessions.	"guess", "birthday", "just because", "random", "favourite number", "lucky"

(d) Game 3 Mechanism Analysis (15 marks)

Game 3 or V10B enhanced students' understanding of Nash equilibrium and mechanism design. Unlike the previous games, where players aimed to minimise their chosen number relative to the group average, Game 3 required them to maximise their number to 1.5 times the group's average. This reversal helped students differentiate among various kinds of equilibrium and realise how mechanisms influence outcomes. The majority of winning players chose 10 and justified it using the "1.5 × average" rule, implying that students understood the new rule and modified their strategy as necessary. This portrays applied learning of equilibrium behaviour. Additionally, the requirement to pair up with a classmate to play encouraged interpersonal interaction, contributing to the goal of fostering classroom engagement and connection. For further improvements, adjusting the game rules (mechanism) itself can be proposed. For example, implementing multiple rounds with different rules such as varying the multiplier or introducing penalties for overestimation could deepen students' understanding of mechanism design and enhance their ability to adapt strategically in changing environments.

Task 2

Write a short report (up to 2 pages) on game theory's applications in AI.

Game theory is a core framework in artificial intelligence that enables systems to analyse and anticipate behaviour in both competitive and collaborative environments, helping AI agents adjust their strategies to reach optimal decisions across a wide range of scenarios. It plays a critical role in designing intelligent systems, mainly as AI agents become more autonomous and interactive. One main application is in multi-agent systems, where AI agents must coordinate, negotiate, or compete to reach their own or shared goals. This is necessary in areas such as autonomous vehicles, robotic swarms, and AI-powered traffic management systems, where agents must constantly respond to others' actions. In strategic environments such as chess or tic-tac-toe, game theory establishes self-play reinforcement learning, allowing agents to train by competing against themselves and converge towards optimal strategies. AlphaGo is one good example of how game theory was used to exceed human capabilities by engaging in repetitive self-play. Game theory also contributes to advancements in adversarial AI, such as Generative Adversarial Networks (GANs), where two neural networks called the generator and the discriminator compete with each other to improve the quality and realism of generated data. Furthermore, game theory supports ethical decision-making, allowing AI to reason about fairness, social trade-offs, and accountability in complex situations like life-or-death scenarios such as autonomous driving decisions. In economics and automation, game theory enhances AI-driven business strategies by guiding automated auctions, pricing models, and resource allocation using mechanisms that ensure individual decision-making contributes to desirable collective outcomes. In conclusion, game theory serves as a critical foundation for allowing AI systems to operate effectively in changing, multi-agent settings. It enables AI to anticipate others' actions, adapt its strategies through learning, and make informed decisions based on the surrounding environment. This not only improves individual agent performance but also supports interactions that are guided by shared objectives within complex systems.