Universitat Rovira i Virgili

Complex Networks

Practical Report

Report of the Exercise 1: Implementation of complex network models

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Contents

Motivation

This exercise is motivated by the importance of understanding the different models studied for networks generation and their characteristics, in order to be able to distinguish in a proper way which model should be used on each oncoming facing problem on the future.

The models presented on this report have been all implemented in python following the algorithms in the literature reviewed (Newman (2003), Boccaletti et al. (2006) and Clauset et al. (2009)).

The models presented in this report are:

1. Erdös-Rényi (ER) $G(N, p)$: random networks of any degree distribution.
2. Configuration Model (CM) using the power-law distribution: scale-free networks of any degree distribution.
3. Barabási-Albert (BA) preferential attachment model: is a network growth procedure which was introduced to show how a simple rule can generate degree distributions similar to the ones observed in many real networks.

For the testing part of this report it was decided to perform two different kinds of experiments:

- Using low number of nodes: for showing some plots on the generated networks to discuss their properties.
- Using high number of nodes: for discussing the properties of the degrees distribution of the generated networks and visualising them in 2D plots, in order to discuss among them.

Here we show a plot of this three model resulting networks with 200 nodes and similar settings.

In figure 1 we can observe a sample of the different generated networks using the models exposed for this exercise.

Generation of Erdös-Rényi (ER) random networks

For this section it was requested to implement one of the two models presented on the literature for generating random networks, the $G(N, K)$ or the $G(N, p)$. Moreover, it was required to present some testings on the implemented code, giving visual results and discussing them according to the literature theoretical descriptions.

Decisions on the model

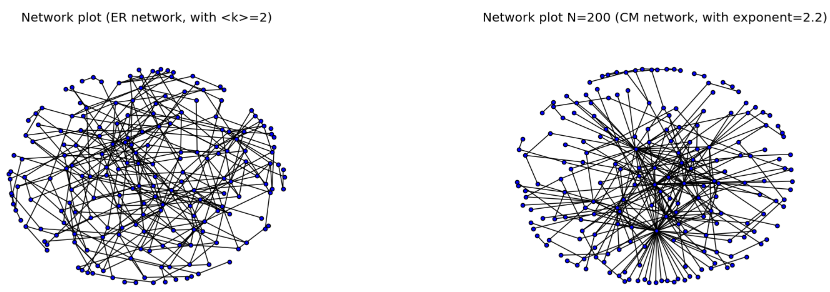
I decided to implement the $G(N, p)$, since it was more interesting being able to generate networks where the connectivity between nodes was expressed as a provability, rather than having a priory the total number of connections and just plugging them randomly. In the literature was also mentioned that the $G(N, p)$ model is more widely used rather than the $G(N, K)$ model in practice.

Testing the $G(N, p)$ model implementation

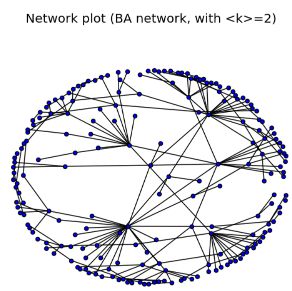
In this section the results from implementing the $G(N, p)$ model are presented, together with some their detailed descriptions.

In figure 2 we can observe the different generated networks using the Erdös-Rényi $G(N, p)$ model for random networks generation.

It was decided to use the configurations of $N = 200$ in order to better appreciate the increase in the number of connection on the network, since with larger networks there where too many nodes to distinguish the edges in the plot.



(a) Erdös-Rényi (ER) $G(N, p)$ (b) Configuration Model (CM)



(c) Barabási-Albert (BA)

Figure 1: ER random generated networks with 200 Nodes.

From now on in this section I will present the different networks built using this model, with their characteristics exposed. However, we will not present the results obtained with the networks with less than $N = 1000$ nodes, since analysing networks larger than the mentioned threshold was a requirement of this exercise.

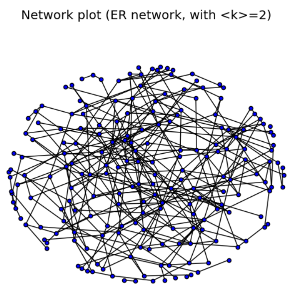
The structure of the following subsections will be:

1. Characteristics of the built network.
2. Plots of the degree distribution.
3. Plots of the complementary cumulative degree distribution.

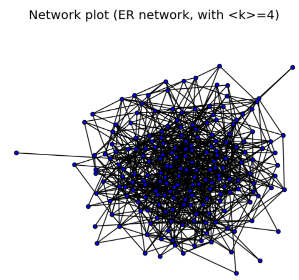
Random Networks $< k >= 2$

For the average degree $< k >= 2$ we will present three networks which will differ on the number of nodes that composes them.

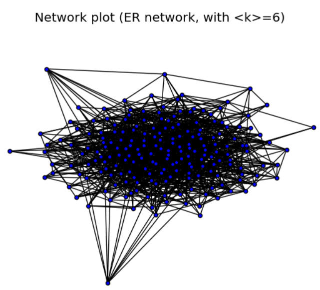
1. $N = 1000$ **Characteristics of the built network:**
   - Size: 1000
   - Average degree $< k >$: 2
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.002



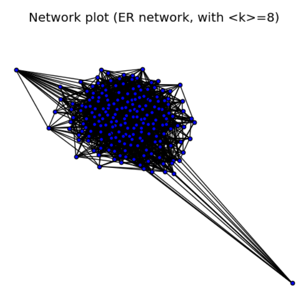
(a) $<k>=2$



(b) $<k>=4$



(c) $<k>=6$



(d) $<k>=8$

Figure 2: ER random generated networks with 200 Nodes.

The degree distribution:

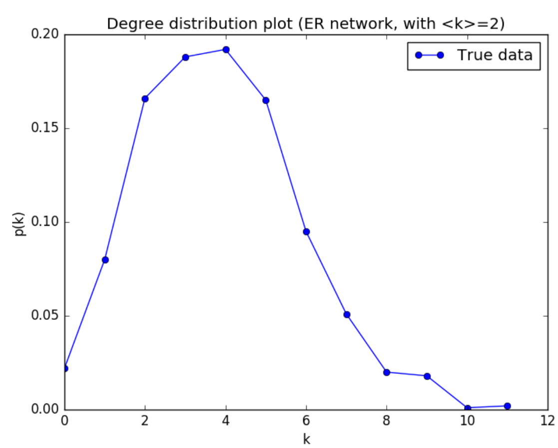


Figure 3: Degree distribution plot $<k>=2$, $N=1000$.

In figure 3 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

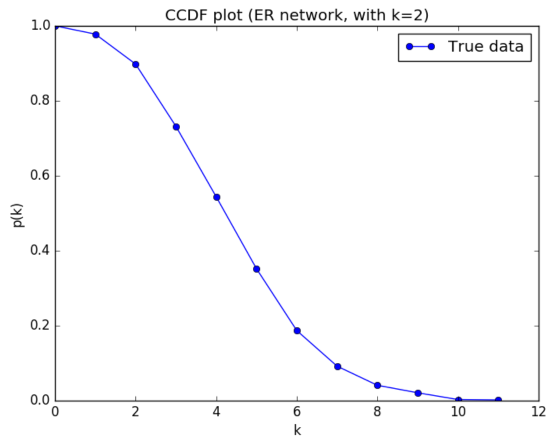


Figure 4: Degree distribution plot $< k >= 2$, $N = 1000$.

In figure 3 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

2. $N = 2000$ **Characteristics of the built network:**

   - Size: 2000
   - Average degree $< k >$: 2
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.001

   The degree distribution:

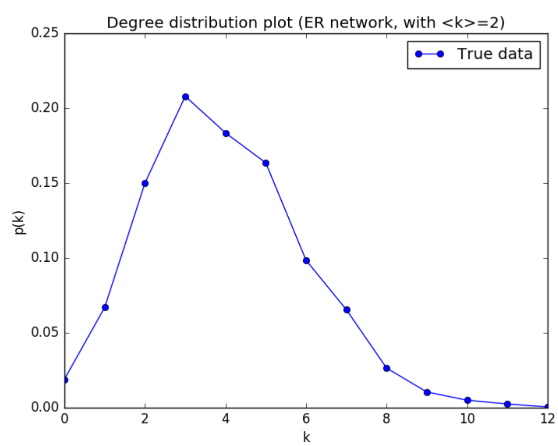


Figure 5: Degree distribution plot $< k >= 2$, $N = 2000$.

In figure 5 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

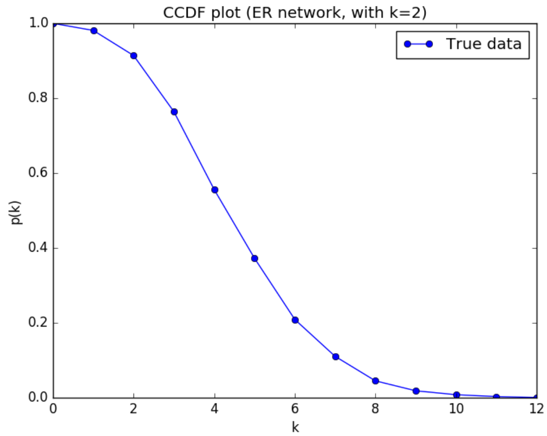


Figure 6: Degree distribution plot $< k >= 2$, $N = 2000$.

In figure 5 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

3. $N = 3000$ **Characteristics of the built network:**

- Size: 3000
- Average degree $< k >$: 2
- Model used: $G(N, p)$
- Correspondent probability p: 0.00067

The degree distribution:

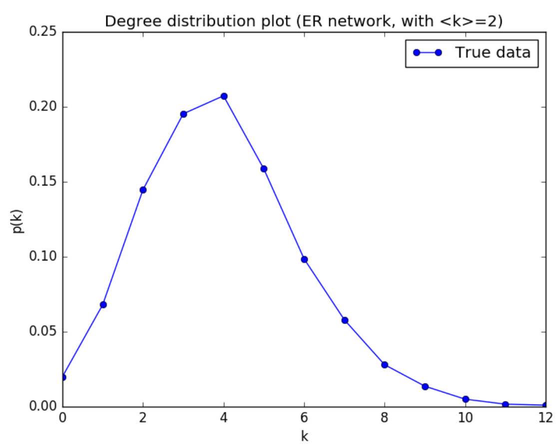


Figure 7: Degree distribution plot $< k >= 2$, $N = 3000$.

In figure 7 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

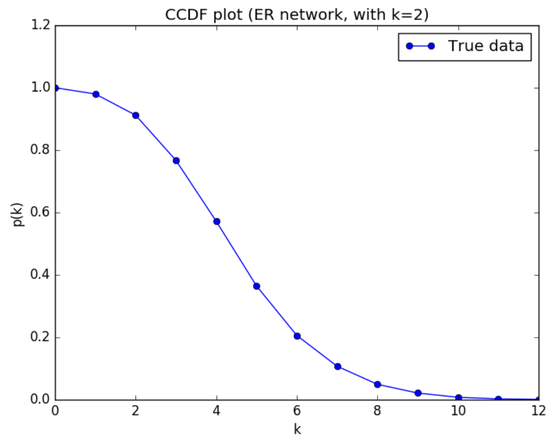


Figure 8: Degree distribution plot $< k >= 2$, $N = 3000$.

In figure 3 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure. And moreover, it can be observed that with more data (nodes) the resemblance of the distribution to the *Poisson* one, increases and better matches the desired pattern.

Random Networks $< k >= 4$

For the average degree $< k >= 4$ we will present three networks which will differ on the number of nodes that composes them.

1. $N = 1000$ **Characteristics of the built network:**
   - Size: 1000
   - Average degree $< k >$: 4
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.004

The degree distribution:

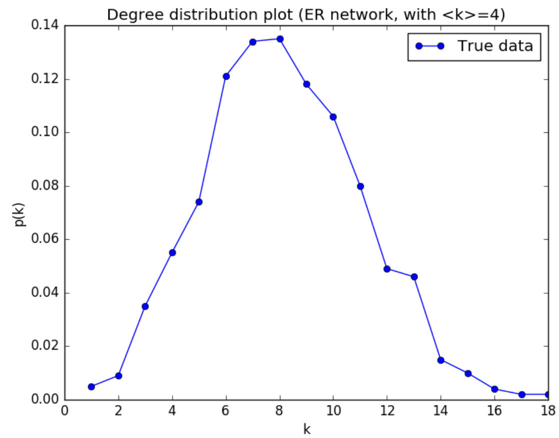


Figure 9: Degree distribution plot $< k >= 4$, $N = 1000$.

In figure 9 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

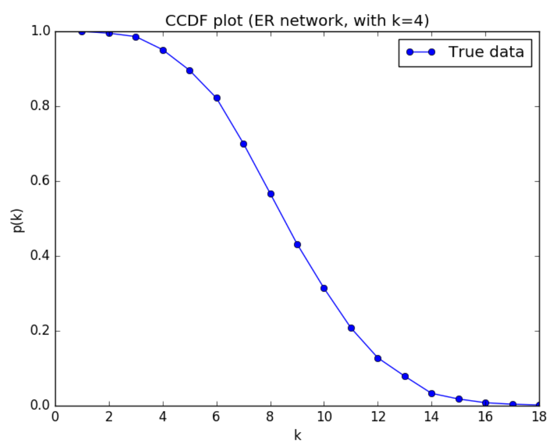


Figure 10: Degree distribution plot $< k >= 4$, $N = 1000$.

In figure 9 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

2. $N = 2000$ **Characteristics of the built network:**
   - Size: 2000
   - Average degree $< k >$: 4
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.002

The degree distribution:

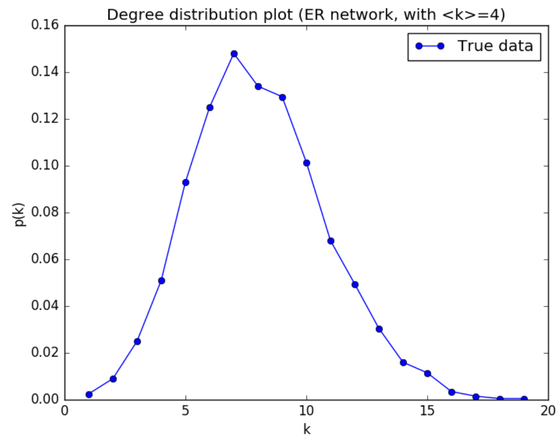


Figure 11: Degree distribution plot $< k >= 4$, $N = 2000$.

In figure 11 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

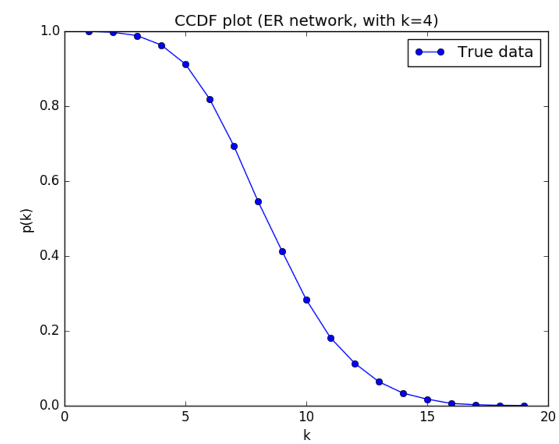


Figure 12: Degree distribution plot $< k >= 4$, $N = 2000$.

In figure 11 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

3. $N = 3000$ **Characteristics of the built network:**

   - Size: 3000
   - Average degree $< k >$: 4
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.0013

The degree distribution:

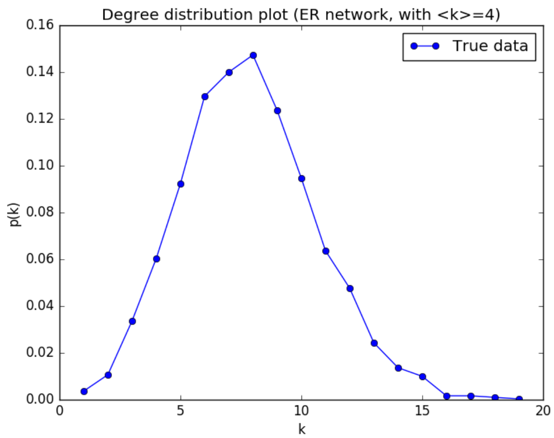


Figure 13: Degree distribution plot $< k >= 4$, $N = 3000$.

In figure 13 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

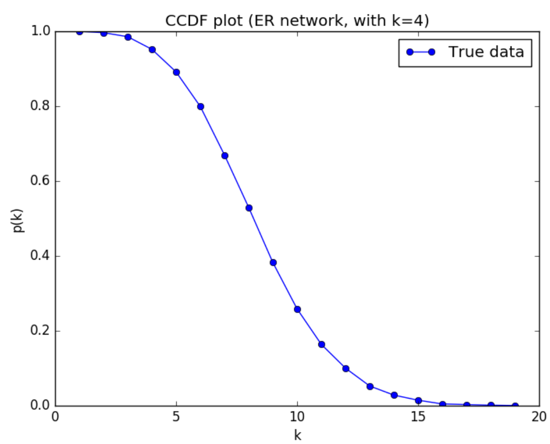


Figure 14: Degree distribution plot $< k >= 4$, $N = 3000$.

In figure 13 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

Random Networks $< k >= 6$

For the average degree $< k >= 4$ we will present three networks which will differ on the number of nodes that composes them.

1. $N = 1000$

   Characteristics of the built network:

   - Size: 1000
   - Average degree $< k >$: 6
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.006

   The degree distribution:

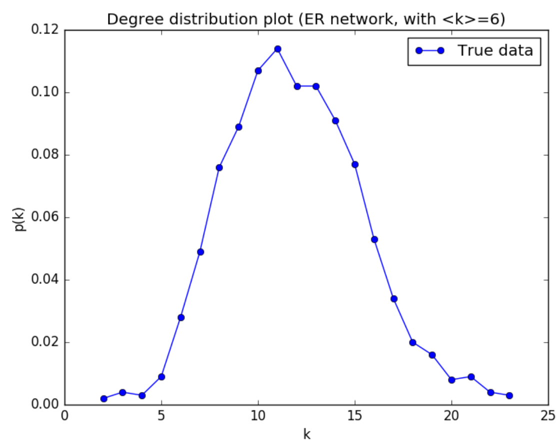


Figure 15: Degree distribution plot $< k >= 6$, $N = 1000$.

In figure 15 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

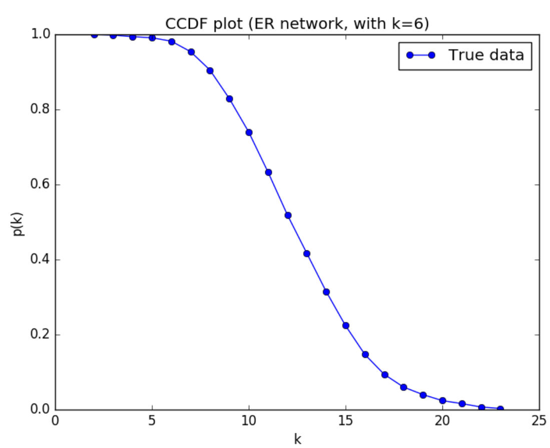


Figure 16: Degree distribution plot $< k >= 6$, $N = 1000$.

In figure 15 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

2. $N = 2000$

Characteristics of the built network:

- Size: 2000
- Average degree $< k >$: 6
- Model used: $G(N, p)$
- Correspondent probability p: 0.003

The degree distribution:

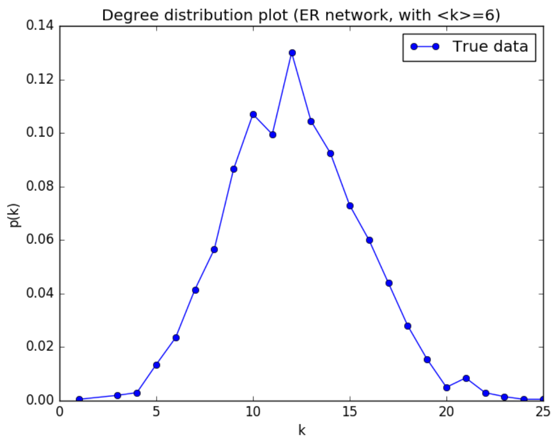


Figure 17: Degree distribution plot $< k >= 6$, $N = 2000$.

In figure 17 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

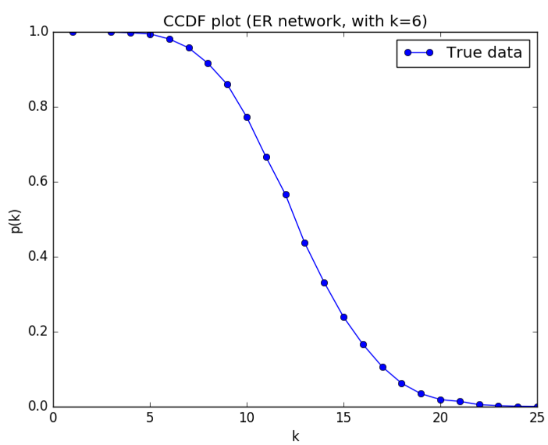


Figure 18: Degree distribution plot $< k >= 6$, $N = 2000$.

In figure 17 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

3. $N = 3000$ **Characteristics of the built network:**

- Size: 3000
- Average degree $< k >$: 6
- Model used: $G(N, p)$
- Correspondent probability p: 0.002

The degree distribution:

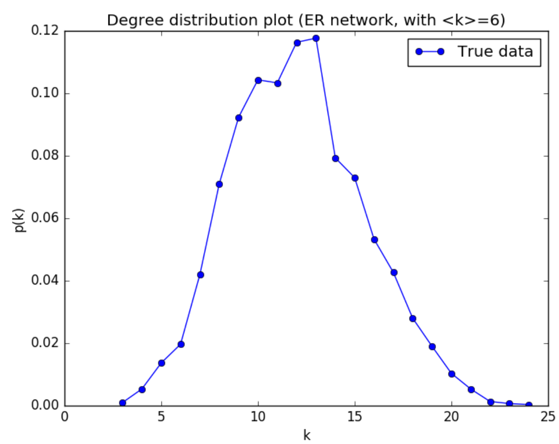


Figure 19: Degree distribution plot $< k >= 6$, $N = 3000$.

In figure 19 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

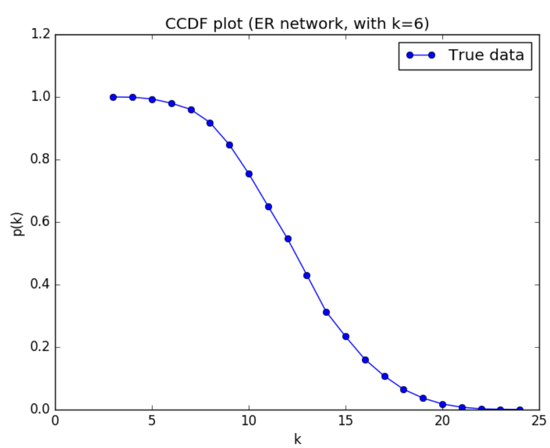


Figure 20: Degree distribution plot $< k >= 6$, $N = 3000$.

In figure 19 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

Random Networks $<k>= 8$

For the average degree $<k>= 4$ we will present three networks which will differ on the number of nodes that composes them.

1. $N = 1000$ **Characteristics of the built network:**

   - Size: 1000
   - Average degree $<k>$: 8
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.008

   The degree distribution:

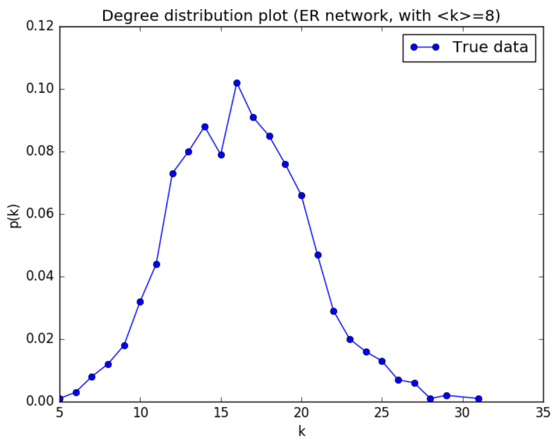


Figure 21: Degree distribution plot $<k>= 8$, $N = 1000$.

In figure 21 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

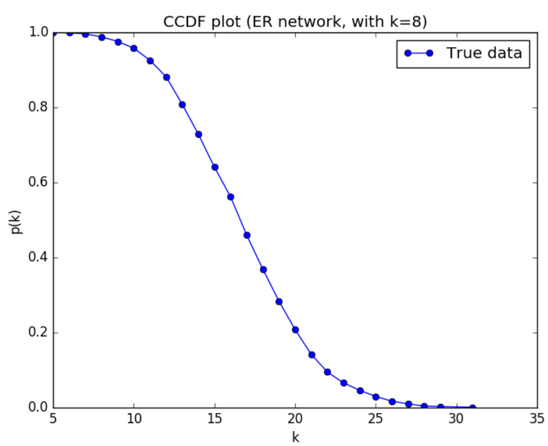


Figure 22: Degree distribution plot $<k>= 8$, $N = 1000$.

In figure 21 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

2. $N = 2000$ **Characteristics of the built network:**

   - Size: 2000
   - Average degree $< k >$: 8
   - Model used: $G(N, p)$
   - Correspondent probability p: 0.004

The degree distribution:

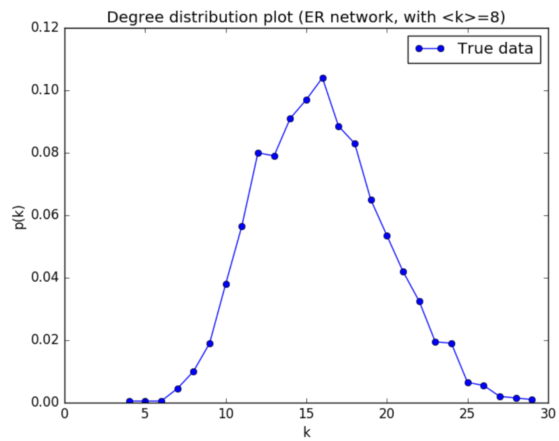


Figure 23: Degree distribution plot $< k >= 8$, $N = 2000$.

In figure 23 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

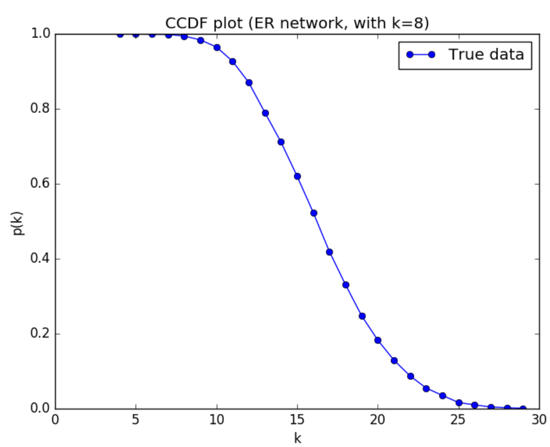


Figure 24: Degree distribution plot $< k >= 8$, $N = 2000$.

In figure 23 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

3. $N = 3000$ **Characteristics of the built network:**

- Size: 3000
- Average degree $< k >$: 8
- Model used: $G(N, p)$
- Correspondent probability p: 0.00267

The degree distribution:

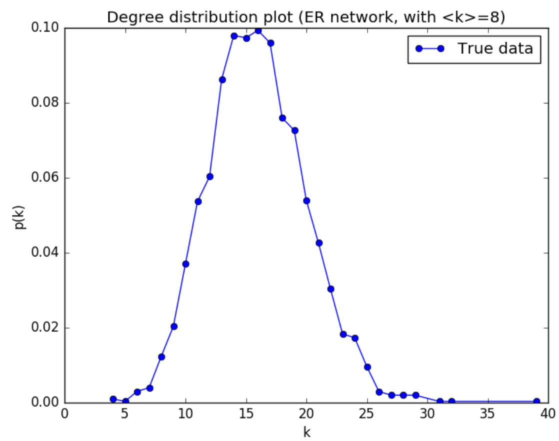


Figure 25: Degree distribution plot $< k >= 8$, $N = 3000$.

In figure 25 it can be observed that the distribution is following a *Poisson* distribution rather than a *Gaussian* one.

The complementary cumulative degree distribution:

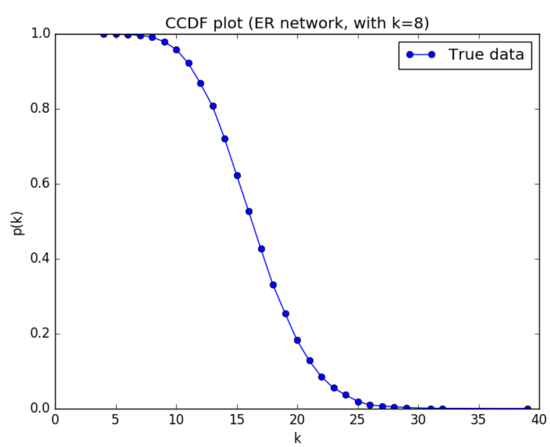


Figure 26: Degree distribution plot $< k >= 8$, $N = 3000$.

In figure 25 it can be observed that the CCDF distribution is following a *Poisson* distribution rather than a *Gaussian* one, as we had been able to observe from the previous figure.

Implementation of the Configuration Model (CM)

For this section it was requested to implement the Configuration Model (CM), and to retrieve the obtained results, showing the degree distributions as plots and histograms in logarithmic scale. Moreover, it was required to present some testings on the implemented code, giving visual results and discussing them according to the literature theoretical descriptions.

It was required also to implement two different function fitting mechanisms:

- Linear regression for fitting a power-law function: $log(y) = \gamma log(x) + c$
- *Maximum Likelihood Estimate* (MLE):

$$\gamma = 1 + \left(\sum_{i=1}^{n} ln \frac{k_i}{k_m in - \frac{1}{2}} \right)^{-1}$$

In the following part of this section we will show the results obtained from running the implemented Configuration Model (CM) over the following different configurations:

- Number of nodes used: 1000, 2000 and 3000.
- Exponents of the power-law distribution: 2.2, 2.5 and 2.7.

Testing the CM model implementation

In this section the results from implementing the CM model are presented, together with some their descriptions.

Pleas notice that some important information of the figures might be found on the legends of the figures themselves rather that in their captions.

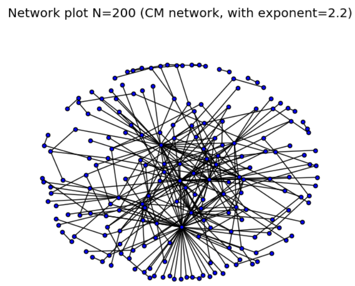
In figure 27 we can observe the different generated networks using the Configuration Model for SF[1] networks generation.

NOTE: From now on the results will be presented for networks of size $N = 3000$ *in order to avoid redundancy on the results, since many figures have been generated for the CM model results interpretation. The rest of the images will be attached to the delivery file together with the generated networks in Pajek format.*

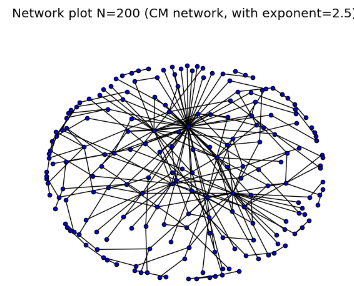
The structure of the following subsections will be:

1. Characteristics of the built network
2. Plots of the degree distribution.
3. Plots of the complementary cumulative degree distribution.

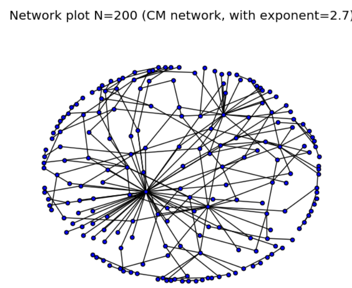
[1] scale-free



(a) $exp = 2.2$



(b) $exp = 2.5$



(c) $exp = 2.7$

Figure 27: CM scale-free generated networks with 200 Nodes.

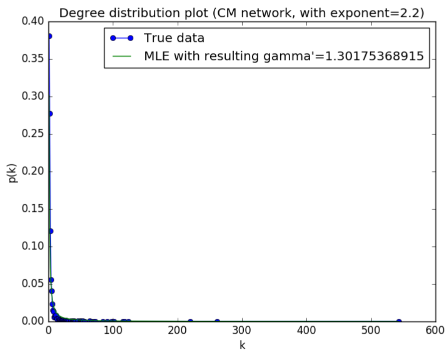
Scale-Free Networks $exp = 2.2$

For the $exp = 2.2$ we will present the network generated with $N = 3000$ nodes.

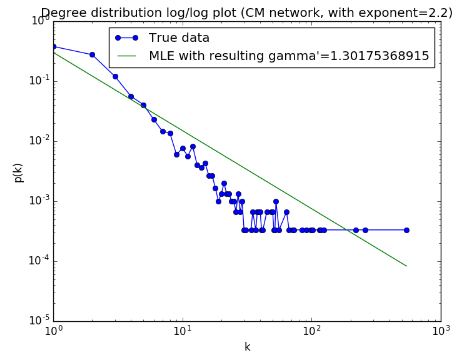
Characteristics of the built network:

- Size: 3000
- Exponent: 2.2
- MLE gamma': 1.302
- Regressed gamma'': 1.368
- Model used: CM

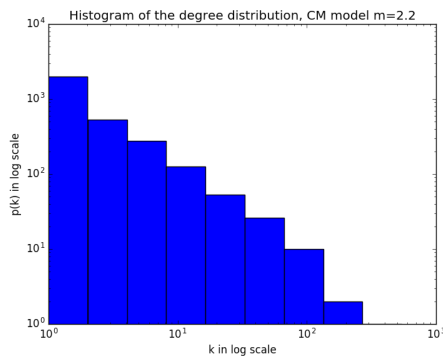
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

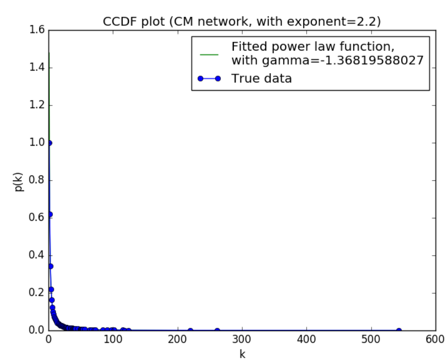


(c) Degree distribution histogram plot in *log-log* scale.

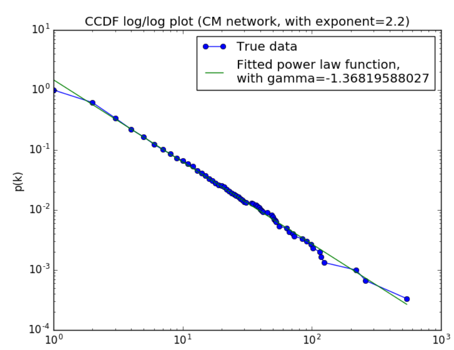
Figure 28: Degree distribution plots $exp = 2.2$, $N = 3000$.

In figure 28a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot.



(b) CCDF plot in *log-log* scale.

Figure 29: Complementary Cumulative Degree distribution plots $exp = 2.2$, $N = 3000$.

In figure 29a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

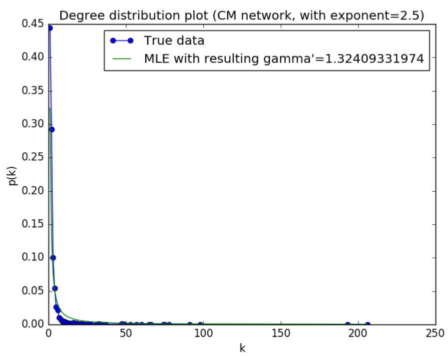
Scale-Free Networks $exp = 2.5$

For the $exp = 2.5$ we will present the network generated with $N = 3000$ nodes.

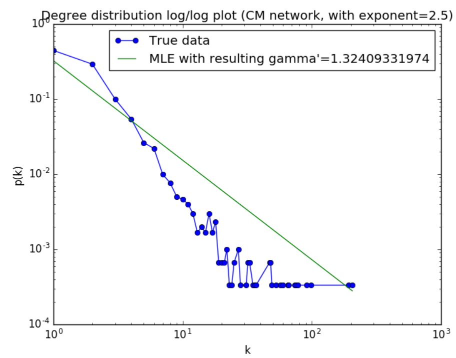
Characteristics of the built network:

- Size: 3000
- Exponent: 2.5
- MLE gamma': 1.324
- Regressed gamma": 1.465
- Model used: CM

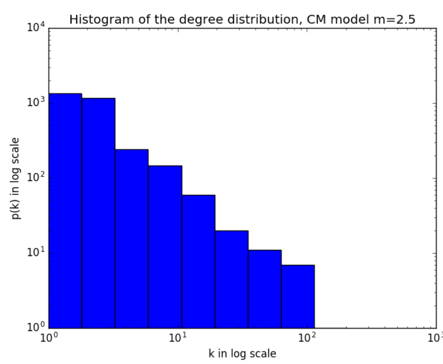
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

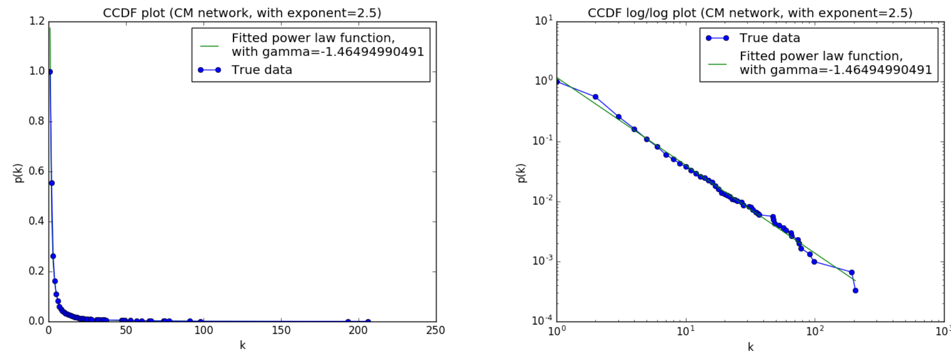


(c) Degree distribution histogram plot in *log-log* scale.

Figure 30: Degree distribution plots $exp = 2.5$, $N = 3000$.

In figure 30a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot.

(b) CCDF plot in *log-log* scale.

Figure 31: Complementary Cumulative Degree distribution plots $exp = 2.5$, $N = 3000$.

In figure 31a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

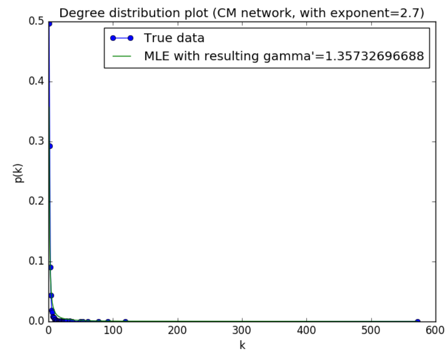
Scale-Free Networks $exp = 2.7$

For the $exp = 2.7$ we will present the network generated with $N = 3000$ nodes.

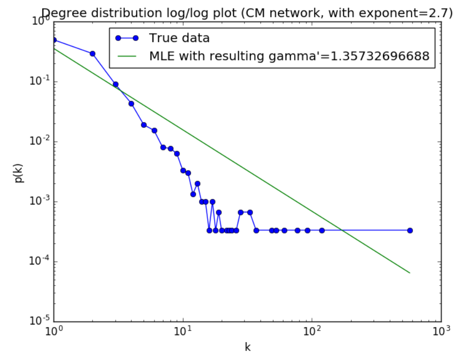
Characteristics of the built network:

- Size: 3000
- Exponent: 2.7
- MLE gamma': 1.357
- Regressed gamma": 1.429
- Model used: CM

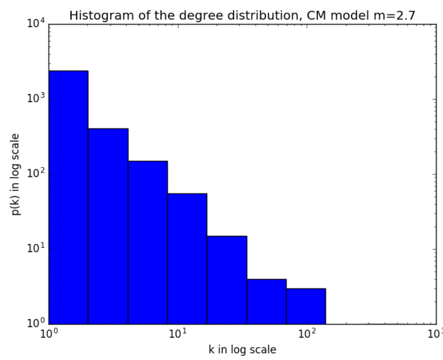
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

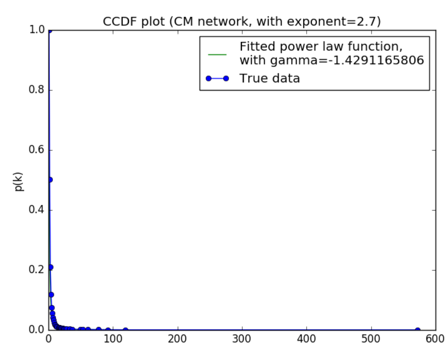


(c) Degree distribution histogram plot in *log-log* scale.

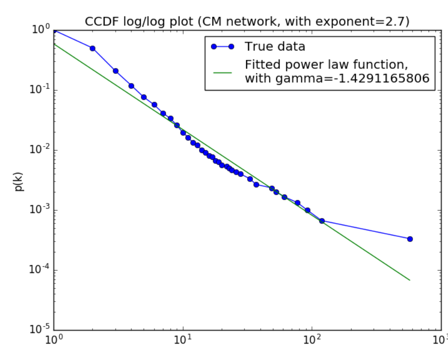
Figure 32: Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 32a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot.



(b) CCDF plot in *log-log* scale.

Figure 33: Complementary Cumulative Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 33a it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

Observations on the Configuration Model (CM) degrees distributions

As can be seen on the plots showed for the Configuration Model results with $N = 3000$. The MLE seams to be working in a proper way, since the slope value retrieved is very similar to the one provided by the linear regression. However, it can be observed that is a bit too flat in all retrieved cases. This fact is not so exaggerated when we analyse smaller networks, as can be seen in figure 34

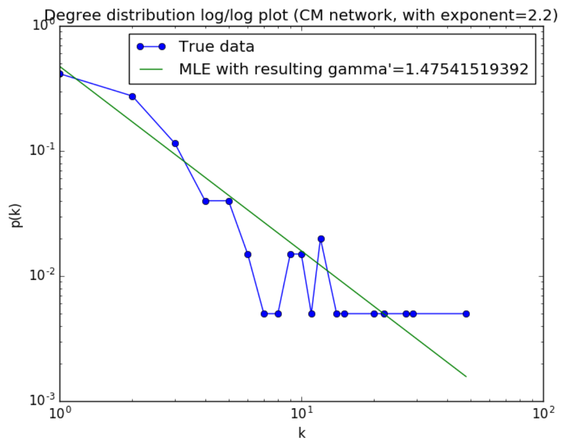


Figure 34: Degree distribution log-log plot and MLE.

Barabási-Albert (BA) preferential attachment model

In this section the results from implementing the BA model are presented, together with some their descriptions.

Pleas notice that some important information of the figures might be found on the legends of the figures themselves rather that in their captions.

In figure 35 we can observe the different generated networks using the Barabási-Albert (BA) preferential attachment model for SF networks incremental generation.

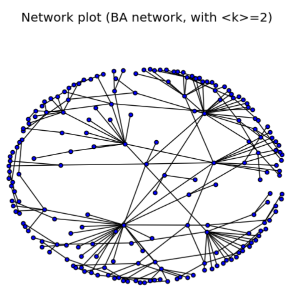
NOTE: From now on the results will be presented for networks of size $N = 3000$ *in order to avoid redundancy on the results, since many figures have been generated for the BA model results interpretation. The rest of the images will be attached to the delivery file together with the generated networks in Pajek format.*

The structure of the following subsections will be:

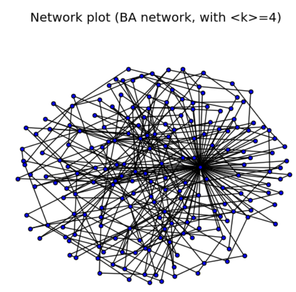
1. Characteristics of the built network
2. Plots of the degree distribution.
3. Plots of the complementary cumulative degree distribution.

Scale-Free Networks $< k >= 2$

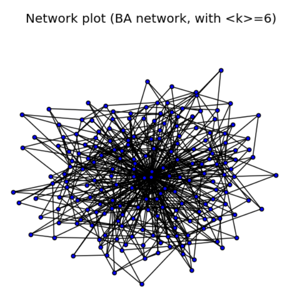
For the $< k >= 2$ we will present the network generated with $N = 3000$ nodes.



(a) $<k>=2$



(b) $<k>=4$



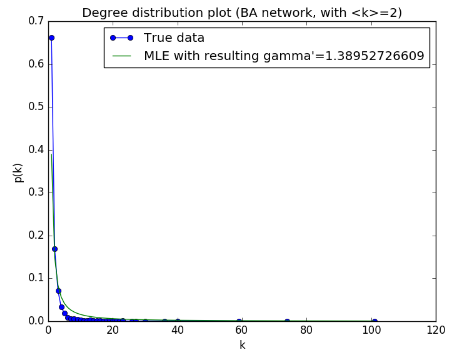
(c) $<k>=6$

Figure 35: CM scale-free generated networks with 200 Nodes.

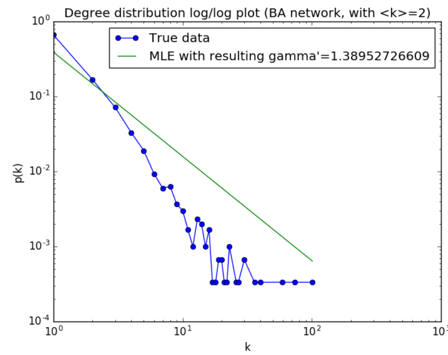
Characteristics of the built network:

- Size: 3000
- Average degree $<k>$: 2
- MLE gamma': 1.39
- Regressed gamma": 1.77
- Model used: BA

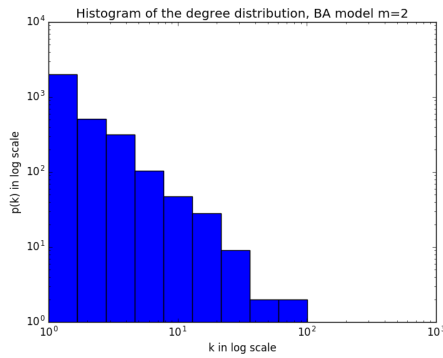
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

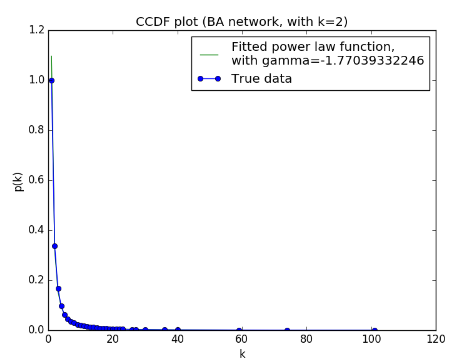


(c) Degree distribution histogram plot in *log-log* scale.

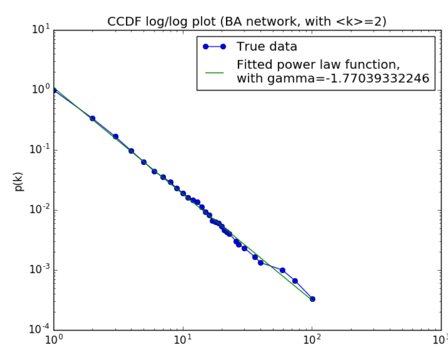
Figure 36: Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 36 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot.



(b) CCDF plot in *log-log* scale.

Figure 37: Complementary Cumulative Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 37 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

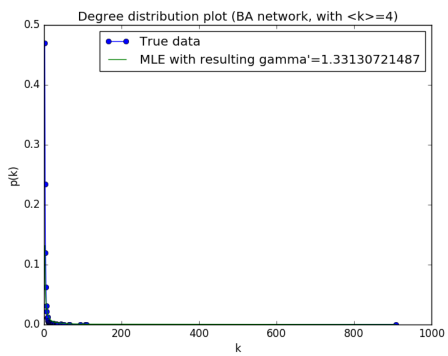
Scale-Free Networks $< k >= 4$

For the $< k >= 4$ we will present the network generated with $N = 3000$ nodes.

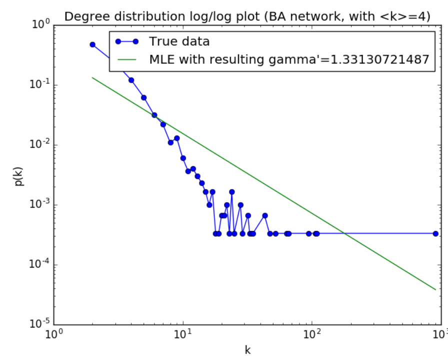
Characteristics of the built network:

- Size: 3000
- Average degree $< k >$: 4
- MLE gamma': 1.33
- Regressed gamma": 1.52
- Model used: BA

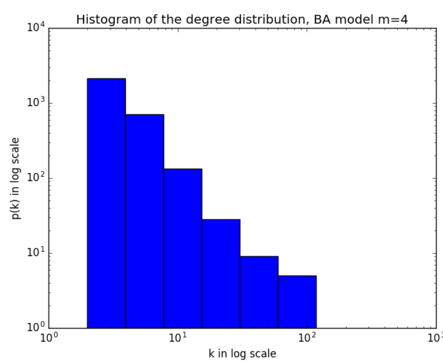
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

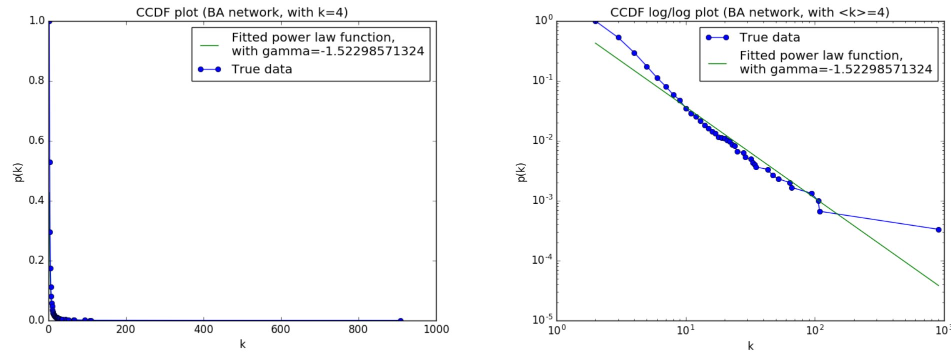


(c) Degree distribution histogram plot in *log-log* scale.

Figure 38: Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 38 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot. (b) CCDF plot in *log-log* scale.

Figure 39: Complementary Cumulative Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 39 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

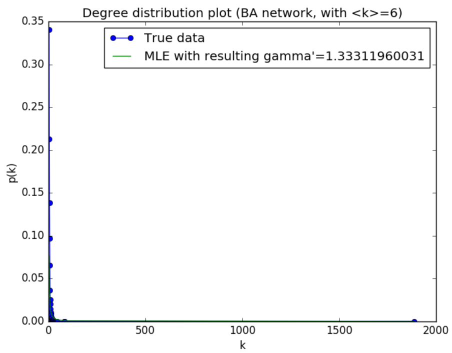
Scale-Free Networks $< k >= 6$

For the $< k >= 6$ we will present the network generated with $N = 3000$ nodes.

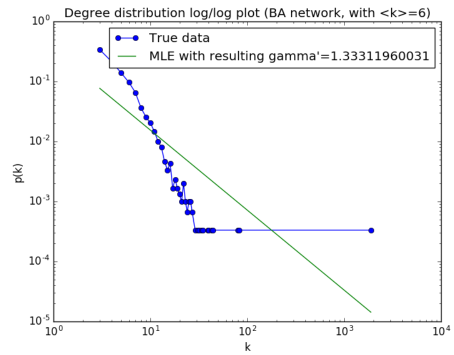
Characteristics of the built network:

- Size: 3000
- Average degree $< k >$: 6
- MLE gamma': 1.33
- Regressed gamma": 1.64
- Model used: BA

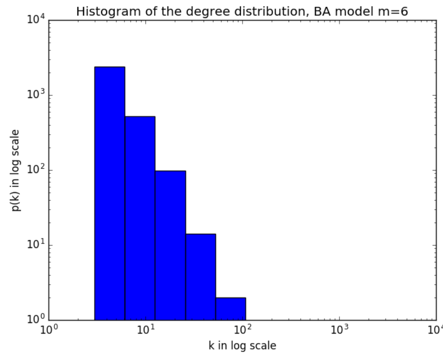
The degree distribution:



(a) Degree distribution plot.



(b) Degree distribution plot in *log-log* scale.

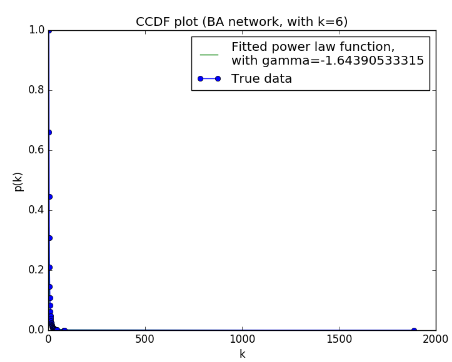


(c) Degree distribution histogram plot in *log-log* scale.

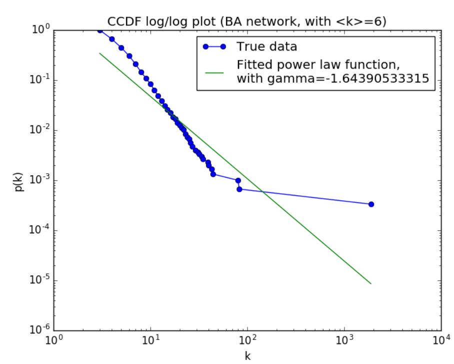
Figure 40: Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 40 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones.

The complementary cumulative degree distribution:



(a) CCDF plot.



(b) CCDF plot in *log-log* scale.

Figure 41: Complementary Cumulative Degree distribution plots $exp = 2.7$, $N = 3000$.

In figure 41 it can be observed that the CCDF distribution is following a *Power-law* distribution rather than *Gaussian* or Poisson ones (as the ER random networks showed to be doing on the previous sections), as we had been able to observe from the previous figure.

Observations on the Barabási-Albert (BA) preferential attachment model degrees distributions

As can be seen on the plots showed for the Barabási-Albert (BA) preferential attachment model results with $N = 3000$. The MLE seams to be working in a proper way, since the slope value retrieved is very similar to the one provided by the linear regression. However, it can be observed that is a bit too flat in all retrieved cases. This fact is not so exaggerated when we analyse smaller networks, as can be seen in figure 42, moreover, we may observe that not only it's not such a dramatic error, but that the retrieved slope is too step, rather than too flat. So we find the inverse effect when analysing small networks with the MLE.

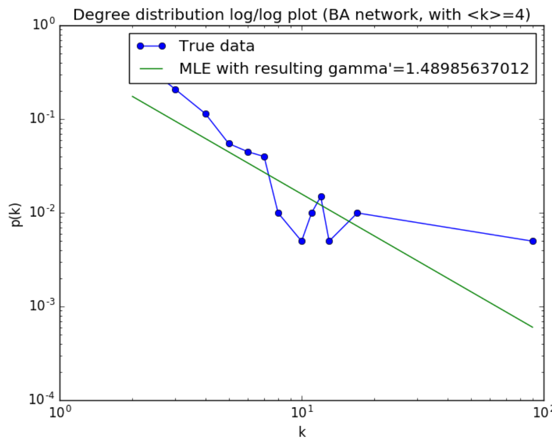


Figure 42: Degree distribution log-log plot and MLE.

Strengths and Weaknesses

In this section I will comment some issues that were not planned and altered the planification of performing the exercise.

Strengths

- The literature provided by the teacher was very complete, although difficult to understand at some point. So the only kind of research needed was for clarification and support to the already available, to double check concepts and algorithms meanings.
- The amount of time provided was widely more than needed, which permitted to get a deeper understanding on the target topics.

Weaknesses

- It showed up that the computer that I have available for working, did not support the installation of the *Scipy* python library. This implied loosing a lot of time trying to get it to work and having to invest extra time in implementing the desired functionalities from scratch.
- The computer did not support networks with more than 3000 nodes. This constrained the experiments that I could perform, but however I could compare results from other sources and verify that the results that I obtained where coherent and expressive enough to discuss among them in this report.

Conclusions and Future Work

- From this exercise we learnt different methods of generating networks with a certain degree distribution, and to analyse the degree distribution of given networks. So from an existing network we could easily distinguish if we are facing a random network or a scale-free network (which as can be seen on the presented plots contain hubs). We could also predict the possible growth of a network following the preferential attachment model (BA).
- As future work, it wold be interesting to analyse more properties of certain given networks, in order to better understand the insights of networks appearing in nature and daily live scenarios.

References

S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. *Physics Reports*, 424(45):175 – 308, 2006. ISSN 0370-1573. doi: http://dx.doi.org/10.1016/j.physrep.2005.10.009. URL `http://www.sciencedirect.com/science/article/pii/S037015730500462X`.

Aaron Clauset, Cosma Rohilla Shalizi, and M. E. J. Newman. Power-law distributions in empirical data. *SIAM Rev.*, 51(4):661–703, November 2009. ISSN 0036-1445. doi: 10.1137/070710111. URL `http://dx.doi.org/10.1137/070710111`.

M. E. J. Newman. The structure and function of complex networks. *SIAM REVIEW*, 45:167–256, 2003.