In[ ]:=
```
P0[p_] = 5 p^0 + -8 p^1 + 8 p^2 + 0 p^3 + 0 p^4 + 0 p^5;
P1[p_] = 5 p^0 + -8 p^1 + 9 p^2 + -1 p^3 + 0 p^4 + 0 p^5;
P2[p_] = 5 p^0 + -8 p^1 + 9 p^2 + 0 p^3 + -2 p^4 + 0 p^5;
P3[p_] = 5 p^0 + -7 p^1 + 8 p^2 + -1 p^3 + 0 p^4 + 0 p^5;
P4[p_] = 5 p^0 + -7 p^1 + 8 p^2 + 0 p^3 + -2 p^4 + 0 p^5;
P5[p_] = 5 p^0 + -7 p^1 + 9 p^2 + -2 p^3 + -2 p^4 + 0 p^5;
P6[p_] = 5 p^0 + -7 p^1 + 6 p^2 + 1 p^3 + 0 p^4 + 0 p^5;
P7[p_] = 5 p^0 + -7 p^1 + 7 p^2 + 0 p^3 + 0 p^4 + 0 p^5;
P8[p_] = 5 p^0 + -7 p^1 + 7 p^2 + 1 p^3 + -2 p^4 + 0 p^5;
P9[p_] = 5 p^0 + -6 p^1 + 6 p^2 + 0 p^3 + 0 p^4 + 0 p^5;
P10[p_] = 5 p^0 + -6 p^1 + 6 p^2 + 1 p^3 + -2 p^4 + 0 p^5;
P11[p_] = 5 p^0 + -6 p^1 + 7 p^2 + -1 p^3 + -2 p^4 + 0 p^5;
P12[p_] = 4 p^0 + -2 p^1 + -3 p^2 + 8 p^3 + -2 p^4 + 0 p^5;
P13[p_] = 4 p^0 + -2 p^1 + -2 p^2 + 7 p^3 + -2 p^4 + 0 p^5;
P14[p_] = 4 p^0 + -2 p^1 + -2 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P15[p_] = 4 p^0 + -1 p^1 + -3 p^2 + 7 p^3 + -2 p^4 + 0 p^5;
P16[p_] = 4 p^0 + -1 p^1 + -3 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P17[p_] = 4 p^0 + -1 p^1 + -2 p^2 + 6 p^3 + -4 p^4 + 0 p^5;
P18[p_] = 5 p^0 + -6 p^1 + 5 p^2 + 1 p^3 + 0 p^4 + 0 p^5;
P19[p_] = 5 p^0 + -5 p^1 + 5 p^2 + 0 p^3 + 0 p^4 + 0 p^5;
P20[p_] = 5 p^0 + -5 p^1 + 5 p^2 + 1 p^3 + -2 p^4 + 0 p^5;
P21[p_] = 5 p^0 + -5 p^1 + 6 p^2 + -1 p^3 + -2 p^4 + 0 p^5;
P22[p_] = 4 p^0 + -1 p^1 + -4 p^2 + 8 p^3 + -2 p^4 + 0 p^5;
P23[p_] = 4 p^0 + 0 p^1 + -4 p^2 + 7 p^3 + -2 p^4 + 0 p^5;
P24[p_] = 4 p^0 + 0 p^1 + -4 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P25[p_] = 4 p^0 + 0 p^1 + -3 p^2 + 6 p^3 + -4 p^4 + 0 p^5;
P26[p_] = 3 p^0 + 3 p^1 + -9 p^2 + 10 p^3 + -2 p^4 + 0 p^5;
P27[p_] = 3 p^0 + 3 p^1 + -8 p^2 + 9 p^3 + -2 p^4 + 0 p^5;
P28[p_] = 3 p^0 + 3 p^1 + -8 p^2 + 10 p^3 + -4 p^4 + 0 p^5;
P29[p_] = 3 p^0 + 4 p^1 + -9 p^2 + 9 p^3 + -2 p^4 + 0 p^5;
P30[p_] = 3 p^0 + 4 p^1 + -9 p^2 + 10 p^3 + -4 p^4 + 0 p^5;
P31[p_] = 3 p^0 + 4 p^1 + -8 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P32[p_] = 5 p^0 + -5 p^1 + 5 p^2 + -1 p^3 + -2 p^4 + 0 p^5;
P33[p_] = 4 p^0 + 0 p^1 + -4 p^2 + 6 p^3 + -4 p^4 + 0 p^5;
P34[p_] = 3 p^0 + 4 p^1 + -9 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P35[p_] = 2 p^0 + 6 p^1 + -10 p^2 + 9 p^3 + -2 p^4 + 0 p^5;
P36[p_] = 2 p^0 + 6 p^1 + -10 p^2 + 10 p^3 + -4 p^4 + 0 p^5;
P37[p_] = 2 p^0 + 6 p^1 + -9 p^2 + 8 p^3 + -4 p^4 + 0 p^5;
P38[p_] = 2 p^0 + 6 p^1 + -10 p^2 + 8 p^3 + -4 p^4 + 0 p^5;

d = {P0[p], P1[p], P2[p], P3[p], P4[p], P5[p], P6[p], P7[p], P8[p], P9[p],
  P10[p], P11[p], P12[p], P13[p], P14[p], P15[p], P16[p], P17[p], P18[p], P19[p],
```

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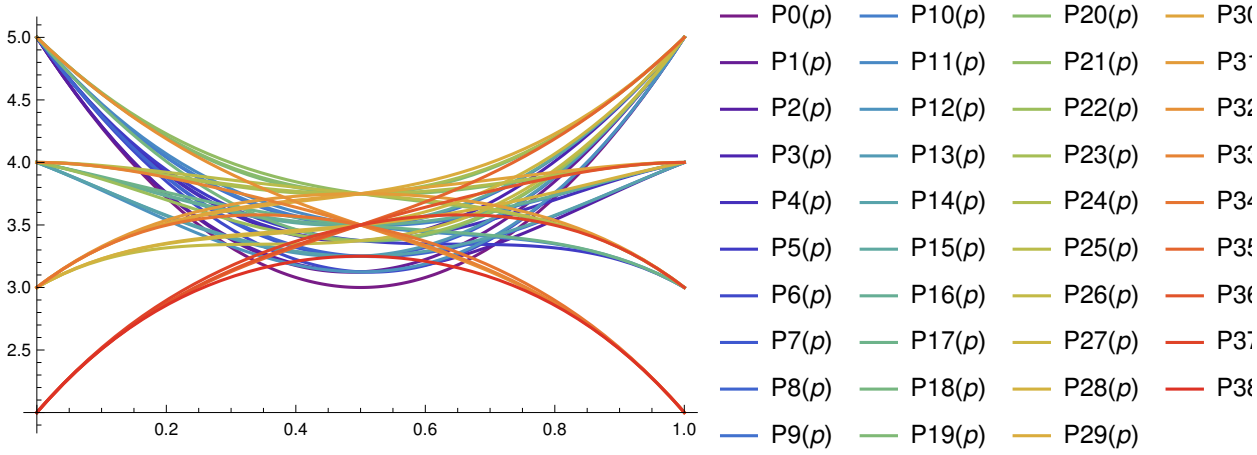
  P20[p], P21[p], P22[p], P23[p], P24[p], P25[p], P26[p], P27[p], P28[p], P29[p],
  P30[p], P31[p], P32[p], P33[p], P34[p], P35[p], P36[p], P37[p], P38[p]}
Plot[{P0[p], P1[p], P2[p], P3[p], P4[p], P5[p], P6[p], P7[p], P8[p], P9[p],
  P10[p], P11[p], P12[p], P13[p], P14[p], P15[p], P16[p], P17[p], P18[p], P19[p],
  P20[p], P21[p], P22[p], P23[p], P24[p], P25[p], P26[p], P27[p], P28[p], P29[p],
  P30[p], P31[p], P32[p], P33[p], P34[p], P35[p], P36[p], P37[p], P38[p]},
 {p, 0, 1}, Axes → True, PlotLegends → "Expressions",
 PlotStyle → ColorData["Rainbow"] /@ (Range[0, Length@d] / Length@d)]

f[p_] = Expand[PiecewiseExpand[FullSimplify[Min[d], 0 < p < 1]]]
FindMaximum[f[p], {p, 0.4}]
FindMaximum[f[p], {p, 0.6}]
Solve[FunctionSingularities[f[p], p, Reals] && 0 < p < 1, p, Reals]
Plot[f[p], {p, 0, 1}, PlotLegends → "Expressions",
 Epilog → {PointSize[Medium], Red, Point[{p, 2}] /. %}]
```

Out[ ]=
$$\{5-8p+8p^2,\ 5-8p+9p^2-p^3,\ 5-8p+9p^2-2p^4,\ 5-7p+8p^2-p^3,\ 5-7p+8p^2-2p^4,$$
$$5-7p+9p^2-2p^3-2p^4,\ 5-7p+6p^2+p^3,\ 5-7p+7p^2,\ 5-7p+7p^2+p^3-2p^4,$$
$$5-6p+6p^2,\ 5-6p+6p^2+p^3-2p^4,\ 5-6p+7p^2-p^3-2p^4,\ 4-2p-3p^2+8p^3-2p^4,$$
$$4-2p-2p^2+7p^3-2p^4,\ 4-2p-2p^2+8p^3-4p^4,\ 4-p-3p^2+7p^3-2p^4,\ 4-p-3p^2+8p^3-4p^4,$$
$$4-p-2p^2+6p^3-4p^4,\ 5-6p+5p^2+p^3,\ 5-5p+5p^2,\ 5-5p+5p^2+p^3-2p^4,$$
$$5-5p+6p^2-p^3-2p^4,\ 4-p-4p^2+8p^3-2p^4,\ 4-4p^2+7p^3-2p^4,\ 4-4p^2+8p^3-4p^4,$$
$$4-3p^2+6p^3-4p^4,\ 3+3p-9p^2+10p^3-2p^4,\ 3+3p-8p^2+9p^3-2p^4,\ 3+3p-8p^2+10p^3-4p^4,$$
$$3+4p-9p^2+9p^3-2p^4,\ 3+4p-9p^2+10p^3-4p^4,\ 3+4p-8p^2+8p^3-4p^4,$$
$$5-5p+5p^2-p^3-2p^4,\ 4-4p^2+6p^3-4p^4,\ 3+4p-9p^2+8p^3-4p^4,\ 2+6p-10p^2+9p^3-2p^4,$$
$$2+6p-10p^2+10p^3-4p^4,\ 2+6p-9p^2+8p^3-4p^4,\ 2+6p-10p^2+8p^3-4p^4\}$$

Out[ ]=




Out[ ]=
$$\begin{cases} 5-8p+8p^2 & 0.356\ldots \le p \le 0.644\ldots \\ -2\left(-1-3p+5p^2-4p^3+2p^4\right) & \text{True} \end{cases}$$

FindMaximum: The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient increase in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[ ]= {3.16553, {p → 0.356158}}

FindMaximum: The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient increase in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[ ]= {3.16553, {p → 0.643842}}

Out[ ]= {{p → 0.356…}, {p → 0.644…}}

Out[ ]=