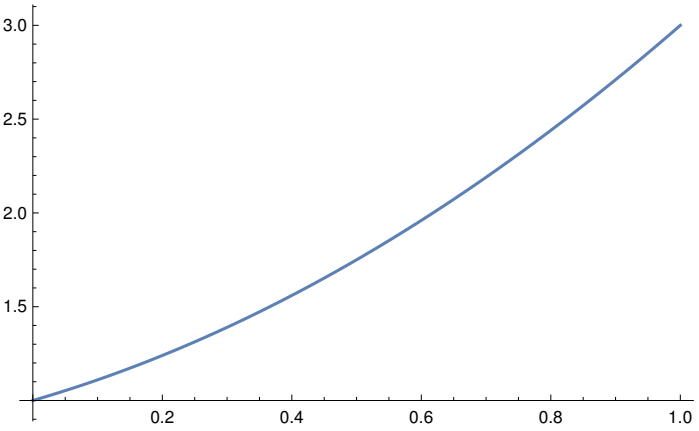
In[98]:=
```
<< MaTeX`
```

4 vertices and 3 edges - all function

In[68]:=
```
D43[p_] = 1 + p + p^2
Plot[D43[p], {p, 0, 1}]
```

Out[68]= $1 + p + p^2$

Out[69]=



4 vertices and 4 edges

In[18]:=
```
D44class1[p_] = 1 + 1 + (1 - (1 - p) ^ 2) + 3 p ^ 2 (1 - p)
Expand[D44class1[p]]
D44class2[p_] = 1 + p (1 + 1 + 2 p (1 - p))
Expand[D44class2[p]]
Plot[{D44class1[p], D44class2[p]}, {p, 0, 1}, PlotLegends → "Expressions"]
FindMaximum[D44class1[p], {p, 0.5}]
```

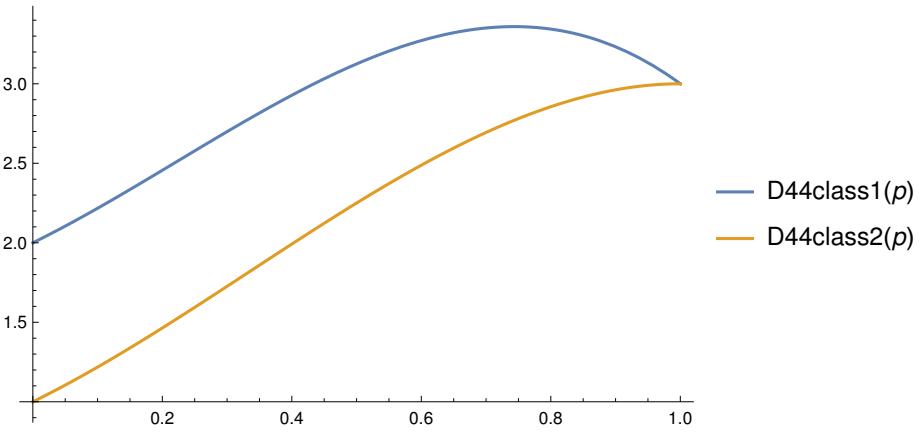
Out[18]= $3 - (1 - p)^2 + 3 (1 - p) p^2$

Out[19]= $2 + 2 p + 2 p^2 - 3 p^3$

Out[20]= $1 + p (2 + 2 (1 - p) p)$

Out[21]= $1 + 2 p + 2 p^2 - 2 p^3$

Out[22]=



Out[23]= {3.35958, {p → 0.74338}}

Influence of x1 (deg 3 deg 2) and x5 (deg 3 deg 3) in graph with 4 nodes and 5 edges

In[ ]:=
```
I1[p_] = 5 p^2 (1 - p)^2 + p^3 (1 - p)
Expand[I1[p]]
I5[p_] = 4 p^2 (1 - p)^2
Expand[I5[p]]
Plot[{I1[p], I5[p]}, {p, 0, 1}, PlotLegends → "Expressions"]
```

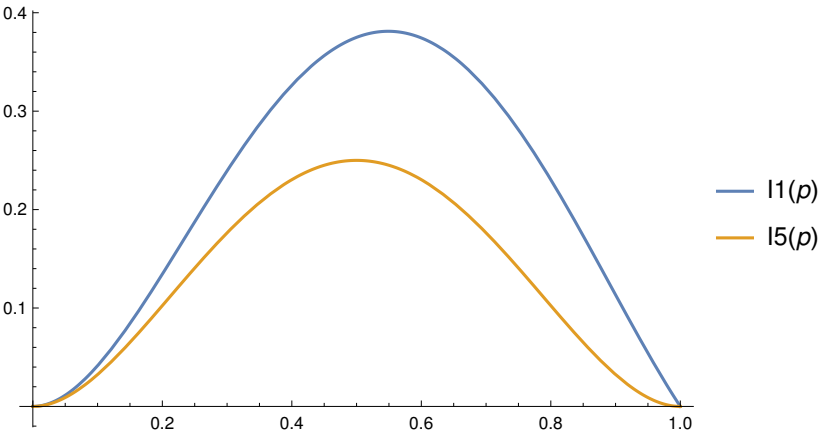
Out[ ]= $5 (1 - p)^2 p^2 + (1 - p) p^3$

Out[ ]= $5 p^2 - 9 p^3 + 4 p^4$

Out[ ]= $4 (1 - p)^2 p^2$

Out[ ]= $4 p^2 - 8 p^3 + 4 p^4$

Out[ ]=



In[ ]:=
```
Export[
 "/home/julia/git/ComplexityOfBooleanFunctions/plots/graphinfluence.png", %228, "PNG"]
```

Out[ ]= /home/julia/git/ComplexityOfBooleanFunctions/plots/graphinfluence.png

In[113]:=
```
A1[p_] = 1 + (1 - p) (D44class2[p]) + p (2 + 2 p (1 - p) (1 + (1 - p)))
Expand[A1[p]]
A2[p_] = 2 + 4 p + 2 p^2 - 9 p^3 + 4 p^4
A3[p_] = 2 + 4 p + 3 p^2 - 10 p^3 + 4 p^4
A5[p_] = 1 + (1 - p) (D44class1[p]) + p (1 + p (1 + (1 - p)) + (1 - p) (1 + p (1 + (1 - p))))
Expand[A5[p]]
LowerBound[p_] = 4 p (1 - p) (4 I1[p] + I5[p])^2
Expand[LowerBound[p]]
plot = Plot[{A1[p], A5[p], LowerBound[p]}, {p, 0, 1},
  PlotLegends → {MaTeX@{"\mathbb{E}_x^{\pi_p}[c(A_1,x)]",
      "\mathbb{E}_x^{\pi_p}[c(A_5,x)]", "4p(1-p)(\mathbf{I}^p(f_{45}))^2"}}]
Export["/home/julia/git/ComplexityOfBooleanFunctions/plots/graph5algs.eps", plot]
intersect1 = NSolve[A1[p] == A5[p] && p ≤ 1 && p > 0]
A1'[p /. intersect1]
A5'[p /. intersect1]
FindMaximum[A1[p], {p, 0.5}]
```

Out[113]= $1 + p\,(2 + 2\,(1 - p)\,(2 - p)\,p) + (1 - p)\,(1 + p\,(2 + 2\,(1 - p)\,p))$

Out[114]= $2 + 3\,p + 4\,p^2 - 10\,p^3 + 4\,p^4$

Out[115]= $2 + 4\,p + 2\,p^2 - 9\,p^3 + 4\,p^4$

Out[116]= $2 + 4\,p + 3\,p^2 - 10\,p^3 + 4\,p^4$

Out[117]= $1 + (1 - p)\left(3 - (1 - p)^2 + 3\,(1 - p)\,p^2\right) + p\,(1 + (2 - p)\,p + (1 - p)\,(1 + (2 - p)\,p))$

Out[118]= $3 + 2\,p + 3\,p^2 - 9\,p^3 + 4\,p^4$

Out[119]= $4\,(1 - p)\,p\,(4\,\mathrm{I1}[p] + \mathrm{I5}[p])^2$

Out[120]= $64\,p\,\mathrm{I1}[p]^2 - 64\,p^2\,\mathrm{I1}[p]^2 + 32\,p\,\mathrm{I1}[p] \times \mathrm{I5}[p] - 32\,p^2\,\mathrm{I1}[p] \times \mathrm{I5}[p] + 4\,p\,\mathrm{I5}[p]^2 - 4\,p^2\,\mathrm{I5}[p]^2$

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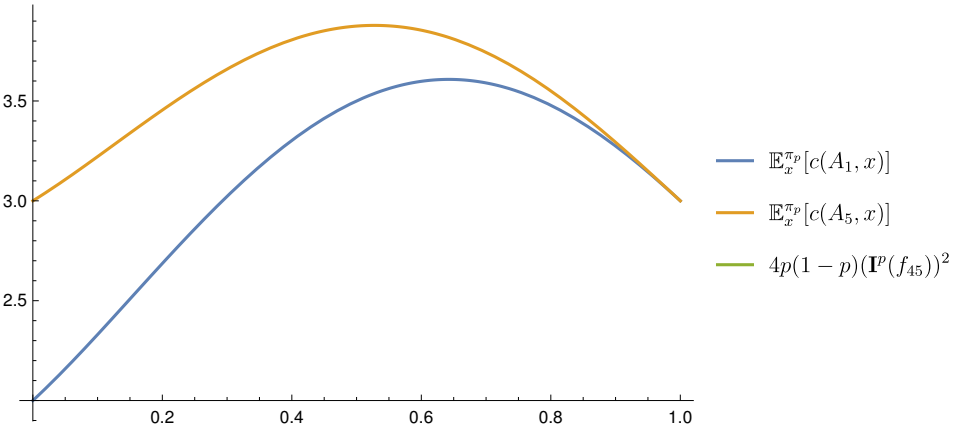
Out[121]=



Out[122]= /home/julia/git/ComplexityOfBooleanFunctions/plots/graph5algs.eps

Out[123]= {{p → 1.}, {p → 1.}}

Out[124]= {-3., -3.}

Out[125]= {-3., -3.}

Out[126]= {3.60808, {p → 0.64245}}

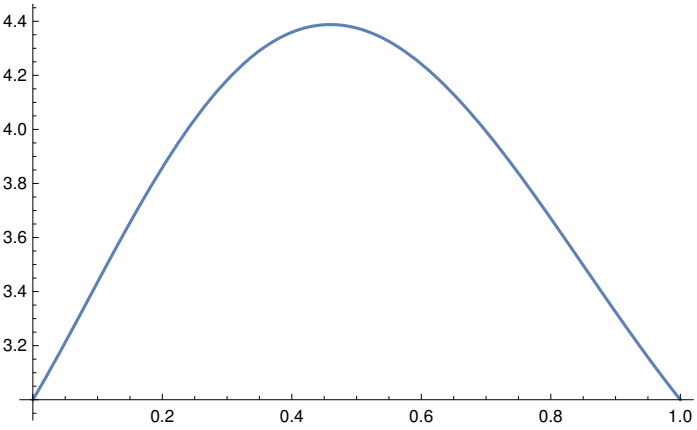
In[127]:=
```
A[p_] = 1 + (1 – p) (1 + (1 – p) (1 + p (1 + (1 – p))) + p (1 + (1 – p))) + p (1 + (1 – p) (1 + (1 – p) (1 + (1 – p))))
A6[p_] = 1 + (1 – p) A1[p] + p A[p]
Expand[A6[p]]
Plot[A6[p], {p, 0, 1}]
FindMaximum[A6[p], {p, 0.5}]
```

Out[127]= $1 + (1 + (1 + (1 - p)\ (2 - p))\ (1 - p))\ p + (1 - p)\ (1 + (2 - p)\ p + (1 - p)\ (1 + (2 - p)\ p))$

Out[128]= $1 + (1 - p)\ (1 + p\ (2 + 2\ (1 - p)\ (2 - p)\ p) + (1 - p)\ (1 + p\ (2 + 2\ (1 - p)\ p))) +$
$p\ (1 + (1 + (1 + (1 - p)\ (2 - p))\ (1 - p))\ p + (1 - p)\ (1 + (2 - p)\ p + (1 - p)\ (1 + (2 - p)\ p)))$

Out[129]= $3 + 4\ p + 6\ p^2 - 27\ p^3 + 23\ p^4 - 6\ p^5$

Out[130]=



Out[131]= {4.38777, {p → 0.459499}}