```
MAJ3
In[901]:= k = 2;
             n = 2k - 1;
             Pdiff[p] = 1 - p^k - (1-p)^k;
             Plot[Pdiff[p], {p, 0, 1}]
              P1[p_{-}] = Sum[Binomial[n, i] * p^i * (1 - p)^(n - i), \{i, k, n\}];
              Plot[P1[p], {p, 0, 1}]
              Palt[p_] = 1 - P1[p]^k - (1 - P1[p])^k;
              pss[p] = p * P1[p] + (1-p)(1-P1[p]);
             A1[p] = (2 + Pdiff[p])(2 + Palt[p]);
             A4[p] =
                    2 + Pdiff[p] + 1 + (pss[p] + P1[p] (1 - P1[p]) (1 - p) + (1 - P1[p]) P1[p] p) * (1 + Pdiff[p]) + (1 + Pdif
                       ((1-pss[p]) + P1[p] p (1-p)^2 + (1-P1[p]) (1-p) p^2) * (2 + Pdiff[p]);
             A3[p] = 2 + Pdiff[p] + 1 + (1 - P1[p](1 - p) + P1[p](1 - P1[p])(1 - p)) * (1 + Pdiff[p]) +
                       (P1[p](1-p) + P1[p]p(1-p)^2 + (1-P1[p])p(1-(1-p)^2) + (1-P1[p])(1-p)p^2)*
                          (2 + Pdiff[p]);
             A2[p] = A3[1-p];
             A2new[p_{-}] = 2 + Pdiff[p] + 1 +
                       (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
                       (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
                       P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
                       P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
              Expand[A2new[p]]
              A2hs[p_{-}] = 4p^{0} + 4p^{1} + 7p^{2} + 6p^{3} + -54p^{4} + 12p^{5} + 75p^{6} + -66p^{7} + 16p^{8} + 0p^{9}
             A3new[p_] = 2 + Pdiff[p] + 1 +
                       (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
                       (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
                       P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
                       P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
              Expand[A3new[p]]
              Expand[A2new[1 - p]]
             A3hs[p_] = 4\,p^{\,0} + 4\,p^{\,1} + 8\,p^{\,2} + 4\,p^{\,3} + -59\,p^{\,4} + 28\,p^{\,5} + 61\,p^{\,6} + -62\,p^{\,7} + 16\,p^{\,8} + 0\,p^{\,9}
             A4new[p_{-}] = 2 + Pdiff[p] + 1 +
                       (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
                       (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
                       P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
                       P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
```

Expand[Atest5[p]]

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Expand[A4new[p]]
A4hs[p] = 4p^0 + 4p^1 + 7p^2 + 6p^3 + -57p^4 + 20p^5 + 68p^6 + -64p^7 + 16p^8 + 0p^9
A4hs[1/2]
Plot[{A1[p], A2new[p], A3new[p], A4new[p]}, {p, 0, 1}]
Maximize[\{A1[p], 0 ]
Maximize[\{A2new[p], 0 ]
Maximize[\{A3new[p], 0 ]
Maximize[\{A4new[p], 0 ]
Atest[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest[p]]
Atest2[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest2[p]]
Atest3[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest3[p]]
Atest4[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest4[p]]
Atest5[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
```

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Atest6[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
    P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
Expand[Atest6[p]]
Atest7[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
Expand[Atest7[p]]
Atest8[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
    P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest8[p]]
Atest9[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
    (1 - P1[p]) p (2 + Pdiff[p] + P1[p] (1 + (1 - p))) +
    P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest9[p]]
Atest10[p] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
    (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
    P1[p](1-p)(2+Pdiff[p]+(1-P1[p])(1+p))+
    P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest10[p]]
Atest11[p_] = 2 + Pdiff[p] + 1 +
    (1 - P1[p]) (1 - p) (2 + Pdiff[p] + P1[p] (1 + p)) +
    (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
    P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
    P1[p] p (1 + (1 - p) + (1 - p)^2 (2 + Pdiff[p]));
Expand[Atest11[p]]
```

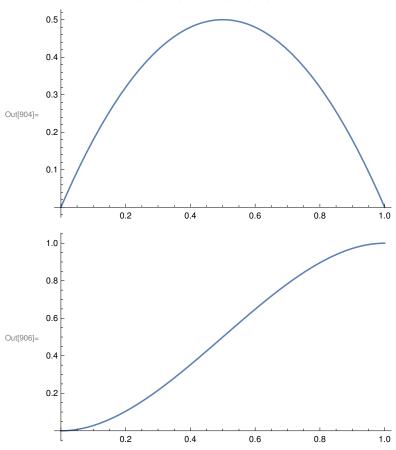
```
Atest12[p] = 2 + Pdiff[p] + 1 +
        (1 - P1[p]) (1 - p) (1 + p + p^2 (2 + Pdiff[p])) +
        (1 - P1[p]) p (1 + (1 - p) + (1 - (1 - p)^2) (2 + Pdiff[p])) +
       P1[p](1-p)(1+p+(1-p^2)(2+Pdiff[p]))+
        P1[p] p (2 + Pdiff[p] + (1 - P1[p]) (1 + (1 - p)));
Expand[Atest12[p]]
b = {A1[p], A2new[p], A3new[p], A4new[p], Atest[p], Atest2[p], Atest3[p], Atest4[p], Atest5[p],
        Atest6[p], Atest7[p], Atest8[p], Atest9[p], Atest10[p], Atest11[p], Atest12[p]);
Plot[{A1[p], A2new[p], A3new[p], A4new[p], Atest[p], Atest2[p], Atest3[p], Atest4[p],
     Atest5[p], Atest6[p], Atest7[p], Atest8[p], Atest9[p], Atest10[p], Atest11[p], Atest12[p]},
  {p, 0, 1}, PlotStyle → ColorData["Rainbow"]/@(Range[0, Length@b]/Length@b),
  PlotLegends → "Expressions"]
c = Subtract[{A1[p], A2new[p], A3new[p], A4new[p], Atest[p],
          Atest2[p], Atest3[p], Atest4[p], Atest5[p], Atest6[p], Atest7[p],
          Atest8[p], Atest9[p], Atest10[p], Atest11[p], Atest12[p]}, A4hs[p]];
Plot[c, \{p, 0, 1\}, PlotStyle \rightarrow ColorData["Rainbow"]/@(Range[0, Length@c]/Length@c),
  PlotLabel → "All algorithms subtracted by the minimal one"]
NSolve[Atest2[p] == A4hs[p] && p < 1 && p > 0]
NSolve[Atest10[p] == A4hs[p] \&\& p < 1 \&\& p > 0]
NSolve[A3hs[p] == A4hs[p] && p < 1 && p > 0]
Expand[A1[p]]
A1[1/2]
Expand[A2[p]]
Expand[A3[p]]
Expand[A4[p]]
A4[1/2]
d = {A1[p], A2[p], A3[p], A4[p], A2hs[p], A4hs[p], Atest[p], Atest2[p]};
Plot[{A1[p], A2[p], A3[p], A4[p], A2hs[p], A4hs[p], Atest[p], Atest2[p]}, {p, 0, 1},
  PlotStyle → ColorData["Rainbow"]/@(Range[0, Length@d]/Length@d),
  PlotLegends → "Expressions"]
e = \{A1[p] - A4[p], A2[p] - A4[p], A3[p] - A4[p], A4[p] - A4[p], A4[p]
        A2hs[p] - A4[p], A3hs[p] - A4[p], A4hs[p] - A4[p], Atest2[p] - A4[p];
Plot[A1[p] - A4[p], A2[p] - A4[p], A3[p] - A4[p], A4[p] - A4[p],
     A2hs[p] - A4[p], A3hs[p] - A4[p], A4hs[p] - A4[p], Atest2[p] - A4[p], \{p, 0, 1\},
  PlotStyle → ColorData["Rainbow"]/@(Range[0, Length@e]/Length@e),
  PlotLegends → "Expressions"]
Expand[A1[1 - eps]]
Expand[A2[1 - eps]]
Expand[A3[1 - eps]]
```

Expand[A4[1 - eps]]

intersect1 = NSolve[A3[p] == A4[p] && p < 1 && p > 0]A4'[p/. intersect1] A3'[p/. intersect1] intersect2 = NSolve[A2[p] == A4[p] && p < 1 && p > 0]A4'[p/. intersect2] A2'[p/. intersect2]

intersect3 = $NSolve[A2[p] == A1[p] \&\& p \le 1 \&\& p > 0]$ Expand[A1[eps]] Expand[A3[eps]]

Plot[Min[A3[p], A2[p], A4[p]], {p, 0, 1}]



$$\text{Out} [\text{914}] = \ \ 4 + 4 \ p + 7 \ p^2 + 6 \ p^3 - 54 \ p^4 + 12 \ p^5 + 75 \ p^6 - 66 \ p^7 + 16 \ p^8$$

Out[915]=
$$4 + 4 p + 7 p^2 + 6 p^3 - 54 p^4 + 12 p^5 + 75 p^6 - 66 p^7 + 16 p^8$$

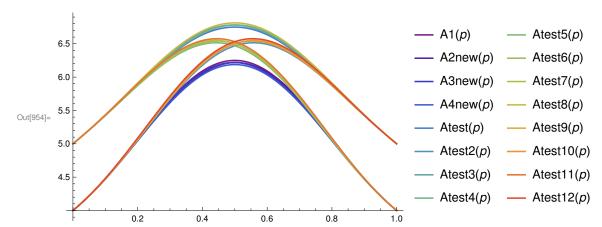
$$\mathsf{Out}[\mathsf{917}] = \ 4 + 4\ p + 8\ p^2 + 4\ p^3 - 59\ p^4 + 28\ p^5 + 61\ p^6 - 62\ p^7 + 16\ p^8$$

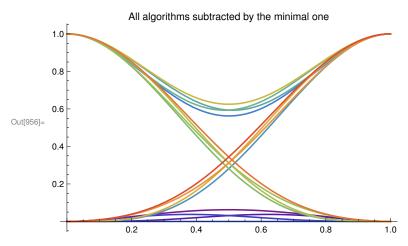
Out[918]=
$$4 + 4 p + 8 p^2 + 4 p^3 - 59 p^4 + 28 p^5 + 61 p^6 - 62 p^7 + 16 p^8$$

Out[948]= $4 + 4 p + 8 p^2 + 4 p^3 - 47 p^4 + 8 p^5 + 72 p^6 - 64 p^7 + 16 p^8$

Out[950]= $5 + 4 p + 3 p^2 + 6 p^3 - 49 p^4 + 16 p^5 + 65 p^6 - 62 p^7 + 16 p^8$

Out[952]= $4 + 4 p + 8 p^2 + 4 p^3 - 44 p^4 + 79 p^6 - 66 p^7 + 16 p^8$





Out[957]= {}

Out[958]= {}

Out[959]= {}

Out[960]= $4 + 4 p + 8 p^2 + 4 p^3 - 56 p^4 + 20 p^5 + 68 p^6 - 64 p^7 + 16 p^8$

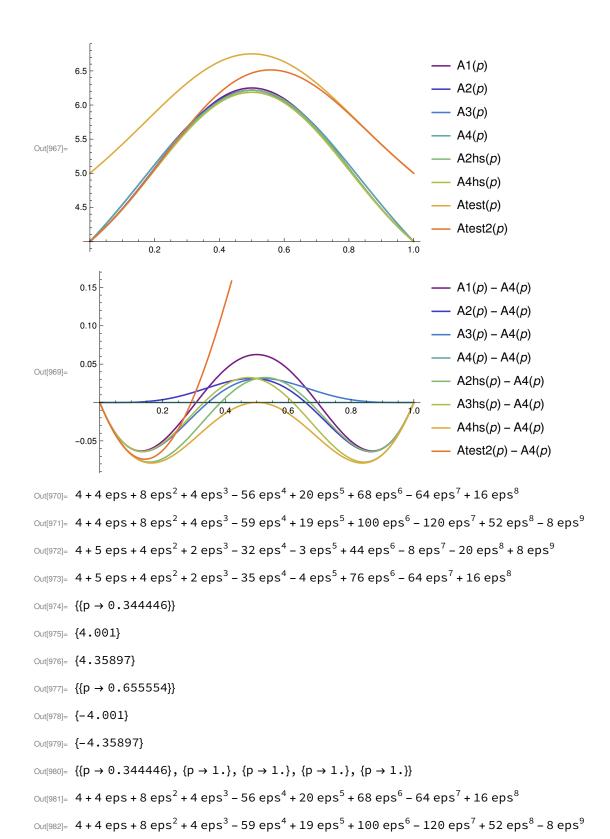
Out[961]= $\frac{25}{4}$

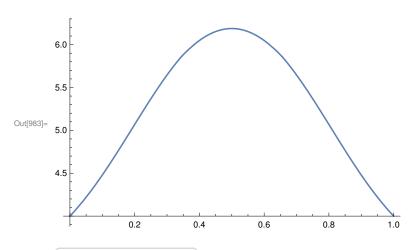
 $Out[962] = 4 + 5 p + 4 p^{2} + 2 p^{3} - 32 p^{4} - 3 p^{5} + 44 p^{6} - 8 p^{7} - 20 p^{8} + 8 p^{9}$

Out[963]= $4 + 4 p + 8 p^2 + 4 p^3 - 59 p^4 + 19 p^5 + 100 p^6 - 120 p^7 + 52 p^8 - 8 p^9$

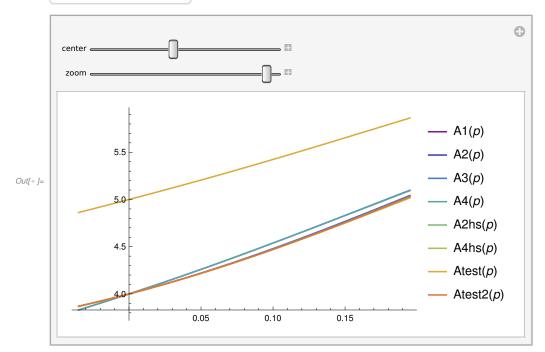
Out[964]= $4 + 5 p + 4 p^2 + 2 p^3 - 35 p^4 - 4 p^5 + 76 p^6 - 64 p^7 + 16 p^8$

Out[965]=





interactive plot %716



- ... Infinity: Indeterminate expression 0 ComplexInfinity encountered.
- ... Infinity: Indeterminate expression 0 ComplexInfinity encountered.
- ••• General: Further output of Power::infy will be suppressed during this calculation.
- ••• Infinity: Indeterminate expression 0 ComplexInfinity encountered.

- ••• General: Further output of Infinity::indet will be suppressed during this calculation.
- ••• Power: Infinite expression $\frac{1}{0}$ encountered.
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- General: Further output of Power::infy will be suppressed during this calculation.
- ... Infinity: Indeterminate expression 0 ComplexInfinity encountered.
- ••• General: Further output of Infinity::indet will be suppressed during this calculation.