Electric Power Markets   Spring 2025   ENV 717

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Assignment # 5
****Submit a word or pdf document with your answers to all questions***
The total points for this assignment are 102 – this means you can loose 2 and still get the perfect score of 100.

1. (42%) Unit commitment problem[1]:
A balancing authority needs to solve the UC problem for a 3-hours planning horizon. Three thermal generating units are used to supply demands of 160MW, 500MW and 400MW in time periods 1, 2 and 3, respectively. Data on technical limits and economic parameters of the generating units is provided in Table 1. Required spinning reserves during each of the three periods are 160MW, 50MW, and 40MW but **we will ignore them** in the formulation of this UC problem.

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)
1	5	0.5	20	0.1	350	50	300	300	200	200
2	7	0.3	18	0.125	200	80	150	150	100	100
3	6	1	5	0.15	140	40	100	100	100	100

Table 1: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h).

Assume that at time t=0, units #1 and # 2 are off-line while generating unit# 3 is online and producing 100MW.

Formulate the unit commitment (UC) problem spelling out all the values and constraints.

*For a) through g) spell out all the decision variables without writing sigmas or foralls.

a) (2%) Write down a list of all the decision variables, spelling out each of them (i.e., do not use the indices to refer to types of decision variables, but rather list them all). The first decision variable in your list is: $p_{1,1}$=power produced by generator # 1 in hour1. The second element in the list is: $p_{1,2}$=power produced by generator # 1 in hour 2, and so on.
b) (2%) Write down the parameters representing the initial conditions of the 3 electricity generating units. (You can denote these initial conditions with capital/uppercase letters to distinguish them from the closely related decision variables.

[1] For this problem and the next one you can submit handwritten formulation -inserted in your word document- as long as everything is neat and clear.

c) (5%) Write down the objective function spelling out all the terms and writing down the values of all the parameters. Explain this function in words

d) (7%) Write down all the constraints that relate the operating status of the plants with the indicator decision variables for shutting down and starting up. These are 18 equations with logical conditions that state that any generator that is online can be shut-down but not started-up and that any generator that is off-line can be started –up but not shut-down.

e) (2%) Write down all the equations that ensure that binary decision variables take the value 0 or 1.

f) (3%) Write down all the power limit constraints (i.e., constrain power generation to be within the min and the maximum technical limits of the generator). Please write one constraint for the minimum power generation and another for the maximum power generation.

g) (8%) Write down all the ramping limits constraints.

h) (3%) Write down the power balance constraints.

i) (10%) Now **re-write the formulation** in the most succinct and clear way. Start by listing all the indices and sets, then list all the parameters, the decision variables (referring to the indices), and write expressions that represent several constraints by using the sigma and forall operators.

Please note that to be rigorous, for the ramping constraints we need to write one expression for t=1 and another for t=2 and t=3.

Here is the constraint of t=1 for ramping up. (You need to write an expression for the ramp-up constraint for t=2,3, and two expressions for the ramp-down constraints).

where $P_{g,0}, U_{g,0}$ are parameters representing initial conditions

$$p_{g,1} - P_{g,0} \le R_g^U U_{g,0} + R_g^{SU} y_{g,1} \quad \forall g$$

2. (10%) For each of the following solutions of the UC problem described in above, determine feasibility. If the solution is determined to be not feasible then explain which constraints are being violated.

a)
$p_{1,1} = 0, p_{2,1} = 160, p_{3,1} = 0$

b)
$p_{1,1}=160,\ p_{2,1}=0,\ p_{3,1}=0,\ p_{1,2}=350,\ p_{2,2}=50,\ p_{3,2}=100,$
$u_{1,1}=1,\ u_{2,1}=0,\ u_{3,1}=0,\ u_{1,2}=1,\ u_{2,2}=1,\ u_{3,2}=1$
$y_{1,1}=1,\ y_{2,1}=0,\ y_{3,1}=0,\ y_{1,2}=0,\ y_{2,2}=1,\ y_{3,2}=1$
$z_{1,1}=0,\ z_{2,1}=0,\ z_{3,1}=1,\ z_{1,2}=0,\ z_{2,2}=0,\ z_{3,2}=0$

c)
$p_{1,1}=160,\ p_{2,1}=0,\ p_{3,1}=0,\ p_{1,2}=350,\ p_{2,2}=50,\ p_{3,2}=100,$
$u_{1,1}=1,\ u_{2,1}=0,\ u_{3,1}=0,\ u_{1,2}=1,\ u_{2,2}=1,\ u_{3,2}=1$
$y_{1,1}=1,\ y_{2,1}=0,\ y_{3,1}=1,\ y_{1,2}=0,\ y_{2,2}=1,\ y_{3,2}=1$
$z_{1,1}=0,\ z_{2,1}=0,\ z_{3,1}=0,\ z_{1,2}=0,\ z_{2,2}=0,\ z_{3,2}=0$

d)
$p_{1,1}=60,\ p_{2,1}=10,\ p_{3,1}=90,\ p_{1,2}=300,\ p_{2,2}=110,\ p_{3,2}=90,$
$u_{1,1}=1,\ u_{2,1}=1,\ u_{3,1}=1,\ u_{1,2}=1,\ u_{2,2}=1,\ u_{3,2}=1$
$y_{1,1}=1,\ y_{2,1}=1,\ y_{3,1}=0,\ y_{1,2}=0,\ y_{2,2}=1,\ y_{3,2}=1$
$z_{1,1}=0,\ z_{2,1}=0,\ z_{3,1}=0,\ z_{1,2}=0,\ z_{2,2}=0,\ z_{3,2}=0$

3. (40%) Self-scheduling problem:
An **independent power producer** owns two power generating resources with characteristics as described in Table1.

Generating Unit	C_g^F (1)	C_g^{SD} (2)	C_g^{SU} (3)	C_g^V (4)	$P_g^{\max}$ (5)	$P_g^{\min}$ (6)	R_g^D (7)	R_g^{SD} (8)	R_g^{SU} (9)	R_g^U (10)
1	5	400	10,700	24.0	300	30	150	180	200	100
2	5	300	10,000	22.9	150	50	50	25	75	25

Table 1: Technical characteristics of generators. (1) Online (fixed) cost of generating unit g ($/h). (2) shut-down cost of generating unit g ($). (3) Start-up cost of generating unit g ($). (4) Variable cost of generating unit g ($/MWh). (5) Power generation capacity of unit g (MW). (6) minimum power output of unit g (MW). (7) ramping-down limit of generating unit g (MW/h). (8) shut-down ramping limit of generating unit g (MW/h). (9) start-up ramping limit of generating unit g(MW/h). (10) ramping-up limit of generating unit g (MW/h).

Assume this power producer is a price-taker (i.e., has no power to influence prices in the market) and will always be dispatched if its marginal cost is lower than the market price.
Formulate the optimization problem the power producer can use to find the optimal scheduling of the generating units to maximize profits during the next 5 hours (estimated prices given in table below) (There is no need to find a solution to this problem. Please ignore the constraints for

the minimum uptime and minimum down time, and assume that the generating units are off-line and hence have an initial power output of zero).

t (hour)	λ_t Forecast market Price ($/MWh)
1	27
2	30
3	35
4	33
5	26

Table 2: Forecasted prices

a) (5%) Write the decision variables of this problem. Feel free to define indices and sets.

b) (5%) Write the objective function (Feel free to use sigma notation).

c) (20%) Write down all the constraints of this optimization problem. Write all constraints in canonical form* (i.e., have all decision variables in the left hand-side LHS of the constraint and any constant on the right hand-side RHS). Writing constraints in canonical form for a maximization problem means that all decision variables need to be to the left of "less than or equal" sign. Please write the maximum and minimum power output constraints and the ramp rate constraints for each generator using for all t (but spell out one set of constraints for each generator). For the rest of the constraints use sigma notation and forall).

d) (10%) What is the difference between this model and the unit commitment model? Please explain who runs these models, how these models are used, what are the uncertainties faced and how are these uncertainties tackled.

3. (10%) Begin reading the paper and companion supporting information material[2]: Cornelius, A.; Bandyopadhyay, R.; Patiño-Echeverri, D. Assessing Environmental, Economic, and Reliability Impacts of Flexible Ramp Products in Midcontinent ISO. Renewable and Sustainable Energy Reviews, 2018 (81) Part 2, 2291-2298, https://doi.org/10.1016/j.rser.2017.06.037

   Answer the following questions:

   a) (5%) What is the purpose of this study?

   b) (5%) What are the research questions?

[2] Both Adam and Rubenka are my former students. Adam graduated from the MEM program in 2014 and Rubenka from the Ph.D. in 2016. This work started with Adams's MP in 2012 when Flexi ramp products were just being discussed. Although Flexible-ramp products were thought to be a temporary solution, all these years later they still exist in MISO and CAISO and have now been adopted by SPP as well. For A5 you will be asked to describe the methods used in this paper and to compare the models with those discussed in class.