ENV 790.30 - Time Series Analysis for Energy Data | Spring 2024 Assignment 3 - Due date 02/01/24

Julia Kagiliery

Directions

You should open the .rmd file corresponding to this assignment on RStudio. The file is available on our class repository on Github.

Once you have the file open on your local machine the first thing you will do is rename the file such that it includes your first and last name (e.g., "LuanaLima_TSA_A02_Sp24.Rmd"). Then change "Student Name" on line 4 with your name.

Then you will start working through the assignment by **creating code and output** that answer each question. Be sure to use this assignment document. Your report should contain the answer to each question and any plots/tables you obtained (when applicable).

Please keep this R code chunk options for the report. It is easier for us to grade when we can see code and output together. And the tidy opts will make sure that line breaks on your code chunks are automatically added for better visualization.

When you have completed the assignment, **Knit** the text and code into a single PDF file. Submit this pdf using Sakai.

Questions

Consider the same data you used for A2 from the spreadsheet "Table_10.1_Renewable_Energy_Production_and_Consumption The data comes from the US Energy Information and Administration and corresponds to the December 2022 Monthly Energy Review. Once again you will work only with the following columns: Total Biomass Energy Production, Total Renewable Energy Production, Hydroelectric Power Consumption. Create a data frame structure with these three time series only.

R packages needed for this assignment: "forecast", "tseries", and "Kendall". Install these packages, if you haven't done yet. Do not forget to load them before running your script, since they are NOT default packages.\

```
#Load/install required package here
library(forecast)

## Registered S3 method overwritten by 'quantmod':
## method from
## as.zoo.data.frame zoo

library(tseries)
library(Kendall)
library(ggplot2)
library(lubridate)

## ## Attaching package: 'lubridate'
```

```
## The following objects are masked from 'package:base':
##
       date, intersect, setdiff, union
##
library(readxl)
library(cowplot)
##
## Attaching package: 'cowplot'
## The following object is masked from 'package:lubridate':
##
##
DataPandC <-
  read_excel(
    "~/Julia_Kagiliery_TSA_Sp24/Data/Table_10.1_Renewable_Energy_Production_and_Consumption_by_Source.x
    skip = 12,
    sheet = "Monthly Data",
    col_names = FALSE
  )
## New names:
## * `` -> `...1`
## * `` -> `...2`
## * `` -> `...3`
## * `` -> `...4`
## * `` -> `...5`
## * `` -> `...6`
## * `` -> `...7`
## * `` -> `...8`
## * `` -> `...9`
## * `` -> `...10`
## * `` -> `...11`
## * `` -> `...12`
## * `` -> `...13`
## * `` -> `...14`
DataPandCNAMES <-
  read_excel(
    "~/Julia_Kagiliery_TSA_Sp24/Data/Table_10.1_Renewable_Energy_Production_and_Consumption_by_Source.x
    sheet = "Monthly Data", n_max = 1,
    col_names = FALSE)
## New names:
## * `` -> `...1`
## * `` -> `...2`
## * `` -> `...3`
## * `` -> `...4`
## * `` -> `...5`
## * `` -> `...6`
## * `` -> `...7`
## * `` -> `...8`
## * `` -> `...9`
## * `` -> `...10`
## * `` -> `...11`
```

```
## * `` -> `...12`
## * `` -> `...13`
## * `` -> `...14`

colnames(DataPandC) <- DataPandCNAMES

DataPandC <- DataPandC[,c(1,4:6)]

##Trend Component</pre>
```

$\mathbf{Q}\mathbf{1}$

For each time series, i.e., Renewable Energy Production and Hydroelectric Consumption create three plots: one with time series, one with the ACF and with the PACF. You may use the some code form A2, but I want all the three plots side by side as in a grid. (Hint: use function plot_grid() from the cowplot package)

#Set Up:

```
year1 <- year(DataPandC$Month[1])
month1 <- month(DataPandC$Month[1])

DataPandCTS <- DataPandC[,-1] |>
    ts(start = c(year1,month1), frequency = 12)
```

#Biomass Enegry:

inspiration for plot = FLASE from ChatGPT. Prompt: "How do I use plot_grid() with PCF and ACF in R"

```
P1ACF <-
   Acf(
    DataPandCTS[, 1],
    lag.max = 40,
    main = paste("Biomass Production", label_size = 3),
    plot = FALSE
)

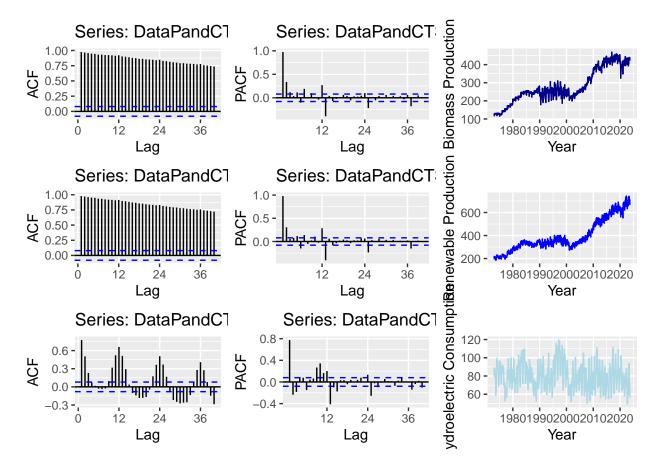
P2PCF <-
Pacf(
   DataPandCTS[, 1],
   lag.max = 40,
   main = paste("Biomass Production", label_size = 3),
   plot = FALSE
)</pre>
```

```
P1 <- autoplot(P1ACF)
P2 <- autoplot(P2PCF)
P3 <- DataPandCTS[, 1] |>
autoplot(color = "darkblue") +
ylab("Biomass Production") +
xlab("Year")
```

#Renewable Energy:

```
P4ACF <-
Acf(
    DataPandCTS[, 2],
    lag.max = 40,
    main = paste("Renewable Production", label_size = 3),
    plot = FALSE</pre>
```

```
)
P5PCF <-
  Pacf(
    DataPandCTS[, 2],
    lag.max = 40,
    main = paste("Renewable Production", label_size = 3),
    plot = FALSE
  )
P4 <- autoplot(P4ACF)
P5 <- autoplot(P5PCF)
P6 <- DataPandCTS[, 2] |>
  autoplot(color = "blue", label_size = 3) +
  ylab("Renewable Production") +
  xlab("Year")
## Warning in ggplot2::geom_line(na.rm = TRUE, ...): Ignoring unknown parameters:
## `label_size`
#Hydroelectric Energy:
P7ACF <-
  Acf(
    DataPandCTS[, 3],
    lag.max = 40,
    main = paste("Hydroelectric Consumption", label_size = 3),
    plot = FALSE
P8PCF <-
  Pacf(
    DataPandCTS[, 3],
    lag.max = 40,
   main = paste("Hydroelectric Consumption", label_size = 3),
    plot = FALSE
  )
P7 <- autoplot(P7ACF)
P8 <- autoplot(P8PCF)
P9 <- DataPandCTS[, 3] |>
  autoplot(color = "lightblue", label_size = 3) +
  ylab("Hydroelectric Consumption") +
 xlab("Year")
## Warning in ggplot2::geom_line(na.rm = TRUE, ...): Ignoring unknown parameters:
## `label_size`
plot_grid(P1, P2, P3, P4, P5, P6, P7, P8, P9, nrow = 3, label_size = 3)
```



$\mathbf{Q2}$

From the plot in Q1, do the series Total Biomass Energy Production, Total Renewable Energy Production, Hydroelectric Power Consumption appear to have a trend? If yes, what kind of trend?

It appears as though the total biomass energy production and the total renewable energy production each have a positive linear trend in that the production increases over time. However, this trend does not seem to be present in total hydroelectric energy consumption. All three time series seem to be influenced by some random and seasonal effects as well. The becomes apparent in the ACF plots in which there is still a relatively high value after many lags.

$\mathbf{Q3}$

Use the lm() function to fit a linear trend to the three time series. Ask R to print the summary of the regression. Interpret the regression output, i.e., slope and intercept. Save the regression coefficients for further analysis.

total number of observations =609

```
t <- c(1:609)

#Trend for Biomass Energy Production:
DataBiomass <- DataPandC$`Total Biomass Energy Production`

DataBiomass <- ts(DataBiomass)

BiomassTrend = lm(DataBiomass ~ t)</pre>
```

```
summary(BiomassTrend)
##
## Call:
## lm(formula = DataBiomass ~ t)
##
## Residuals:
                  1Q
                      Median
        Min
                                     3Q
                                             Max
## -102.344 -23.754
                        5.491
                                31.980
                                          83.154
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 134.27841
                            3.18601
                                       42.15
                                               <2e-16 ***
                 0.47713
                            0.00905
                                       52.72
                                               <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 39.26 on 607 degrees of freedom
## Multiple R-squared: 0.8208, Adjusted R-squared: 0.8205
## F-statistic: 2780 on 1 and 607 DF, p-value: < 2.2e-16
The intercept for this linear trend:
Bbeta0 = as.numeric(BiomassTrend$coefficients[1]) |>
 print()
## [1] 134.2784
The slope for this linear trend:
Bbeta1 = as.numeric(BiomassTrend$coefficients[2]) |>
 print()
## [1] 0.477135
Here, my p-value is less than 0.05 so my coefficient is significant and there is a trend.
#Trend for Renewable Energy Production:
DataRenewable <- DataPandC$ Total Renewable Energy Production
DataRenewable <- ts(DataRenewable) # should this be a time series?
RenewableTrend = lm(DataRenewable ~ t)
summary(RenewableTrend)
##
## Call:
## lm(formula = DataRenewable ~ t)
##
## Residuals:
       Min
                1Q Median
                                3Q
                                        Max
## -148.27 -35.63
                    11.58
                             41.51 144.27
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
```

```
4.90151
## (Intercept) 180.98940
                                      36.92
                                              <2e-16 ***
## t
                 0.70404
                            0.01392 50.57 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 60.41 on 607 degrees of freedom
## Multiple R-squared: 0.8081, Adjusted R-squared: 0.8078
## F-statistic: 2557 on 1 and 607 DF, p-value: < 2.2e-16
The incertcept for this linear trend:
Rbeta0 = as.numeric(RenewableTrend$coefficients[1]) |>
 print()
## [1] 180.9894
The slope for this linear trend:
Rbeta1 = as.numeric(RenewableTrend$coefficients[2]) |>
 print()
## [1] 0.7040391
Here, my p-value is less than 0.05 so my coefficient is significant and there is a trend.
#Trend for Hyrdoelectric Energy Consumption:
DataHydro <- DataPandC$`Hydroelectric Power Consumption`
DataHydro <- ts(DataHydro)</pre>
HydroTrend = lm(DataHydro ~ t)
summary(HydroTrend)
##
## Call:
## lm(formula = DataHydro ~ t)
##
## Residuals:
                1Q Median
                               3Q
                                       Max
## -29.818 -10.620 -0.669
                             9.357 39.528
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 82.734747
                           1.140265 72.557 < 2e-16 ***
              -0.009849
                           0.003239 -3.041 0.00246 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.05 on 607 degrees of freedom
## Multiple R-squared: 0.015, Adjusted R-squared: 0.01338
## F-statistic: 9.247 on 1 and 607 DF, p-value: 0.002461
The incertcept for this linear trend:
Hbeta0 = as.numeric(HydroTrend$coefficients[1]) |>
 print()
```

```
## [1] 82.73475
```

The slope for this linear trend:

```
Hbeta1 = as.numeric(HydroTrend$coefficients[2]) |>
print()
```

```
## [1] -0.009849298
```

Here, my p-value is less than 0.05 so my coefficient is significant and there is a trend.

For all three time series, there apears to be at least some trend so I can fit a somewhat-meaningful model.

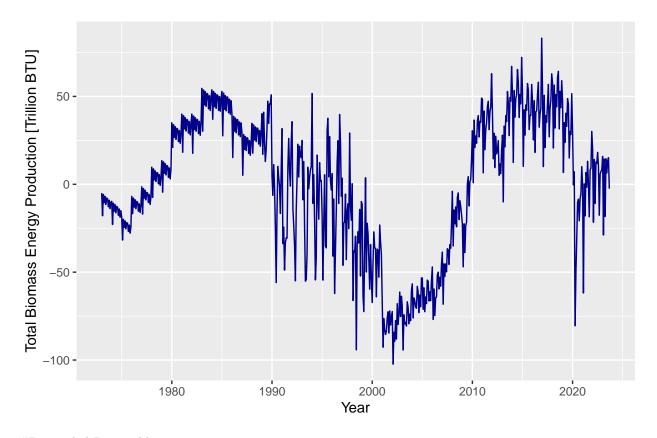
$\mathbf{Q4}$

Use the regression coefficients from Q3 to detrend the series. Plot the detrended series and compare with the plots from Q1. What happened? Did anything change?

Detrended series = the series - (beta0 + beta1*t)

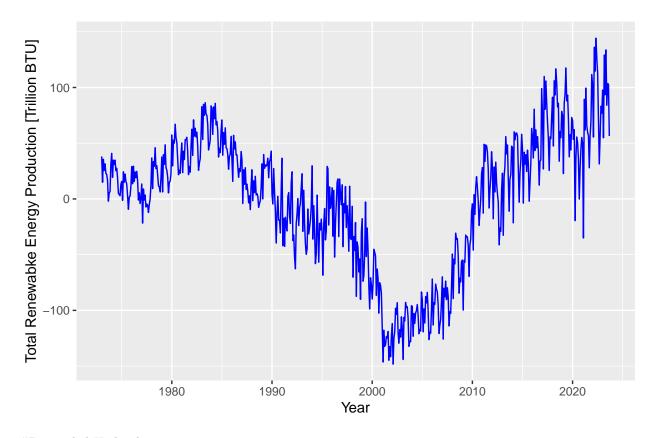
 $\# Detrended \ Biomass$

```
DataBiomassTrend <- Bbeta0 + (Bbeta1 * t)</pre>
TSDataBiomassTrend <-
  ts(DataBiomassTrend,
     start = c(year1, month1),
     frequency = 12)
DataBiomass <- as.numeric(DataBiomass)</pre>
DataBiomassTrend <- as.numeric(DataBiomassTrend)</pre>
DetrendedBiomass <- DataBiomass - DataBiomassTrend
DetrendedBiomass <-
  ts(DetrendedBiomass,
     start = c(year1, month1),
     frequency = 12)
DetrendedBiomass |>
  autoplot(color = "darkblue") +
  ylab("Total Biomass Energy Production [Trillion BTU]") +
 xlab("Year")
```



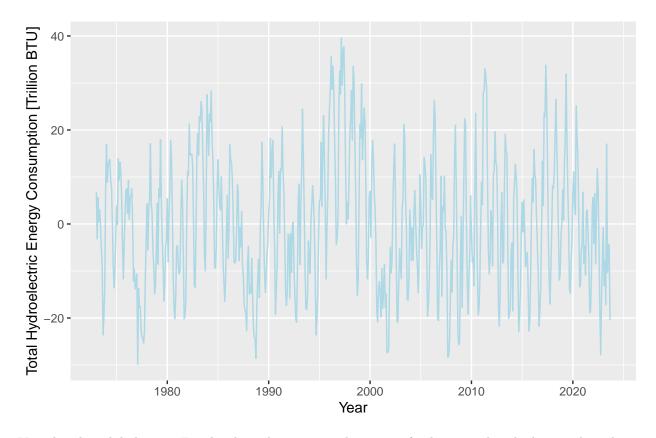
```
#Detrended Renewable
```

```
DataRenewableTrend <- Rbeta0 + (Rbeta1 * t)</pre>
TSDataRenewableTrend <-
  ts(DataRenewableTrend,
     start = c(year1, month1),
     frequency = 12)
DataRenewable <- as.numeric(DataRenewable)</pre>
DataRenewableTrend <- as.numeric(DataRenewableTrend)</pre>
DetrendedRenewable <- DataRenewable - DataRenewableTrend
DetrendedRenewable <-
  ts(DetrendedRenewable,
     start = c(year1, month1),
     frequency = 12)
DetrendedRenewable |>
  autoplot(color = "blue") +
  ylab("Total Renewabke Energy Production [Trillion BTU]") +
  xlab("Year")
```



```
#Detrended Hydroelectric
```

```
DataHydroTrend <- Hbeta0 + (Hbeta1 * t)</pre>
TSDataHydroTrend <-
  ts(DataHydroTrend,
     start = c(year1, month1),
     frequency = 12)
DataHydro <- as.numeric(DataHydro)</pre>
DataHydroTrend <- as.numeric(DataHydroTrend)</pre>
DetrendedHydro <- DataHydro - DataHydroTrend
DetrendedHydro <-
  ts(DetrendedHydro,
     start = c(year1, month1),
     frequency = 12)
DetrendedHydro |>
  autoplot(color = "lightblue") +
  ylab("Total Hydroelectric Energy Consumption [Trillion BTU]") +
  xlab("Year")
```



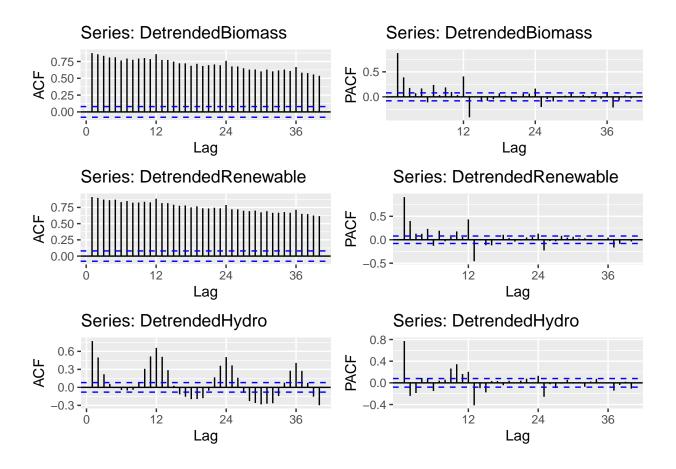
Yes, the plots did change. For the three data points, the range of values is reduced; that is, the ovbious increase in energy BTU as x (time) approaches September of 2023 is largely eliminated and the data more closely oscilates around a horizontal line.

$\mathbf{Q5}$

Plot ACF and PACF for the detrended series and compare with the plots from Q1. You may use plot_grid() again to get them side by side. nut mot mandatory. Did the plots change? How?

```
DBACF <-
  Acf(
    DetrendedBiomass,
    lag.max = 40,
    main = paste("Total Biomass Energy \n Production"),
    plot = FALSE
  )
DBPCF <-
  Pacf(
    DetrendedBiomass,
    lag.max = 40,
    main = paste("Total Biomass Energy \n Production"),
    plot = FALSE
  )
DRACF <-
  Acf(
```

```
DetrendedRenewable,
    lag.max = 40,
    main = paste("Total Biomass Energy \n Production"),
    plot = FALSE
  )
DRPCF <-
 Pacf(
   DetrendedRenewable,
   lag.max = 40,
   main = paste("Total Biomass Energy \n Production"),
   plot = FALSE
  )
DHACF <-
 Acf(
    DetrendedHydro,
    lag.max = 40,
   main = paste("Total Biomass Energy \n Production"),
   plot = FALSE
  )
DHPCF <-
 Pacf(
   DetrendedHydro,
   lag.max = 40,
   main = paste("Total Biomass Energy \n Production"),
    plot = FALSE
  )
DB1 <- autoplot(DBACF)</pre>
DB2 <- autoplot(DBPCF)</pre>
DR1 <- autoplot(DRACF)</pre>
DR2 <- autoplot(DRPCF)</pre>
DH1 <- autoplot(DHACF)</pre>
DH2 <- autoplot(DHPCF)</pre>
plot_grid(DB1, DB2, DR1, DR2, DH1, DH2, nrow = 3)
```



Seasonal Component

Set aside the detrended series and consider the original series again from Q1 to answer Q6 to Q8.

Q6

Just by looking at the time series and the acf plots, do the series seem to have a seasonal trend? No need to run any code to answer your question. Just type in you answer below.

Yes! They all have some kind of repetative oscilations which leads me to believe that there is a seasonal component to my data set. These oscilations seem to occur at regular intervals which leads me to believe they occur at some sort of temporal pattern.

Q7

Use function lm() to fit a seasonal means model (i.e. using the seasonal dummies) the two time series. Ask R to print the summary of the regression. Interpret the regression output. From the results which series have a seasonal trend? Do the results match you answer to Q6?

```
Biomass <- DataPandCTS[, 1]
Renewable <- DataPandCTS[, 2]
Hydro <- DataPandCTS[, 3]

BiomassDummies <- seasonaldummy(Biomass)
RenewableDummies <- seasonaldummy(Renewable)
HydroDummies <- seasonaldummy(Hydro)
```

```
SeasonalBiomass = lm(Biomass ~ BiomassDummies)
  summary(SeasonalBiomass)
##
## Call:
## lm(formula = Biomass ~ BiomassDummies)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -163.19 -55.46 -26.30
                             98.54 178.89
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                     290.4666
                                 13.1583
                                         22.075
                                                   <2e-16 ***
## BiomassDummiesJan -1.6748
                                 18.5171
                                         -0.090
                                                   0.9280
## BiomassDummiesFeb -31.2863
                                 18.5171
                                         -1.690
                                                   0.0916
## BiomassDummiesMar -8.8523
                                 18.5171
                                         -0.478
                                                   0.6328
## BiomassDummiesApr -21.6024
                                 18.5171
                                         -1.167
                                                   0.2438
## BiomassDummiesMay -13.9313
                                 18.5171 -0.752
                                                   0.4521
## BiomassDummiesJun -19.3220
                                 18.5171 -1.043
                                                   0.2972
## BiomassDummiesJul -3.5675
                                 18.5171
                                         -0.193
                                                   0.8473
## BiomassDummiesAug -0.4953
                                 18.5171
                                         -0.027
                                                   0.9787
## BiomassDummiesSep -13.1780
                                 18.5171
                                         -0.712
                                                   0.4770
## BiomassDummiesOct -4.0129
                                 18.6086
                                         -0.216
                                                   0.8293
## BiomassDummiesNov -9.6626
                                 18.6086
                                         -0.519
                                                   0.6038
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 93.04 on 597 degrees of freedom
## Multiple R-squared: 0.01007,
                                    Adjusted R-squared: -0.008173
## F-statistic: 0.5519 on 11 and 597 DF, p-value: 0.8676
SeasonalRenewable = lm(Renewable ~ RenewableDummies)
    summary(SeasonalRenewable)
##
## Call:
## lm(formula = Renewable ~ RenewableDummies)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
## -199.19 -86.35 -48.84 113.18 331.58
##
## Coefficients:
##
                       Estimate Std. Error t value Pr(>|t|)
                                    19.574 20.666
## (Intercept)
                        404.526
                                                     <2e-16 ***
## RenewableDummiesJan
                          2.962
                                    27.546
                                             0.108
                                                      0.914
## RenewableDummiesFeb
                       -34.476
                                    27.546 -1.252
                                                      0.211
## RenewableDummiesMar
                          3.929
                                    27.546
                                            0.143
                                                      0.887
## RenewableDummiesApr
                         -8.695
                                    27.546
                                           -0.316
                                                      0.752
## RenewableDummiesMay
                          6.645
                                    27.546
                                             0.241
                                                      0.809
## RenewableDummiesJun
                                    27.546
                         -4.198
                                           -0.152
                                                      0.879
## RenewableDummiesJul
                         2.460
                                    27.546
                                            0.089
                                                      0.929
```

0.855

27.546 -0.182

RenewableDummiesAug

-5.026

```
## RenewableDummiesSep
                        -29.119
                                    27.546
                                            -1.057
                                                      0.291
## RenewableDummiesOct
                        -20.068
                                    27.682
                                            -0.725
                                                      0.469
## RenewableDummiesNov
                        -20.346
                                    27.682
                                            -0.735
                                                      0.463
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 138.4 on 597 degrees of freedom
## Multiple R-squared: 0.009296,
                                    Adjusted R-squared:
## F-statistic: 0.5093 on 11 and 597 DF, p-value: 0.8976
SeasonalHydro = lm(Hydro ~ HydroDummies)
    summary(SeasonalHydro)
##
## Call:
##
  lm(formula = Hydro ~ HydroDummies)
##
  Residuals:
##
       Min
                1Q
                   Median
                                3Q
                                       Max
           -5.849
                    -0.468
                             6.243
                                    32.290
##
   -31.323
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                     80.282
                                        54.601
## (Intercept)
                                 1.470
                                               < 2e-16 ***
## HydroDummiesJan
                      4.807
                                 2.069
                                         2.323
                                                0.02050
## HydroDummiesFeb
                     -2.725
                                 2.069
                                        -1.317
                                                0.18831
## HydroDummiesMar
                      6.825
                                 2.069
                                         3.298 0.00103 **
## HydroDummiesApr
                      5.319
                                 2.069
                                         2.571
                                                0.01039 *
## HydroDummiesMay
                                 2.069
                     13.922
                                         6.729 4.02e-11 ***
## HydroDummiesJun
                     10.650
                                 2.069
                                         5.147 3.60e-07 ***
## HydroDummiesJul
                                         1.891
                                               0.05914
                      3.912
                                 2.069
## HydroDummiesAug
                     -5.677
                                 2.069
                                        -2.744 0.00626 **
## HydroDummiesSep
                    -16.797
                                 2.069
                                        -8.118 2.72e-15 ***
## HydroDummiesOct
                                 2.079
                                        -7.920 1.17e-14 ***
                    -16.468
## HydroDummiesNov
                    -10.885
                                 2.079
                                        -5.235 2.29e-07 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 10.4 on 597 degrees of freedom
## Multiple R-squared: 0.4697, Adjusted R-squared: 0.4599
## F-statistic: 48.07 on 11 and 597 DF, p-value: < 2.2e-16
```

The hydroelectric consumption data is the only data set where p-value < 0.05 so this is the only place where we have a significant seasonal trend. This does not really match my Q6 answer but it is very ovbious that there is a significant seasonal commponent in the hydroelectric data. It is possible that the data can be mostly explianed in the first two plots by random variablity and trend rather than seasonality. Furthermore, there are many different kinds of possible models for seasonality that may produce better results.

The intercept and coefficients for the seasonal model for the hydroelectric consumption data set are both printed below.

```
Hbeta_int = as.numeric(SeasonalHydro$coefficients[1]) |>
    print()
```

[1] 80.28176

```
Hbeta_coeff = as.numeric(SeasonalHydro$coefficients[2:12]) |>
    print()
```

```
## [1] 4.807299 -2.725270 6.825024 5.319044 13.922220 10.649985
## [7] 3.912260 -5.676917 -16.797387 -16.467980 -10.884780
```

$\mathbf{Q8}$

Use the regression coefficients from Q7 to deseason the series. Plot the deseason series and compare with the plots from part Q1. Did anything change?

Yes, the plot that is deseasoned looks far more uniform than the natural data.

```
nobs <- 609

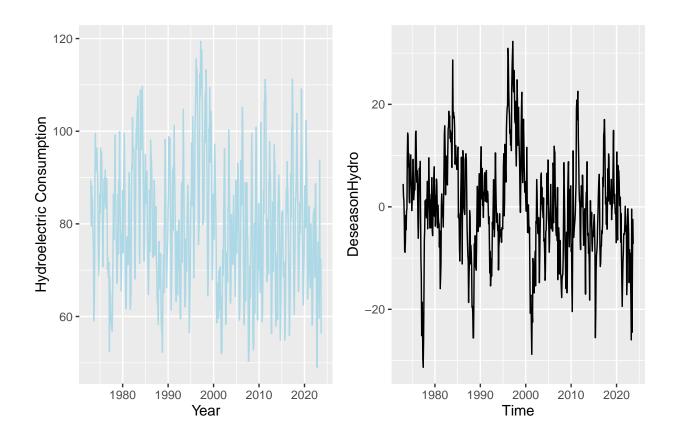
seas_component_Hydro=array(0,nobs)
for(i in 1:nobs){
    seas_component_Hydro[i]=(Hbeta_int + Hbeta_coeff%*%HydroDummies[i,])
}

ts_SeasonalHydro <- ts(seas_component_Hydro,start=c(year1,month1),frequency=12)

DeseasonHydro <- Hydro - ts_SeasonalHydro

SeasP <- autoplot(DeseasonHydro)

plot_grid(P9, SeasP)</pre>
```



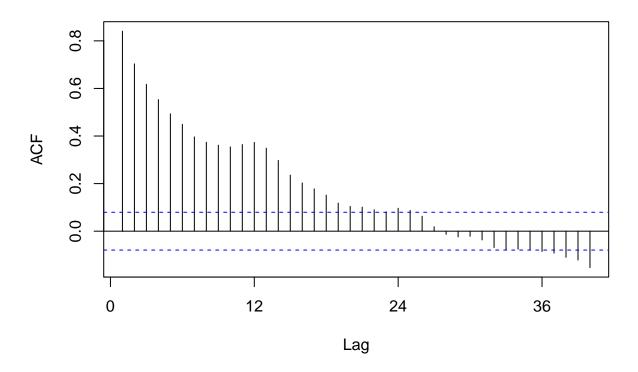
$\mathbf{Q}\mathbf{9}$

Plot ACF and PACF for the deseason series and compare with the plots from Q1. You may use plot_grid() again to get them side by side. nut mot mandatory. Did the plots change? How?

Yes the plots changes! The value of the ACF and PACF are significantly decreased and approach 0 as the lags go on which is a huge change.

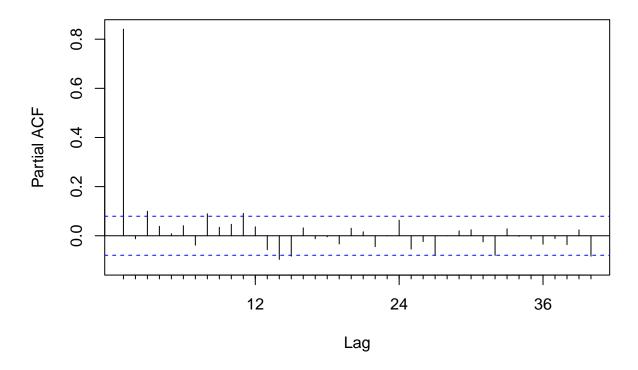
```
Acf(DeseasonHydro,
    lag.max = 40,
    main = paste("Hydro Consumption")
)
```

Hydro Consumption



```
Pacf(DeseasonHydro,
    lag.max = 40,
    main = paste("Hydro Consumption")
)
```

Hydro Consumption



ACF: there is a trend PACF: value of the coefficient