Exploring Consensus Robustness In Swarms with Disruptive Individuals

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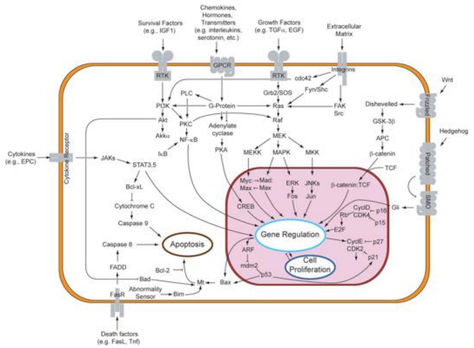
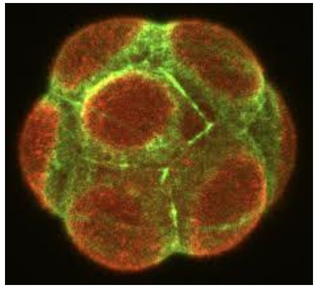
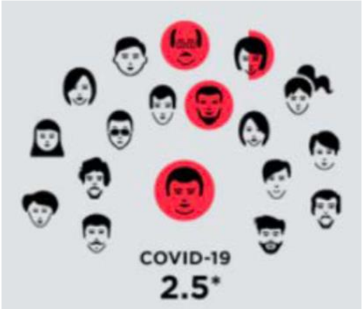
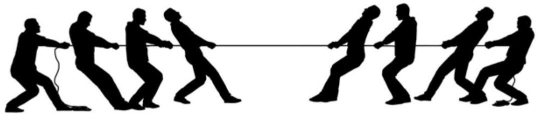
Swarms (collectives)



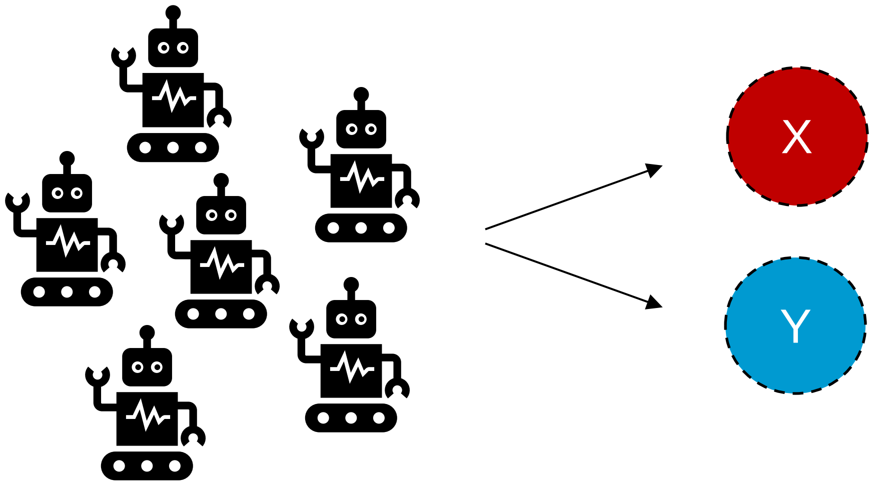


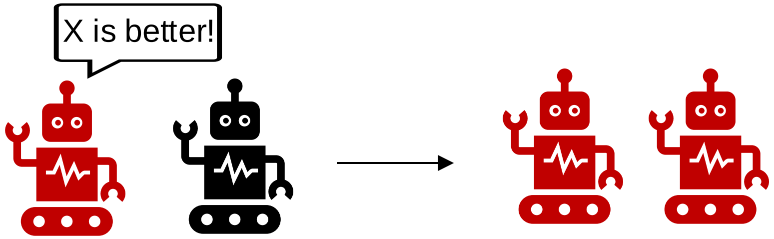


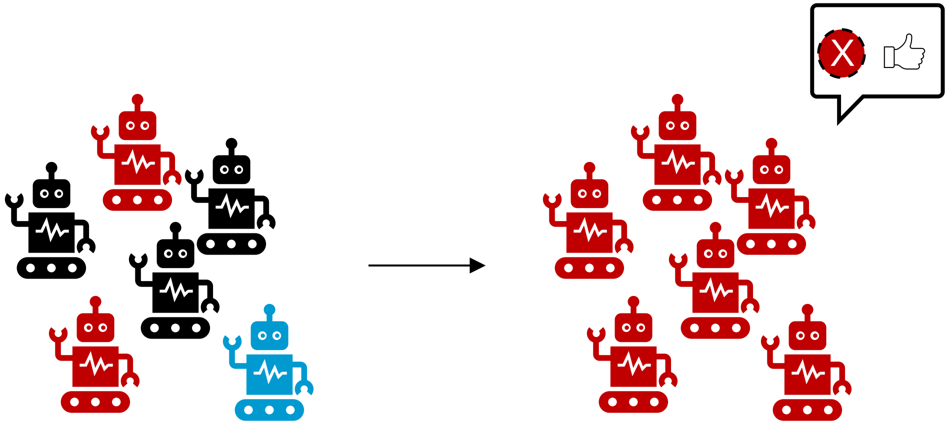


→ **How do swarms agree on decisions?**

Collective decision making

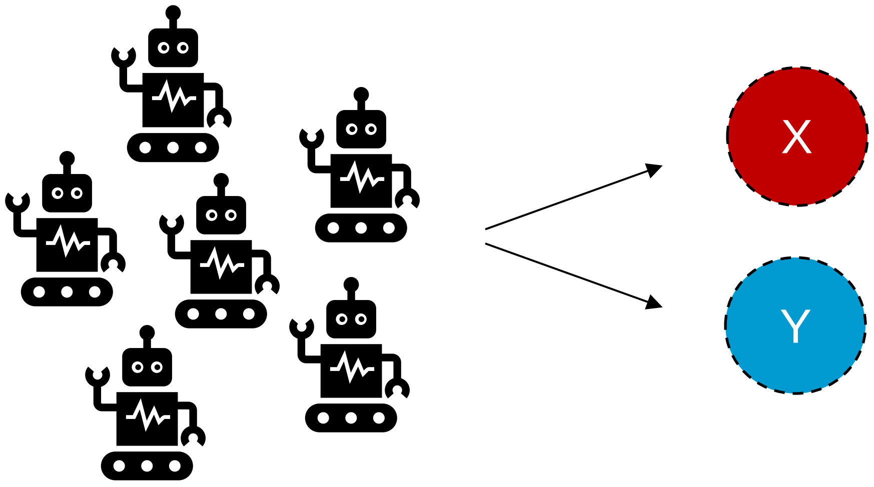


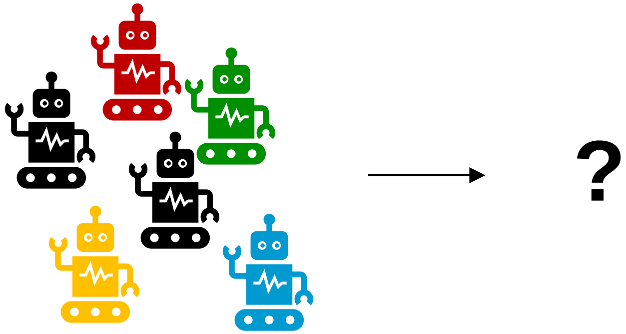


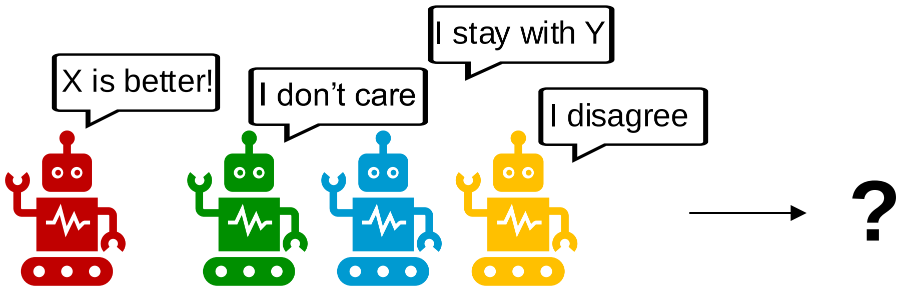


Ideally, consensus can easily be achieved with certain speed

Collective decision making



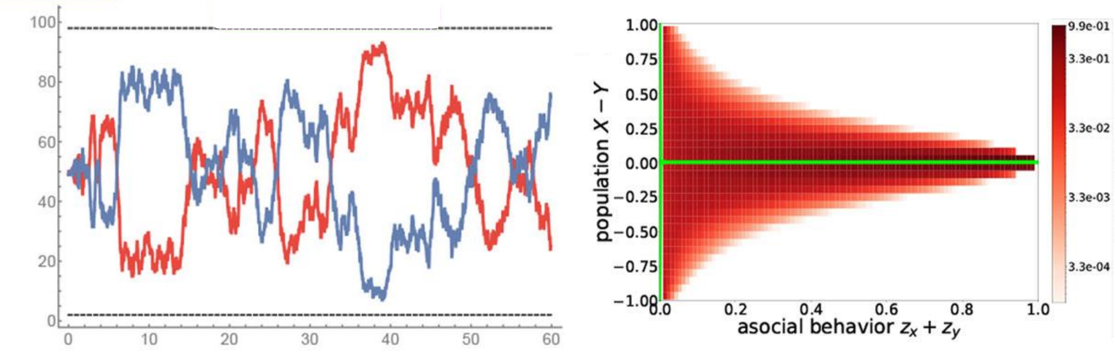




What happens in presence of disruptive (asocial) individuals?

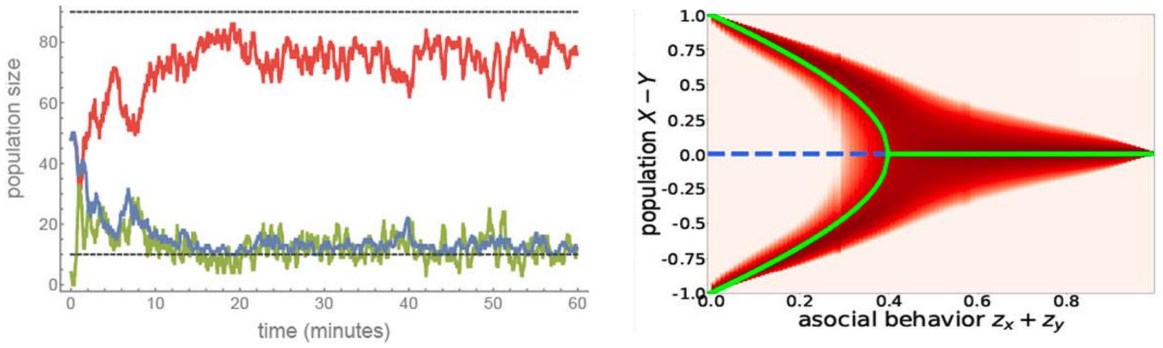
Collective decision making – disruptive individuals

Voter model with zealots



- **Permanent indecision** with already 4% zealots
- Swarm gets quickly locked into indecision state

Cross-inhibition model with zealots

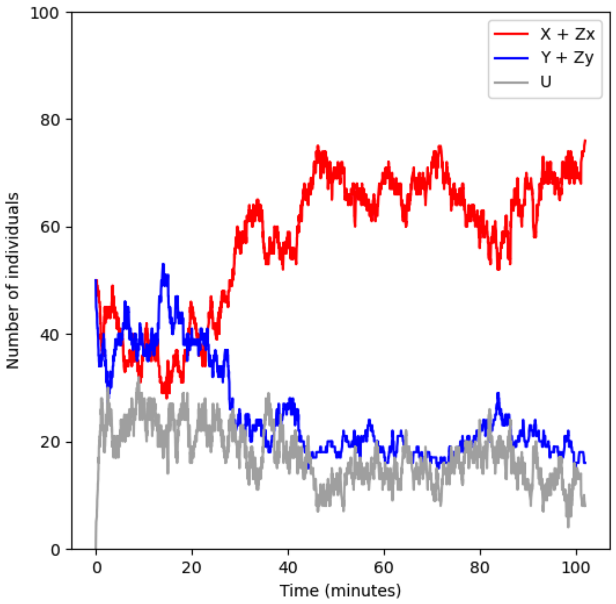


- **Stable dynamics** for 20% zealots!
- Swarm demonstrates resilience against relatively high levels of asocial behaviour

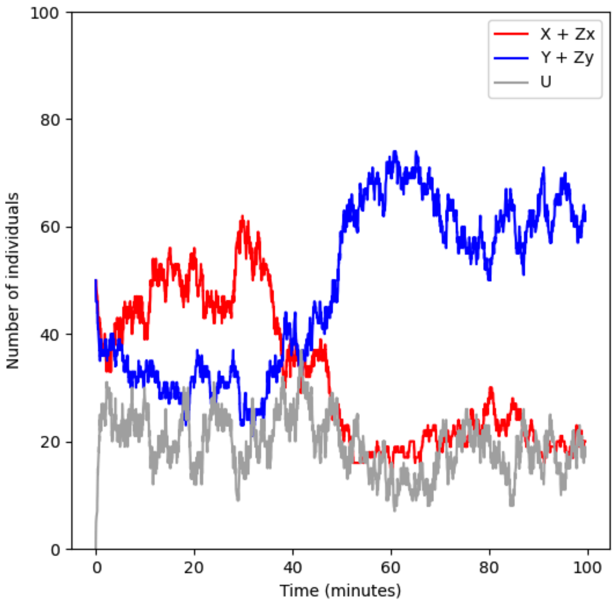
Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

Motivation – group dynamics

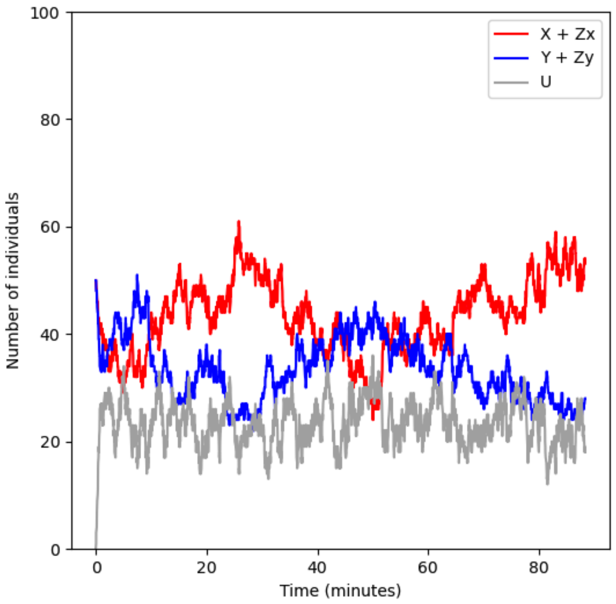
Cross-inhibition model with zealots
N = 100, initially X = Y = 35, U = 0, $\mathbf{Z_x = Z_y = 15}$
→ leads to 3 qualitatively different scenarios!



stable consensus



switching consensus



no consensus

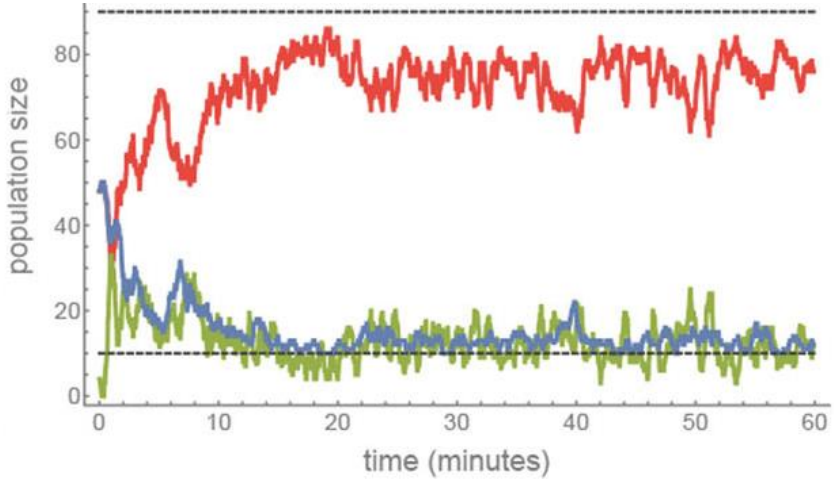
Cross-inhibition model

$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$

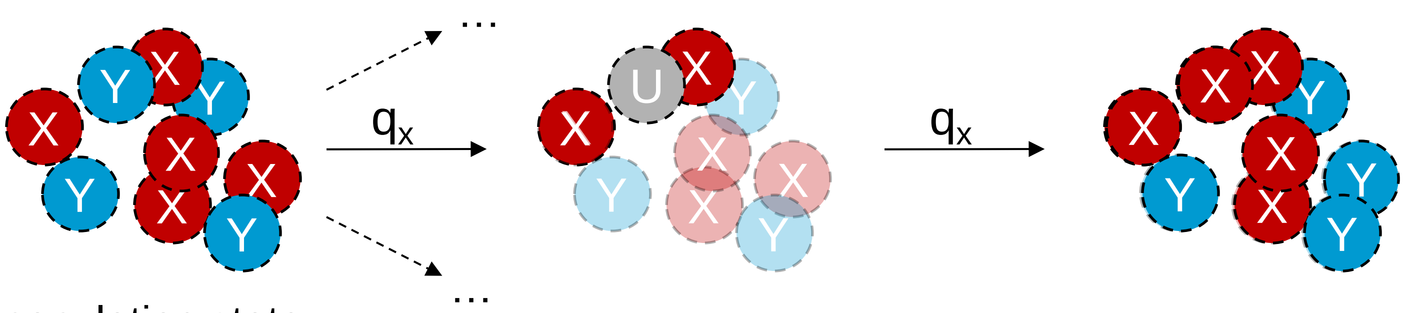
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$

$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$

$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$

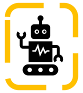


Swarm state evolves **stochastically** as a continuous-time Markov chain

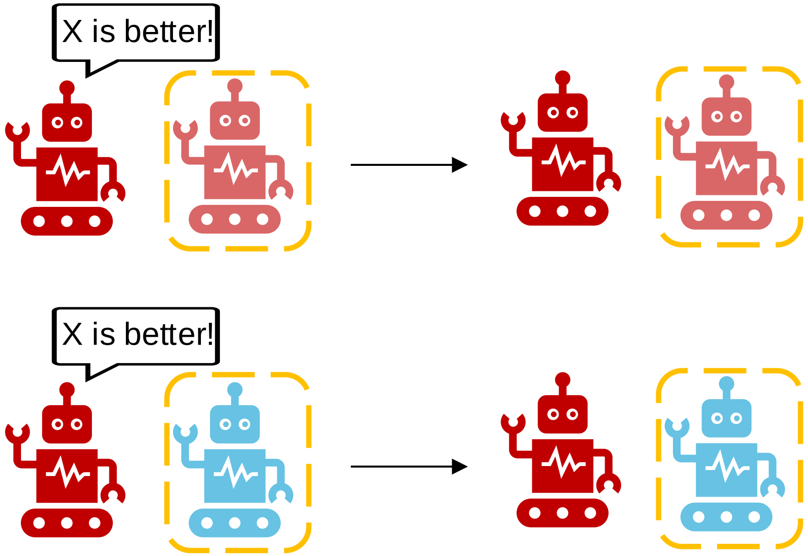


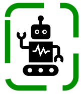
population state

Disruptive individuals

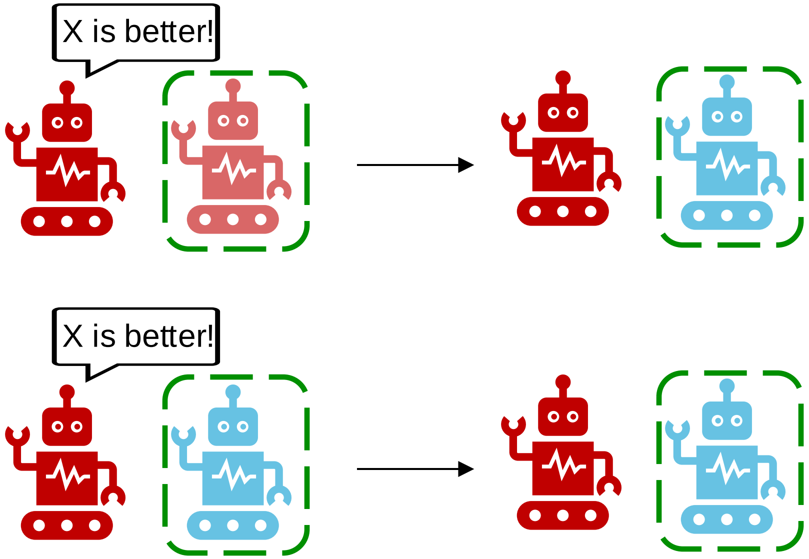
Zealots 

Stubborn individuals which never change their own opinion



Contrarians 

Individuals which counter the opinion of the individual they interact with



Cross-Inhibition model with disruptive individuals

$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$
$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$
$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$

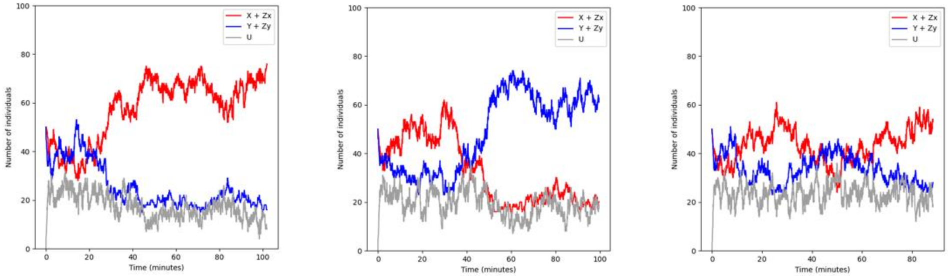
with Zealots

$$(5)\ X + ZY \overset{q_y}{\rightarrow} U + Zy$$
$$(6)\ U + ZY \overset{q_y}{\rightarrow} Y + ZY$$
$$(7)\ Y + ZX \overset{q_x}{\rightarrow} U + ZX$$
$$(8)\ U + Z_X \overset{q_x}{\rightarrow} X + ZX$$

with Contrarians

$$(5)\ X + C_Y \overset{q_y}{\rightarrow} U + C_y$$
$$(6)\ U + C_Y \overset{q_y}{\rightarrow} Y + C_Y$$
$$(7)\ X + C_X \overset{q_x}{\rightarrow} X + C_Y$$
$$(8)\ Y + C_X \overset{q_x}{\rightarrow} U + C_X$$
$$(9)\ U + CX \overset{q_x}{\rightarrow} X + CX$$
$$(10)\ Y + CY \overset{q_y}{\rightarrow} Y + CX$$
$$(11)\ CX + CX \overset{q_x}{\rightarrow} CY + CY$$
$$(12)\ CY + CY \overset{q_y}{\rightarrow} CX + CX$$

Research Questions



1. Robustness analysis

How does the **amount** of disruptive individuals affect consensus reaching/switching?

2. Combined effect

How does the **combination** of zealots and contrarians affect consensus reaching/switching?

3. Group size effect

How does the **group size** affect consensus reaching/switching?

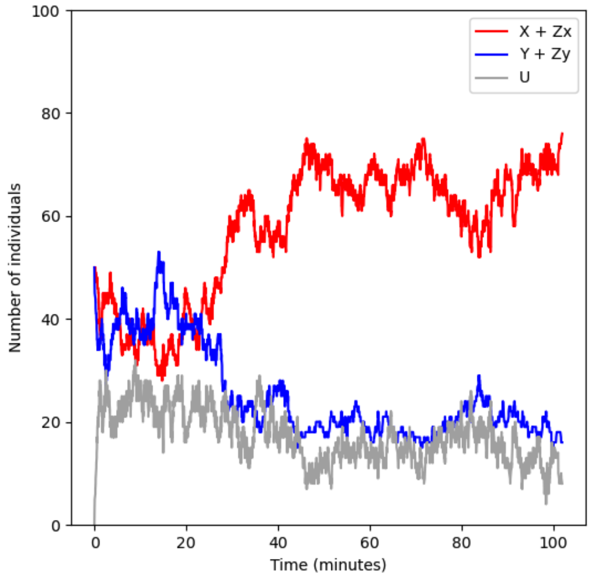
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \ \wedge \ ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee$$
$$((y + Z_y + C_y \geq min_m) \ \wedge \ ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$



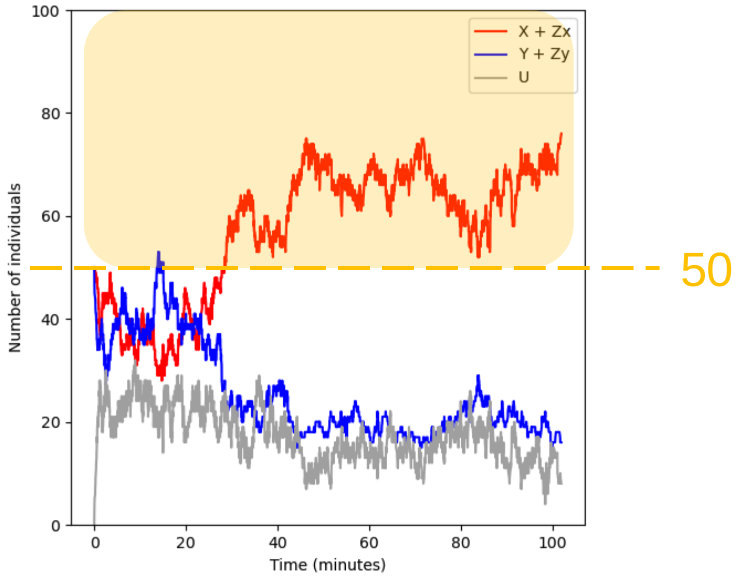
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: **majority *m***, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq \boxed{min_m}) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq \boxed{min_m}) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))))$$



m = 50 (min_m = 1/m * N)

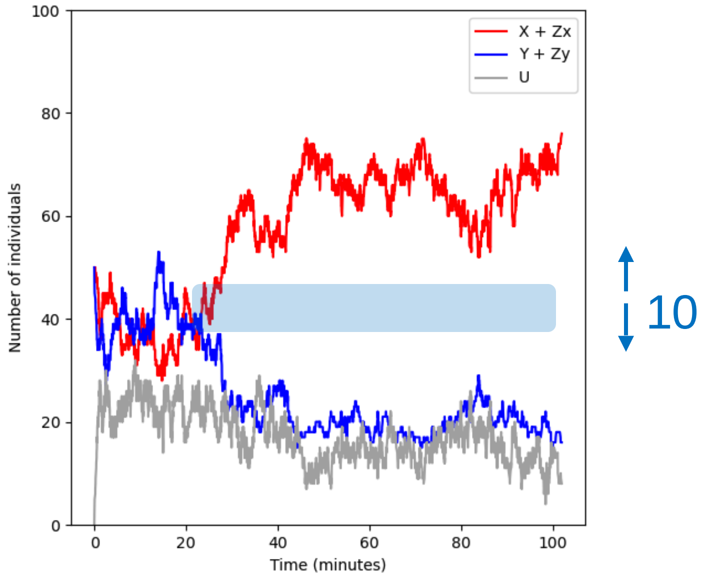
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority *m*, **distance *d***, reaching time *t*, holding time *h*, switching time *s*

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \ \wedge \ ((x + Z_x + C_x) - (y + Z_y + C_y) \boxed{\geq d}) \vee$$
$$((y + Z_y + C_y \geq min_m) \ \wedge \ ((y + Z_y + C_y) - (x + Z_x + C_x) \boxed{\geq d}))))$$



m = 50 (min_m = 1/m * N)

d = 10

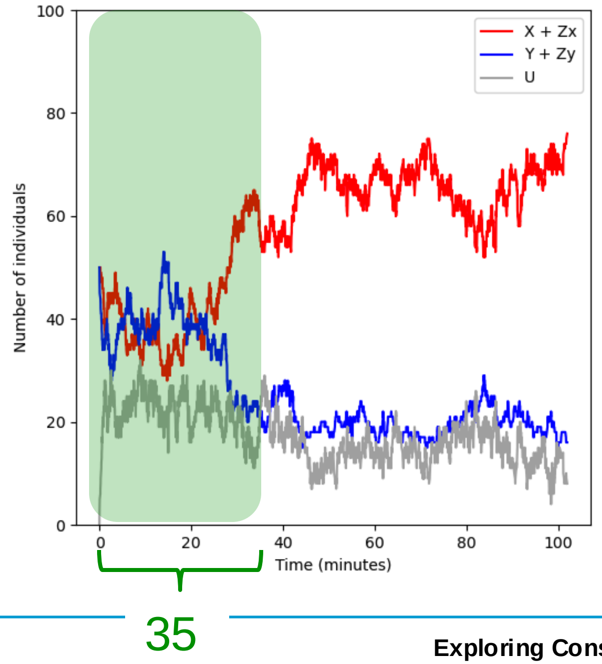
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority *m*, distance *d*, **reaching time *t***, holding time *h*, switching time *s*

$$\boxed{F_{\leq t}}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq min_m) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))))$$



m = 50 (min_m = 1/m * N)

d = 10

t = 35

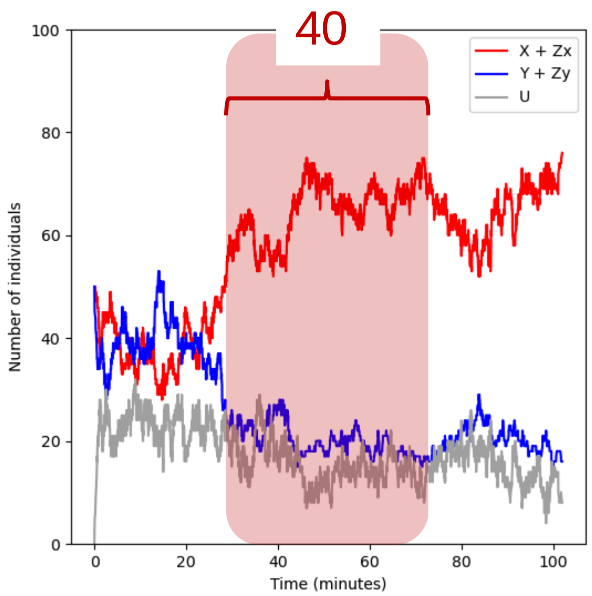
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, **holding time *h***, switching time *s*

$$F_{\leq t}(\boxed{G_{\leq h}}(((x + Z_x + C_x \geq min_m) \ \wedge \ ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \ \vee \\ ((y + Z_y + C_y \geq min_m) \ \wedge \ ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))))$$



m = 50 (min_m = 1/m * N)

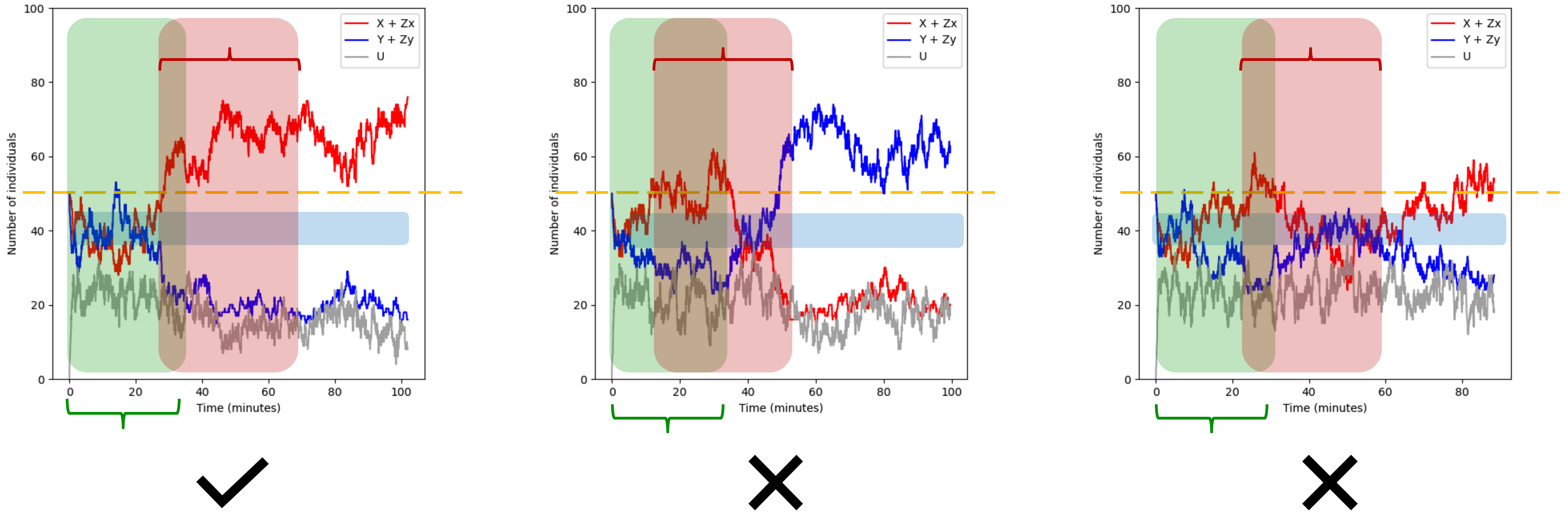
d = 10

t = 35

h = 40

Approach

Is this a stable consensus?



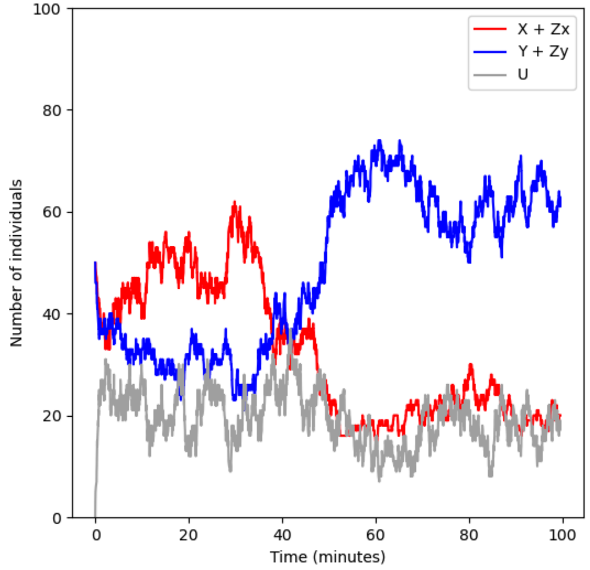
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe stable consensus and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$
\begin{aligned}
F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \ \wedge \\
(true\ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee \\
(((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \ \wedge \\
(true\ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))
\end{aligned}
$$



Approach

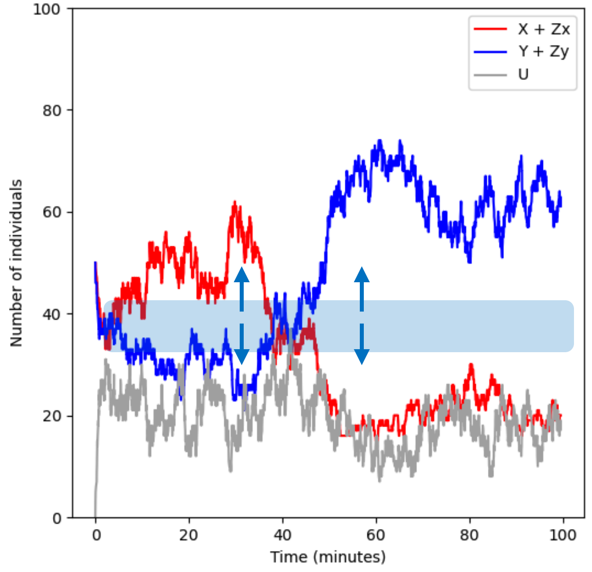
Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe stable consensus and **switching consensus** in BLTL

- Five parameters: majority *m*, ***distance d***, reaching time *t*, holding time *h*, switching time *s*

$$
\begin{aligned}
F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \boxed{\geq d}) \;\wedge \\
(true\; U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \boxed{\geq d}))) \;\vee \\
(((y + Z_y + C_y) - (x + Z_x + C_x) \boxed{\geq d}) \;\wedge \\
(true\; U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \boxed{\geq d})))
\end{aligned}
$$

d = 10



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Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

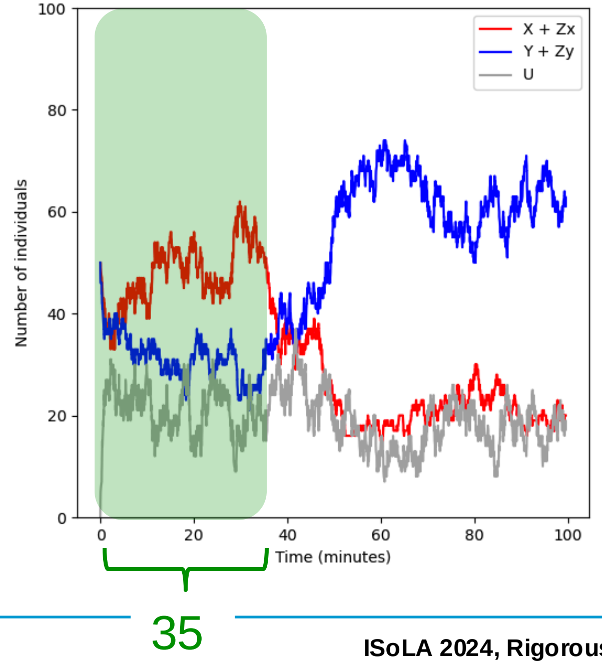
STEP 1: Formally describe stable consensus and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, **reaching time *t***, holding time *h*, switching time *s*

$$
\begin{aligned}
\boxed{F_{\le t}}((((x + Z_x + C_x) - (y + Z_y + C_y) \ge d) \;\wedge \\
(true\ U_{\le s}((y + Z_y + C_y) - (x + Z_x + C_x) \ge d))) \vee \\
(((y + Z_y + C_y) - (x + Z_x + C_x) \ge d) \;\wedge \\
(true\ U_{\le s}((x + Z_x + C_x) - (y + Z_y + C_y) \ge d))))
\end{aligned}
$$

d = 10

t = 35



Approach

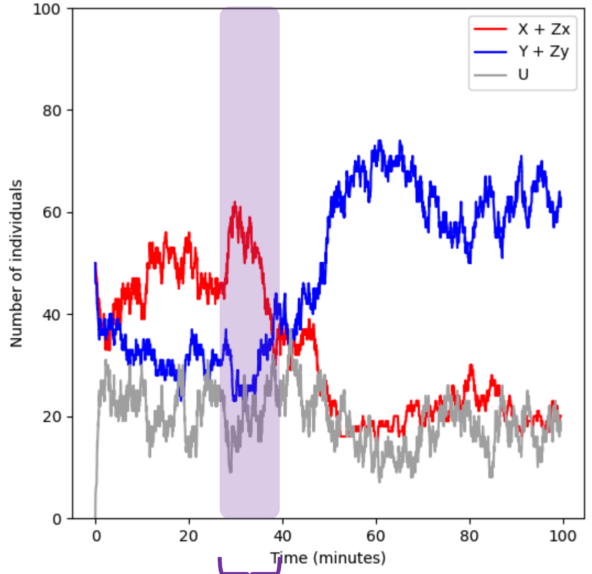
Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe stable consensus and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, **switching time *s***

$$
\begin{aligned}
&F_{\leq t}((((x+Z_x+C_x)-(y+Z_y+C_y)\geq d) \;\wedge \\
&(true\ \boxed{U_{\leq s}}((y+Z_y+C_y)-(x+Z_x+C_x)\geq d))) \;\vee \\
&(((y+Z_y+C_y)-(x+Z_x+C_x)\geq d) \;\wedge \\
&(true\ \boxed{U_{\leq s}}((x+Z_x+C_x)-(y+Z_y+C_y)\geq d))))
\end{aligned}
$$

d = 10
t = 35
s = 10



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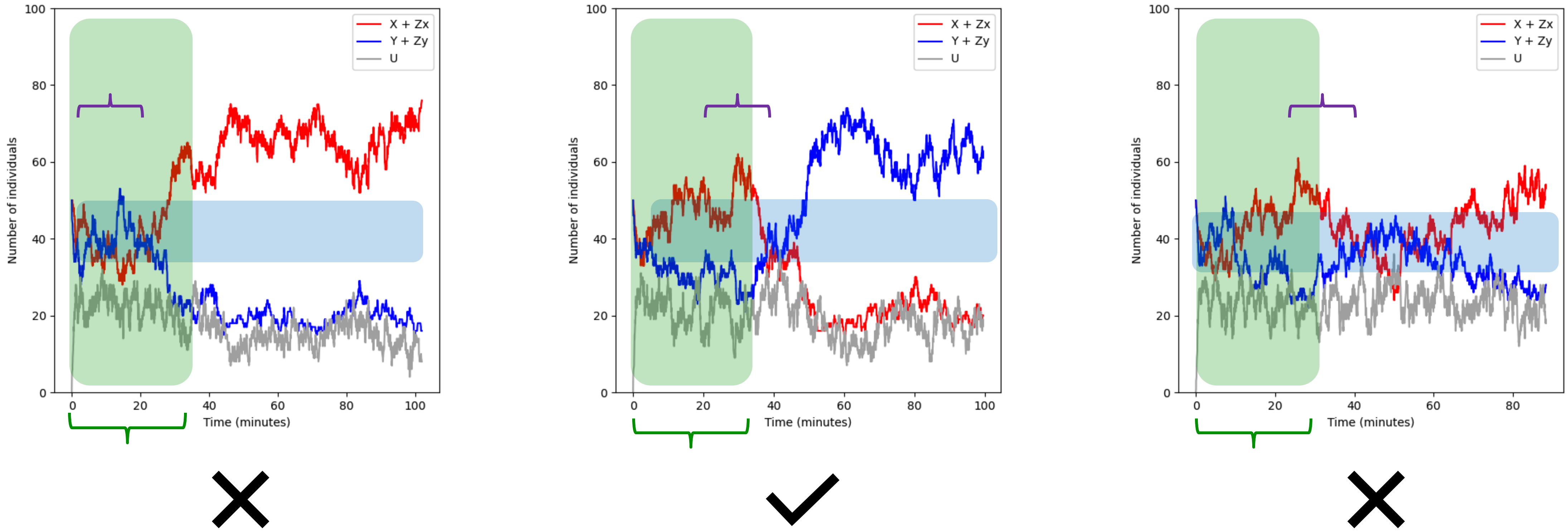
Approach

Is this a switching consensus?



Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

STEP 2: Apply model checking tools (*PRISM* and *PlasmaLab*) to explore the relevant scenarios:

  - Varying number of zealots and contrarians to explore robustness
  - Varying number of both to explore combined effect
  - Varying total group size to explore group size effect

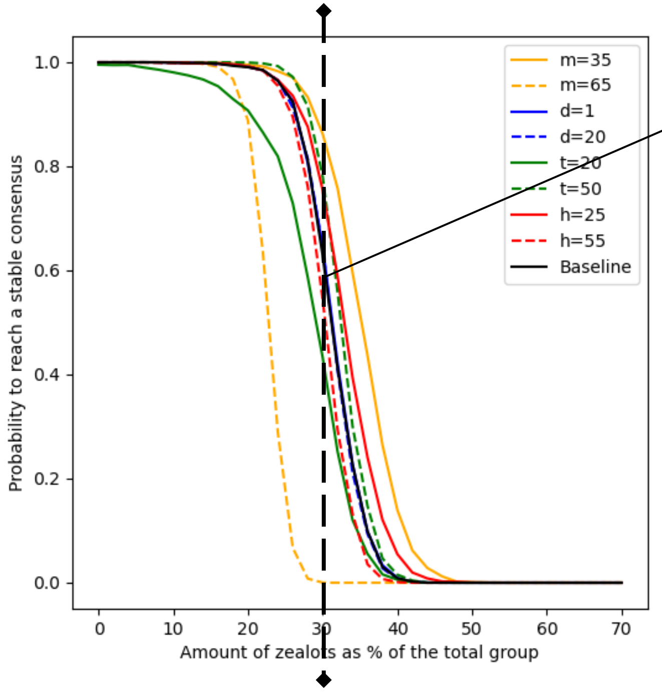
- **Monte Carlo** algorithm to estimate **satisfaction probability**
- Error margin ε=0.025, confidence bound Δ=0.01

Plasma Lab
Statistical Model-Checker
PRISM
model checker

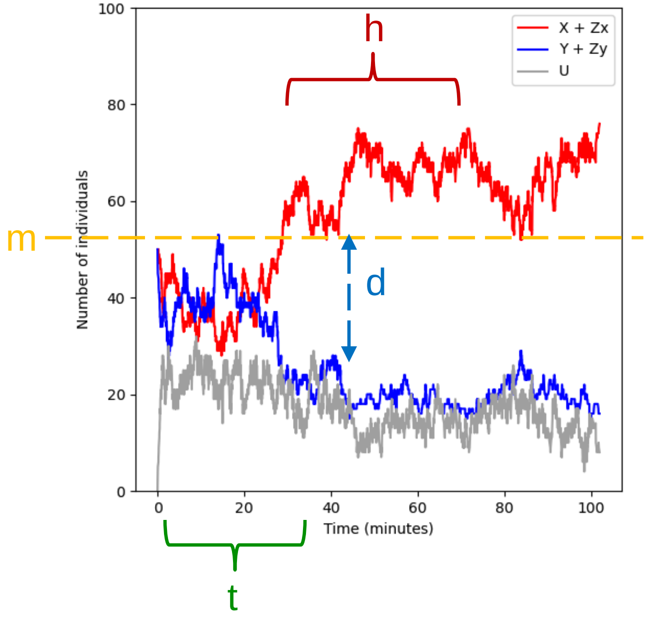
Results – robustness of stable consensus

- Scenario: N = 100 robots, equivalent options X and Y (q_x=q_y), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: m=50, d=10, t=35, h=40

Robustness – Zealots



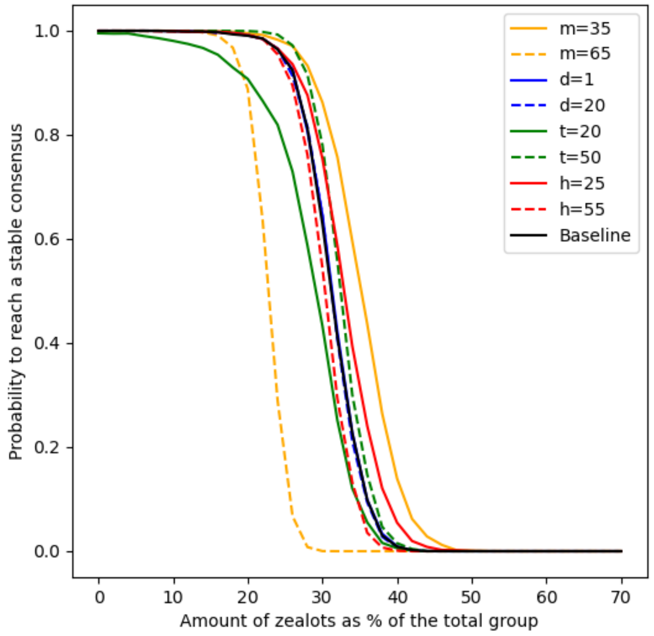
30% zealots → p = 0.629



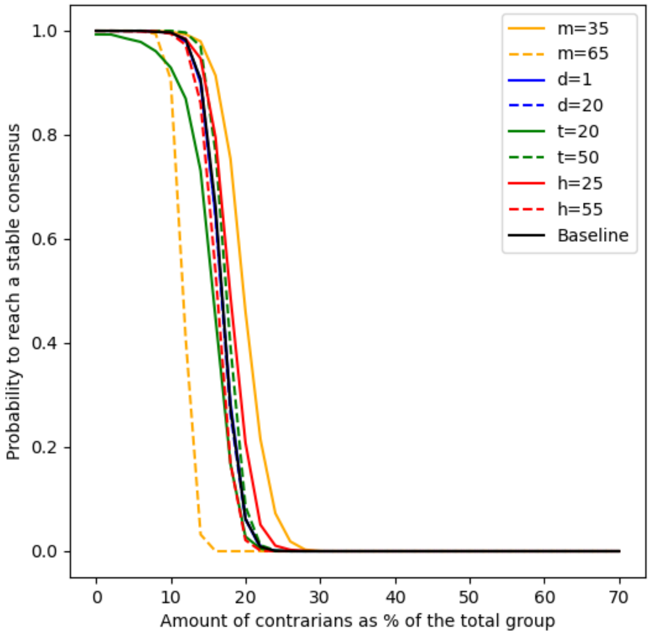
Results – robustness of stable consensus

- Scenario: N = 100 robots, equivalent options X and Y (q_x=q_y), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: m=50, d=10, t=35, h=40

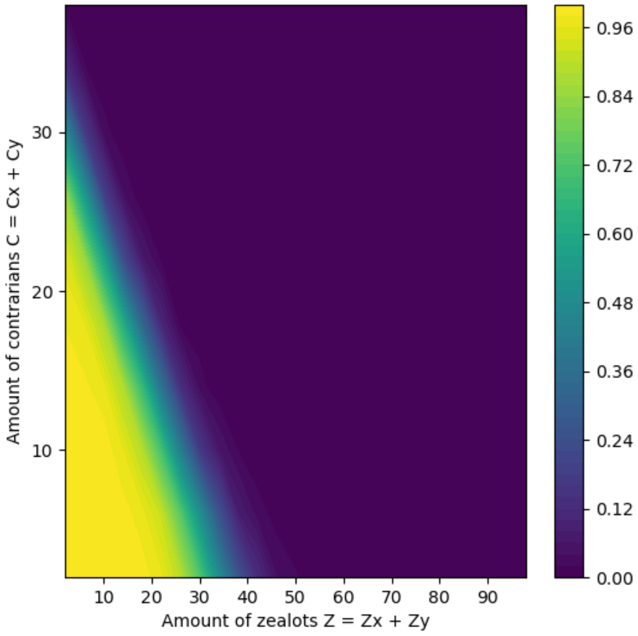
Robustness – Zealots



Robustness – Contrarians



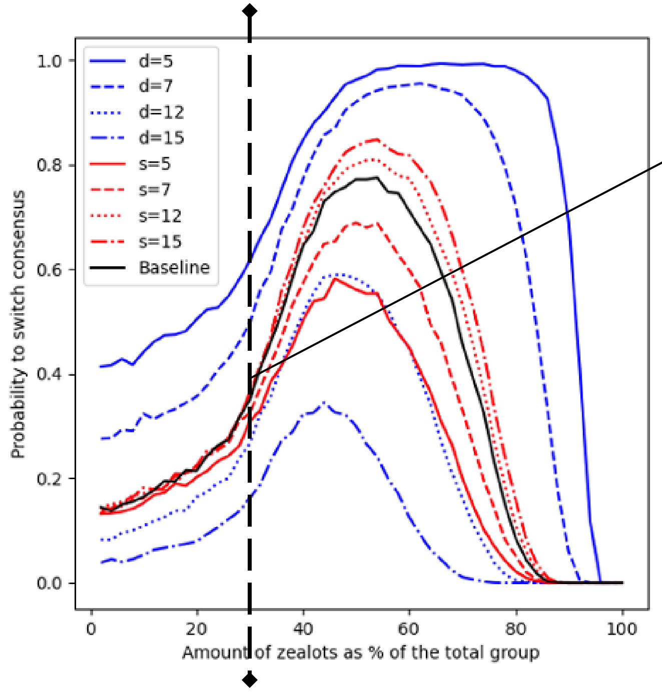
Combined effect



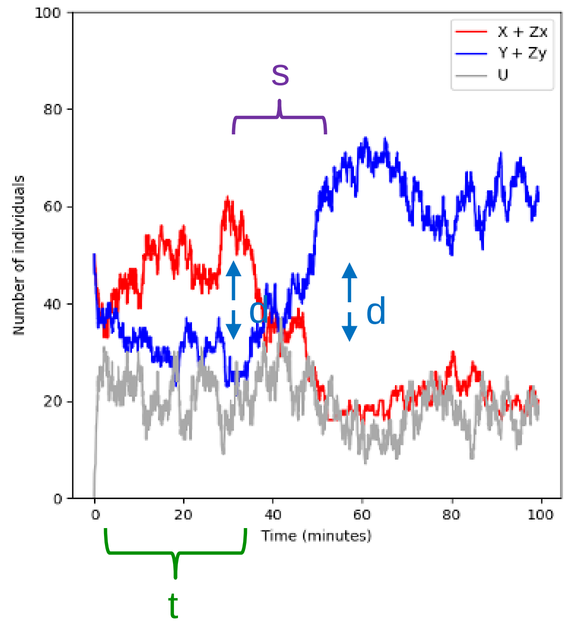
Results – robustness of switching consensus

- Scenario: N = 100 robots, equivalent options X and Y (q_x=q_y), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: d=10, t=35, s=10

Robustness – Zealots



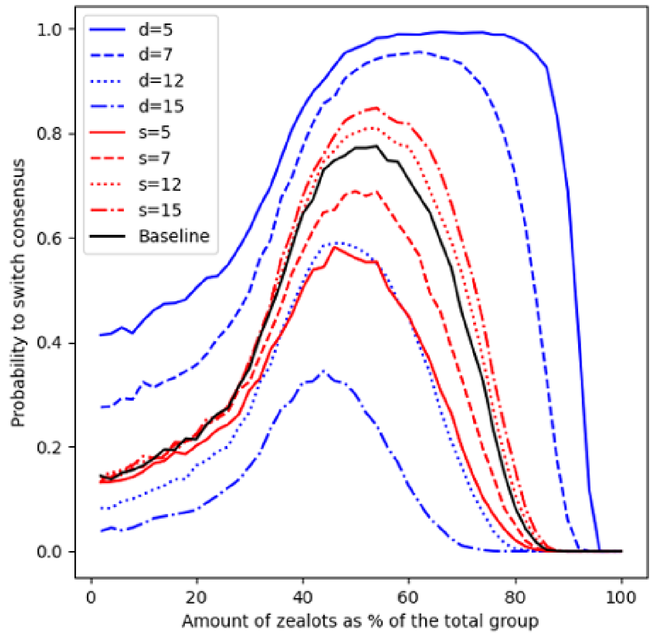
30% zealots → p = 0.349



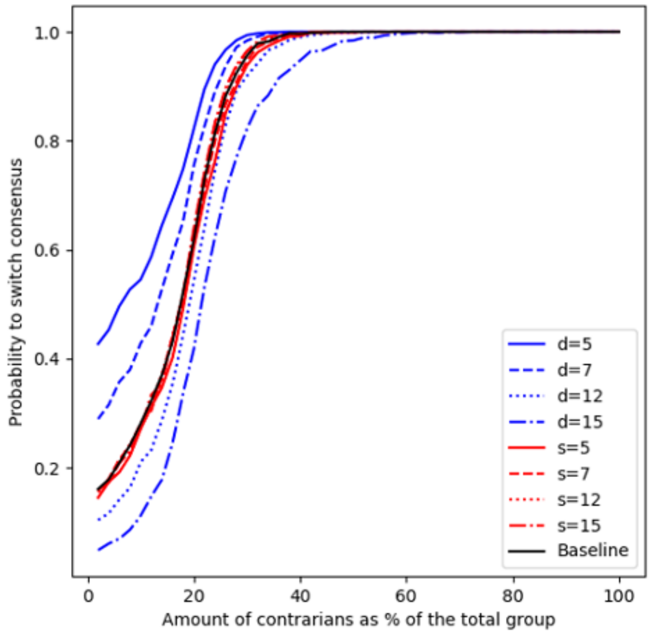
Results – robustness of switching consensus

- Scenario: N = 100 robots, equivalent options X and Y (q_x=q_y), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: d=10, t=35, s=10

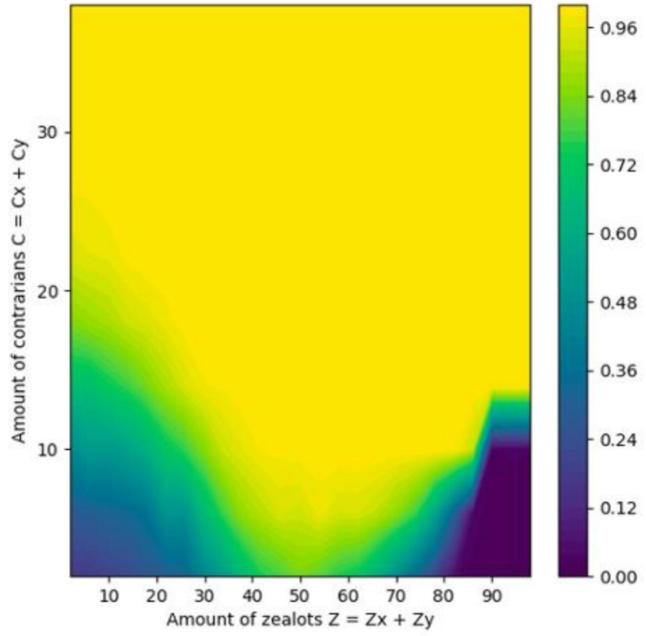
Robustness – Zealots



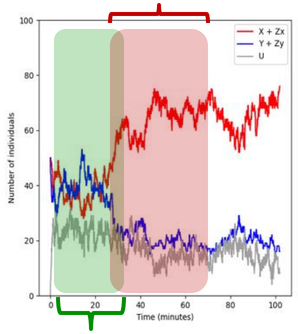
Robustness – Contrarians



Combined effect

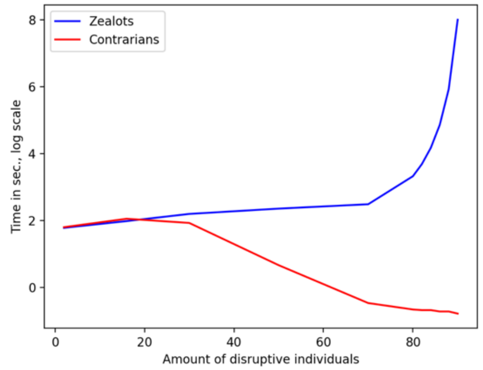


Results – expected times



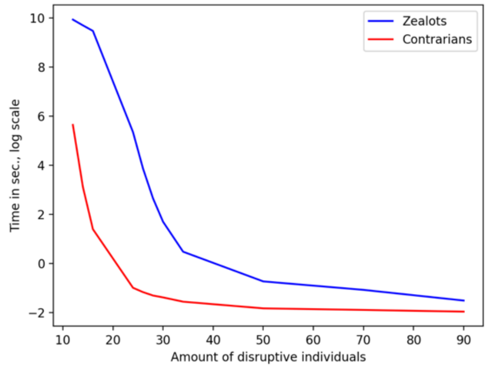
Expected times to reach consensus

#	2	16	30	50	70	80	82	84	86	88	90
Zealots	5.95	7.28	9.02	10.57	12.04	27.82	39.94	64.95	128.85	374.04	2975.68
Contrarians	6.07	7.81	6.89	1.95	0.63	0.52	0.51	0.51	0.49	0.49	0.46



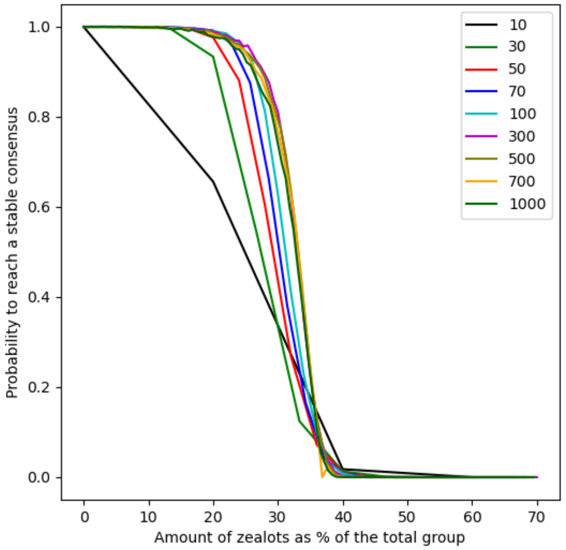
Expected times to hold consensus

#	12	14	16	24	26	28	30	34	50	70	90
Zealots	20686.51	16368.28	13047.85	210.98	47.71	14.13	5.46	1.61	0.48	0.34	0.22
Contrarians	283.57	22.53	4.03	0.37	0.31	0.27	0.25	0.21	0.16	0.15	0.14

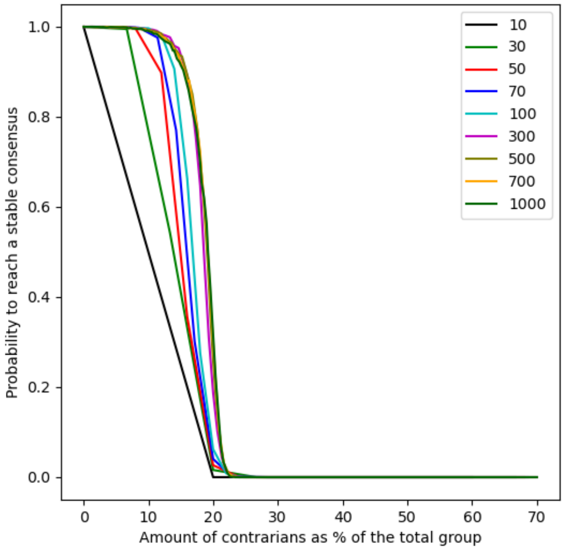


Results – group size effect

Stable consensus

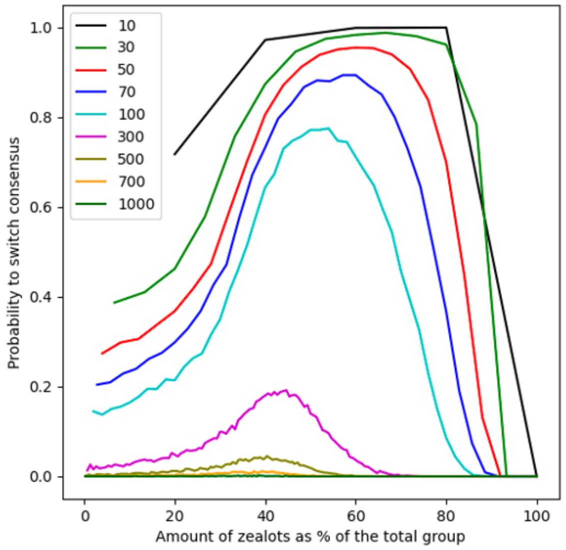


Zealots

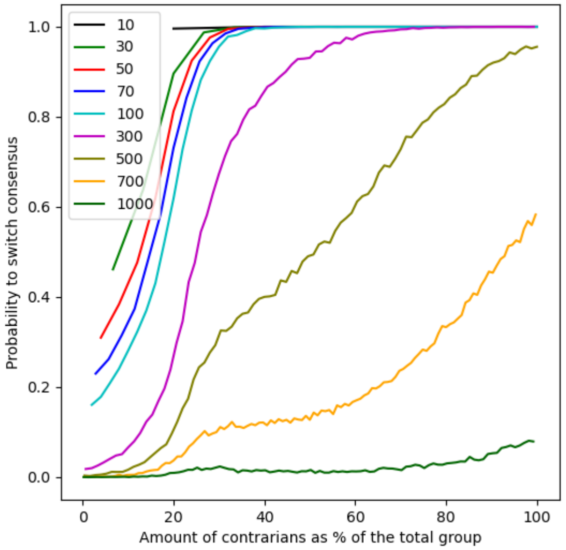


Contrarians

Switching consensus



Zealots



Contrarians

… **robust** to group size scaling!

… **sensitive** to group size scaling!

Conclusion and outlook

- A small increase of disruptive individuals can drastically affect consensus dynamics
- Our method with SMC allows to explore consensus beyond mean-field analysis or single simulation

- **Stable consensus**
  - Cross-inhibition model robust up to certain fraction of zealots/contrarians, then rapid phase transition
  - Zealots are less harmful for reaching consensus than contrarians

- **Switching consensus**
  - Range of zealots for which such trajectories occur with non-negligible probability, but very rare for high number of zealots
  - Contrarians promote switching dynamics

- Future work
  - Group size effect: characterisation of a class of stochastic systems for which consensus reaching is robust to scaling
  - Asymmetric model: what if only one decision is correct?
  - Control theory: interventions over individuals for a global outcome (e.g. vaccination policy)

Thank you very much!

Julia Klein
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University of Konstanz

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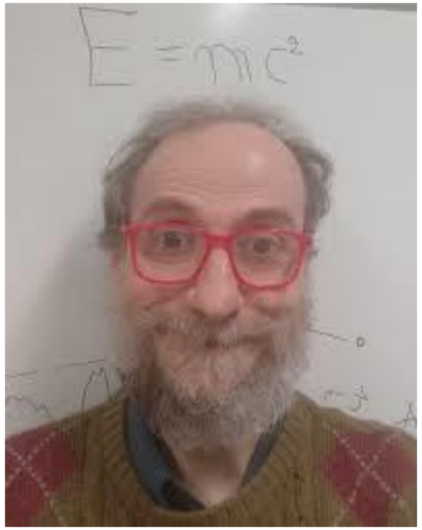
Tatjana Petrov

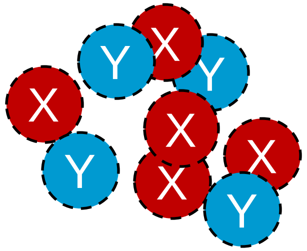


Alberto d'Onofrio

Decision making: Voter model

$$(1)\ X + Y \xrightarrow{q_x} X + X$$
$$(2)\ X + Y \xrightarrow{q_y} Y + Y$$

Swarm state evolves as a continuous-time Markov chain
→ classical **population model**



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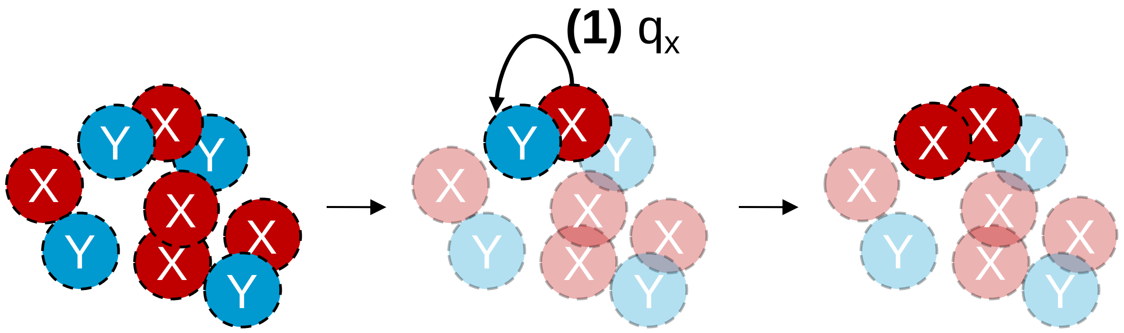

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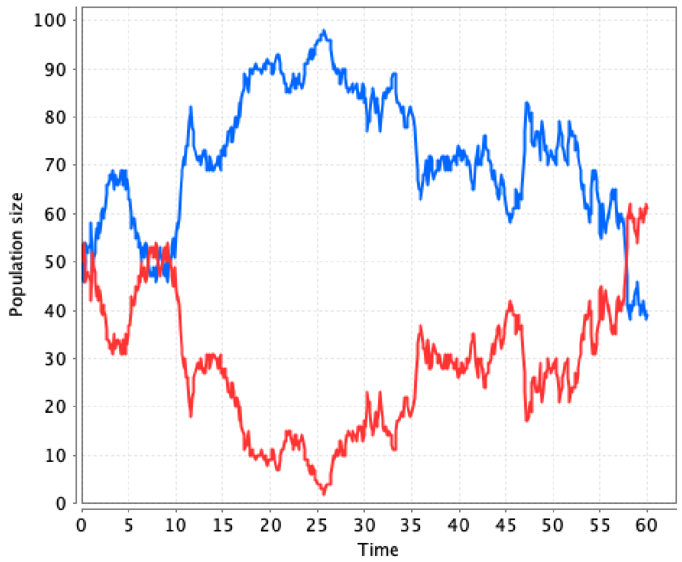

Swarm state evolves as a continuous-time Markov chain
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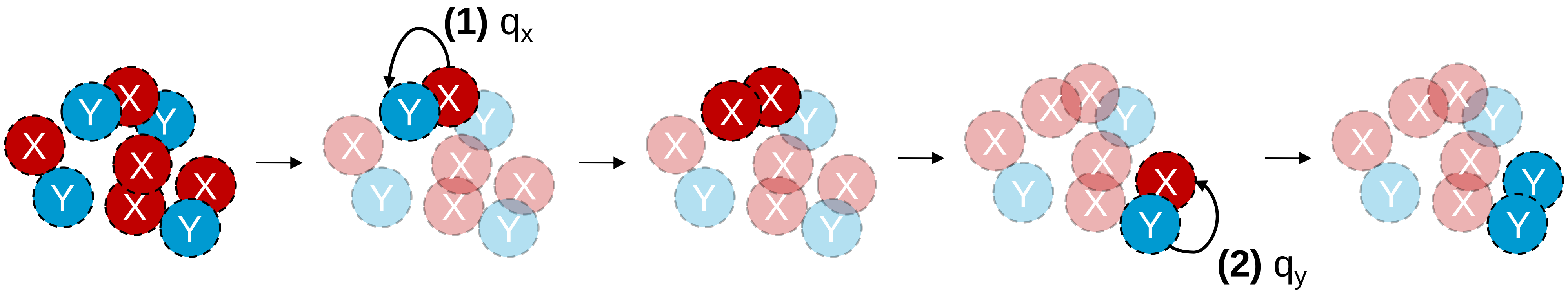
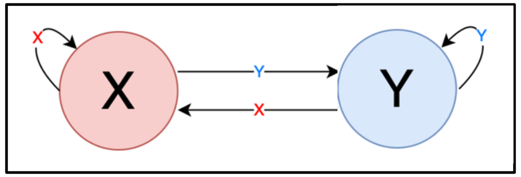


Decision making: Voter model



$$(1)\ X + Y \overset{q_x}{\rightarrow} X + X$$

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Swarm state evolves as a continuous-time Markov chain
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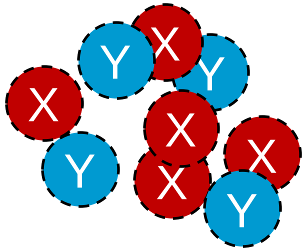
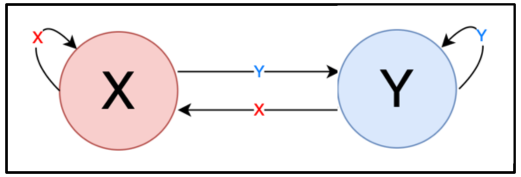


Decision making: Voter model



$$(1)\ \boldsymbol{X} + \boldsymbol{Y} \overset{\boldsymbol{q_x}}{\rightarrow} \boldsymbol{X} + \boldsymbol{X}$$

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Swarm state evolves as a continuous-time Markov chain
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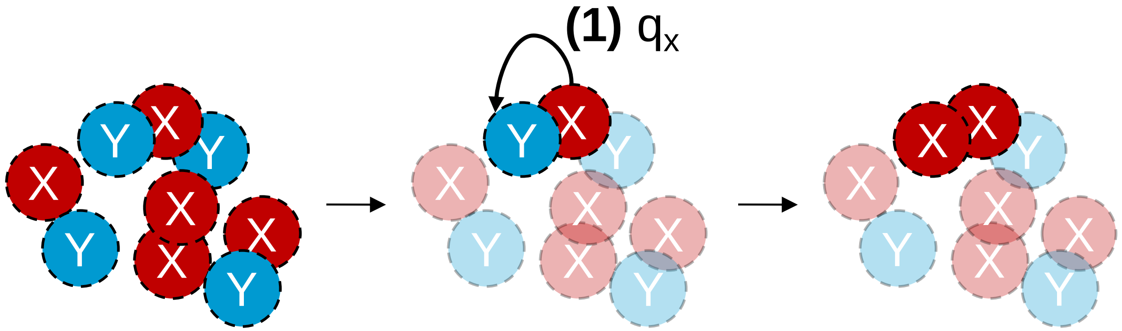
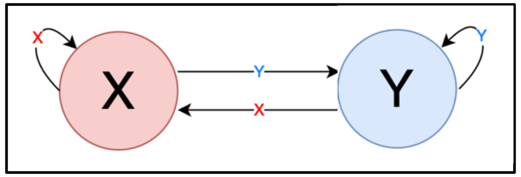
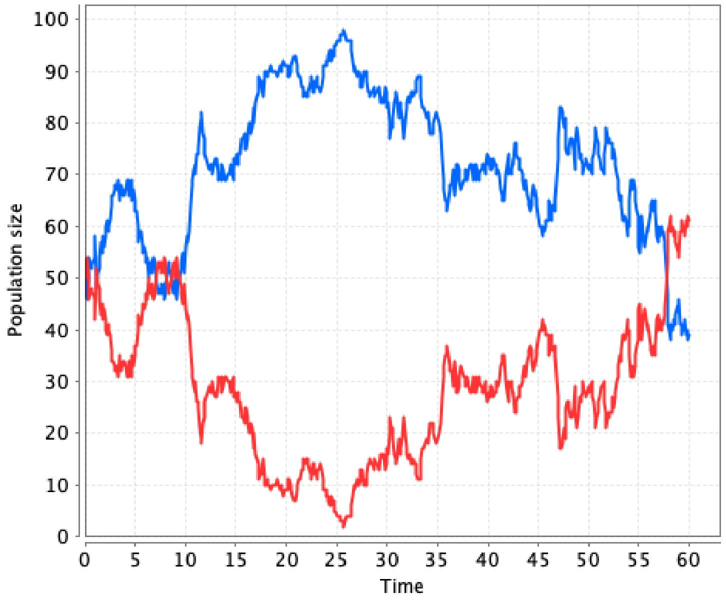


Decision making: Voter model

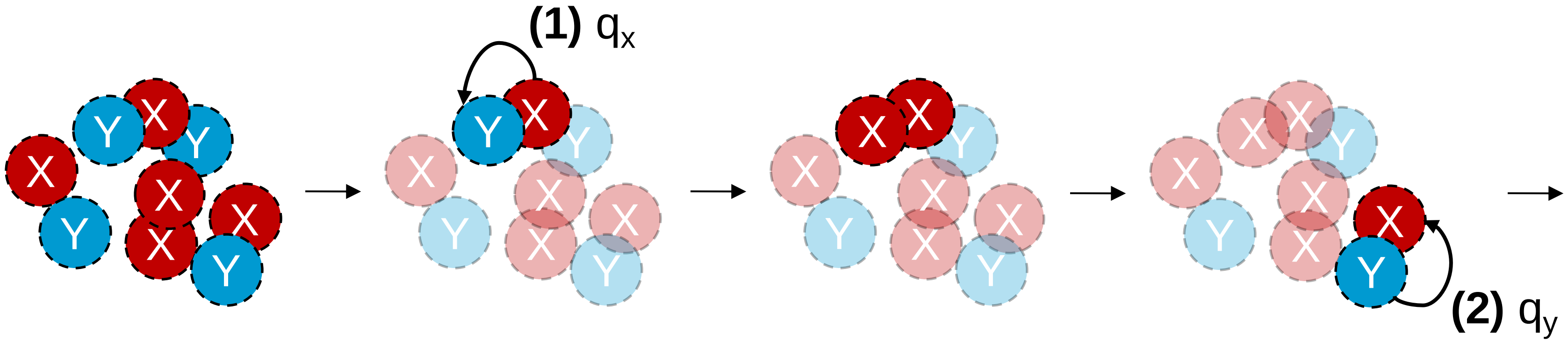


$$(1)\ X + Y \xrightarrow{q_x} X + X$$

$$(2)\ \boldsymbol{X + Y \xrightarrow{q_y} Y + Y}$$



Swarm state evolves as a continuous-time Markov chain
→ classical **population model**



Decision making: Voter model

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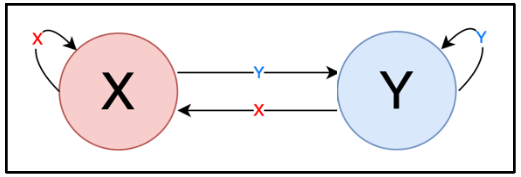
$$(2)\ X + Y \xrightarrow{q_y} Y + Y$$



Swarm state evolves as a continuous-time Markov chain
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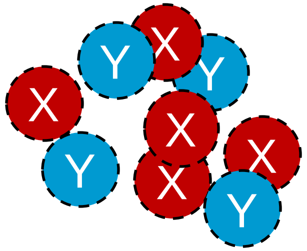

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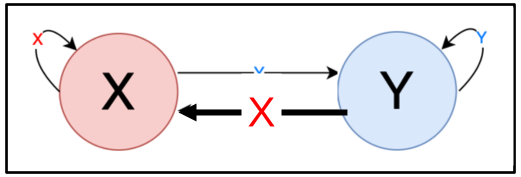
$$(1)\ \boldsymbol{X} + \boldsymbol{Y} \overset{q_x}{\rightarrow} \boldsymbol{X} + \boldsymbol{X}$$

$$(2)\ X + Y \overset{q_y}{\rightarrow} Y + Y$$



Swarm state evolves as a continuous-time Markov chain
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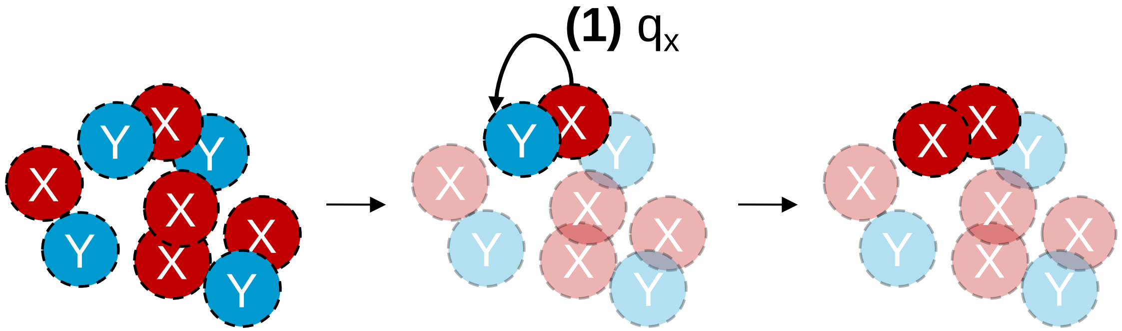

Decision making: Voter model

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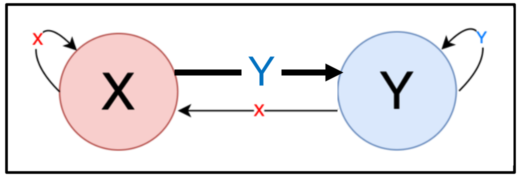
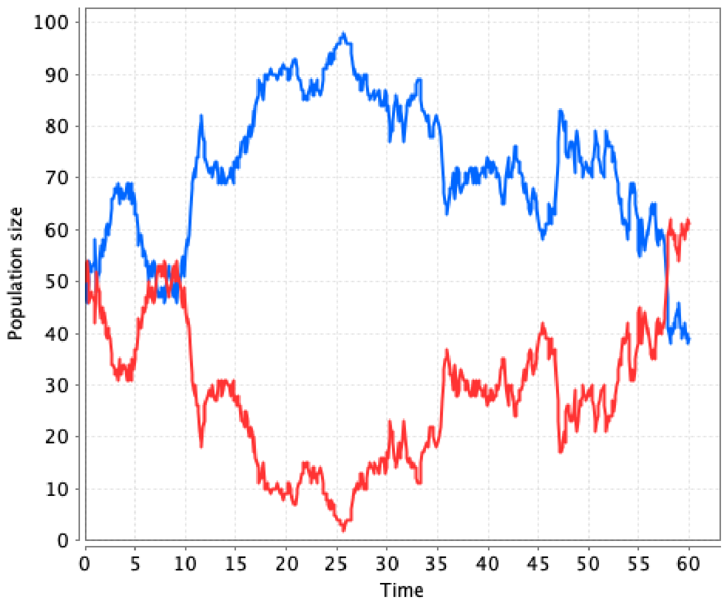
$$(2)\ \boldsymbol{X} + \boldsymbol{Y} \xrightarrow{\boldsymbol{q_y}} \boldsymbol{Y} + \boldsymbol{Y}$$





Swarm state evolves as a continuous-time Markov chain
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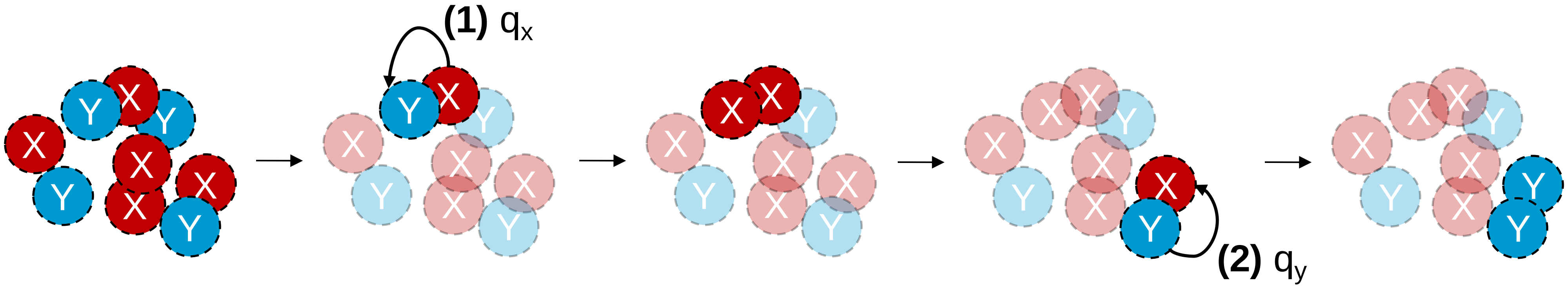
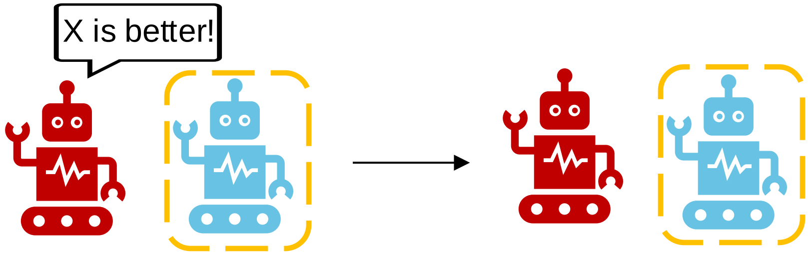


Disruptive individuals

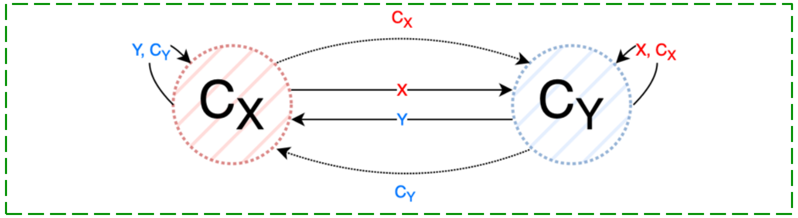
Zealots

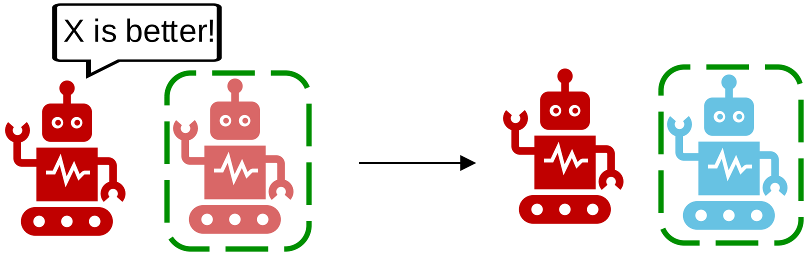
- Stubborn individuals which never change their own opinion



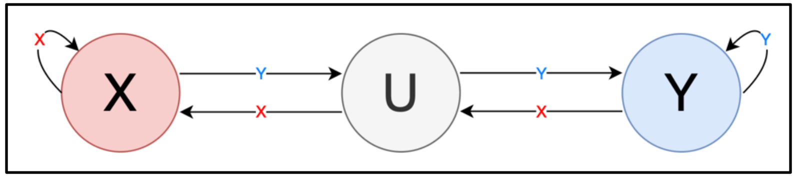
Contrarians

- Individuals which counter the opinion of the individual they interact with





Decision making: cross-inhibition model

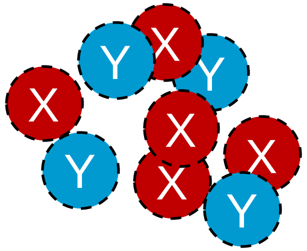


$$(1)\ X + Y \xrightarrow{q_x} X + U$$

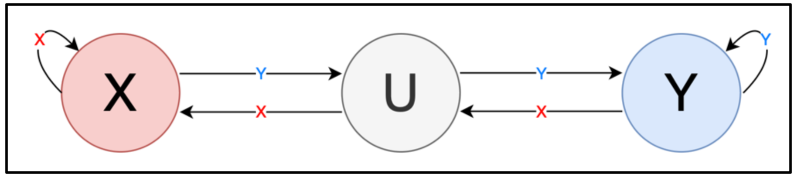
$$(2)\ X + Y \xrightarrow{q_y} U + Y$$

$$(3)\ X + U \xrightarrow{q_x} 2X$$

$$(4)\ Y + U \xrightarrow{q_y} 2Y$$



Decision making: cross-inhibition model

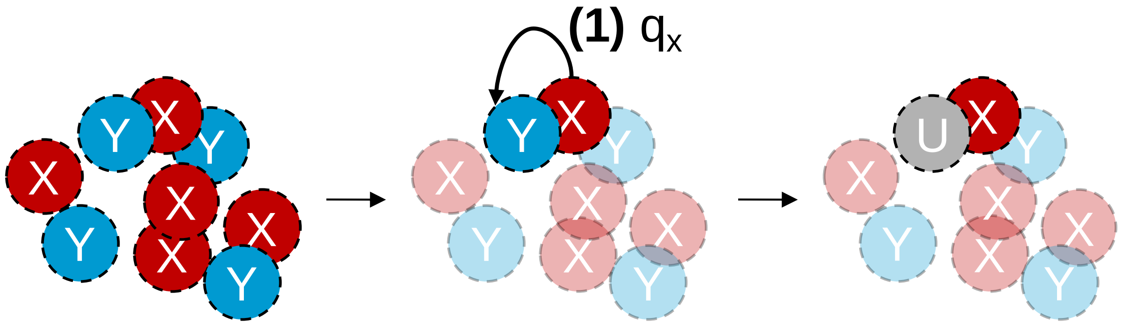


$$(\mathbf{1})\ \boldsymbol{X}+\boldsymbol{Y} \overset{\boldsymbol{q_x}}{\rightarrow} \boldsymbol{X}+\boldsymbol{U}$$

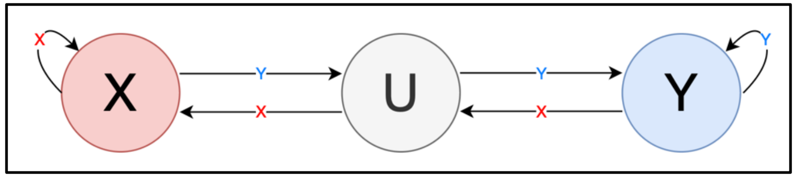
$$(2)\ X+Y \overset{q_y}{\rightarrow} U+Y$$

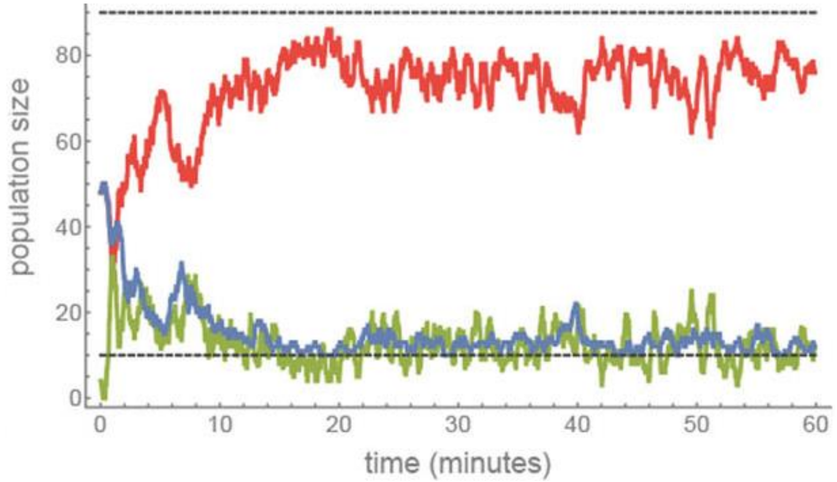
$$(3)\ X+U \overset{q_x}{\rightarrow} 2X$$

$$(4)\ Y+U \overset{q_y}{\rightarrow} 2Y$$



Decision making: cross-inhibition model



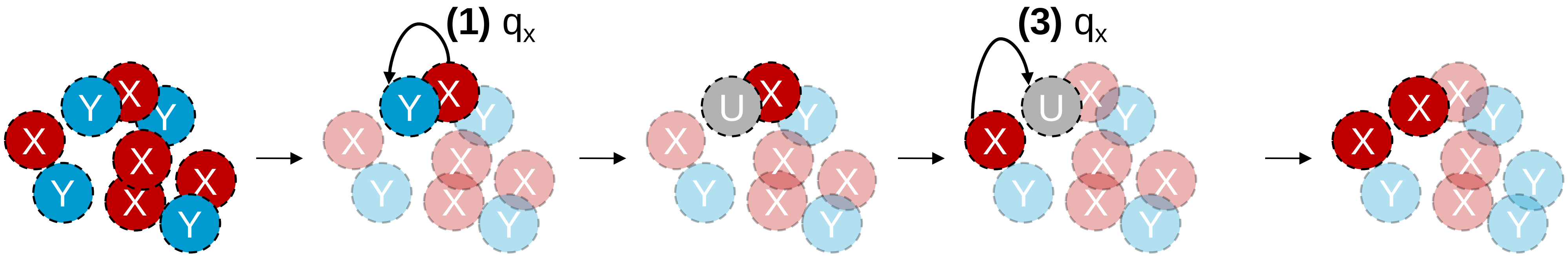


$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$

$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$

$$\mathbf{(3)\ X + U \overset{q_x}{\rightarrow} 2X}$$

$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$



Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

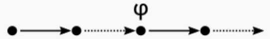
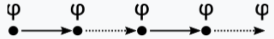
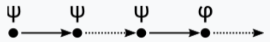
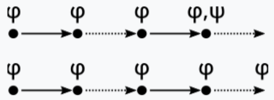
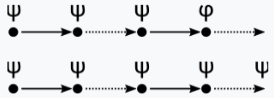
- based on **Linear Temporal Logic (LTL)**

$$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

where $a \in AP$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

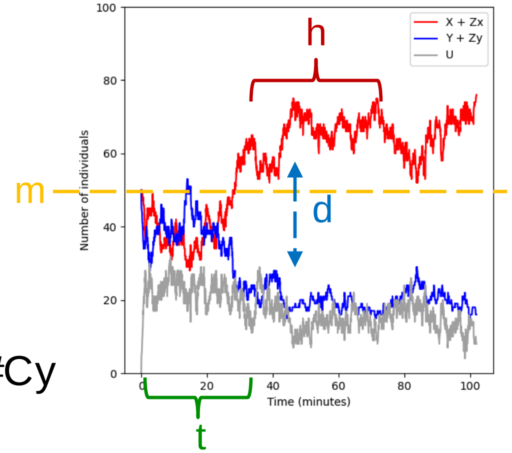
Textual	Symbolic	Explanation	Diagram
Unary operators:			
X ϕ	$\bigcirc\varphi$	ne**X**t: ϕ has to hold at the next state.	
F ϕ	$\Diamond\varphi$	**F**inally: ϕ eventually has to hold (somewhere on the subsequent path).	
G ϕ	$\Box\varphi$	**G**lobally: ϕ has to hold on the entire subsequent path.	
Binary operators:			
ψ **U** ϕ	$\psi\,\mathcal{U}\,\varphi$	**U**ntil: ψ has to hold *at least* until ϕ becomes true, which must hold at the current or a future position.	
ψ **R** ϕ	$\psi\,\mathcal{R}\,\varphi$	**R**elease: ϕ has to be true until and including the point where ψ first becomes true; if ψ never becomes true, ϕ must remain true forever.	
ψ **W** ϕ	$\psi\,\mathcal{W}\,\varphi$	**W**eak until: ψ has to hold *at least* until ϕ; if ϕ never becomes true, ψ must remain true forever.	

- **BLTL: (F, G, X, U, W)** are bounded by temporal bound

$\mathbf{F}_{\leq t}\,\phi$
$\mathbf{G}_{\leq t}\,\phi$
$\mathbf{X}_{\leq t}\,\phi$
$\mathbf{U}_{\leq t}\,\phi$
$\mathbf{W}_{\leq t}\,\phi$

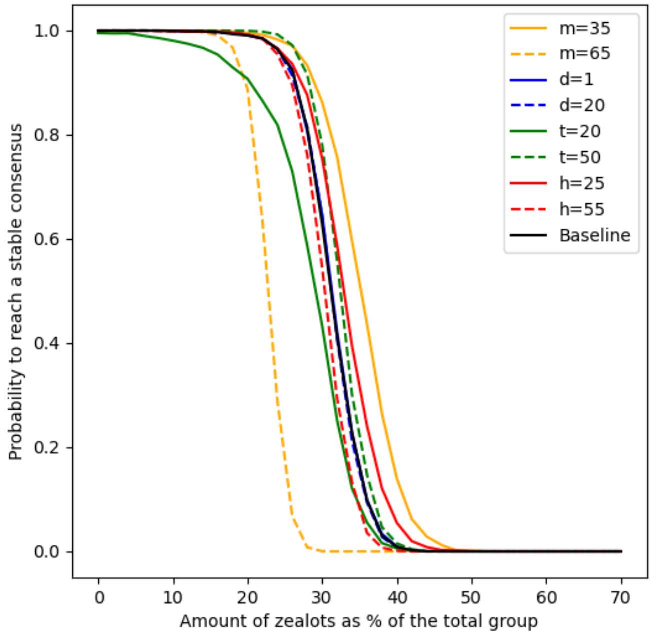
Boyer, B., Corre, K., Legay, A., Sedwards, S.: Plasma-lab: A flexible, distributable statistical model checking library. In: International Conference on Quantitative Evaluation of Systems. pp. 160–164. Springer (2013)

Results – robustness of stable consensus

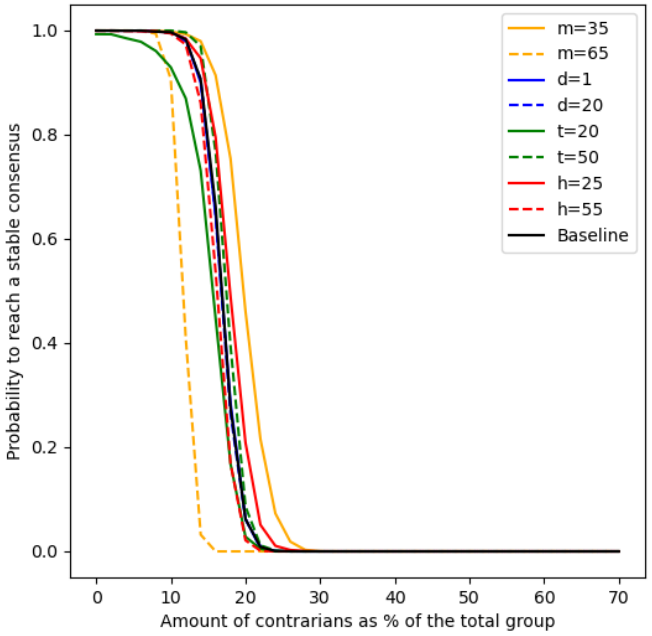


- Scenario: N = 100 robots, equivalent options X and Y (q_x=q_y), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: m=50, d=10, t=35, h=40

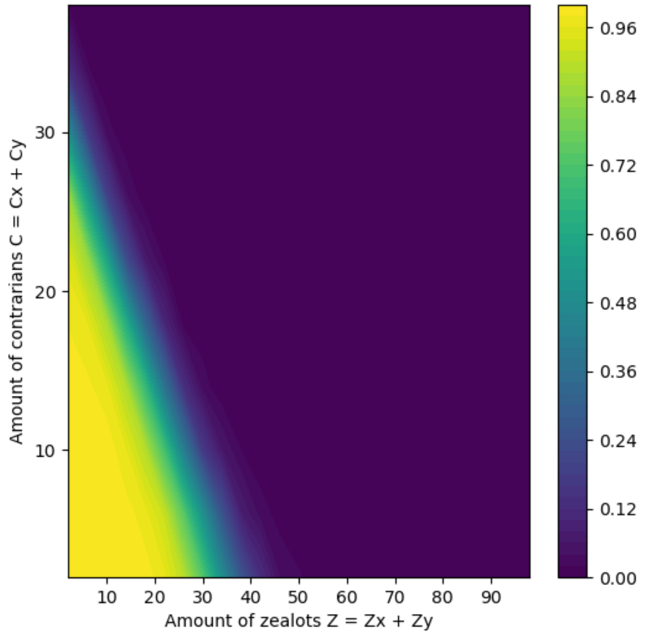
Robustness – Zealots

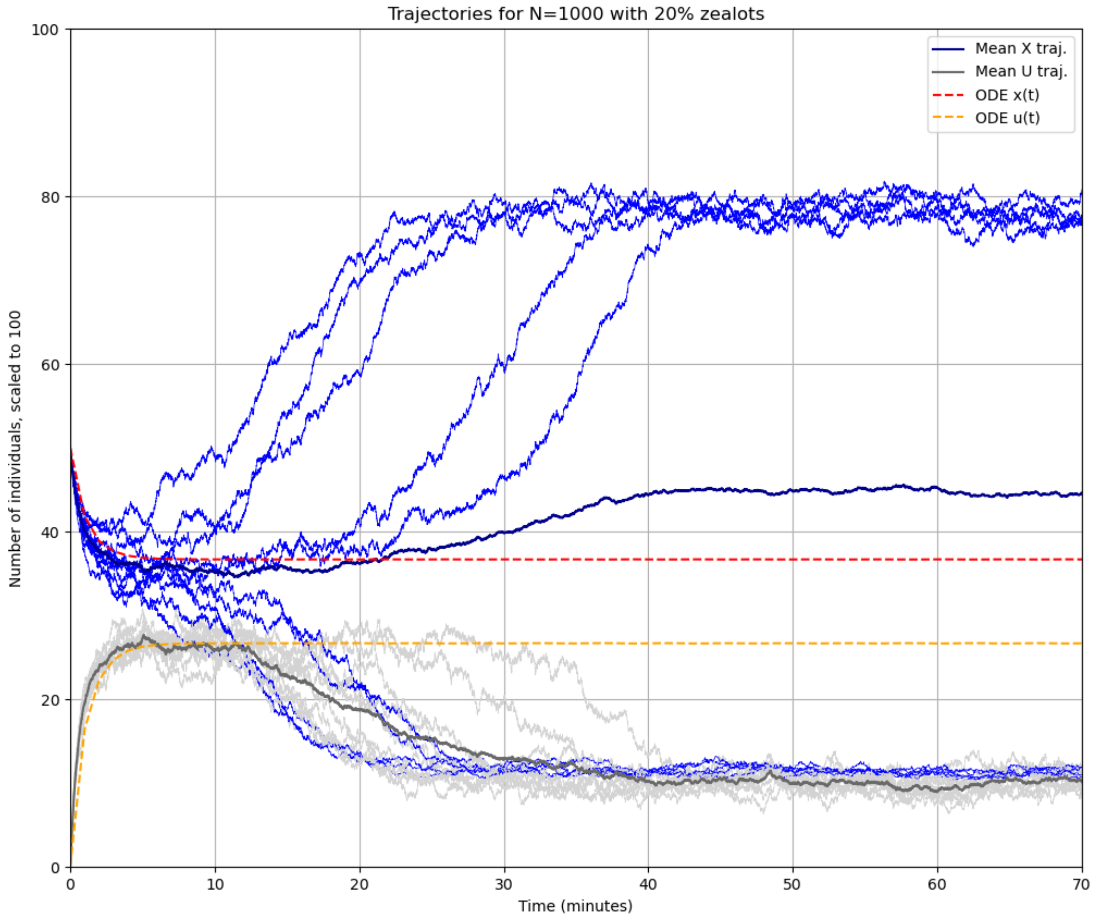


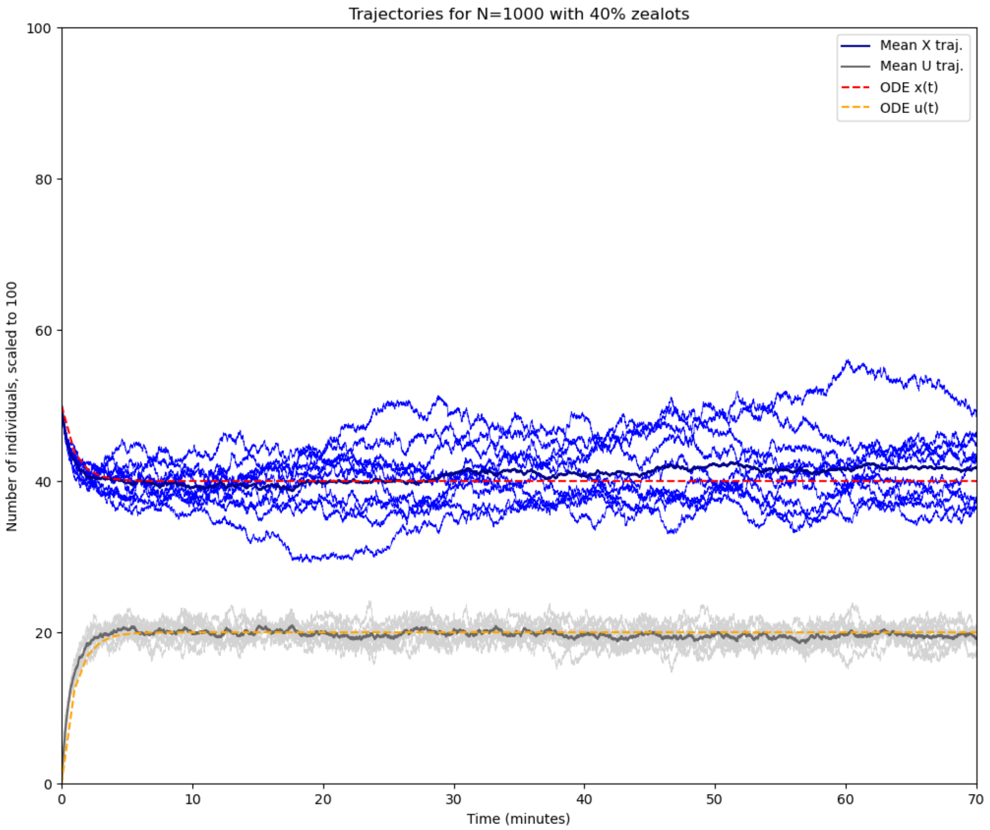
Robustness – Contrarians



Combined effect

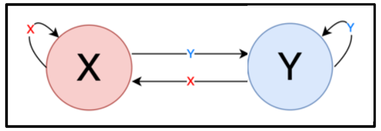


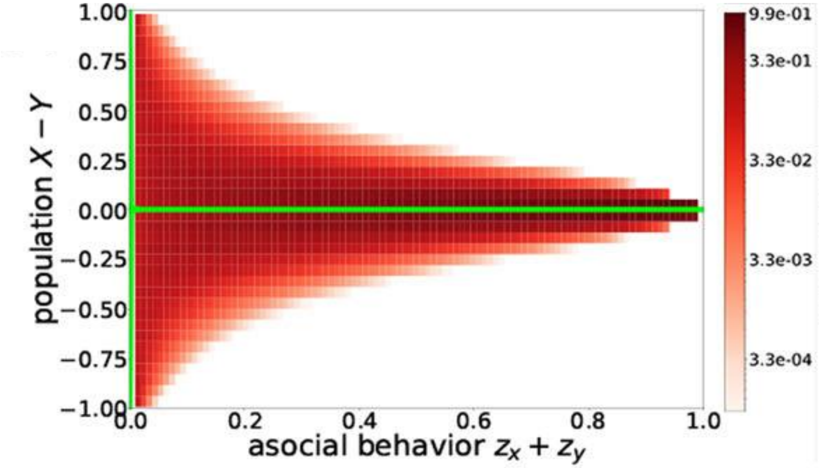

Trajectories for N=1000 with 20% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)


Trajectories for N=1000 with 40% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)

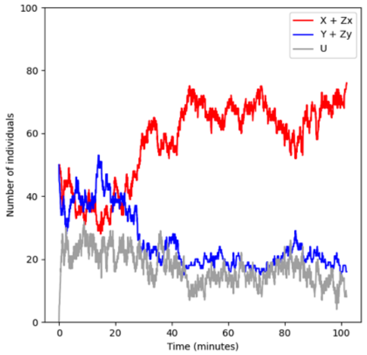
Studied Model of Decision-Making

Voter Model

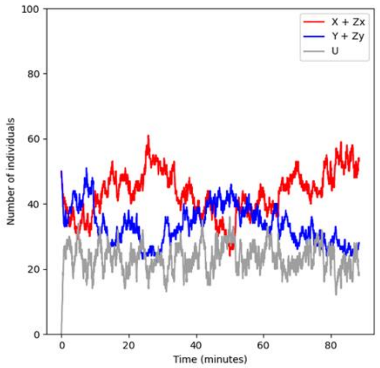




→ in presence of asocial individuals, the swarm gets quickly locked into an indecision state



No zealots
→ quick, stable consensus

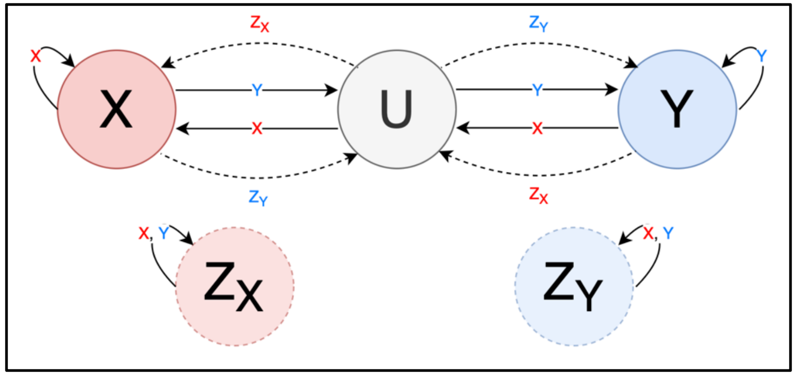


2% zealots
→ permanent indecision

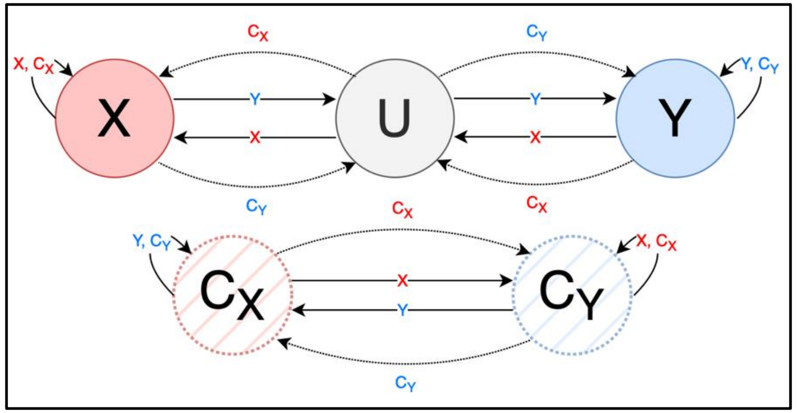
Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

Studied Model with Disruptive Individuals

Cross-Inhibition model with Zealots



Cross-Inhibition model with Contrarians



- **Zealots**: stubborn individuals which never change their own opinion
- Four additional reactions, where ‘pure’ agents interact with zealots & adjust their own states

- **Contrarians**: individuals which counter the opinion of the individual they interact with
- Eight additional reactions, where contrarians influence ‘pure’ individuals & are influenced by others with the same opinion