Exploring Consensus Robustness in Swarms with Disruptive Individuals

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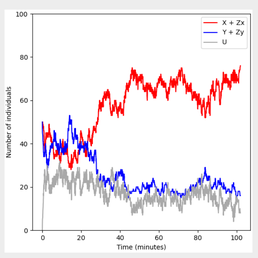
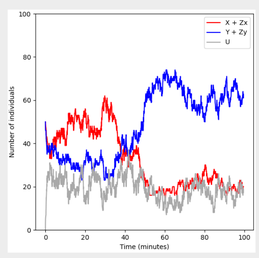
Motivation

- Consensus in collective systems essential for coordinated behaviour
- **Cross-inhibition model** (2a) as underlying model of decision-making is more robust than classic voter model due to 'undecided' state between switching opinion [1]
- Introducing stubborn individuals…
  - **Zealots:** never change their own opinion (2b)
  - **Contrarians:** counter opinion of individual they interact with (2c)

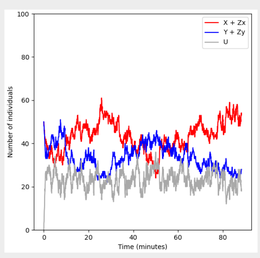
  …can disrupt opinion dynamics!
- Observation of 3 different group dynamics:



(1a) stable consensus



(1b) switching consensus



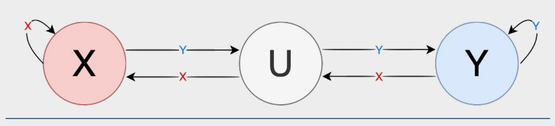
(1c) no consensus

- **Research Questions**
  1. **Robustness:** Quantify how the amount of disruptive individuals affects consensus reaching
  2. **Combined effect:** Quantify how the combination of zealots & contrarians affects consensus reaching
  3. **Group size effect:** Quantify how the group size affects consensus reaching

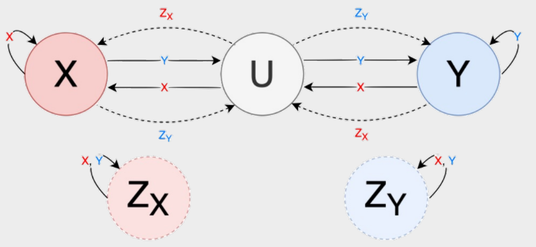
- **Approach**:
  - Investigated scenario: group of N individuals has to decide between two equivalent options X and Y
  - Swarm state evolves as a continuous-time Markov chain (see reaction systems 2e-h)
  - Apply model checking tools (PRISM and PlasmaLab) to explore research questions wrt. different group dynamics

Models

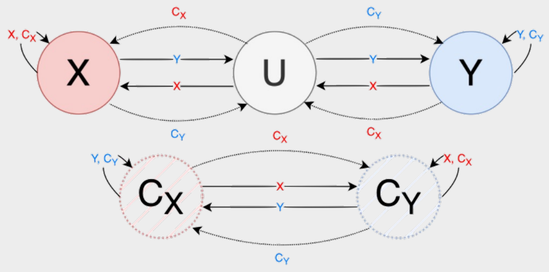
(2a)



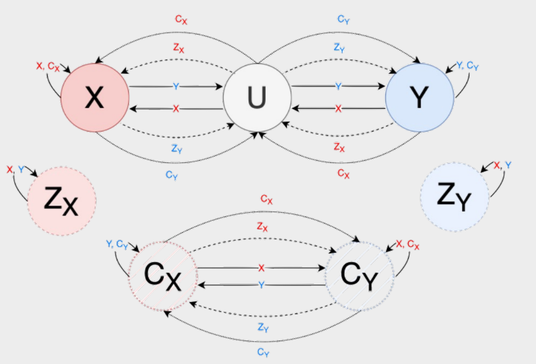
(2b)



(2c)



(2d)



Cross-Inhibition model

$$X + Y \xrightarrow{q_x} X + U$$
$$X + Y \xrightarrow{q_y} Y + U$$
$$X + U \xrightarrow{q_x} 2X$$
$$Y + U \xrightarrow{q_y} 2Y$$

(2e)

Zealots

$$Y + Z_X \xrightarrow{q_x} U + Z_X$$
$$U + Z_X \xrightarrow{q_x} X + Z_X$$
$$X + Z_Y \xrightarrow{q_y} U + Z_Y$$
$$U + Z_Y \xrightarrow{q_y} Y + Z_Y$$

(2f)

Contrarians

$$X + C_Y \xrightarrow{q_y} U + C_Y$$
$$U + C_Y \xrightarrow{q_y} Y + C_Y$$
$$X + C_X \xrightarrow{q_y} X + C_Y$$
$$Y + C_X \xrightarrow{q_x} U + C_X$$
$$U + C_X \xrightarrow{q_x} X + C_X$$
$$Y + C_Y \xrightarrow{q_x} Y + C_X$$
$$C_X + C_X \xrightarrow{q_y} C_Y + C_Y$$
$$C_Y + C_Y \xrightarrow{q_x} C_X + C_X$$

(2g)

Both Zealots & Contrarians

$$C_X + Z_X \xrightarrow{q_y} C_Y + Z_X$$
$$C_Y + Z_Y \xrightarrow{q_x} C_X + Z_Y$$

(2h)

Analysis

1. Formally describe observed scenarios in **Bounded Linear Temporal Logic**
   - Definition of consensus with five parameters: at least *majority m%* of population commits to same decision, *difference* of at least *d* between both groups, consensus is reached within *reaching time t*, consensus is maintained for at least *holding time h*, switch to other opinion happens within *switching time s*
   - Baseline: m=50, d=10, t=35, h=40, s=10
   - **Stable consensus** (1a) in BLTL:

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq min_m) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$

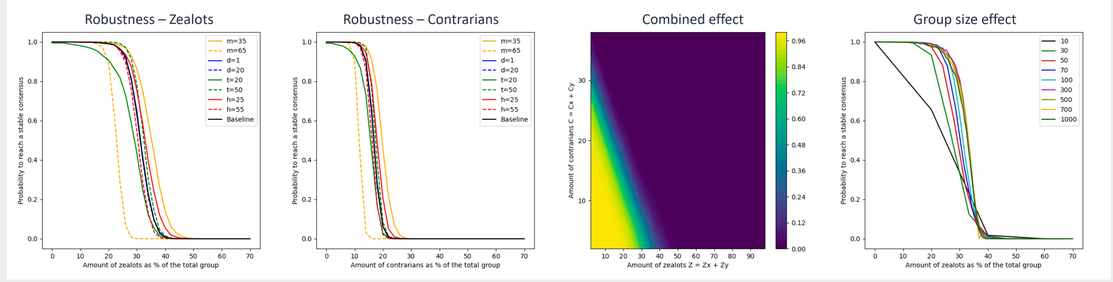
   - **Switching consensus** (1b) in BLTL:

$$F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \wedge (true\ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee (((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \wedge (true\ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))$$

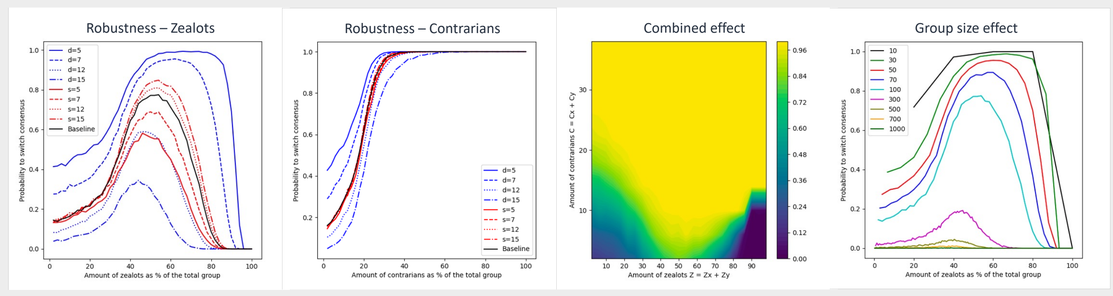
2. **Statistical model checking** to estimate satisfaction probability and quantify robustness under perturbations of the amount of disruptive individuals

Results

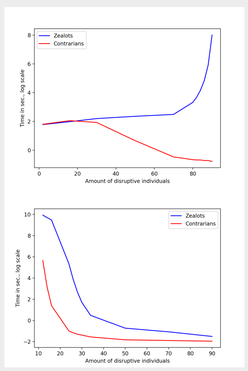
- Robustness of reaching a stable consensus (1a)



- Robustness of switching consensus (1b)



- Expected times



…to reach consensus

#	2	16	30	50	70	80	82	84	86	88	90
Zealots	5.95	7.28	9.02	10.57	12.04	27.82	39.94	64.95	128.85	374.04	2975.68
Contrarians	6.07	7.81	6.89	1.95	0.63	0.52	0.51	0.51	0.49	0.49	0.46

…to hold consensus

#	12	14	16	24	26	28	30	34	50	70	90
Zealots	20686.51	16368.28	13047.85	210.98	47.71	14.13	5.46	1.61	0.48	0.34	0.22
Contrarians	283.57	22.53	4.03	0.37	0.31	0.27	0.25	0.21	0.16	0.15	0.14

Conclusion

- Disruptive individuals can change opinion dynamics
- Precise, **quantitative** analysis of robustness
- **Stable consensus**: robust up to certain #zealots/#contrarians, then rapid phase transition; zealots are less harmful for reaching consensus
- **Switching consensus**: only range of zealots for which switching occurs with high probability; contrarians promote switching dynamics

- **Future work**: formal analysis of group-size effect, variations of current scenario (e.g. non-symmetric decision quality), control theory

References

[1] Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

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