Centre for the Advanced Study of Collective Behaviour

UNIVERSITÀ DEGLI STUDI DI TRIESTE

MAX-PLANCK-GESELLSCHAFT

Universität Konstanz

# Exploring Consensus Robustness In Swarms with Disruptive Individuals

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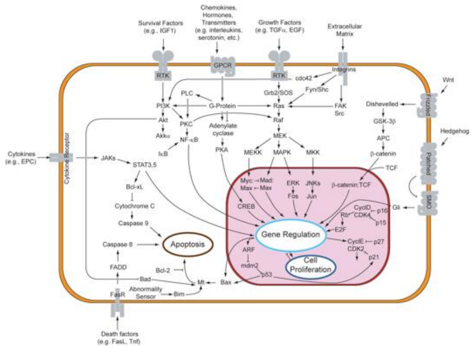
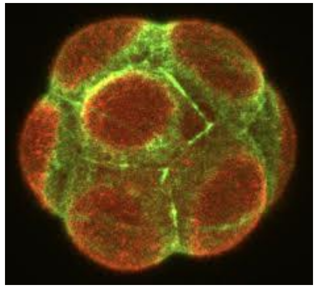
*Centre for the Advanced Study of Collective Behaviour*

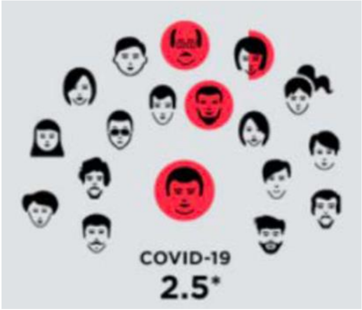
# Swarms (collectives)

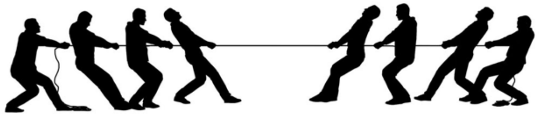






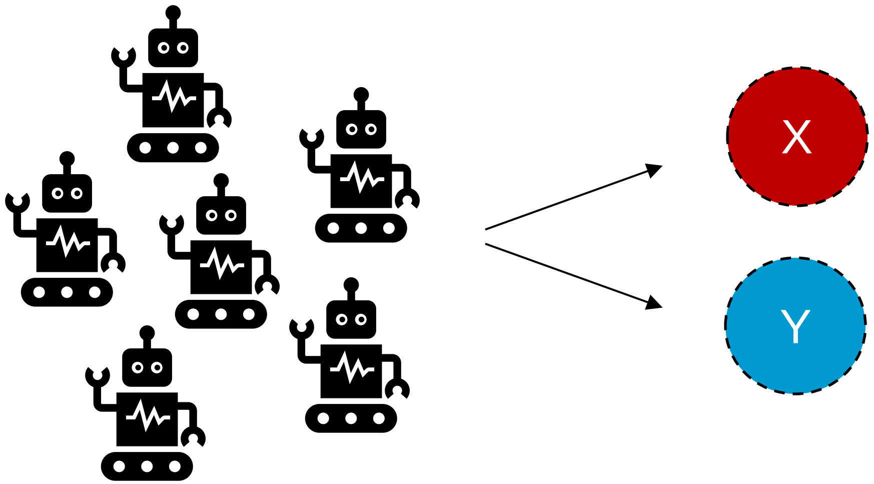


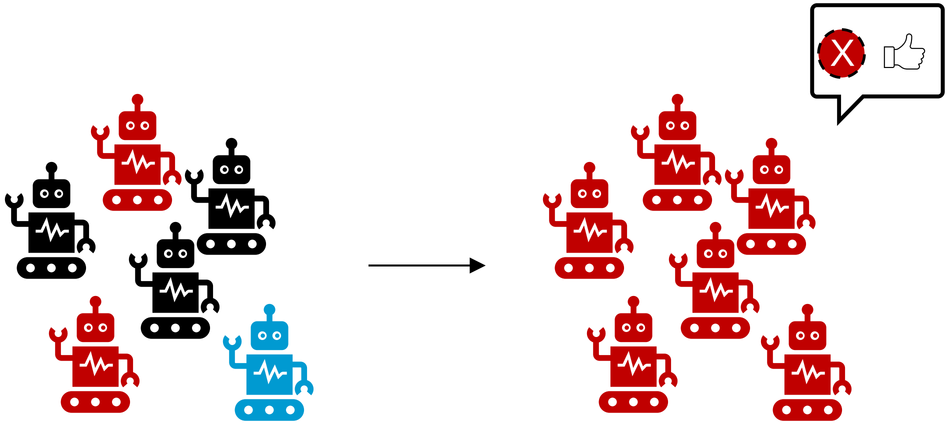


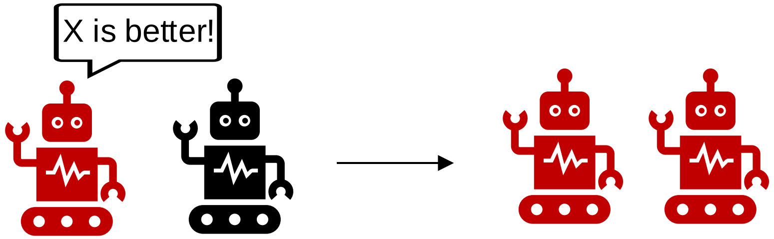



**→ How do swarms agree on decisions?**

# Collective decision making

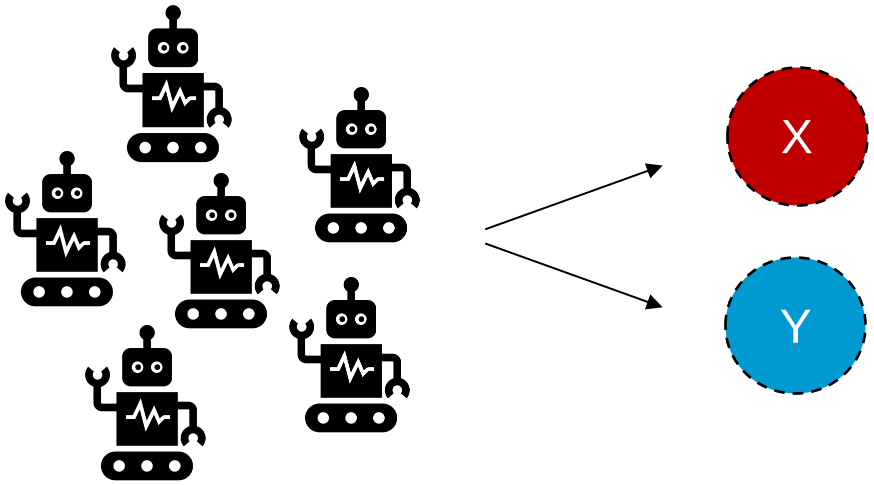


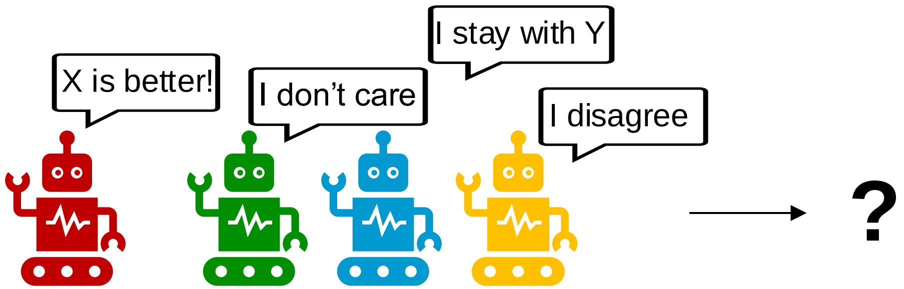



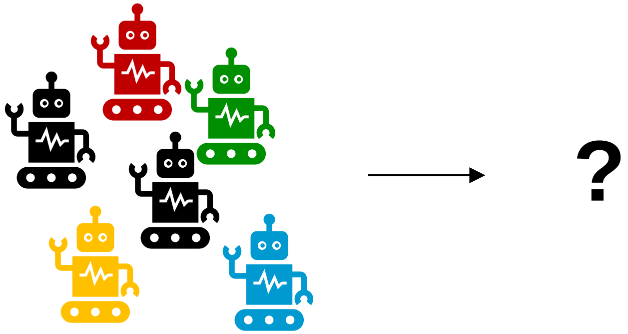




Ideally, consensus will be achieved with certain speed and accuracy

# Collective decision making



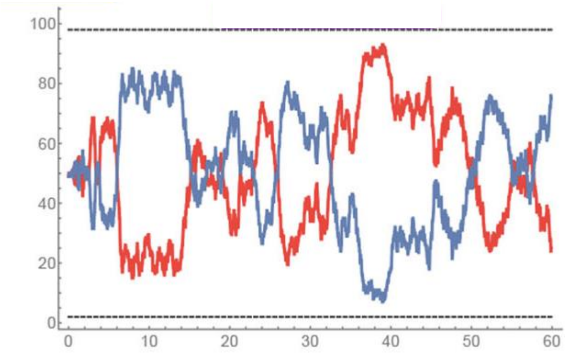



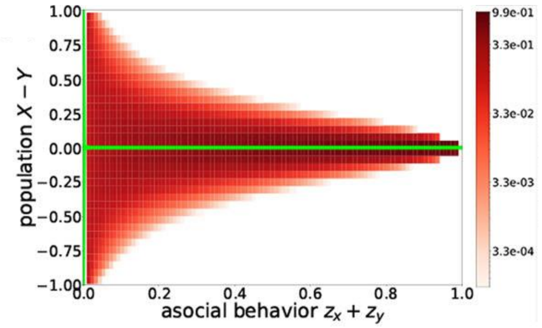

*What happens in presence of disruptive (asocial) individuals?*

# Collective decision making – disruptive individuals

### Voter model with zealots

2 species, 2 reactions

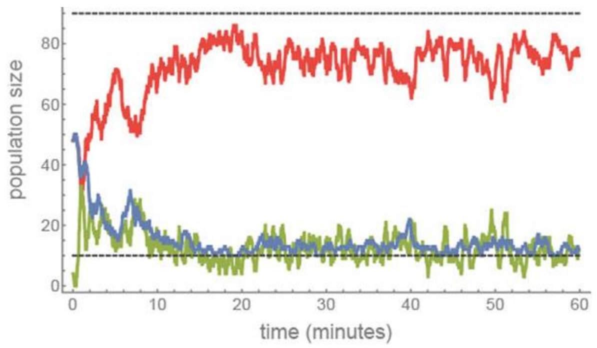


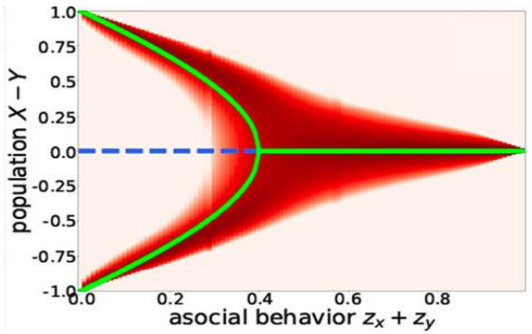




- **Permanent indecision** with already 4% zealots
- Swarm gets quickly locked into indecision state

### Cross-inhibition model with zealots

3 species, 4 reactions



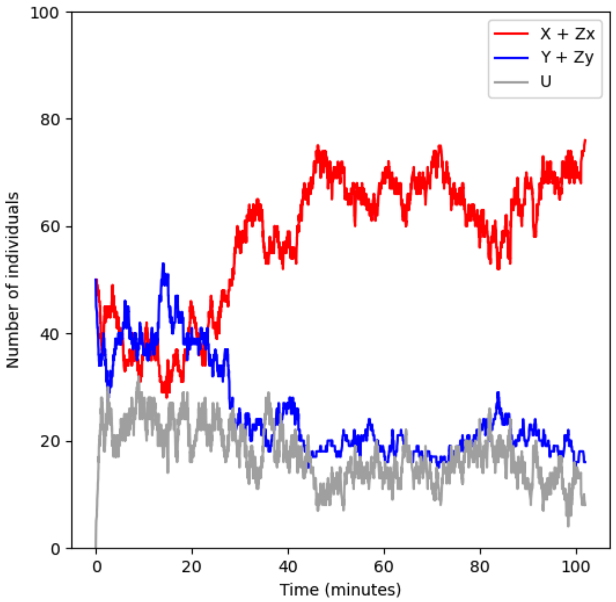




- **Stable dynamics** for 20% zealots!
- Swarm demonstrates resilience against relatively high levels of asocial behaviour

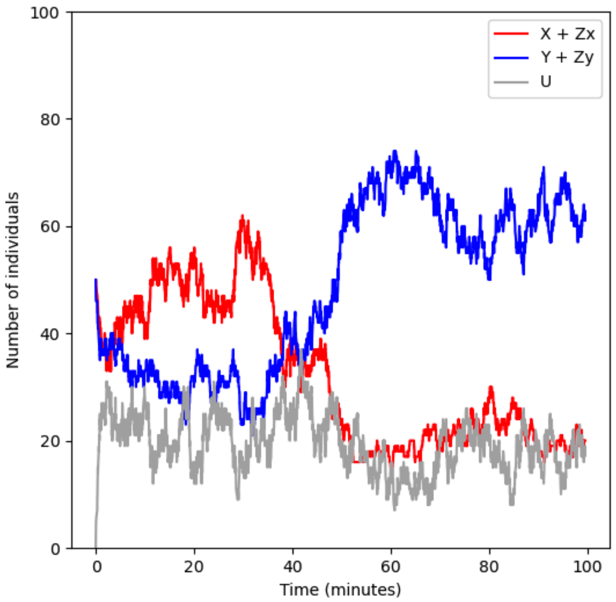
*Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)*

# Motivation – group dynamics

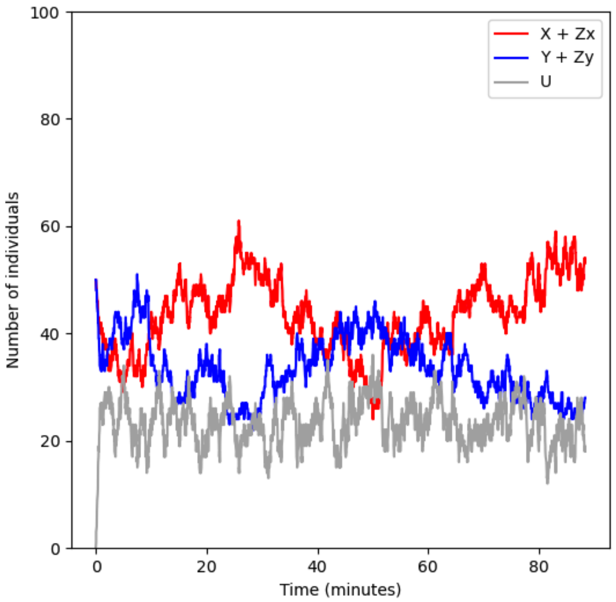
Stochastic **cross-inhibition model with 30% zealots** gives three qualitatively different scenarios!




stable consensus



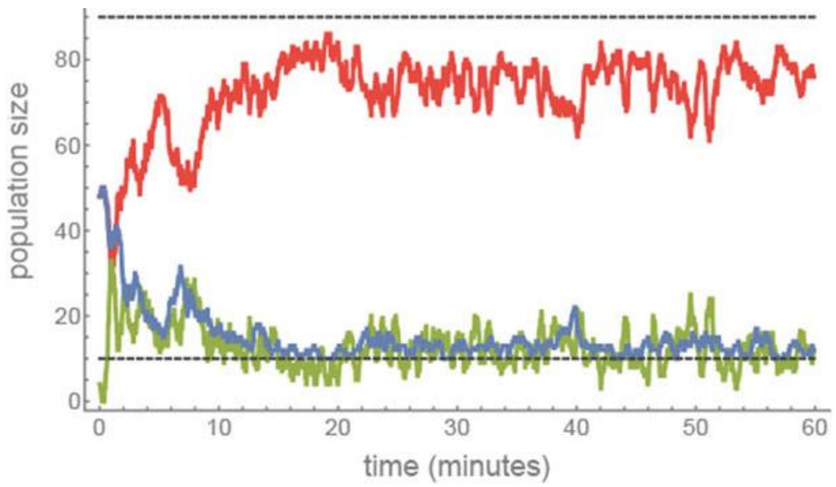

switching consensus



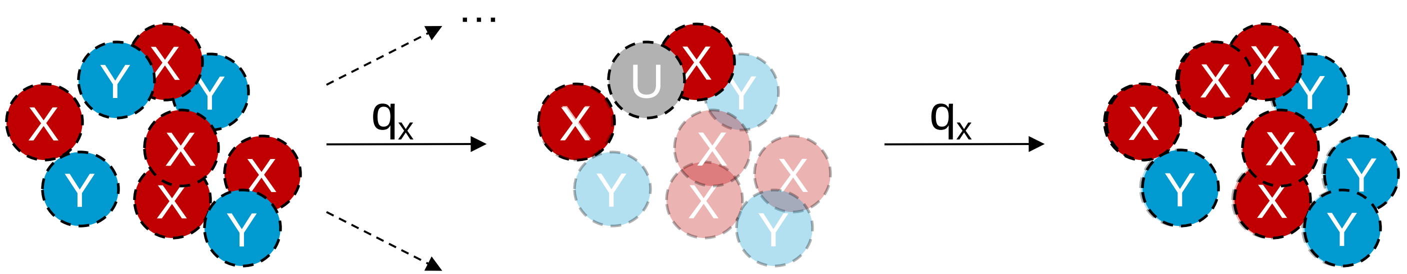

no consensus

# Cross-inhibition model

$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$
$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$
$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$




**Swarm state** evolves **stochastically** as a continuous-time Markov chain

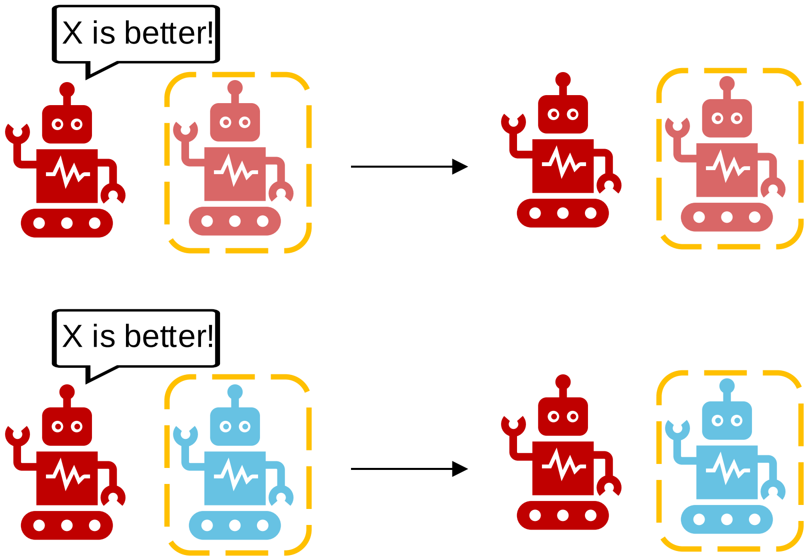



well-mixed population state

# Disruptive individuals

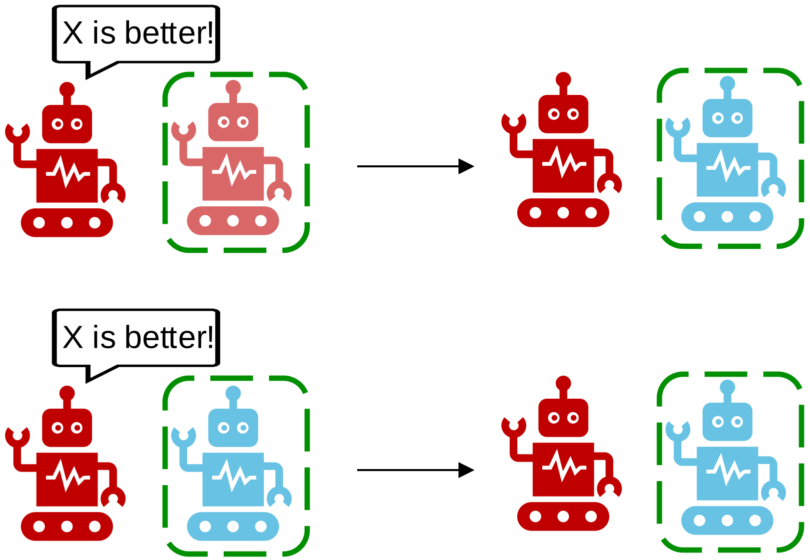
## Zealots

Stubborn individuals which never change their own opinion




## Contrarians

Individuals which counter the opinion of the individual they interact with



# Cross-inhibition model with disruptive individuals

### Zealot dynamics

$$(5)\ X + ZY \overset{q_y}{\rightarrow} U + Zy$$
$$(6)\ U + ZY \overset{q_y}{\rightarrow} Y + ZY$$
$$(7)\ Y + ZX \overset{q_x}{\rightarrow} U + ZX$$
$$(8)\ U + Z_X \overset{q_x}{\rightarrow} X + ZX$$

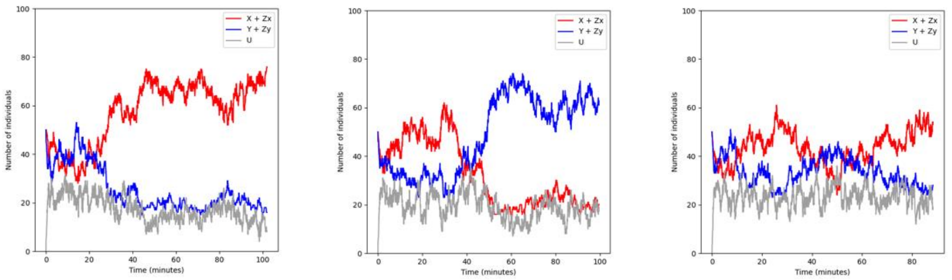
### Cross-inhibition model

$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$
$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$
$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$

### Contrarian dynamics

$$(5)\ X + C_Y \overset{q_y}{\rightarrow} U + C_y$$
$$(6)\ U + C_Y \overset{q_y}{\rightarrow} Y + C_Y$$
$$(7)\ X + C_X \overset{q_x}{\rightarrow} X + C_Y$$
$$(8)\ Y + C_X \overset{q_x}{\rightarrow} U + C_X$$
$$(9)\ U + CX \overset{q_x}{\rightarrow} X + CX$$
$$(10)\ Y + CY \overset{q_y}{\rightarrow} Y + CX$$
$$(11)\ CX + CX \overset{q_x}{\rightarrow} CY + CY$$
$$(12)\ CY + CY \overset{q_y}{\rightarrow} CX + CX$$

# Research Questions




## 1. Robustness analysis

How does the **amount** of disruptive individuals affect consensus reaching/switching?

## 2. Combined effect

How does the **combination** of zealots and contrarians affect consensus reaching/switching?

## 3. Group size effect

How does the **group size** affect consensus reaching/switching?

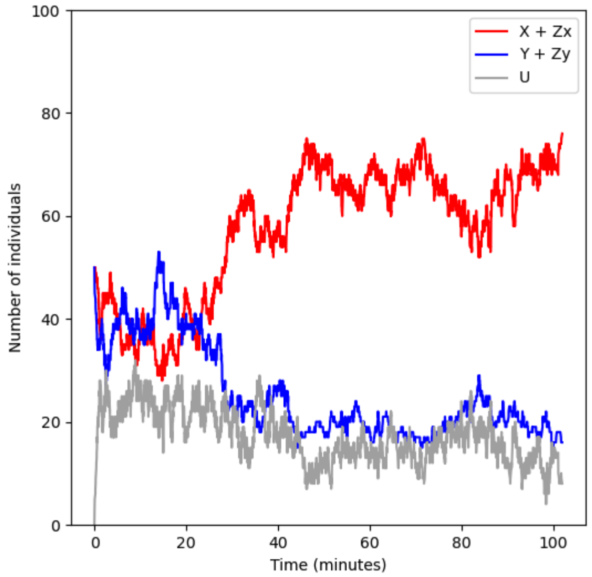
# Approach

**Statistical Model Checking** of properties in **Bounded Linear Temporal Logic (BLTL)**

**STEP 1:** Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \ \wedge \ ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee \\ ((y + Z_y + C_y \geq min_m) \ \wedge \ ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$



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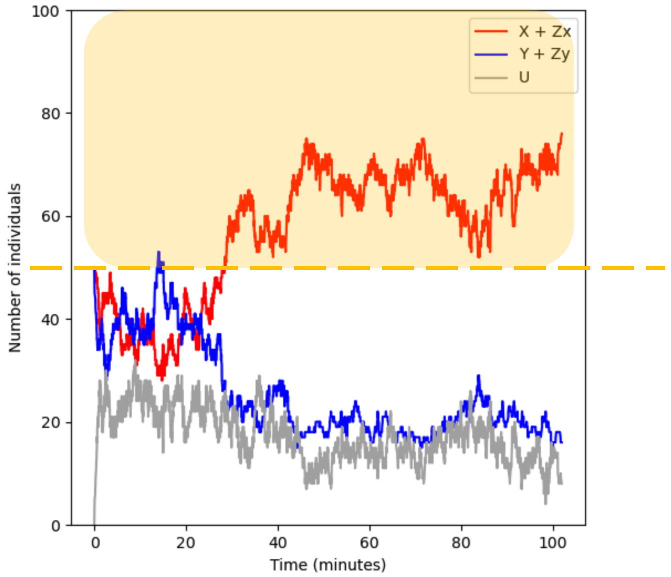

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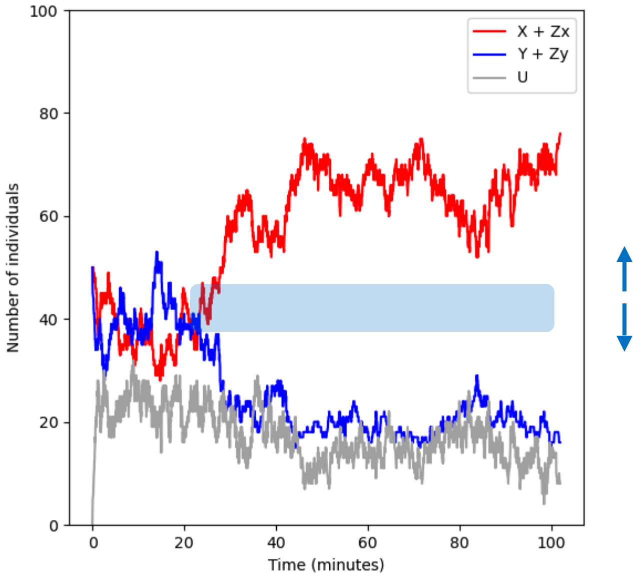
$$F_{\leq t}(G_{\leq h}(((x+Z_x+C_x \geq min_m) \ \wedge \ ((x+Z_x+C_x)-(y+Z_y+C_y)\boxed{\geq d})) \ \vee \\ ((y+Z_y+C_y \geq min_m) \ \wedge \ ((y+Z_y+C_y)-(x+Z_x+C_x)\boxed{\geq d}))))$$



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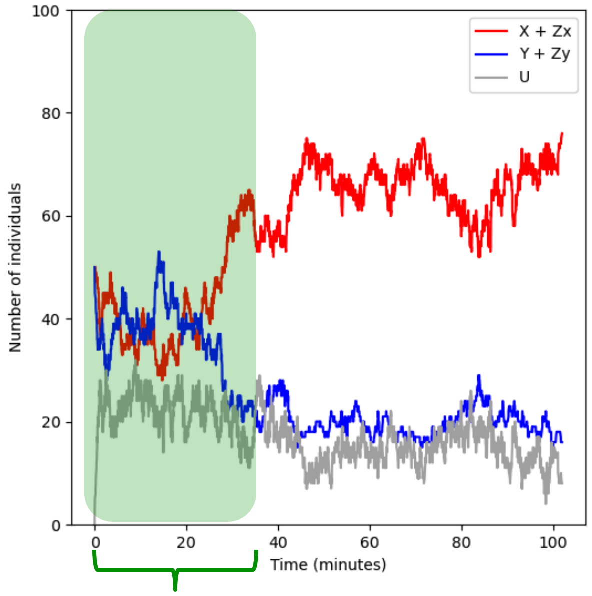

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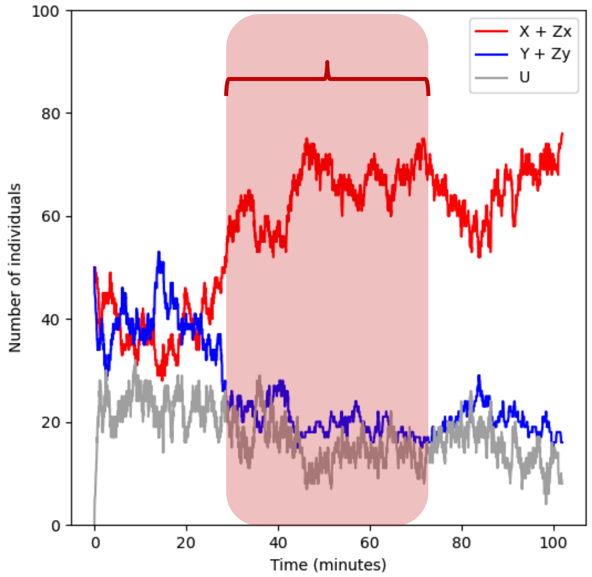
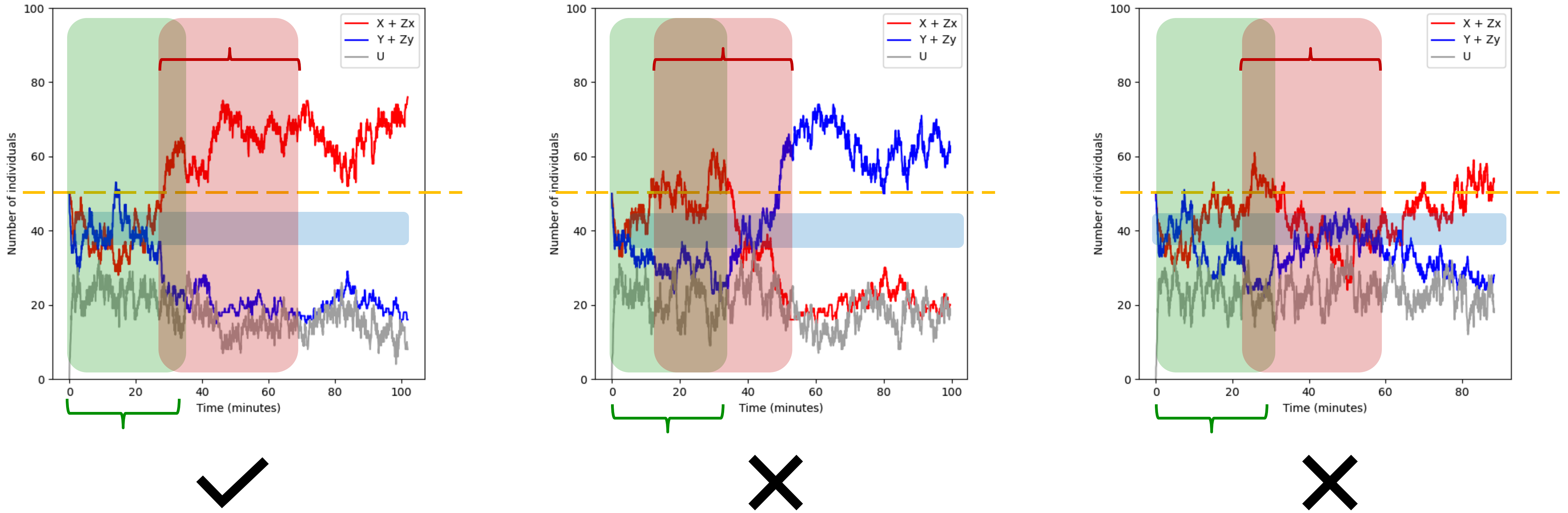


# Approach

*Is this a stable consensus?*



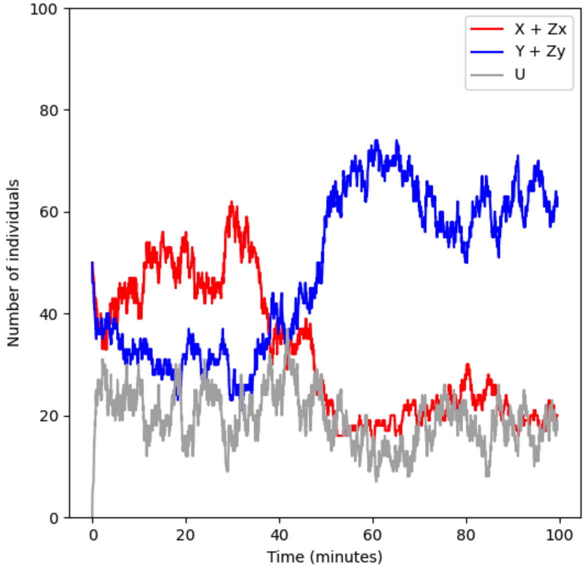
# Approach

**Statistical Model Checking** of properties in **Bounded Linear Temporal Logic (BLTL)**

**STEP 1:** Formally describe stable consensus and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$\begin{aligned} F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \ \wedge \\ (true\ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee \\ (((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \ \wedge \\ (true\ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)))) \end{aligned}$$



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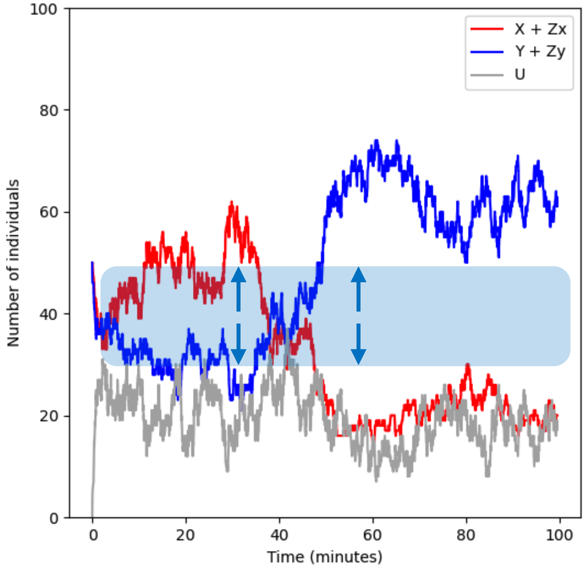
$$
\begin{aligned}
F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \boxed{\geq d}) \ \wedge \\
(true\ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \boxed{\geq d})) \ \vee \\
(((y + Z_y + C_y) - (x + Z_x + C_x) \boxed{\geq d}) \ \wedge \\
(true\ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \boxed{\geq d})))
\end{aligned}
$$



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$$(true\ U_{\leq s}((y+Z_y+C_y)-(x+Z_x+C_x)\geq d))) \;\vee$$
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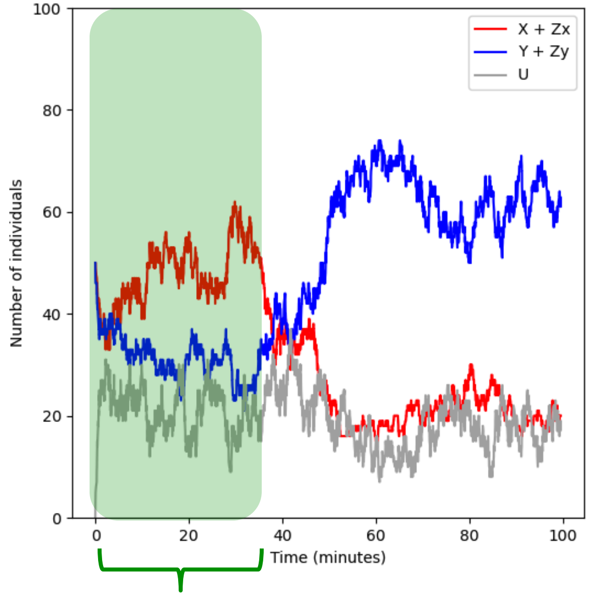

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**Statistical Model Checking** of properties in **Bounded Linear Temporal Logic (BLTL)**

**STEP 1:** Formally describe stable consensus and **switching consensus** in BLTL

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$$
\begin{aligned}
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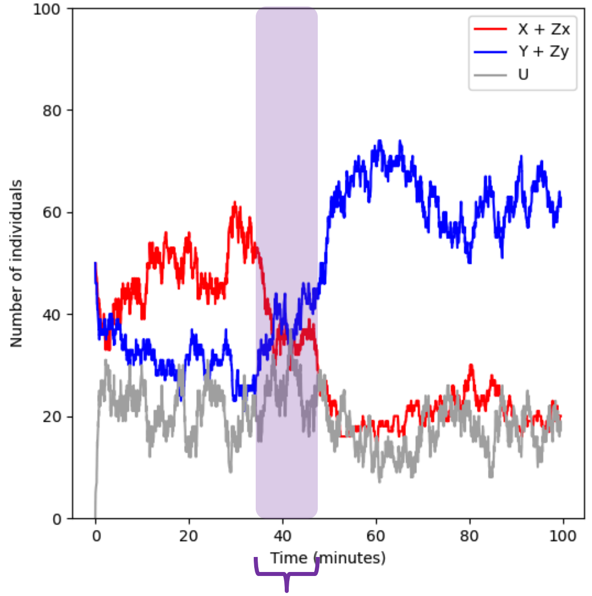
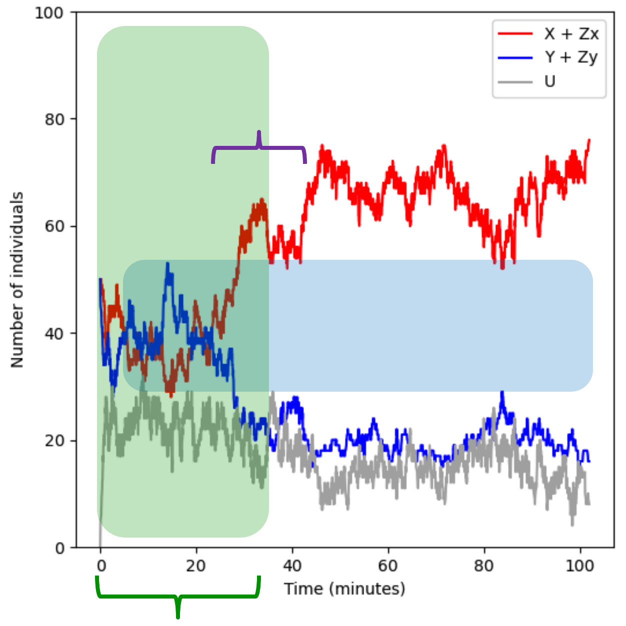
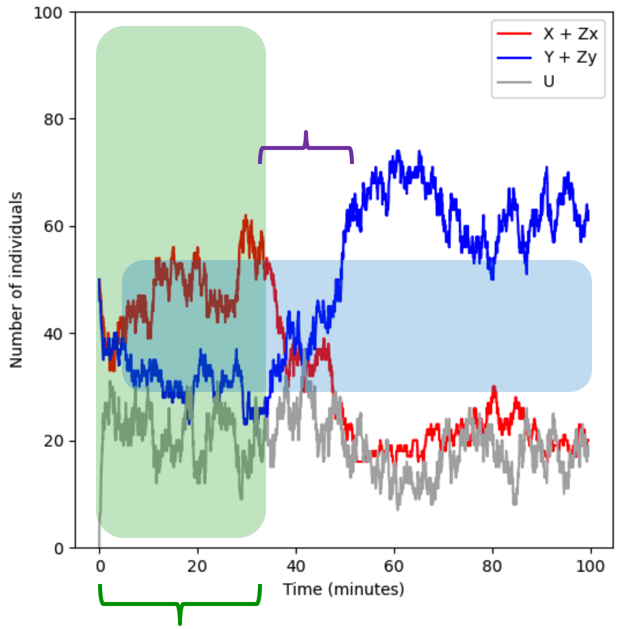
&(((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \;\wedge \\
&(true\; \boxed{U_{\leq s}}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))
\end{aligned}
$$

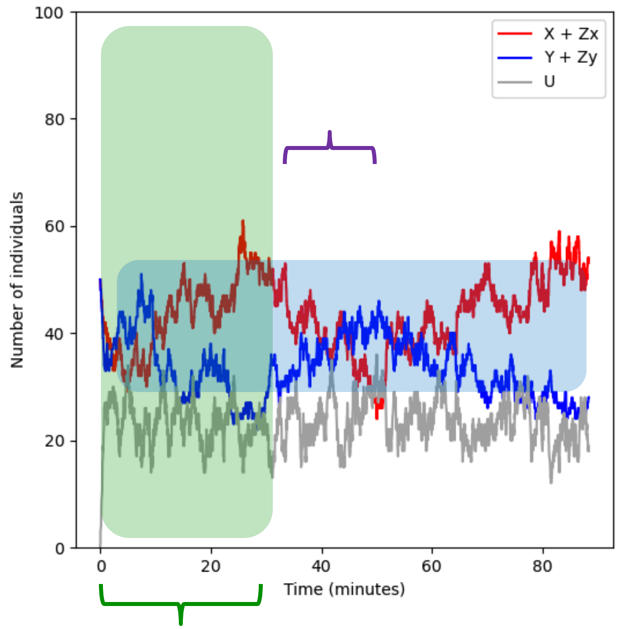


# Approach

*Is this a switching consensus?*



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**Statistical Model Checking** of properties in **Bounded Linear Temporal Logic (BLTL)**

**STEP 1:** Formally describe **stable consensus** and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

**STEP 2:** Apply model checking tools (*PRISM* and *PlasmaLab*) to explore the relevant scenarios:

  - Varying number of zealots and contrarians to explore robustness
  - Varying number of both to explore combined effect
  - Varying total group size to explore group size effect

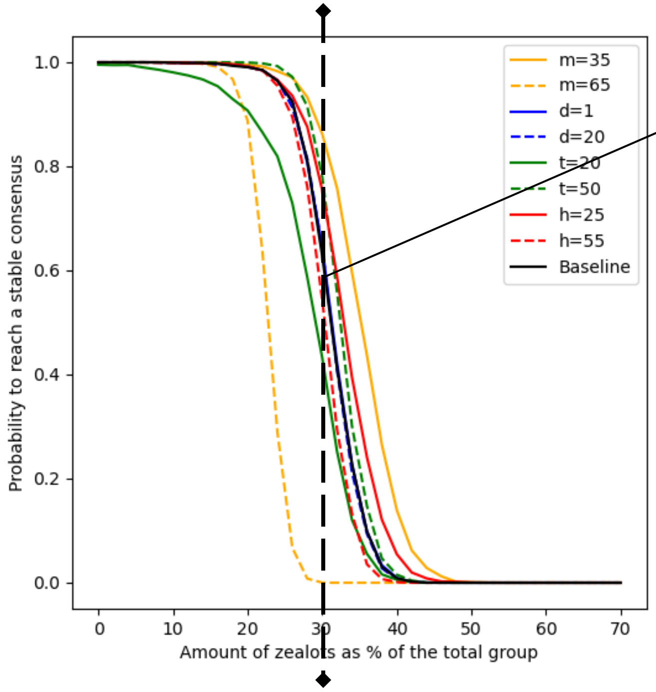
- **Monte Carlo** algorithm to estimate **satisfaction probability**
- Error margin ε=0.025, confidence bound Δ=0.01

Plasma Lab
Statistical Model-Checker
PRISM
model checker

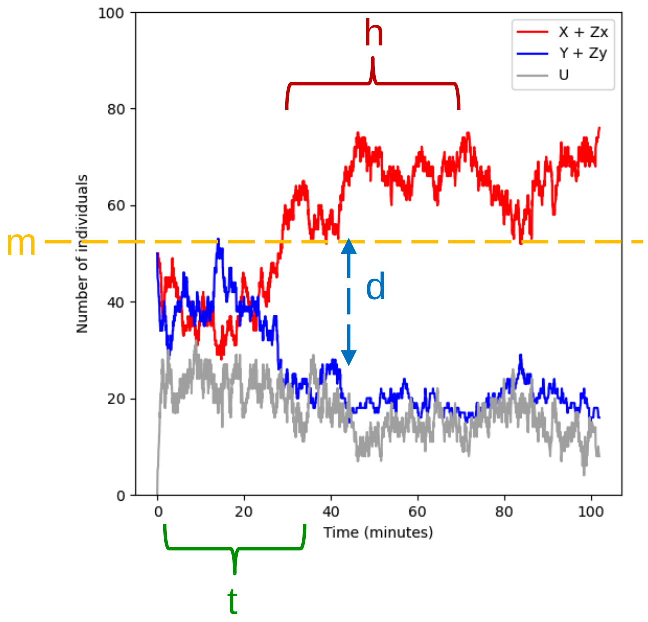
# Results – robustness of stable consensus

- Scenario: N = 100 robots, equivalent options X and Y ($q_x$=$q_y$), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: m=50, d=10, t=35, h=40

Robustness – Zealots




30% zealots → p = 0.629



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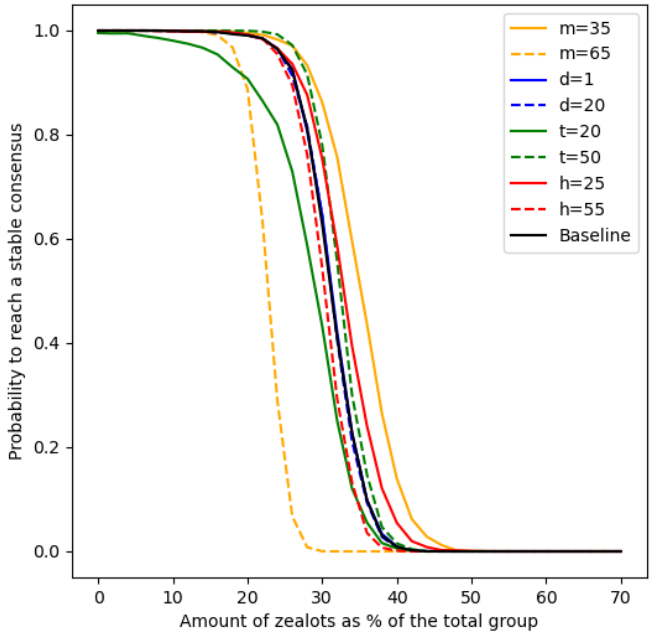
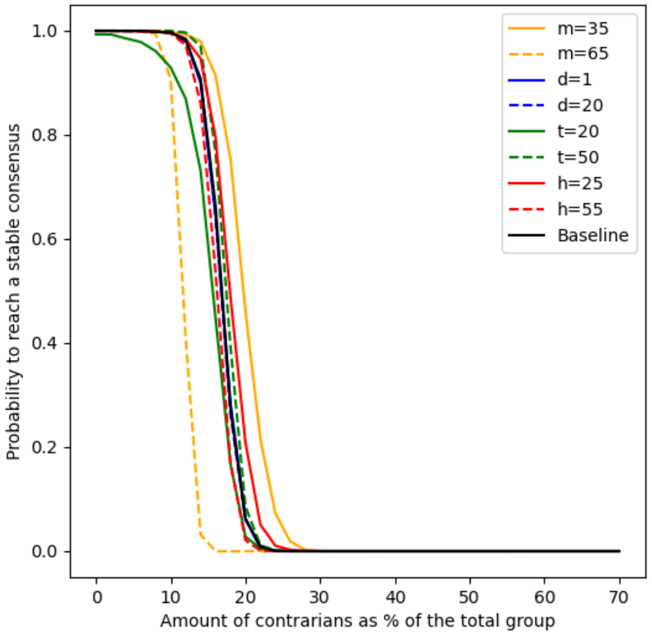
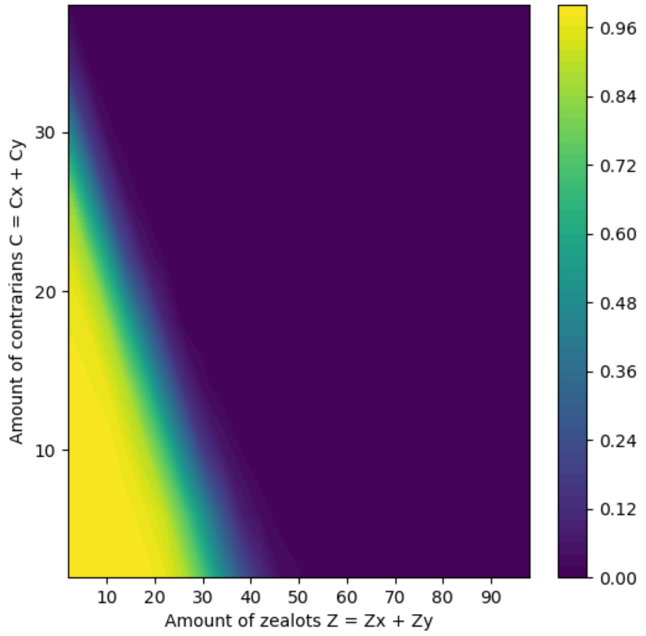
Robustness – Zealots




Robustness – Contrarians



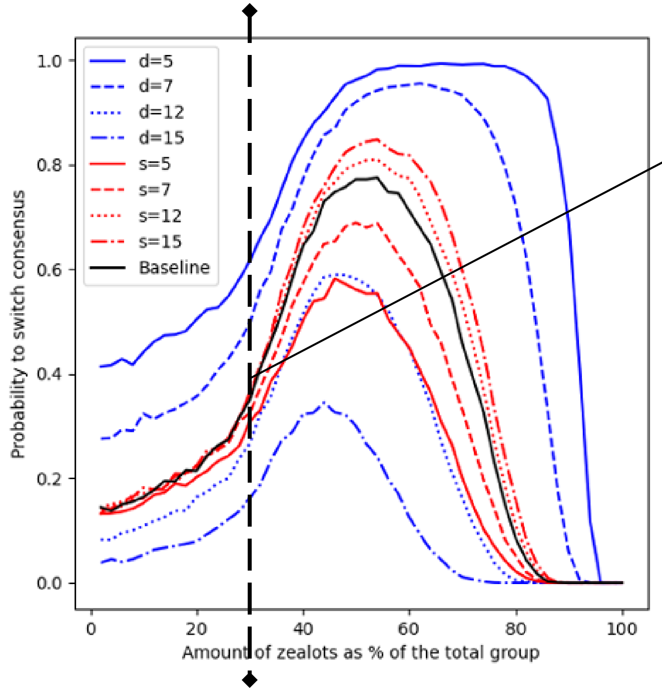

Combined effect



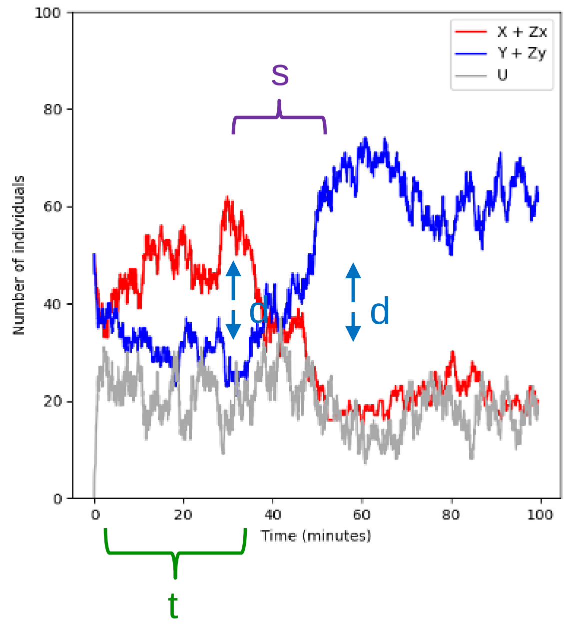
# Results – robustness of switching consensus

- Scenario: N = 100 robots, equivalent options X and Y ($q_x$=$q_y$), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: d=10, t=35, s=10

Robustness – Zealots



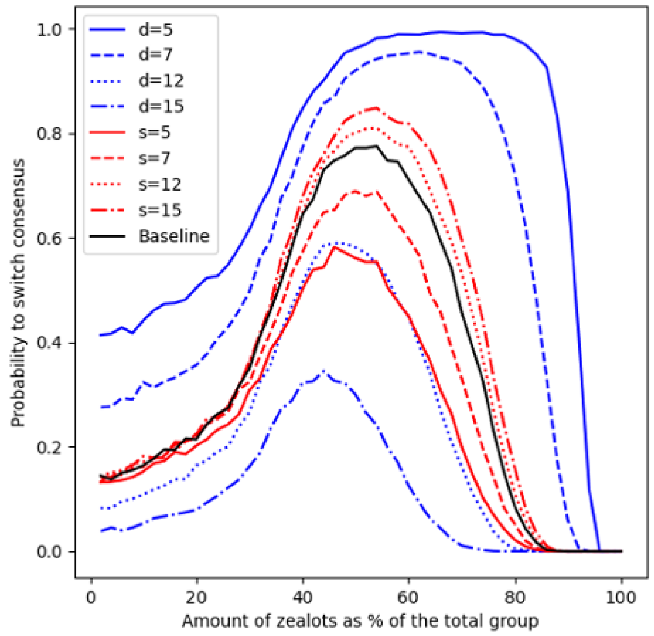

30% zealots → p = 0.349



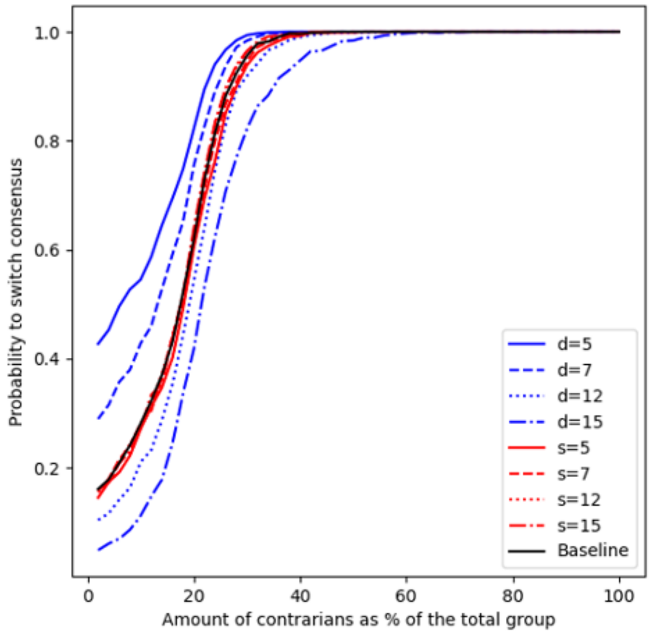
# Results – robustness of switching consensus

- Scenario: N = 100 robots, equivalent options X and Y ($q_x$=$q_y$), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: d=10, t=35, s=10

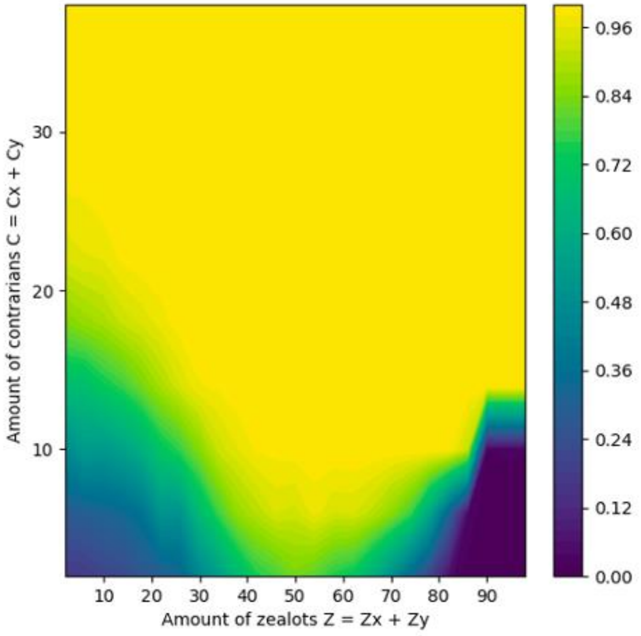
Robustness – Zealots



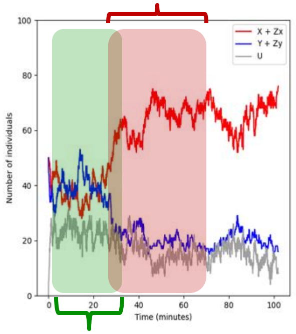

Robustness – Contrarians




Combined effect

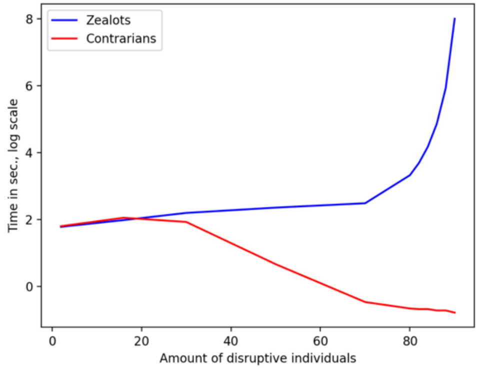


# Results – expected times



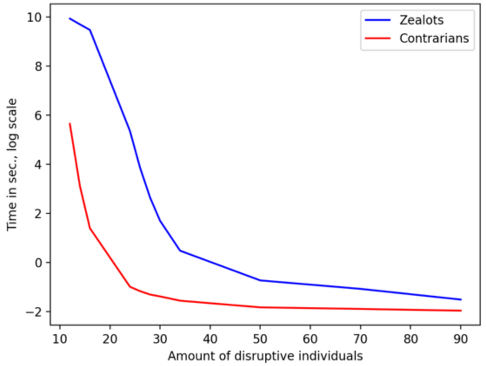

## Expected times to reach consensus

| # | 2 | 16 | 30 | 50 | 70 | 80 | 82 | 84 | 86 | 88 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Zealots | 5.95 | 7.28 | 9.02 | 10.57 | 12.04 | 27.82 | 39.94 | 64.95 | 128.85 | 374.04 | 2975.68 |
| Contrarians | 6.07 | 7.81 | 6.89 | 1.95 | 0.63 | 0.52 | 0.51 | 0.51 | 0.49 | 0.49 | 0.46 |



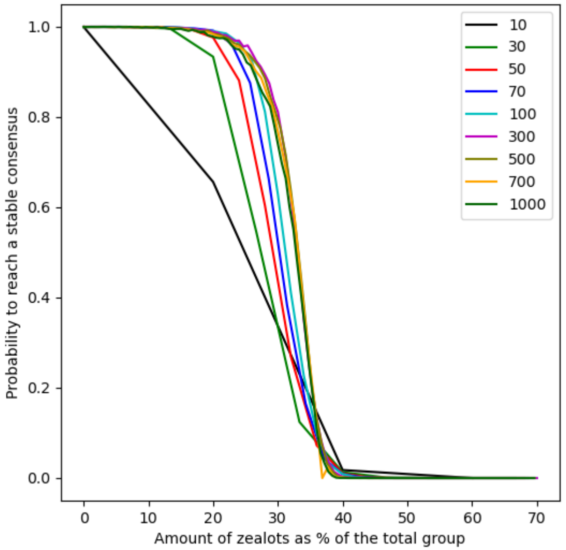

## Expected times to hold consensus

| # | 12 | 14 | 16 | 24 | 26 | 28 | 30 | 34 | 50 | 70 | 90 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Zealots | 20686.51 | 16368.28 | 13047.85 | 210.98 | 47.71 | 14.13 | 5.46 | 1.61 | 0.48 | 0.34 | 0.22 |
| Contrarians | 283.57 | 22.53 | 4.03 | 0.37 | 0.31 | 0.27 | 0.25 | 0.21 | 0.16 | 0.15 | 0.14 |

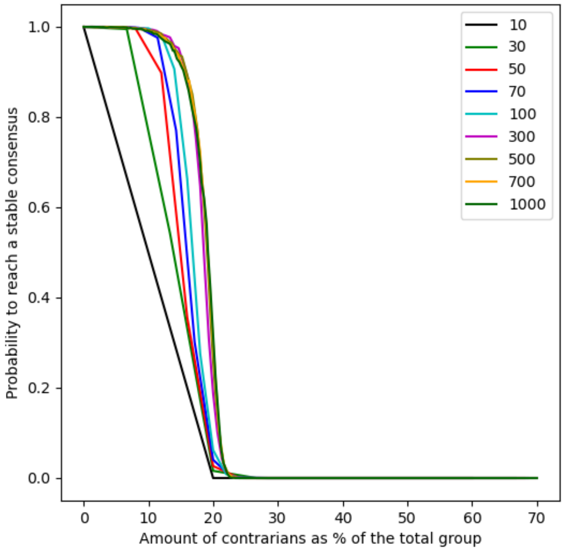


# Results – group size effect

**Stable consensus**

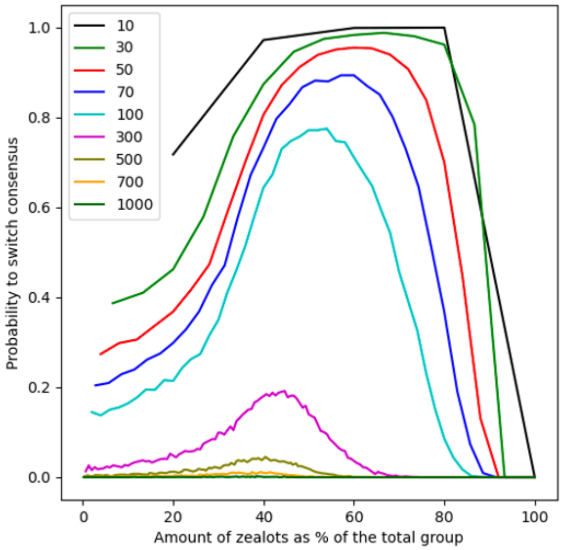



Zealots

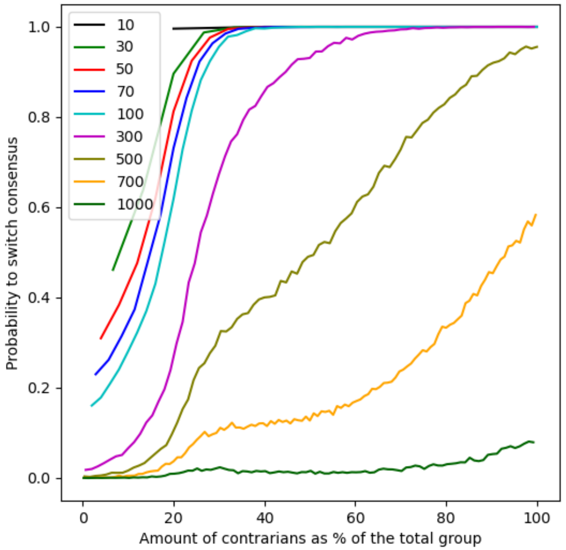



Contrarians

**Switching consensus**




Zealots




Contrarians

… **robust** to group size scaling!

… **sensitive** to group size scaling!

# Conclusion and outlook

- A small increase of disruptive individuals can drastically affect consensus dynamics
- Our method with SMC allows to explore consensus beyond mean-field analysis or single simulation

- **Stable consensus**
  - Cross-inhibition model robust up to certain fraction of zealots/contrarians, then rapid phase transition
  - Zealots are less harmful for reaching consensus than contrarians

- **Switching consensus**
  - Range of zealots for which such trajectories occur with non-negligible probability, but very rare for high number of zealots
  - Contrarians promote switching dynamics

- Future work
  - Group size effect: characterisation of a class of stochastic systems for which consensus reaching is robust to scaling
  - Asymmetric model: what if only one decision is correct?
  - Control theory: interventions over individuals for a global outcome

Centre for the Advanced Study of Collective Behaviour

UNIVERSITÀ DEGLI STUDI DI TRIESTE

MAX-PLANCK-GESELLSCHAFT

Universität Konstanz

# Thank you very much!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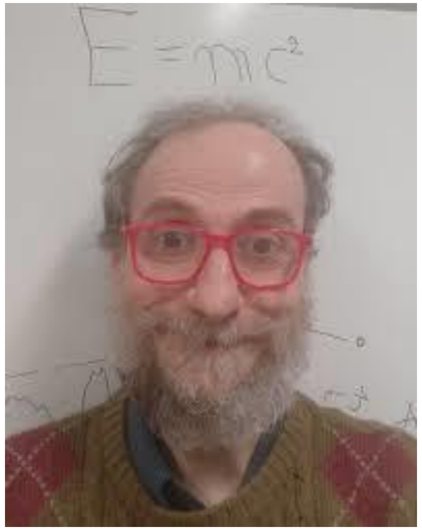
Tatjana Petrov

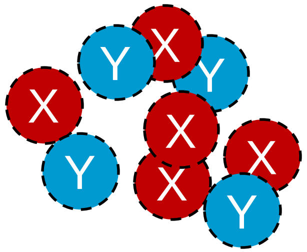



Alberto d‘Onofrio

# Decision making: Voter model

$$(1)\; X + Y \xrightarrow{q_x} X + X$$
$$(2)\; X + Y \xrightarrow{q_y} Y + Y$$

**Swarm state** evolves as a continuous-time Markov chain
→ classical **population model**



# Decision making: Voter model

$$(\mathbf{1})\ \boldsymbol{X} + \boldsymbol{Y} \xrightarrow{\boldsymbol{q_x}} \boldsymbol{X} + \boldsymbol{X}$$

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**Swarm state** evolves as a continuous-time Markov chain
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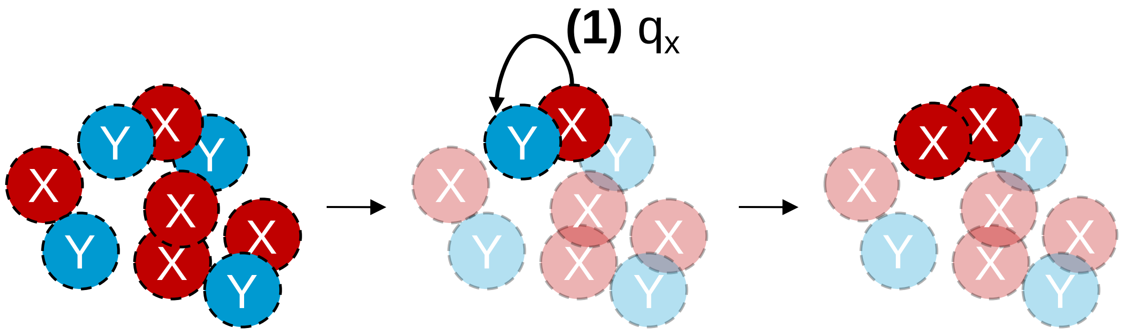
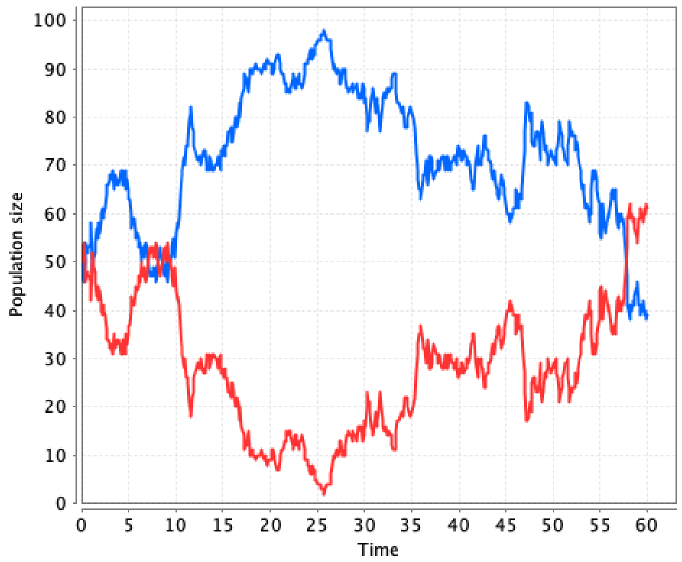


# Decision making: Voter model

$$(1)\ X + Y \xrightarrow{q_x} X + X$$
$$(2)\ \boldsymbol{X + Y \xrightarrow{q_y} Y + Y}$$




**Swarm state** evolves as a continuous-time Markov chain
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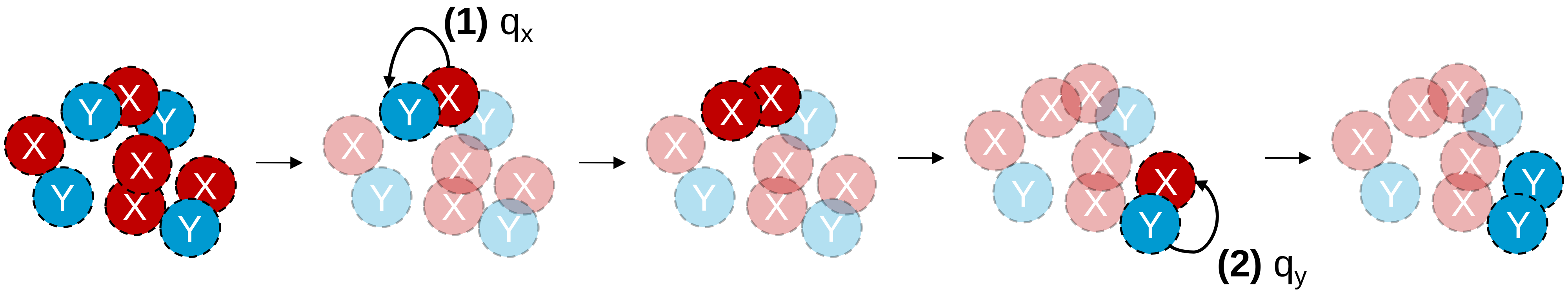
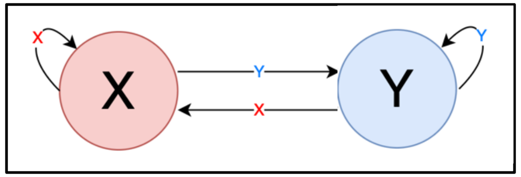


# Decision making: Voter model




$$(1)\ X + Y \xrightarrow{q_x} X + X$$

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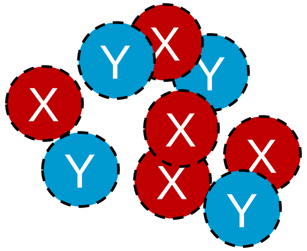
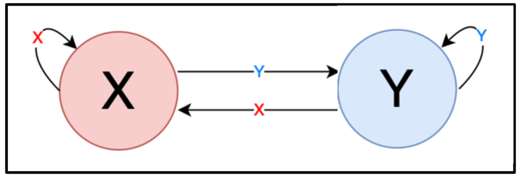


# Decision making: Voter model




$$(1)\ \boldsymbol{X} + \boldsymbol{Y} \overset{\boldsymbol{q_x}}{\rightarrow} \boldsymbol{X} + \boldsymbol{X}$$

$$(2)\ X + Y \overset{q_y}{\rightarrow} Y + Y$$

**Swarm state** evolves as a continuous-time Markov chain
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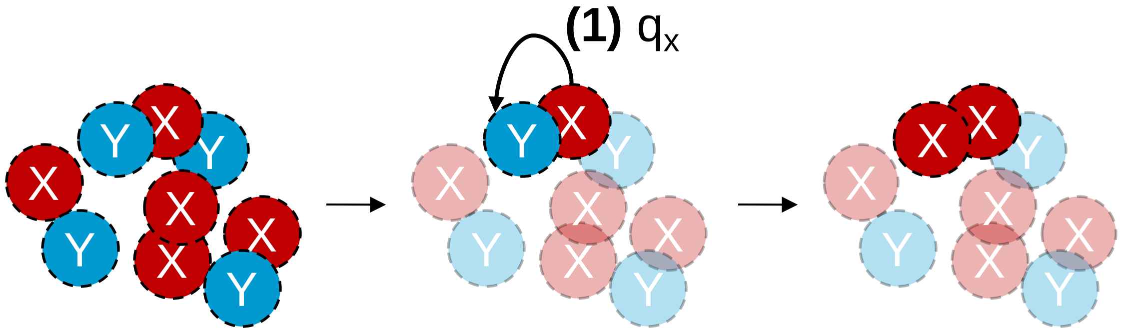
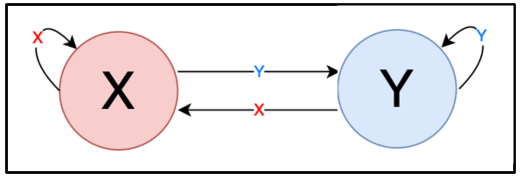
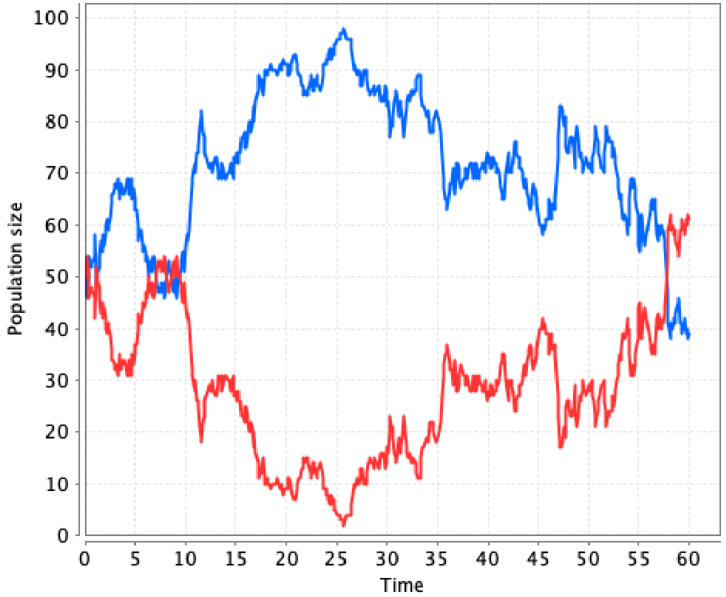


# Decision making: Voter model




$$(1)\ X + Y \xrightarrow{q_x} X + X$$
$$(2)\ \boldsymbol{X + Y \xrightarrow{q_y} Y + Y}$$




**Swarm state** evolves as a continuous-time Markov chain
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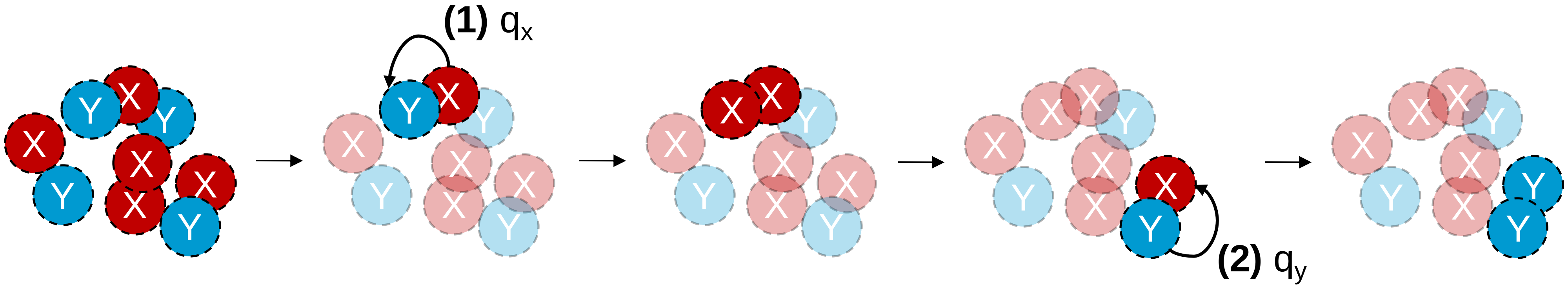
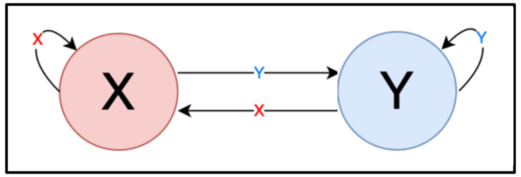


# Decision making: Voter model

$$(1)\ X + Y \overset{q_x}{\rightarrow} X + X$$

$$(2)\ X + Y \overset{q_y}{\rightarrow} Y + Y$$

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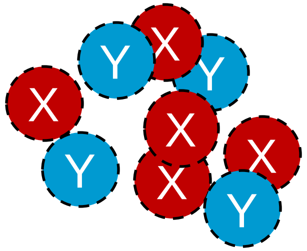
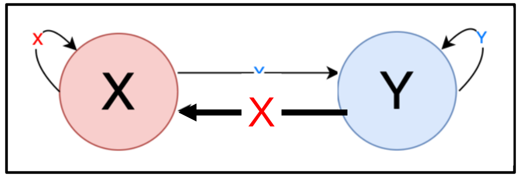


# Decision making: Voter model

$$(1)\ \boldsymbol{X} + \boldsymbol{Y} \xrightarrow{q_x} \boldsymbol{X} + \boldsymbol{X}$$

$$(2)\ X + Y \xrightarrow{q_y} Y + Y$$

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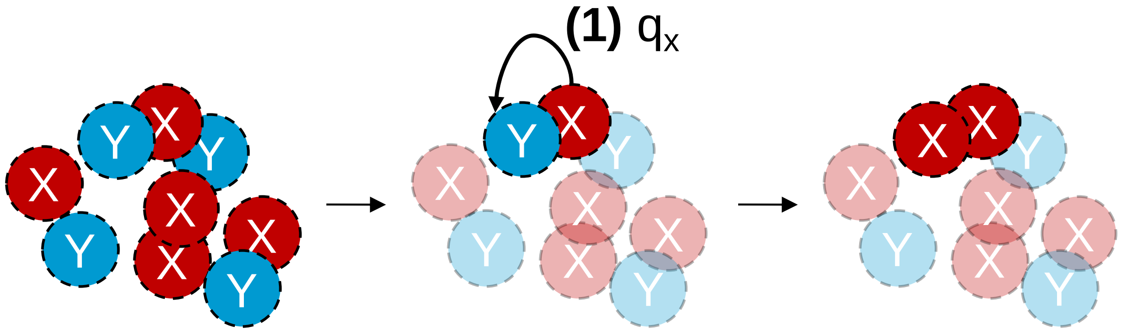

# Decision making: Voter model

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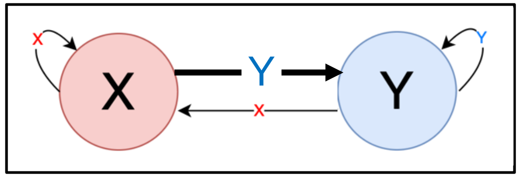


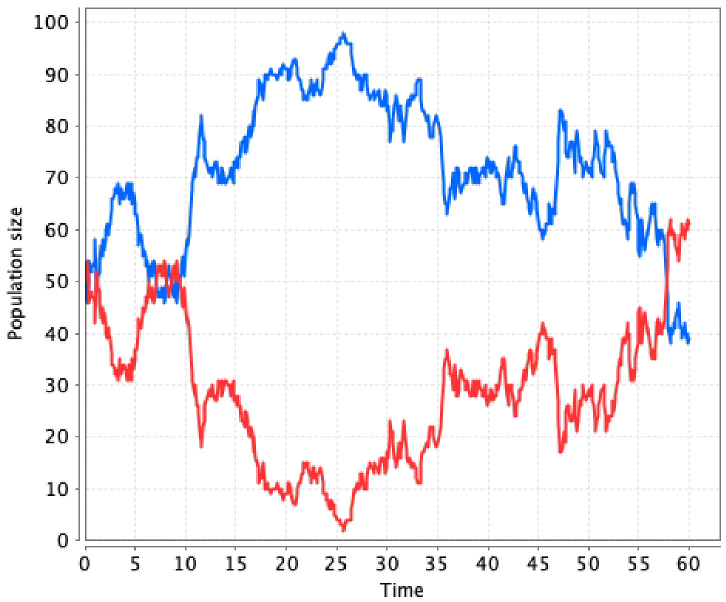


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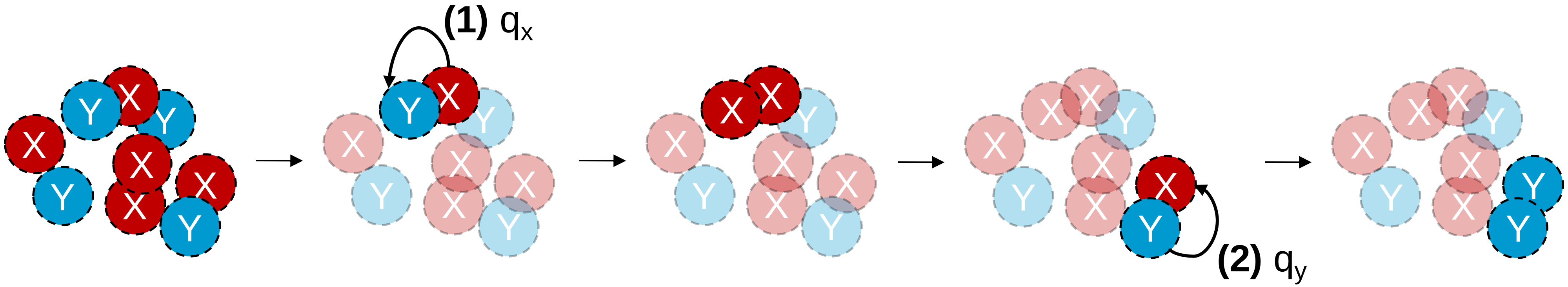
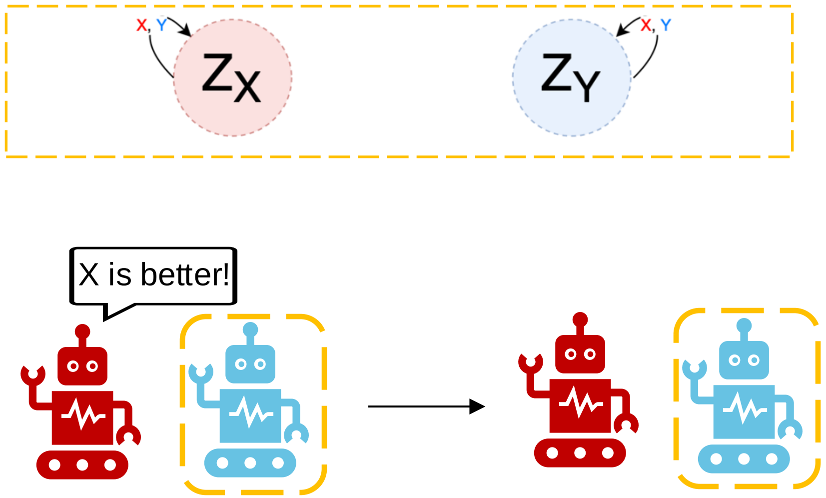


# Disruptive individuals

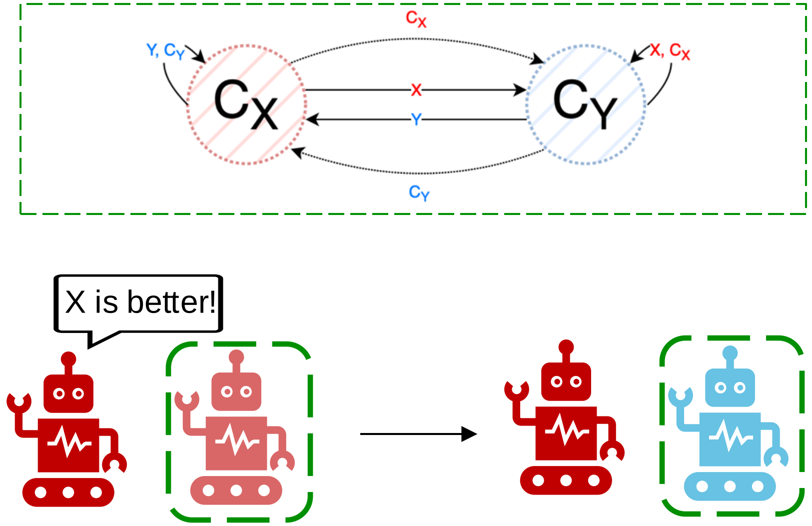
### Zealots

- Stubborn individuals which never change their own opinion

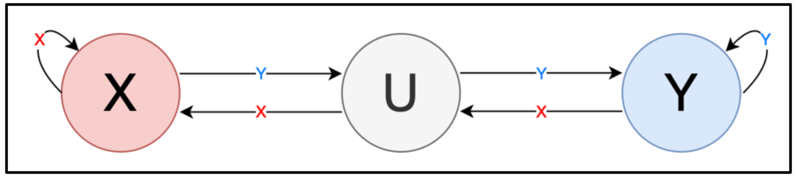



### Contrarians

- Individuals which counter the opinion of the individual they interact with



# Decision making: cross-inhibition model

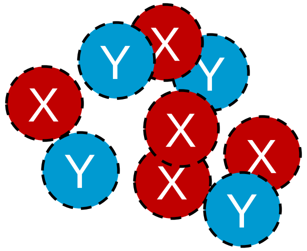



$$(1)\ X + Y \overset{q_x}{\rightarrow} X + U$$

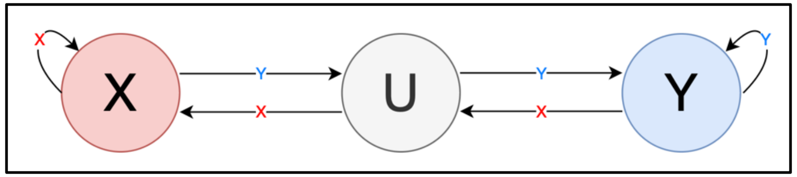
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$

$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$

$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$



# Decision making: cross-inhibition model

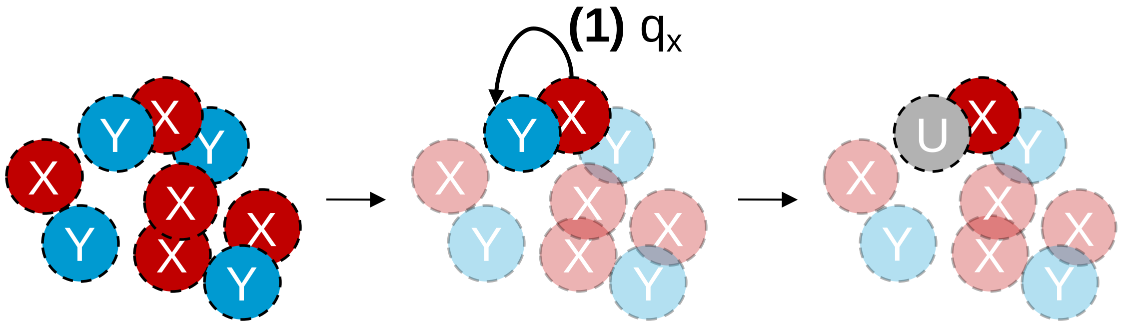



$$(\mathbf{1})\ \boldsymbol{X} + \boldsymbol{Y} \overset{\boldsymbol{q_x}}{\rightarrow} \boldsymbol{X} + \boldsymbol{U}$$

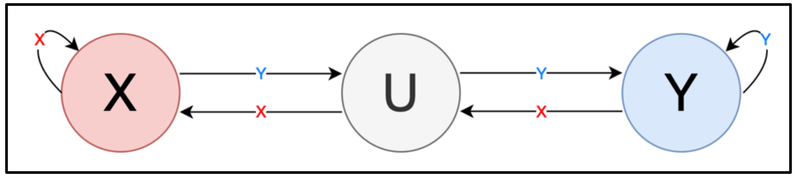
$$(2)\ X + Y \overset{q_y}{\rightarrow} U + Y$$

$$(3)\ X + U \overset{q_x}{\rightarrow} 2X$$

$$(4)\ Y + U \overset{q_y}{\rightarrow} 2Y$$



# Decision making: cross-inhibition model

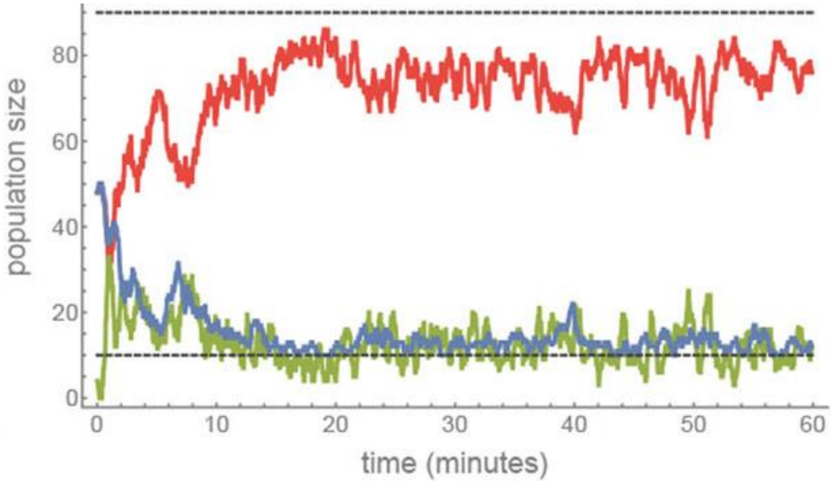


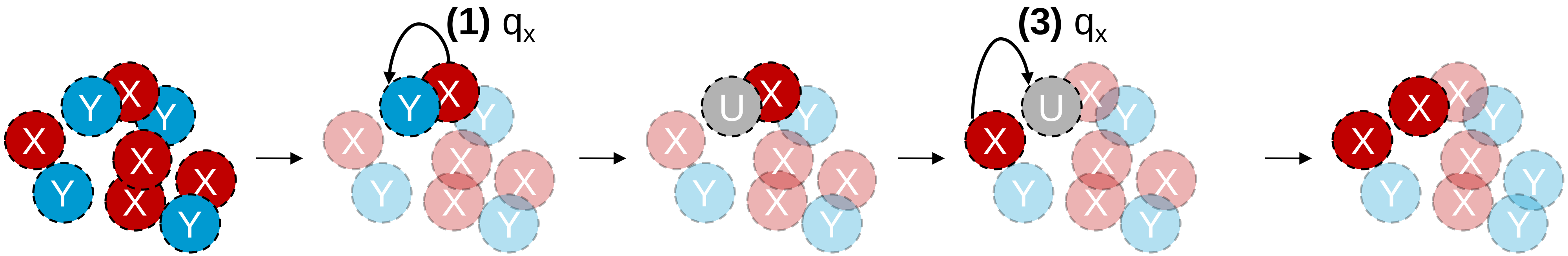

$$(1)\ X + Y \xrightarrow{q_x} X + U$$

$$(2)\ X + Y \xrightarrow{q_y} U + Y$$

$$(3)\ \boldsymbol{X + U \xrightarrow{q_x} 2X}$$

$$(4)\ Y + U \xrightarrow{q_y} 2Y$$



# Approach

**Statistical Model Checking** of properties in **Bounded Linear Temporal Logic (BLTL)**

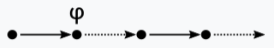
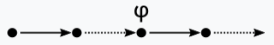
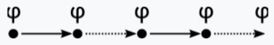
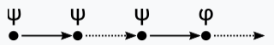
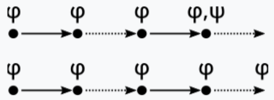
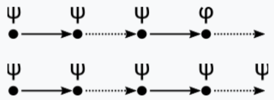
- based on **Linear Temporal Logic (LTL)**

$$\varphi ::= true \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$$

where $a \in AP$

derived operators:

$\vee, \rightarrow, \ldots$ as usual

| Textual | Symbolic | Explanation | Diagram |
|---|---|---|---|
| Unary operators: | | | |
| **X** $\phi$ | $\bigcirc\varphi$ | ne**X**t: $\phi$ has to hold at the next state. |  |
| **F** $\phi$ | $\Diamond\varphi$ | **F**inally: $\phi$ eventually has to hold (somewhere on the subsequent path). |  |
| **G** $\phi$ | $\Box\varphi$ | **G**lobally: $\phi$ has to hold on the entire subsequent path. |  |
| Binary operators: | | | |
| $\psi$ **U** $\phi$ | $\psi\,\mathcal{U}\,\varphi$ | **U**ntil: $\psi$ has to hold *at least* until $\phi$ becomes true, which must hold at the current or a future position. |  |
| $\psi$ **R** $\phi$ | $\psi\,\mathcal{R}\,\varphi$ | **R**elease: $\phi$ has to be true until and including the point where $\psi$ first becomes true; if $\psi$ never becomes true, $\phi$ must remain true forever. |  |
| $\psi$ **W** $\phi$ | $\psi\,\mathcal{W}\,\varphi$ | **W**eak until: $\psi$ has to hold *at least* until $\phi$; if $\phi$ never becomes true, $\psi$ must remain true forever. |  |

- **BLTL: (F, G, X, U, W)** are bounded by temporal bound

$\mathbf{F}_{\leq t}\,\phi$

$\mathbf{G}_{\leq t}\,\phi$

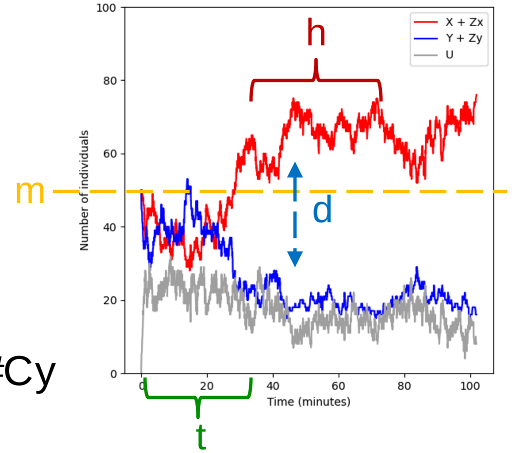
$\mathbf{X}_{\leq t}\,\phi$

$\mathbf{U}_{\leq t}\,\phi$

$\mathbf{W}_{\leq t}\,\phi$

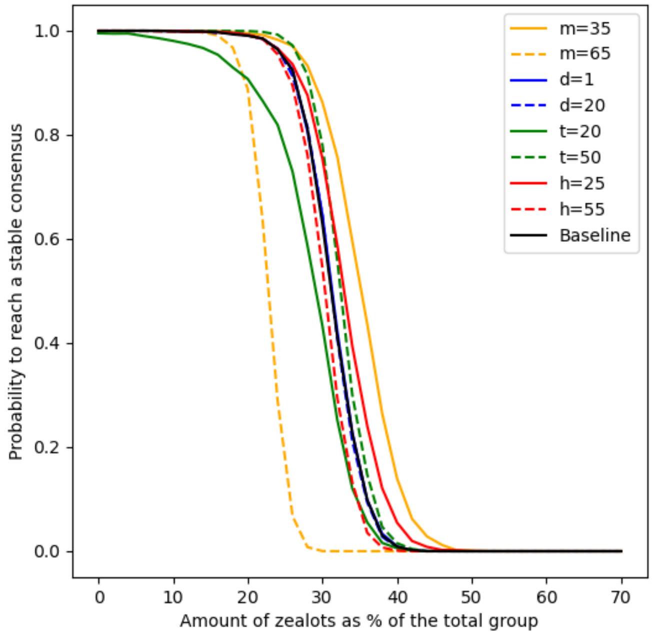
*Boyer, B., Corre, K., Legay, A., Sedwards, S.: Plasma-lab: A flexible, distributable statistical model checking library. In: International Conference on Quantitative Evaluation of Systems. pp. 160–164. Springer (2013)*

# Results – robustness of stable consensus

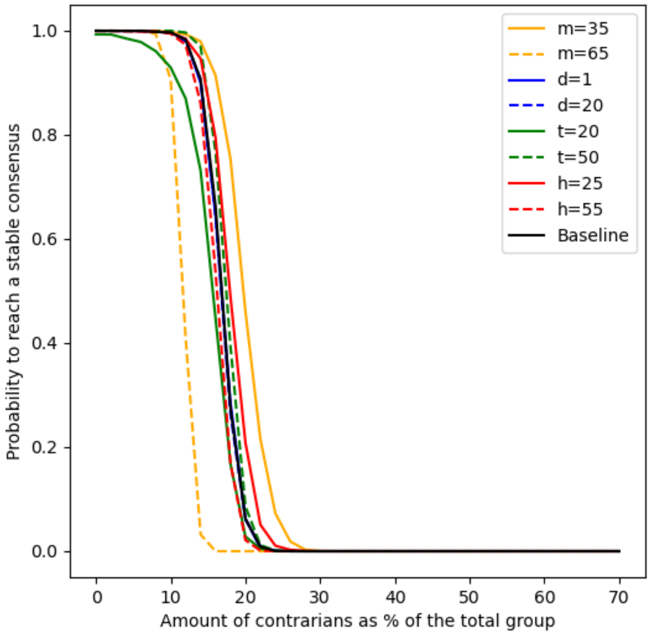



- Scenario: N = 100 robots, equivalent options X and Y ($q_x$=$q_y$), initially #X=#Y, #U=0, #Zx=#Zy, #Cx=#Cy
- Baseline: m=50, d=10, t=35, h=40

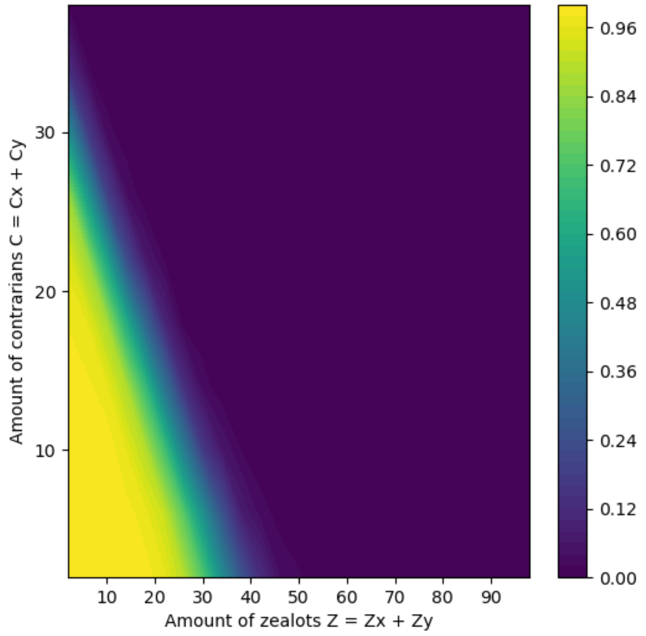
Robustness – Zealots

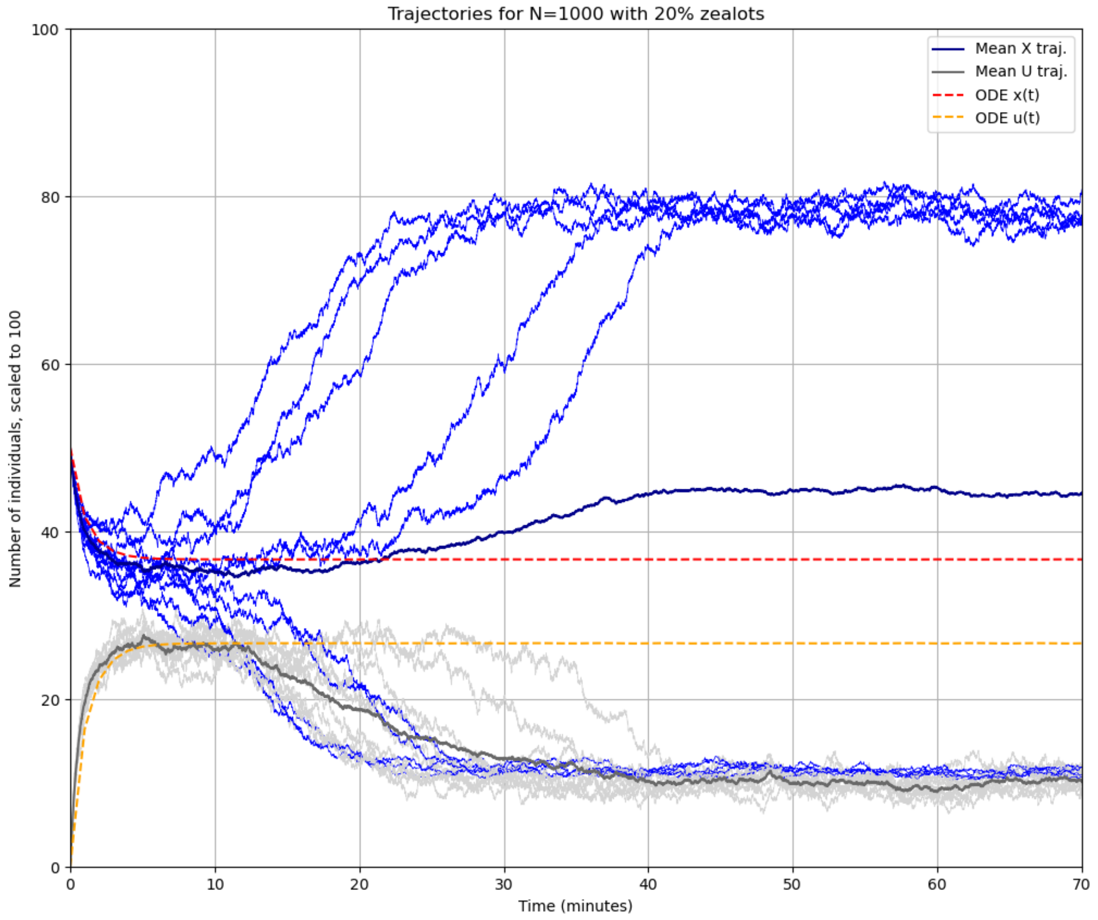



Robustness – Contrarians

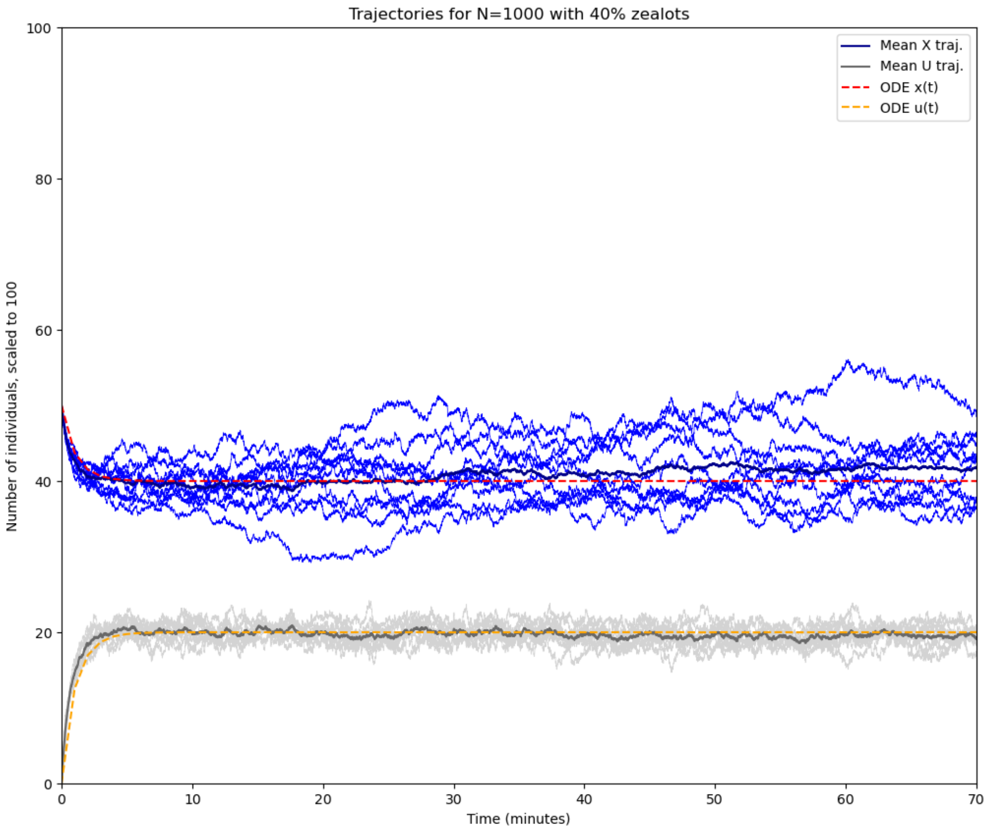



Combined effect





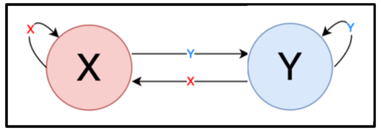
Trajectories for N=1000 with 20% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)
0
20
40
60
80
100
0
10
20
30
40
50
60
70

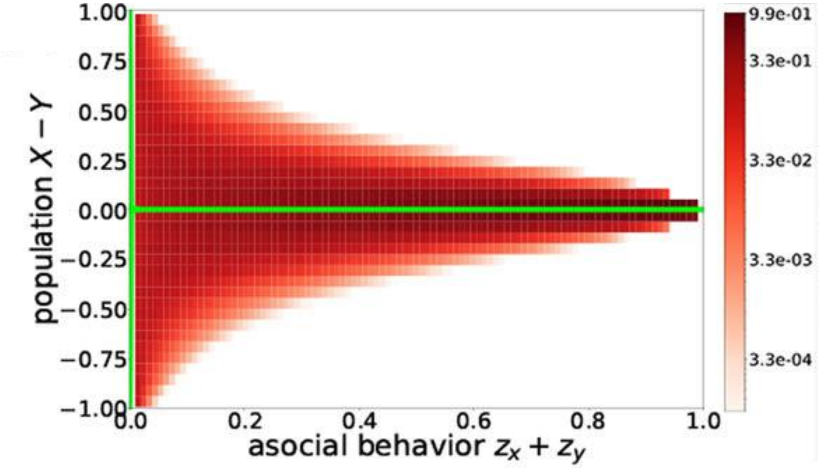



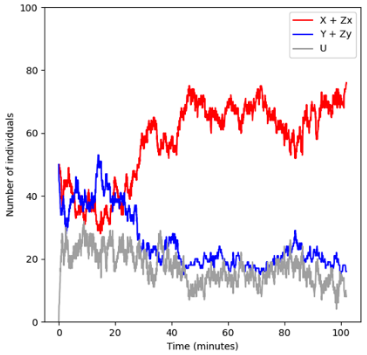
Trajectories for N=1000 with 40% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)
0
20
40
60
80
100
0
10
20
30
40
50
60
70

# Studied Model of Decision-Making

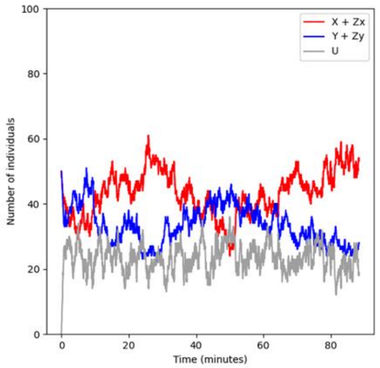
**Voter Model**




**No zealots**
**→ quick, stable consensus**



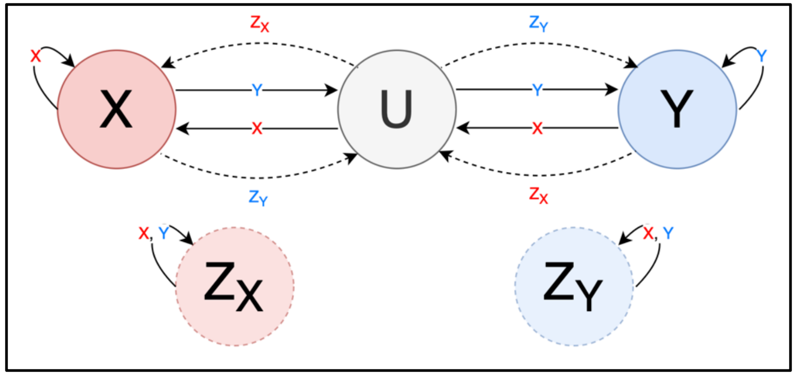

**2% zealots**
**→ permanent indecision**

→ in presence of asocial individuals, the swarm gets quickly locked into an indecision state

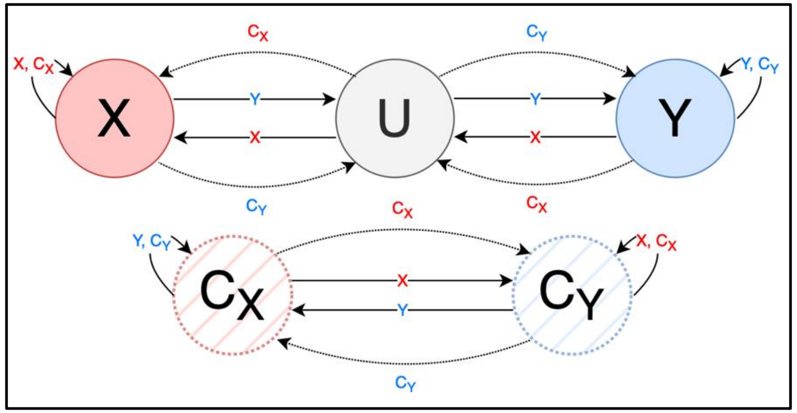

*Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)*

# Studied Model with Disruptive Individuals

### Cross-Inhibition model with Zealots




### Cross-Inhibition model with Contrarians




- **Zealots**: stubborn individuals which never change their own opinion
- Four additional reactions, where ‘pure’ agents interact with zealots & adjust their own states

- **Contrarians**: individuals which counter the opinion of the individual they interact with
- Eight additional reactions, where contrarians influence ‘pure’ individuals & are influenced by others with the same opinion