

Exploring Consensus Robustness In Swarms with Disruptive Individuals

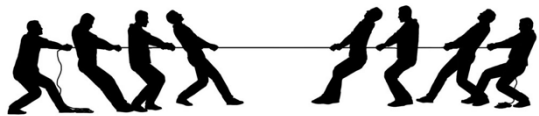
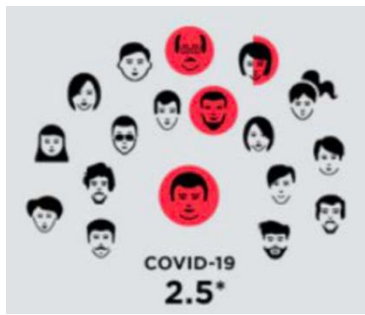
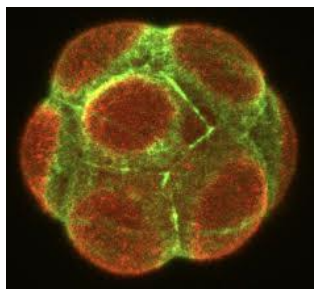
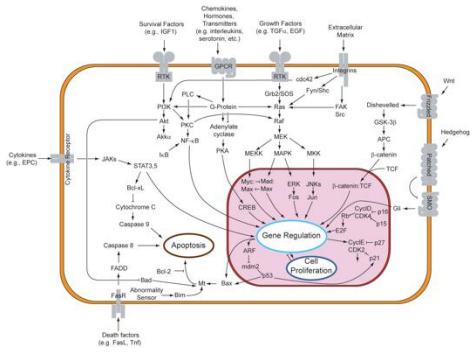


Julia Klein, Tatjana Petrov, Alberto d'Onofrio

University of Konstanz

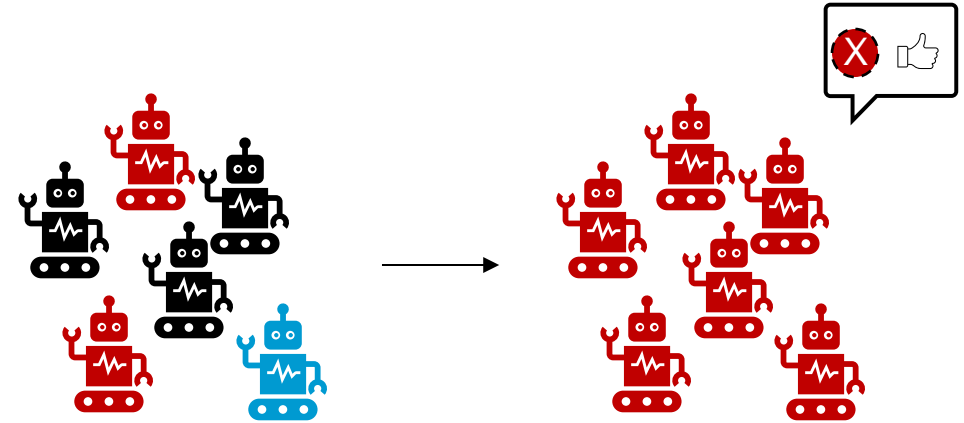
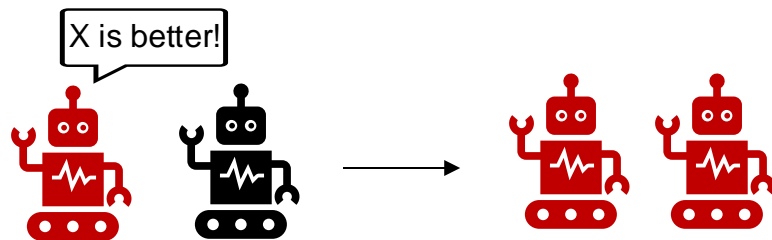
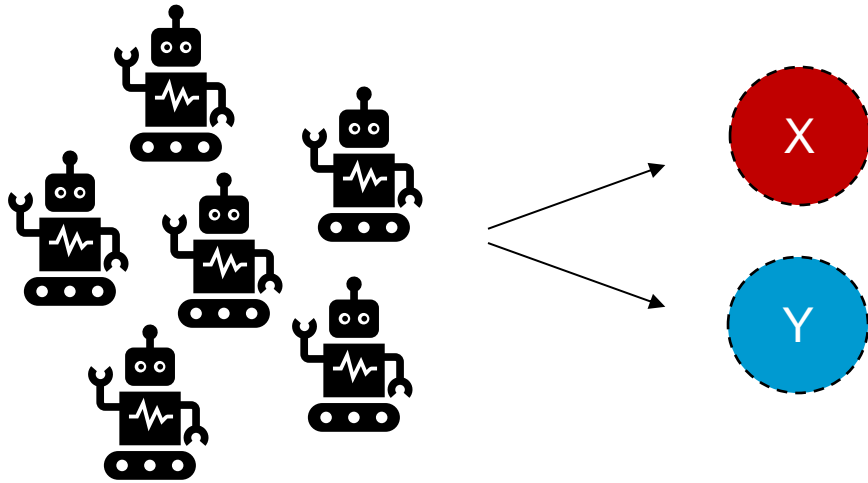
Centre for the Advanced Study of Collective Behaviour

Swarms (collectives)



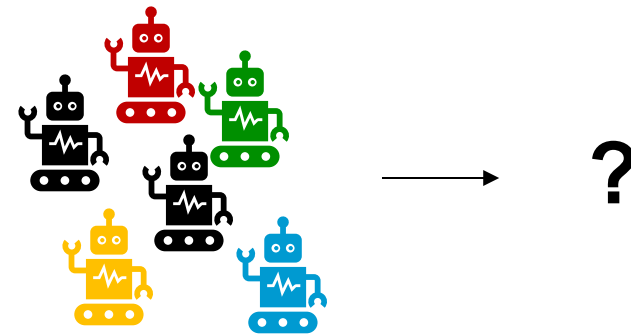
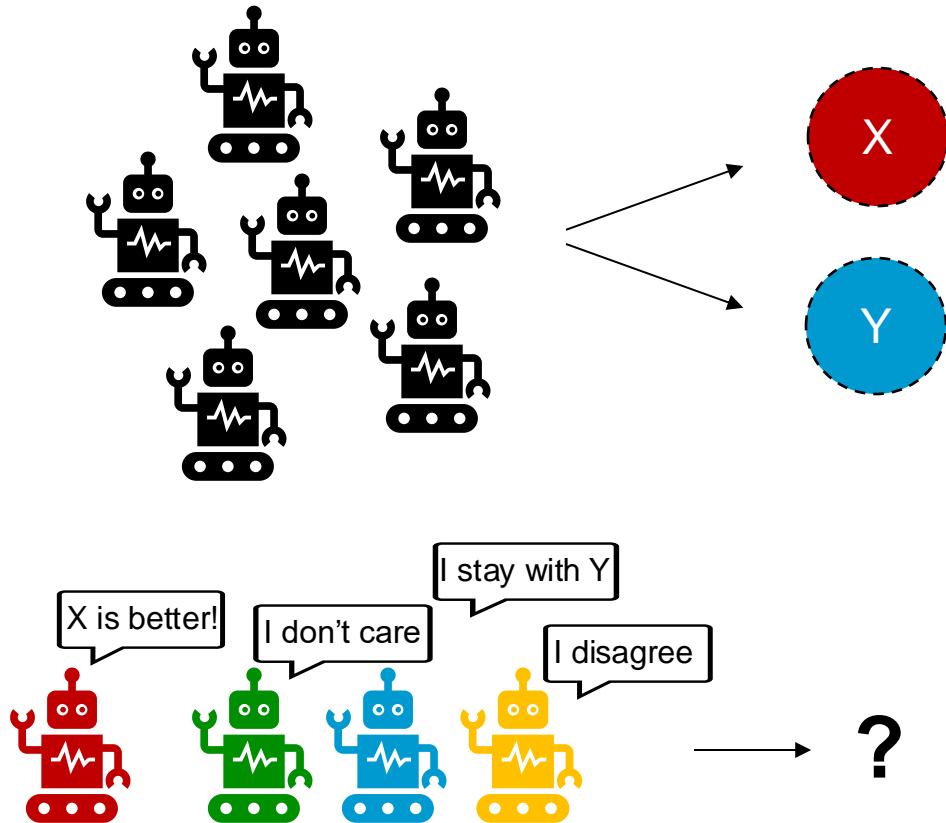
→ How do swarms agree on decisions?

Collective decision making



Ideally, consensus will be achieved with certain speed and accuracy

Collective decision making

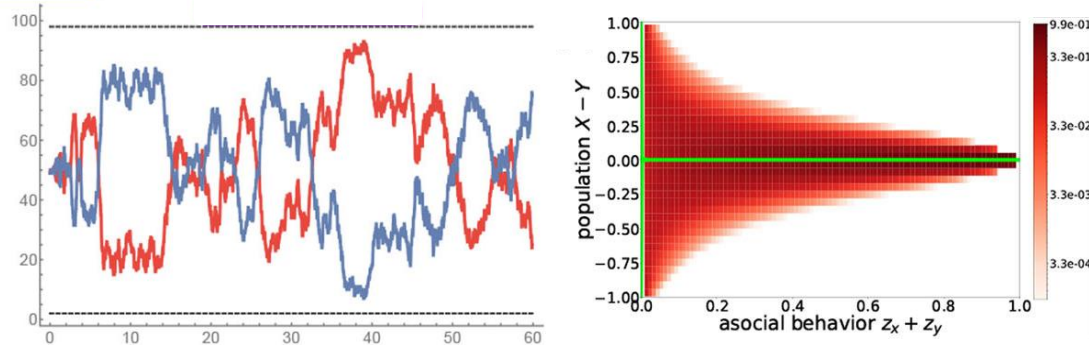


What happens in presence of disruptive (asocial) individuals?

Collective decision making – disruptive individuals

Voter model with zealots

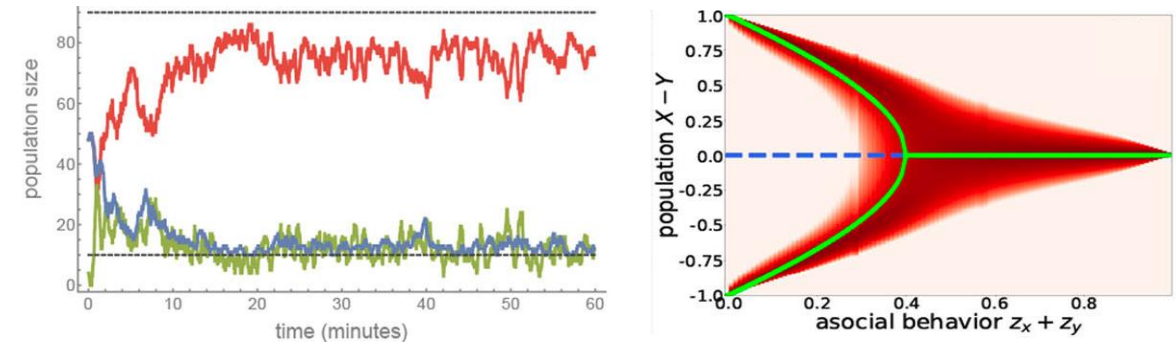
2 species, 2 reactions



- **Permanent indecision** with already 4% zealots
- Swarm gets quickly locked into indecision state

Cross-inhibition model with zealots

3 species, 4 reactions

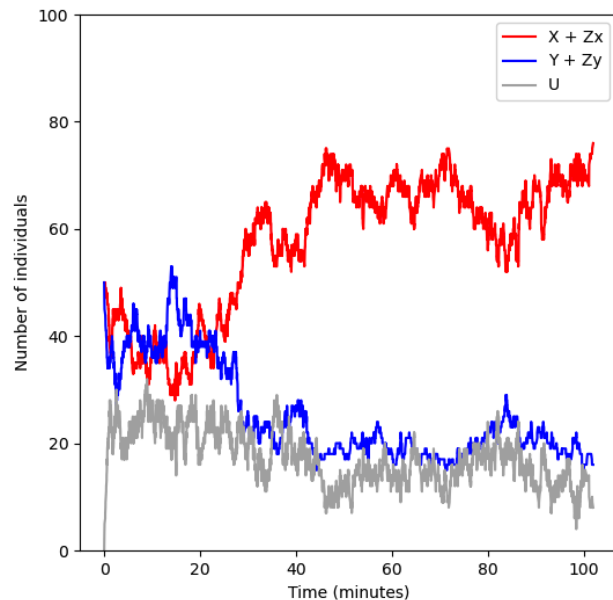


- **Stable dynamics** for 20% zealots!
- Swarm demonstrates resilience against relatively high levels of asocial behaviour

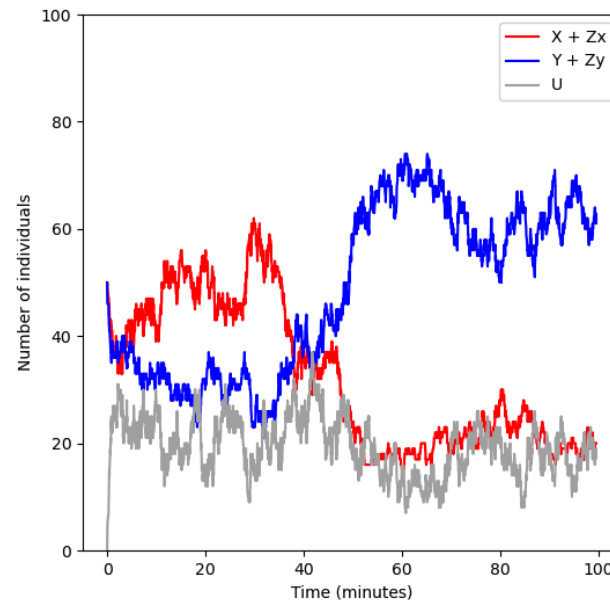
Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. *Communications Physics* 6(1), 236 (2023)

Motivation – group dynamics

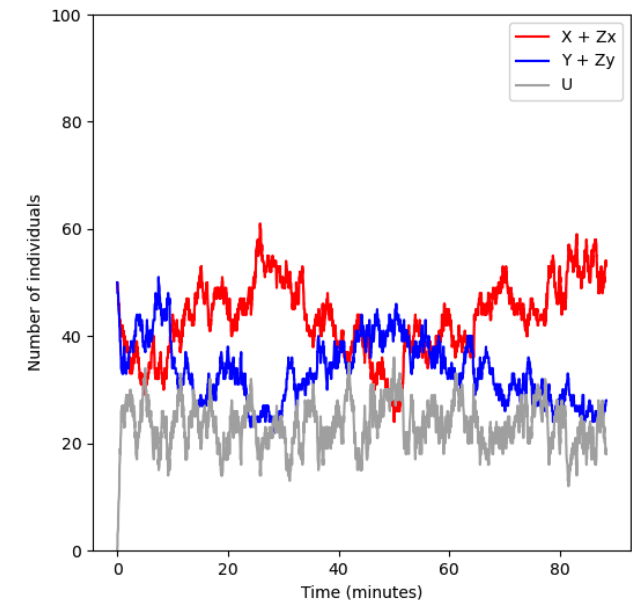
Stochastic **cross-inhibition model with 30% zealots** gives three qualitatively different scenarios!



stable consensus

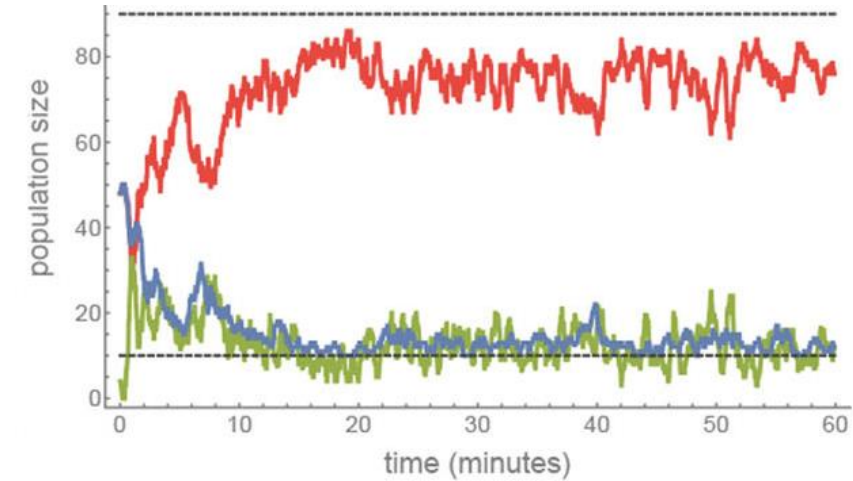
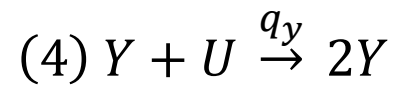
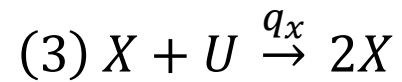
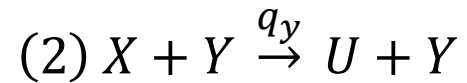
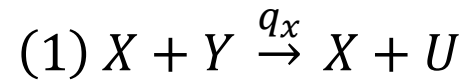


switching consensus

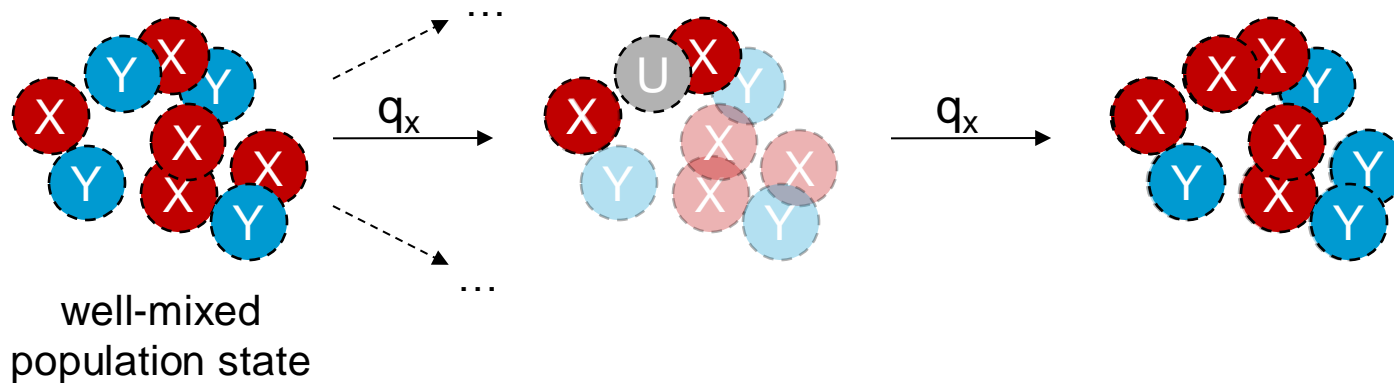


no consensus

Cross-inhibition model



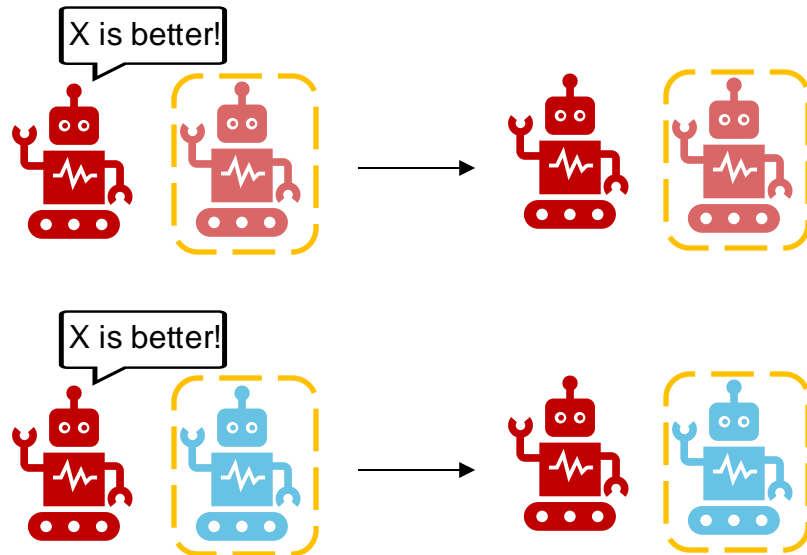
Swarm state evolves **stochastically** as a continuous-time Markov chain



Disruptive individuals

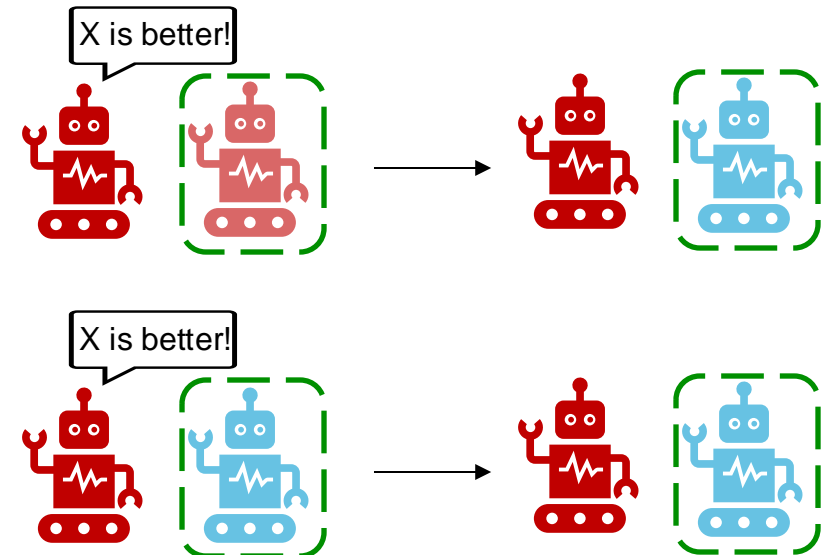
Zealots

Stubborn individuals which never change their own opinion



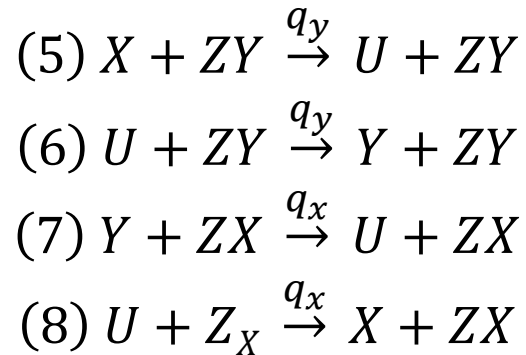
Contrarians

Individuals which counter the opinion of the individual they interact with

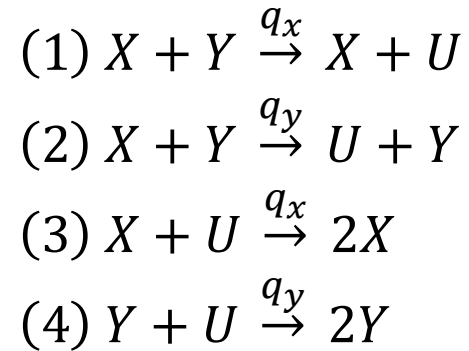


Cross-inhibition model with disruptive individuals

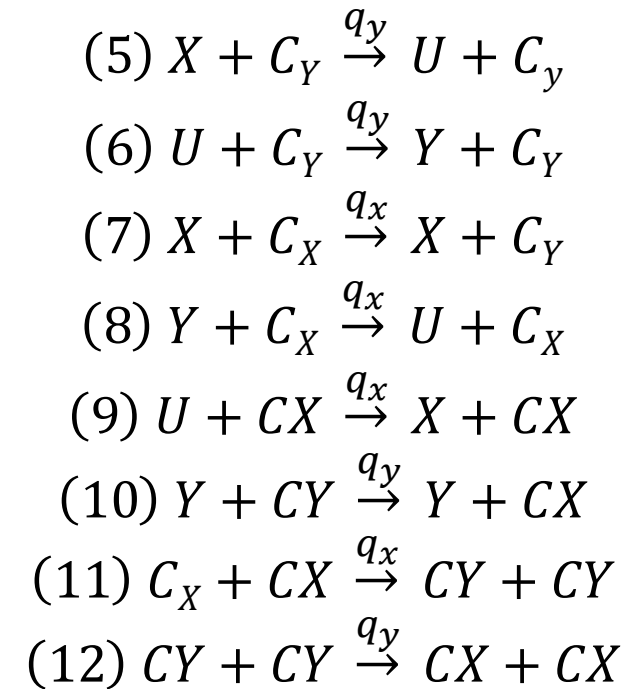
Zealot dynamics



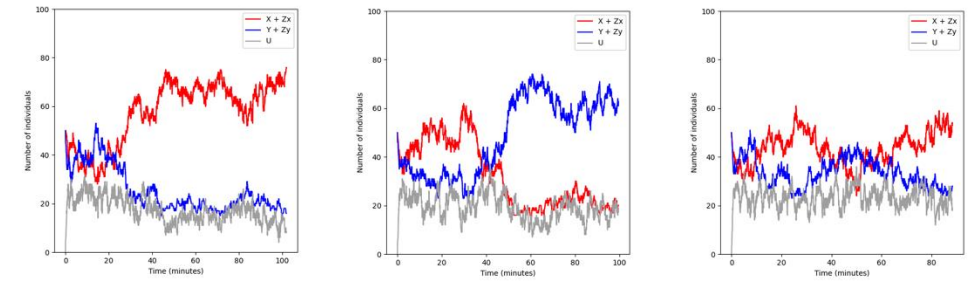
Cross-inhibition model



Contrarian dynamics



Research Questions



1. Robustness analysis

How does the **amount** of disruptive individuals affect consensus reaching/switching?

2. Combined effect

How does the **combination** of zealots and contrarians affect consensus reaching/switching?

3. Group size effect

How does the **group size** affect consensus reaching/switching?

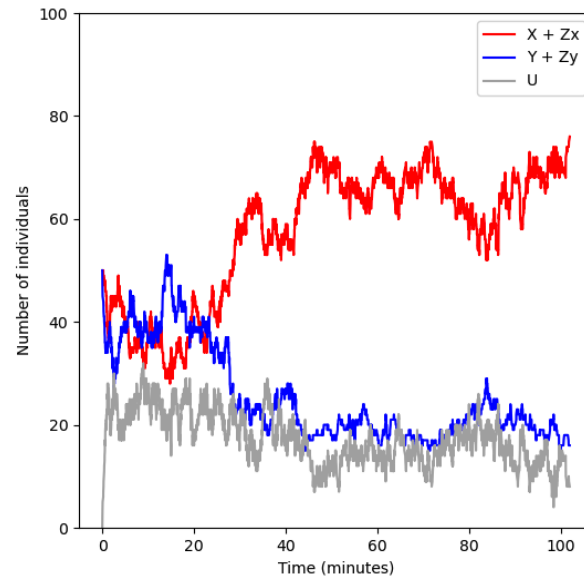
Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and switching consensus in BLTL

- Five parameters: majority m , distance d , reaching time t , holding time h , switching time s

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq \min_m) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq \min_m) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))))))$$



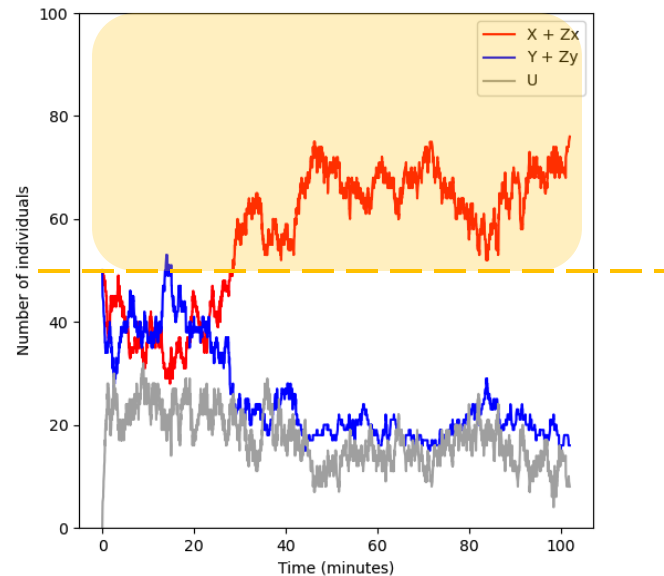
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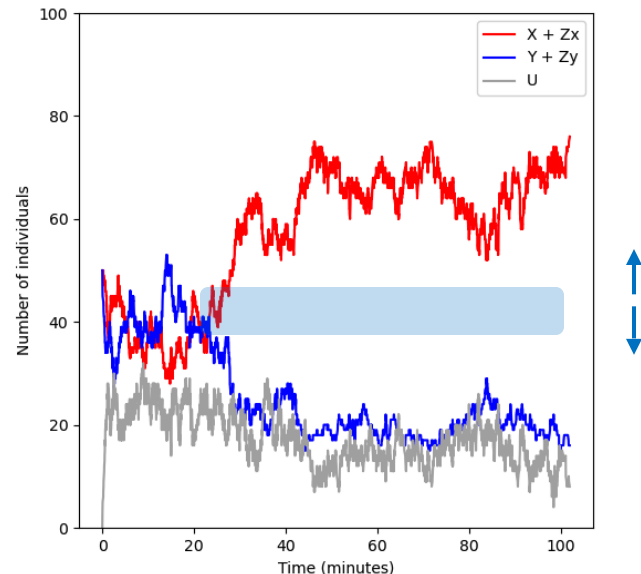
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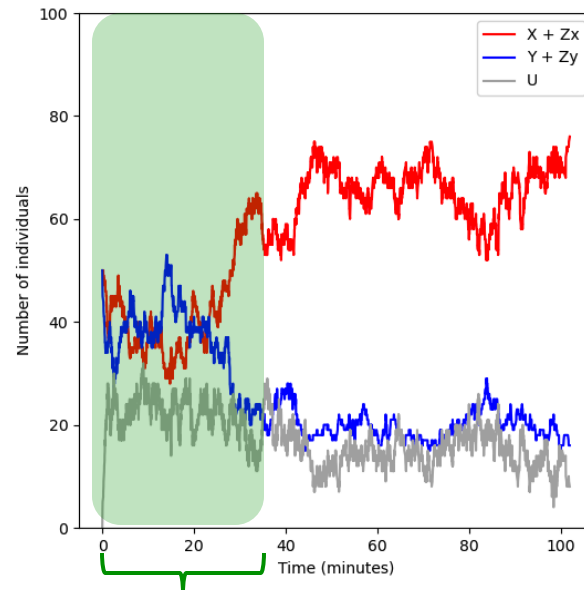
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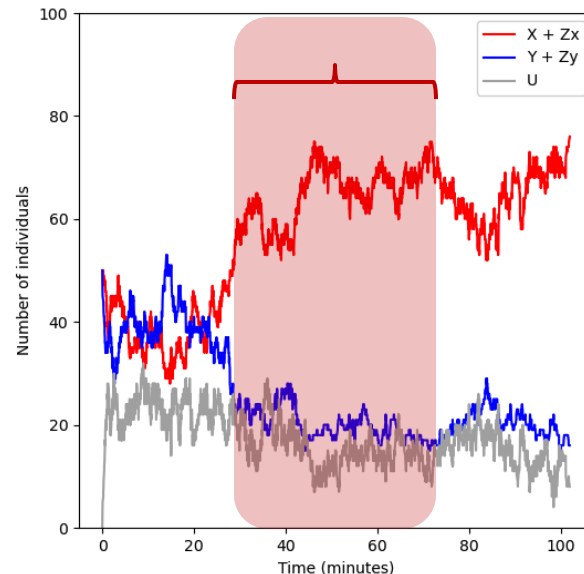
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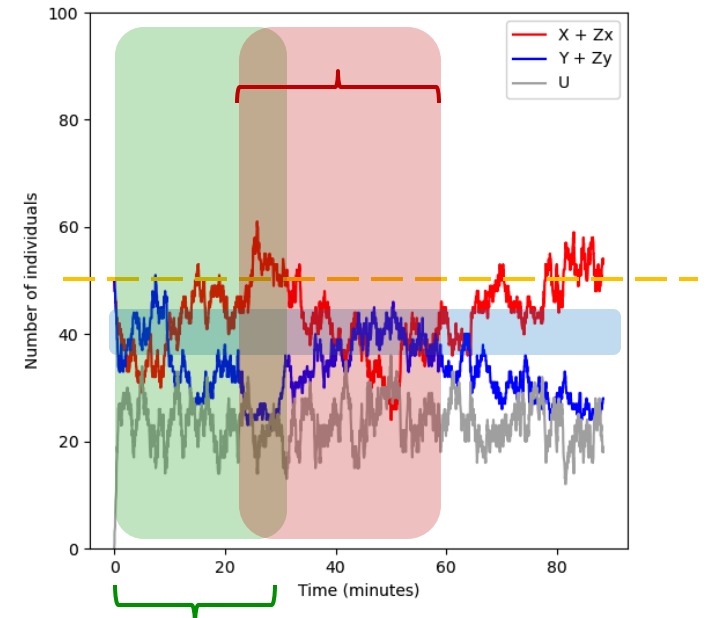
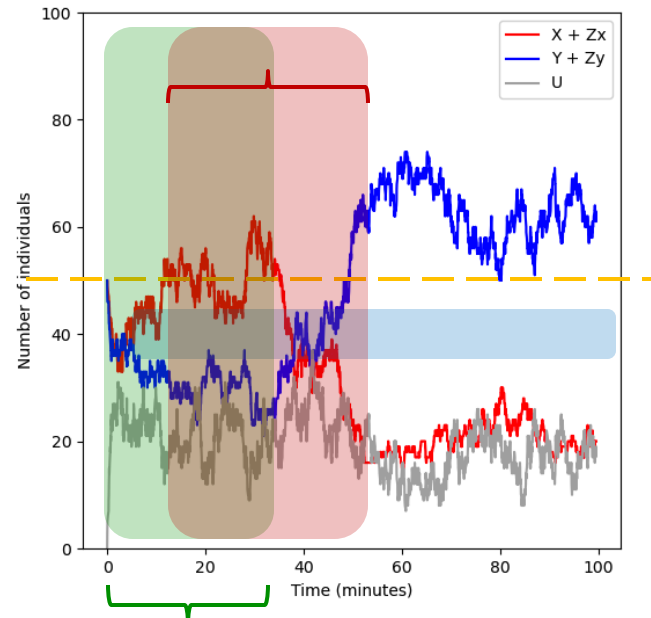
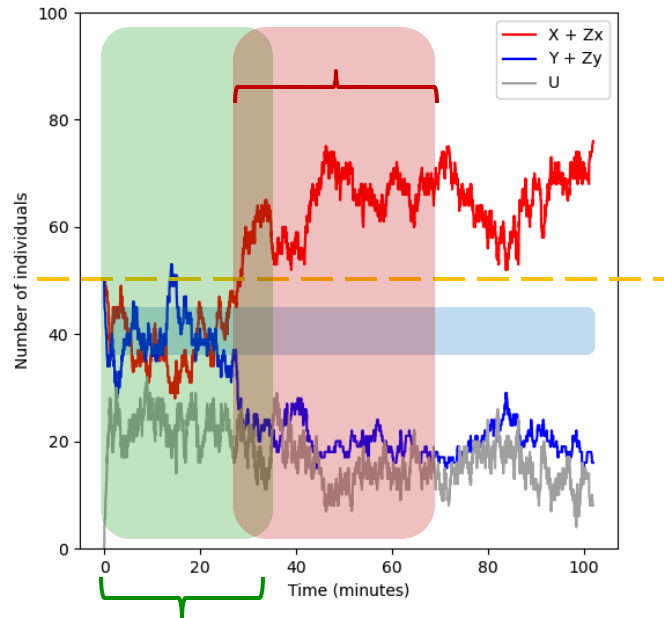
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Approach

Is this a stable consensus?



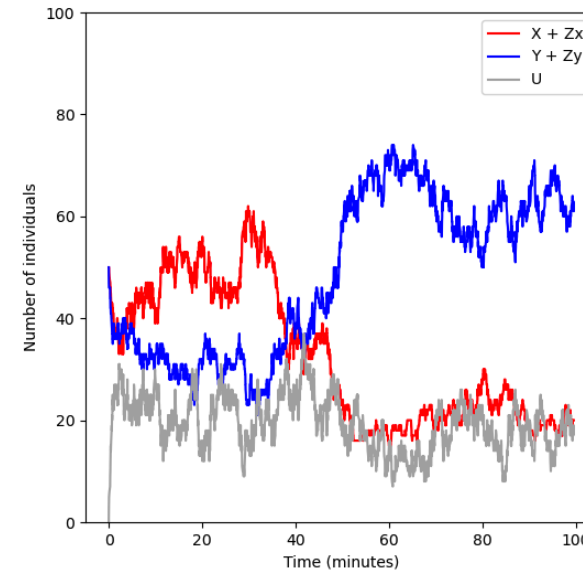
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Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

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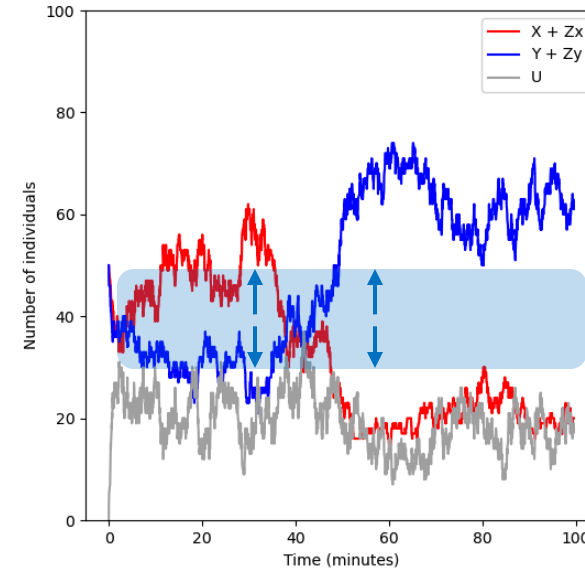
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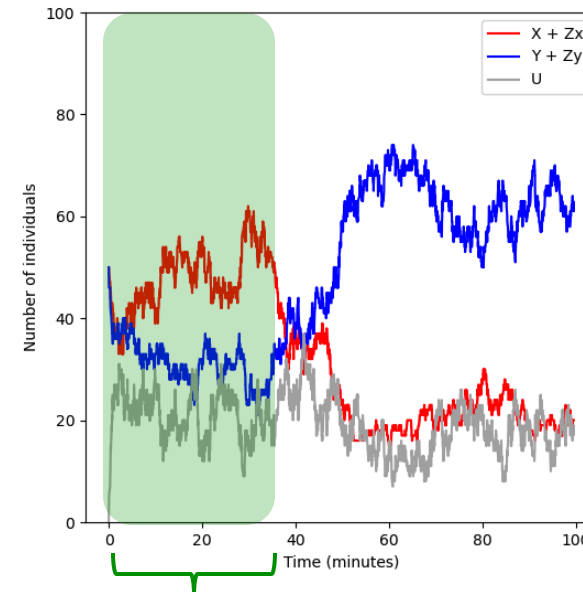
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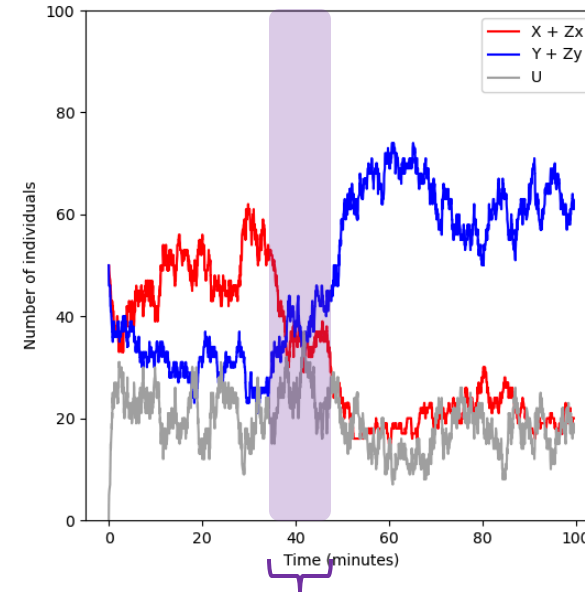
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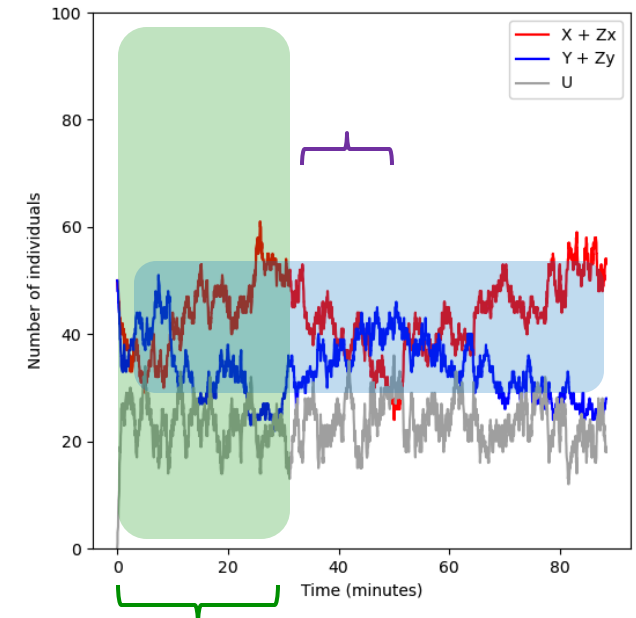
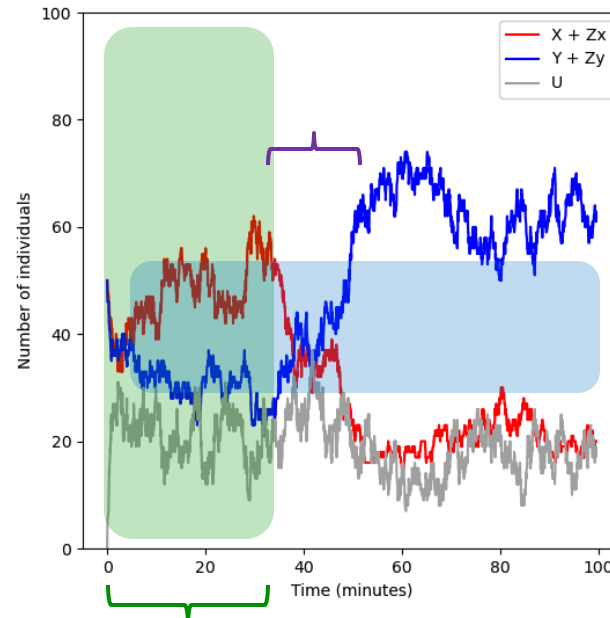
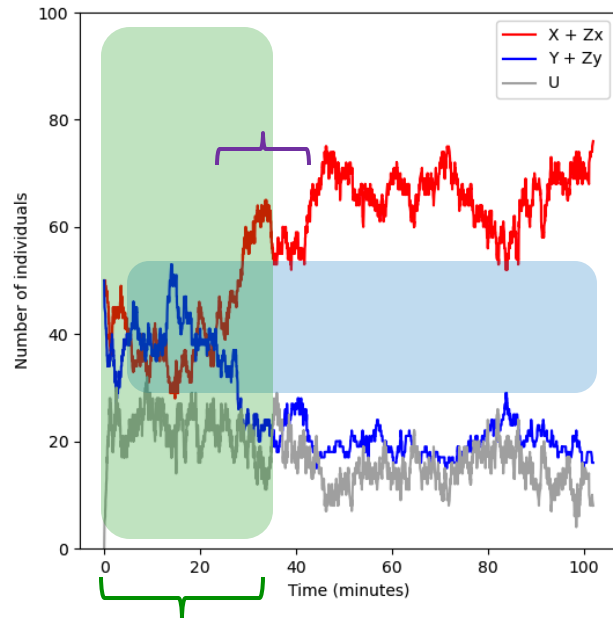
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Approach

Is this a switching consensus?



Approach

Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and **switching consensus** in BLTL

- Five parameters: majority m , distance d , reaching time t , holding time h , switching time s

STEP 2: Apply model checking tools (*PRISM* and *PlasmaLab*) to explore the relevant scenarios:

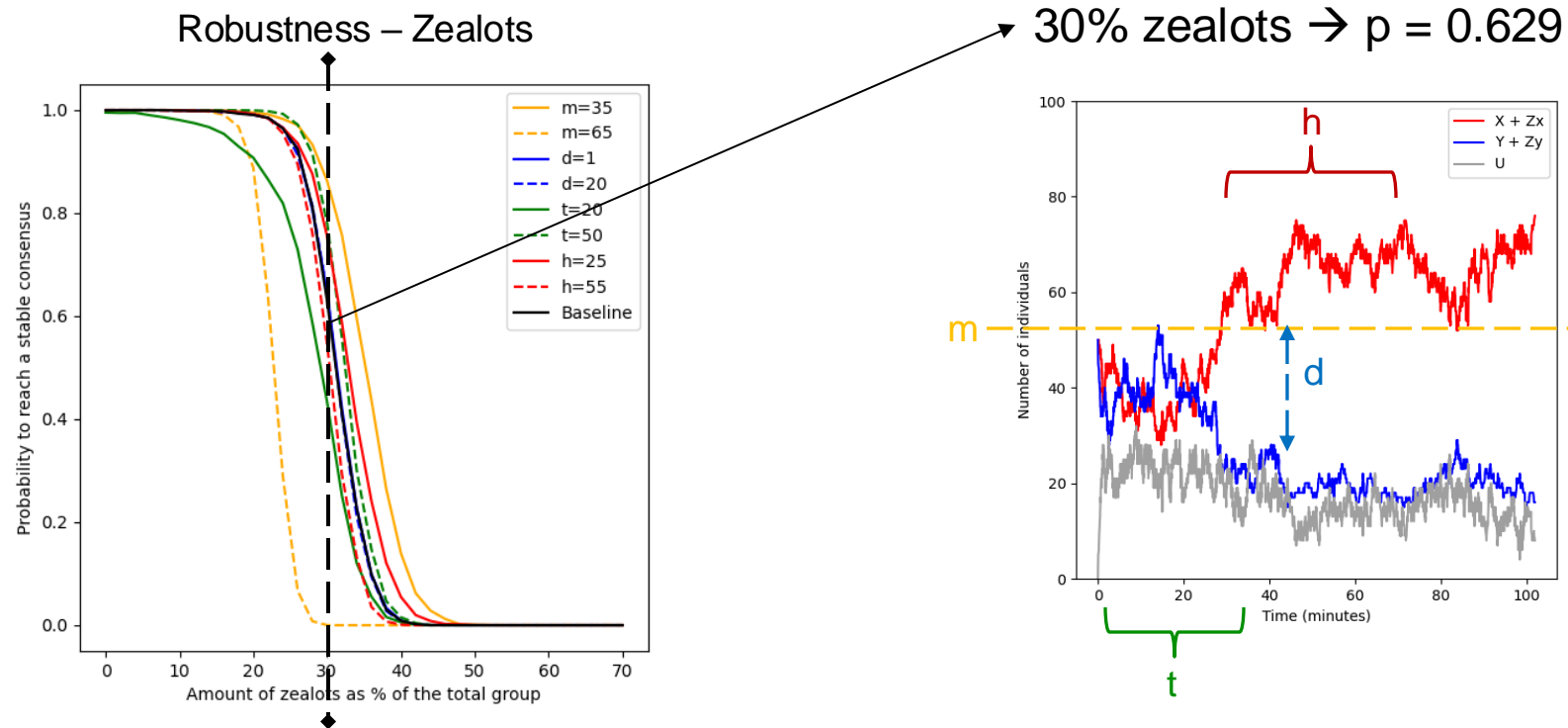
- Varying number of zealots and contrarians to explore robustness
- Varying number of both to explore combined effect
- Varying total group size to explore group size effect
- **Monte Carlo** algorithm to estimate **satisfaction probability**
- Error margin $\varepsilon=0.025$, confidence bound $\Delta=0.01$

Plasma Lab
Statistical Model-Checker



Results – robustness of stable consensus

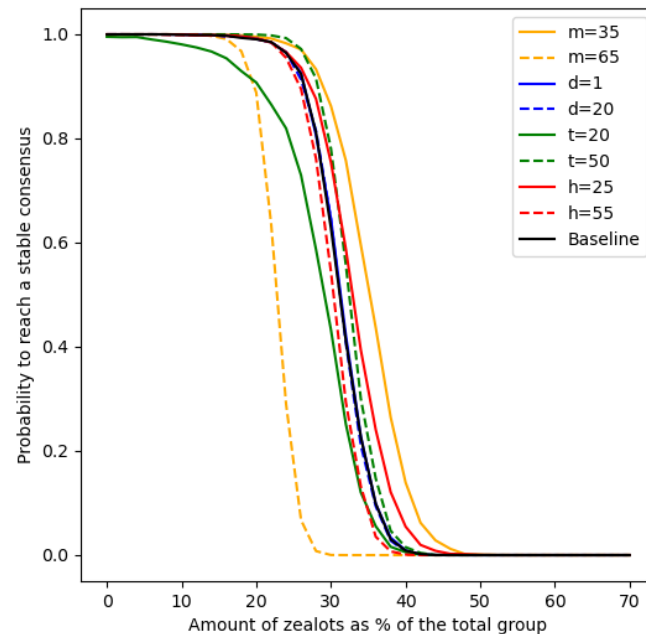
- Scenario: $N = 100$ robots, equivalent options X and Y ($q_x=q_y$), initially $\#X=\#Y$, $\#U=0$, $\#Zx=\#Zy$, $\#Cx=\#Cy$
- Baseline: $m=50$, $d=10$, $t=35$, $h=40$



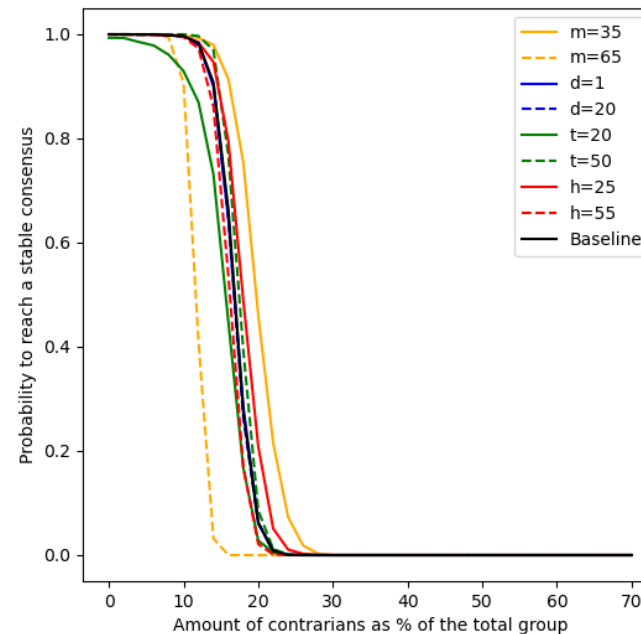
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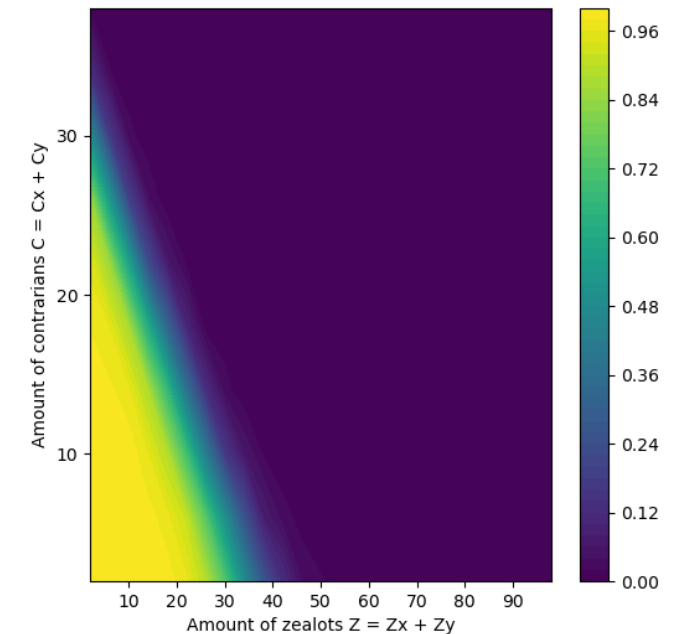
Robustness – Zealots



Robustness – Contrarians

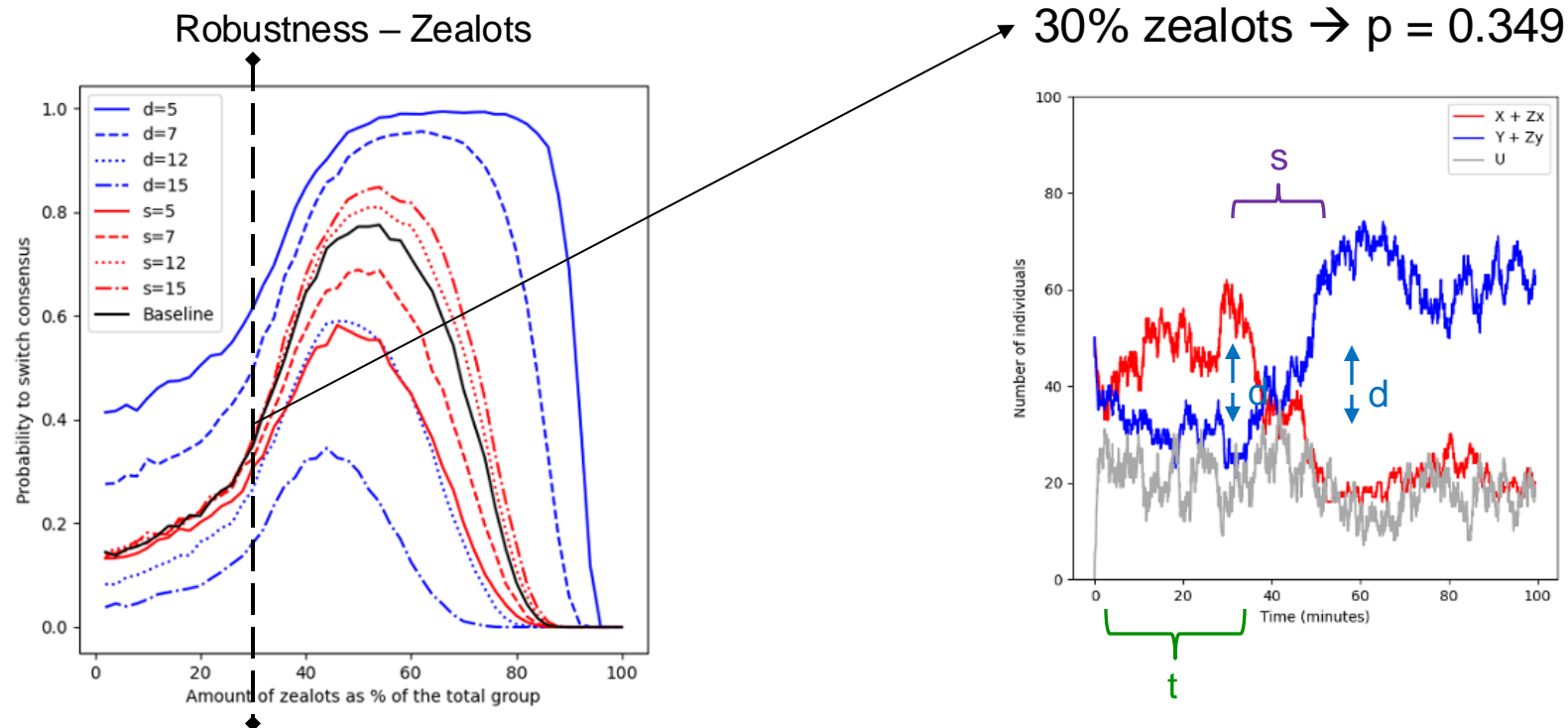


Combined effect



Results – robustness of switching consensus

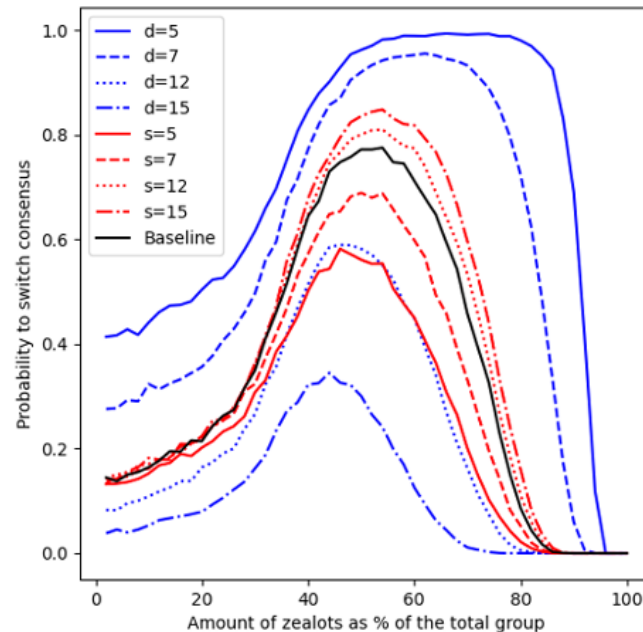
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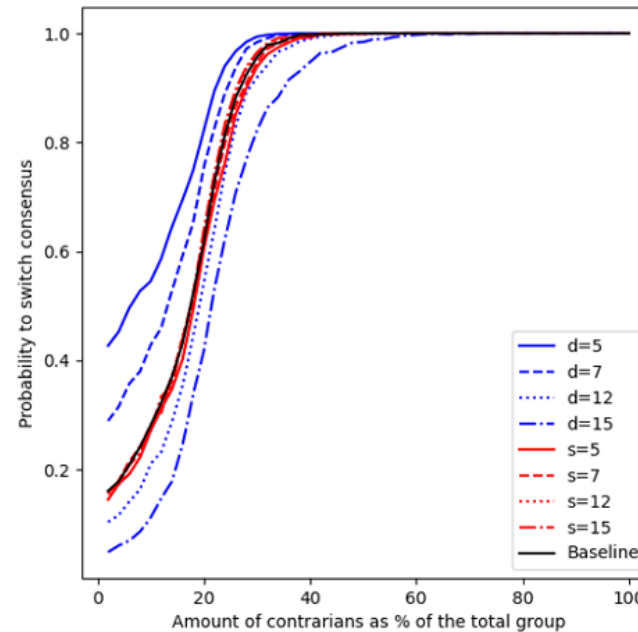
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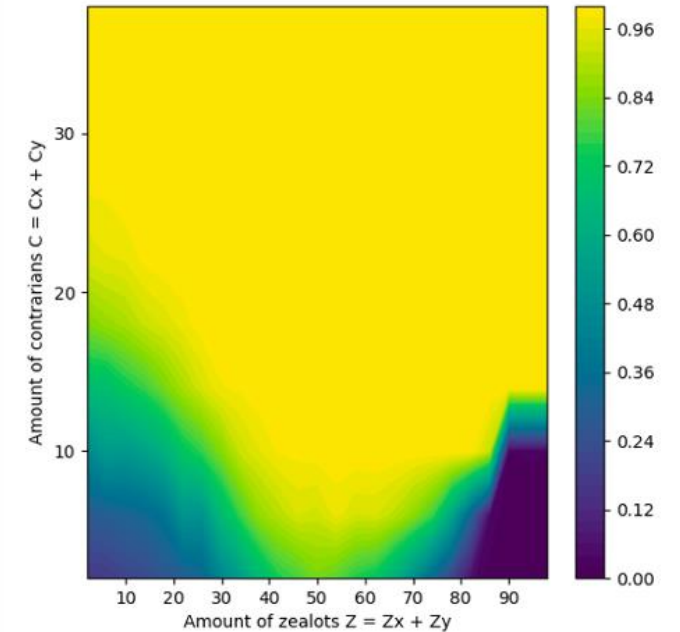
Robustness – Zealots



Robustness – Contrarians



Combined effect



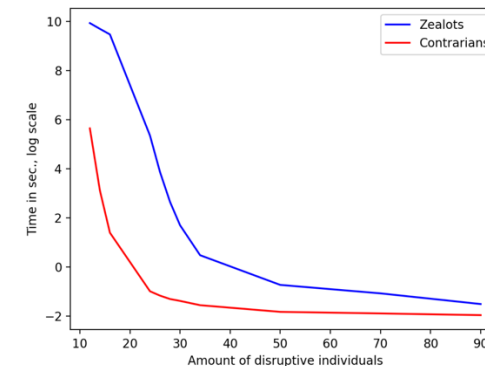
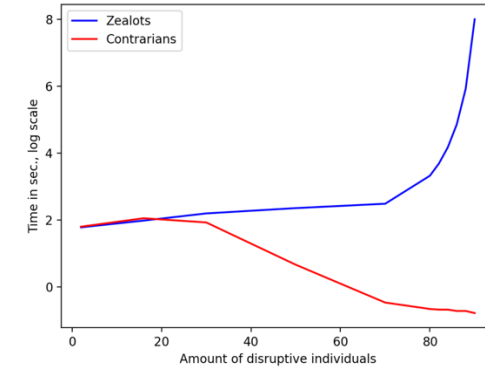
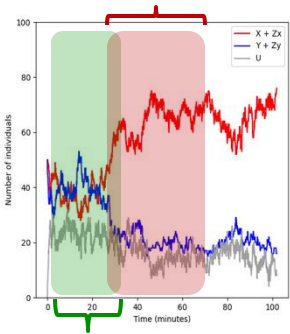
Results – expected times

Expected times to reach consensus

#	2	16	30	50	70	80	82	84	86	88	90
Zealots	5.95	7.28	9.02	10.57	12.04	27.82	39.94	64.95	128.85	374.04	2975.68
Contrarians	6.07	7.81	6.89	1.95	0.63	0.52	0.51	0.51	0.49	0.49	0.46

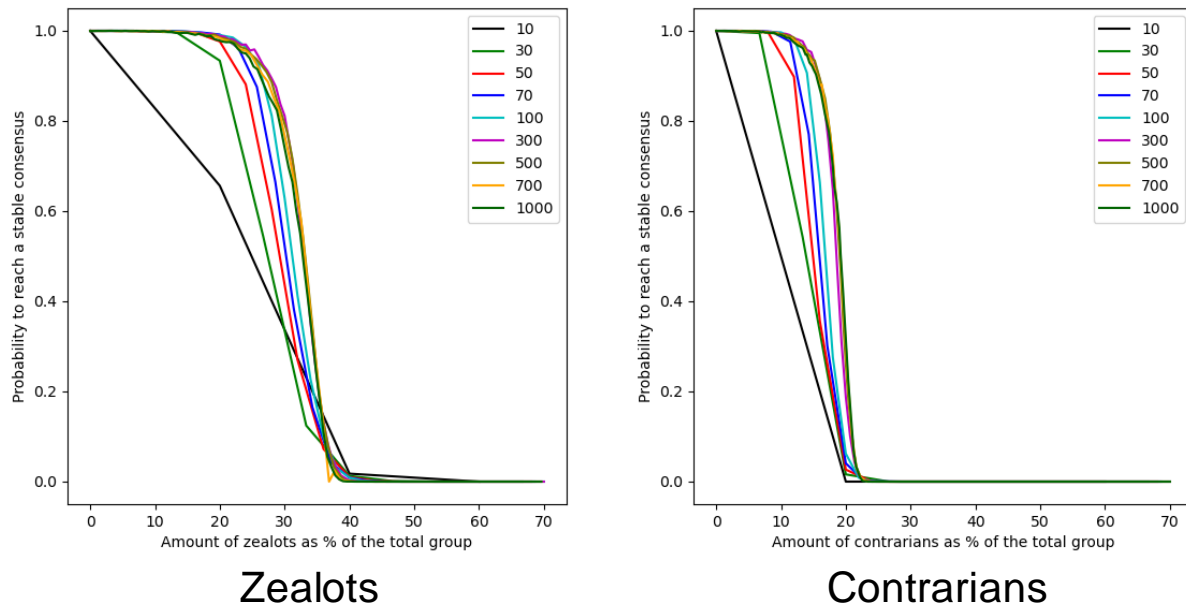
Expected times to hold consensus

#	12	14	16	24	26	28	30	34	50	70	90
Zealots	20686.51	16368.28	13047.85	210.98	47.71	14.13	5.46	1.61	0.48	0.34	0.22
Contrarians	283.57	22.53	4.03	0.37	0.31	0.27	0.25	0.21	0.16	0.15	0.14



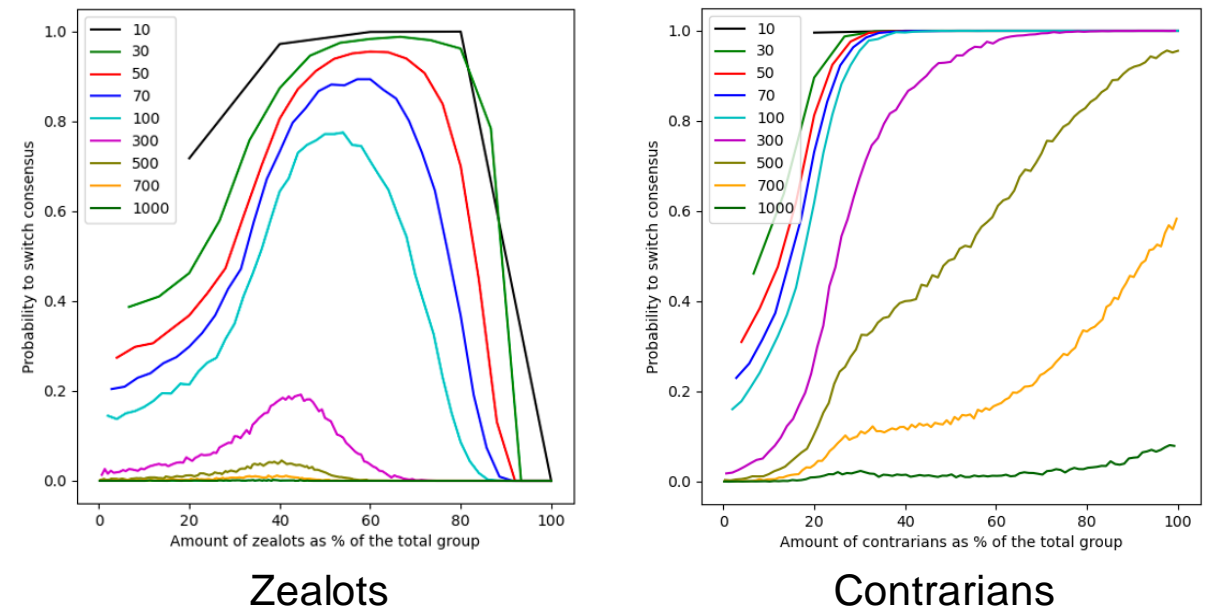
Results – group size effect

Stable consensus



... **robust** to group size scaling!

Switching consensus

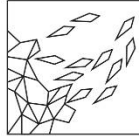


... **sensitive** to group size scaling!

Conclusion and outlook

- A small increase of disruptive individuals can drastically affect consensus dynamics
- Our method with SMC allows to explore consensus beyond mean-field analysis or single simulation
- **Stable consensus**
 - Cross-inhibition model robust up to certain fraction of zealots/contrarians, then rapid phase transition
 - Zealots are less harmful for reaching consensus than contrarians
- **Switching consensus**
 - Range of zealots for which such trajectories occur with non-negligible probability, but very rare for high number of zealots
 - Contrarians promote switching dynamics
- Future work
 - Group size effect: characterisation of a class of stochastic systems for which consensus reaching is robust to scaling
 - Asymmetric model: what if only one decision is correct?
 - Control theory: interventions over individuals for a global outcome

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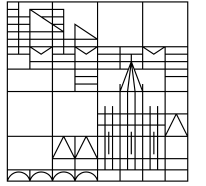


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Konstanz



**Thank you
very much!**

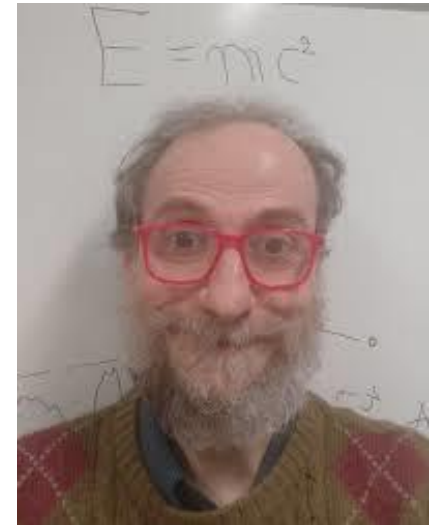
Julia Klein

Centre for the Advanced Study of Collective Behaviour
University of Konstanz

julia.klein@uni-konstanz.de

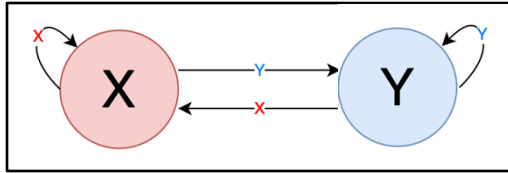


Tatjana Petrov



Alberto d'Onofrio

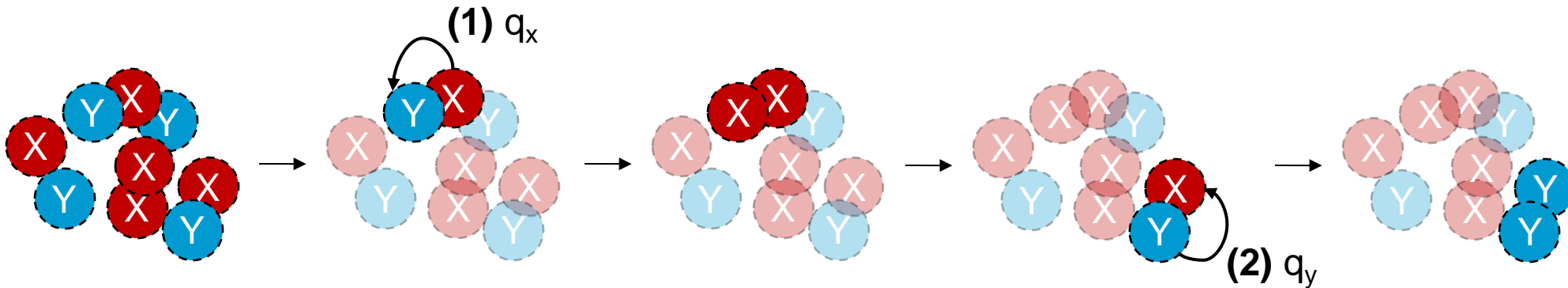
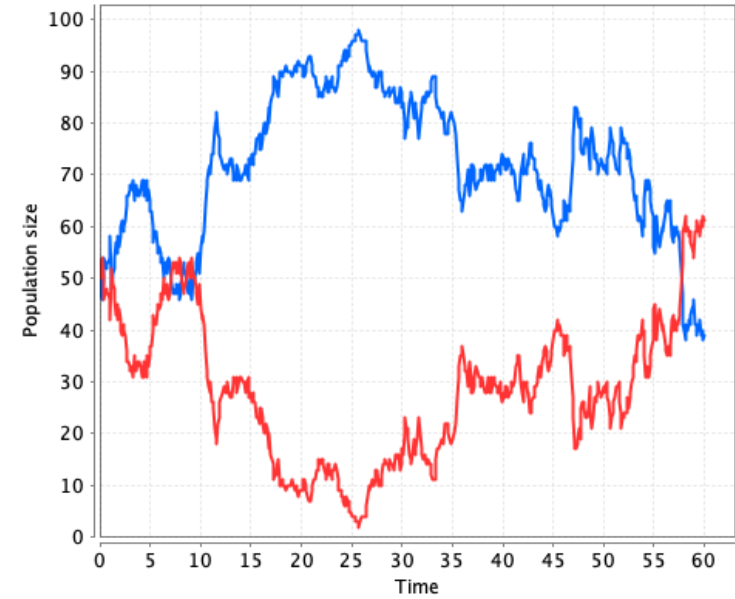
Decision making: Voter model

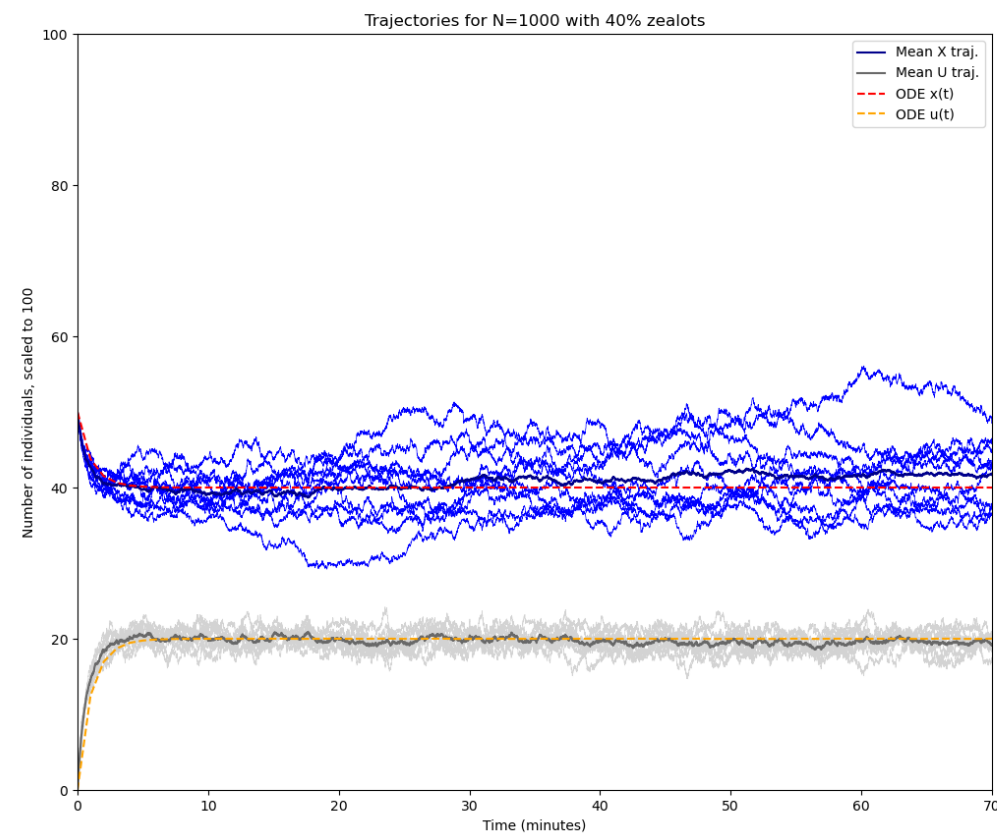
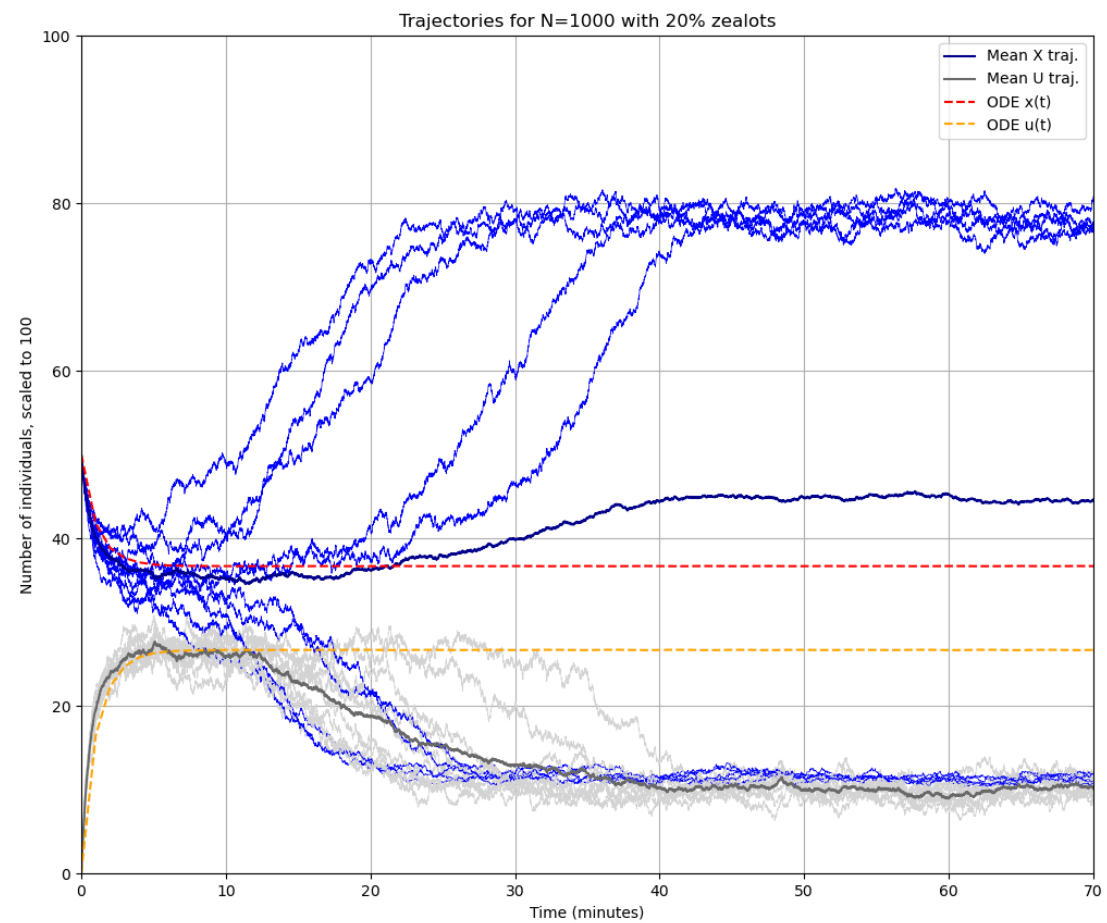


$$(1) X + Y \xrightarrow{q_x} X + X$$

$$(2) X + Y \xrightarrow{q_y} Y + Y$$

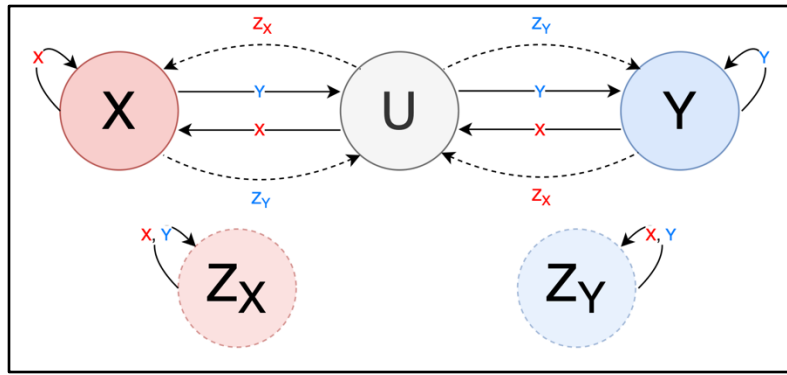
Swarm state evolves as a continuous-time Markov chain
 → classical **population model**





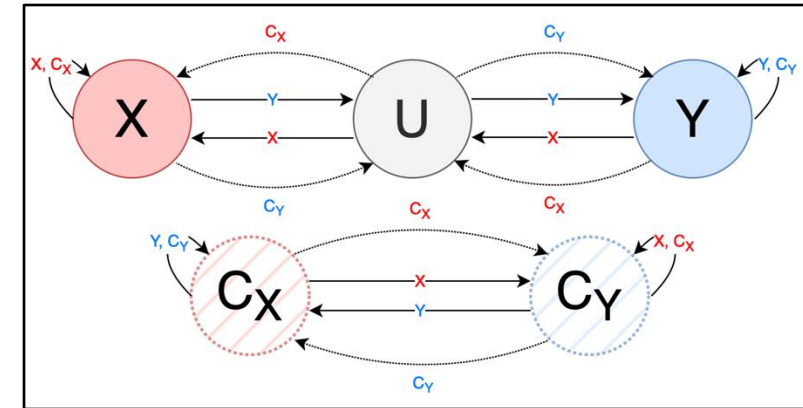
Studied Model with Disruptive Individuals

Cross-Inhibition model with Zealots



- **Zealots**: stubborn individuals which never change their own opinion
- Four additional reactions, where 'pure' agents interact with zealots & adjust their own states

Cross-Inhibition model with Contrarians



- **Contrarians**: individuals which counter the opinion of the individual they interact with
- Eight additional reactions, where contrarians influence 'pure' individuals & are influenced by others with the same opinion