Exploring Consensus Robustness in Swarms with Disruptive Individuals

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Abstract

- Consensus in collective systems essential for coordinated behaviour
- Strongly opinionated minorities can disrupt opinion dynamics

- **Approach**: investigate robustness of consensus-reaching among stubborn individuals (**zealots**) and **contrarians**
  - Statistical model checking to formally quantify **robustness** under perturbations of amount of disruptive individuals
  - Investigate robustness landscape for combinations of different disruptive agents
- Goal: guide design and control of swarm robotics systems with focus on resilience to disruptive agents

Background

- Underlying model of decision-making: **cross-inhibition model** with 'undecided' state between switching opinion (2a)
  - Including zealots: never change their opinion (2b)
  - Including contrarians: counter opinion of individual they interact with (2c)
  - Including both zealots and contrarians (2d)
- Scenario:
  - 2 equivalent options X and Y
  - Group of N=100 individuals
  - Initial state: equally split between X and Y, varying amount of disruptive individuals
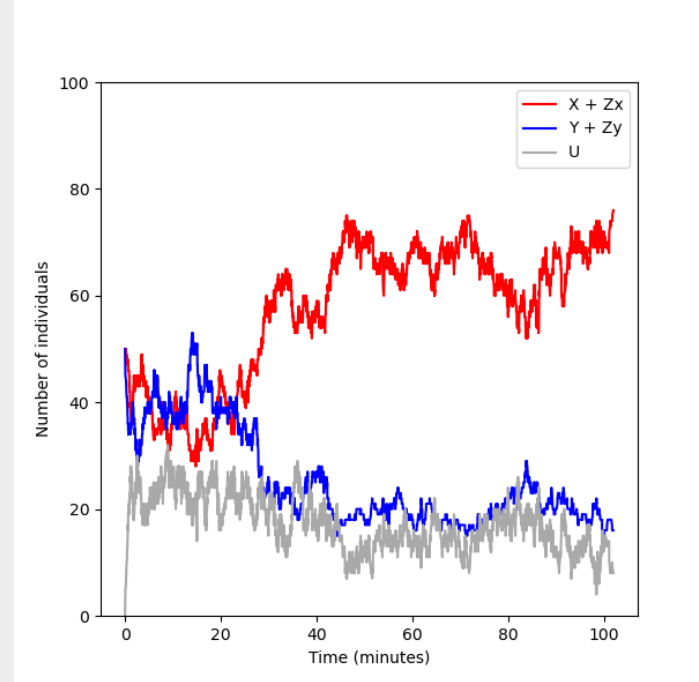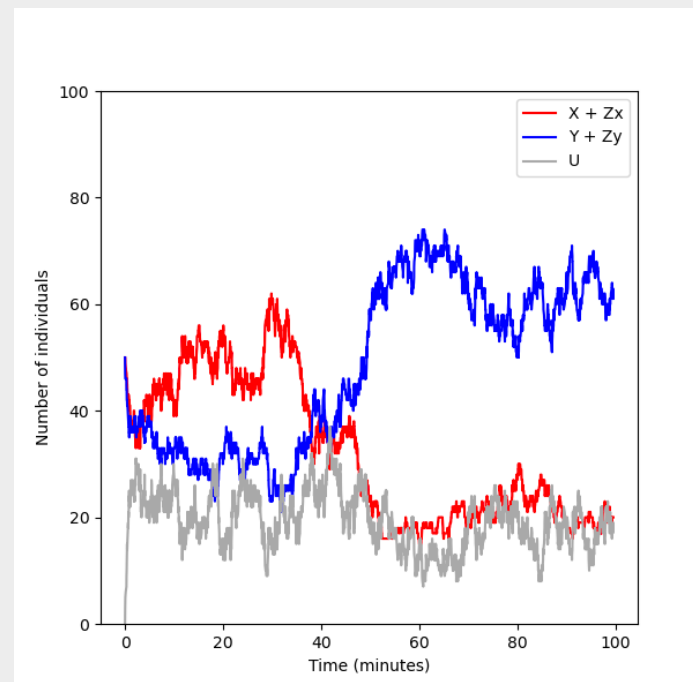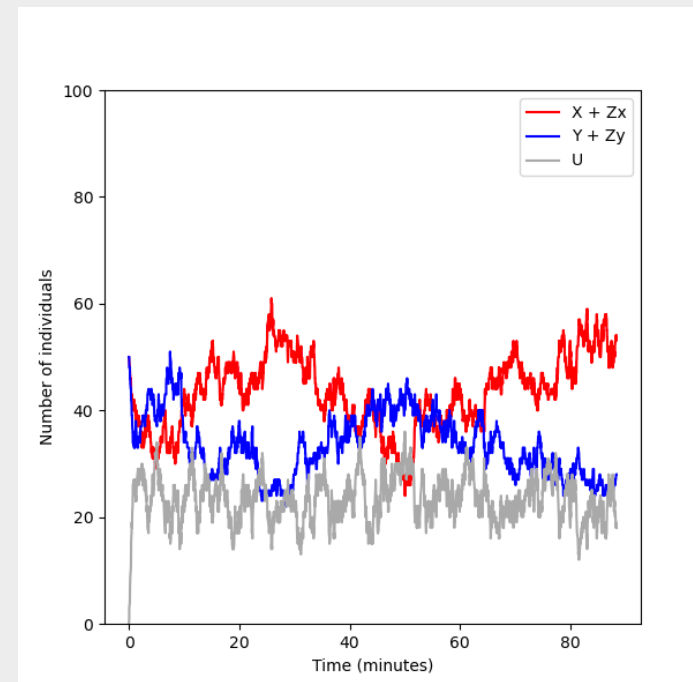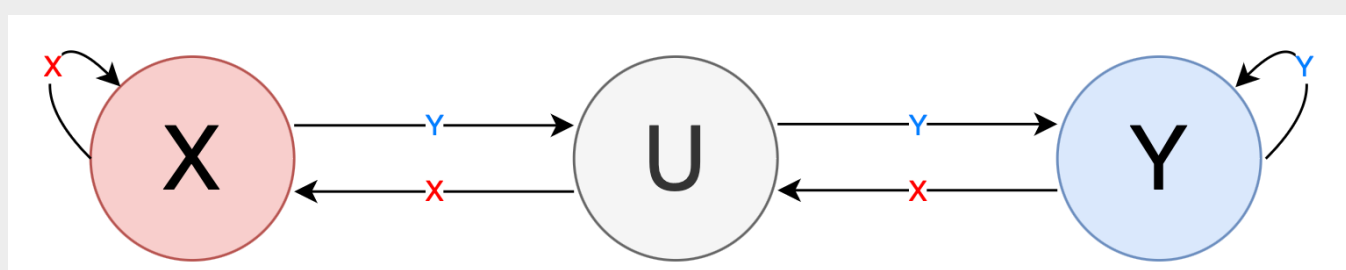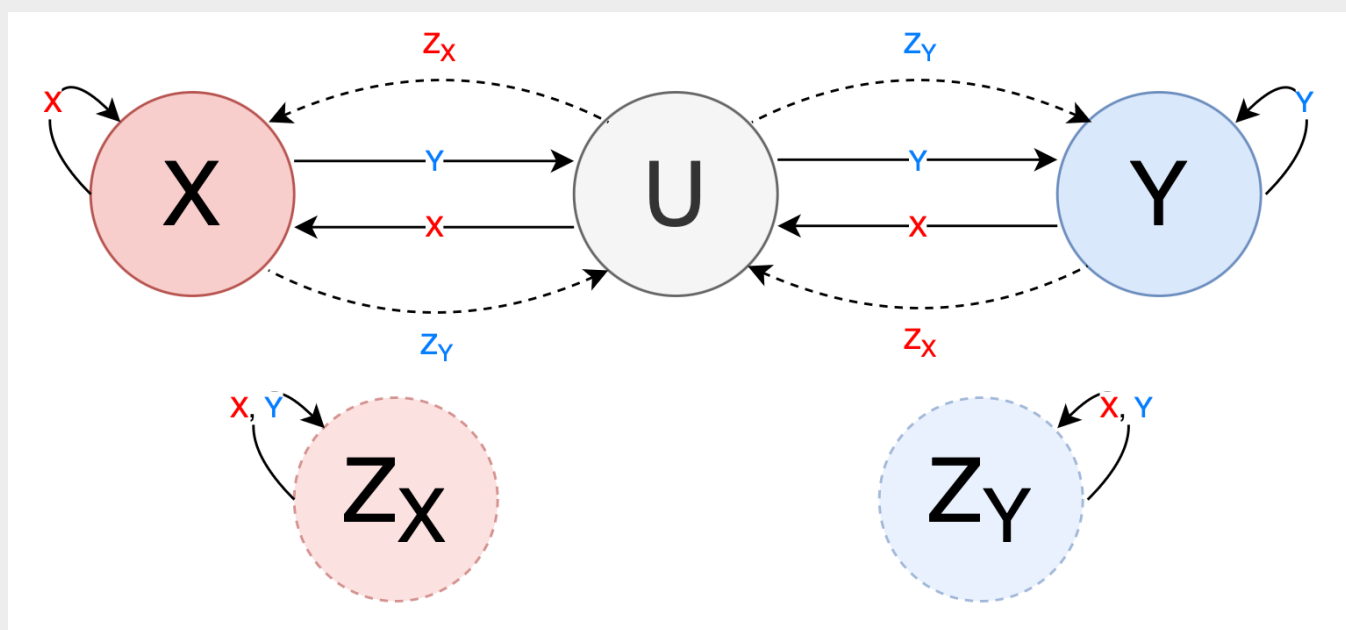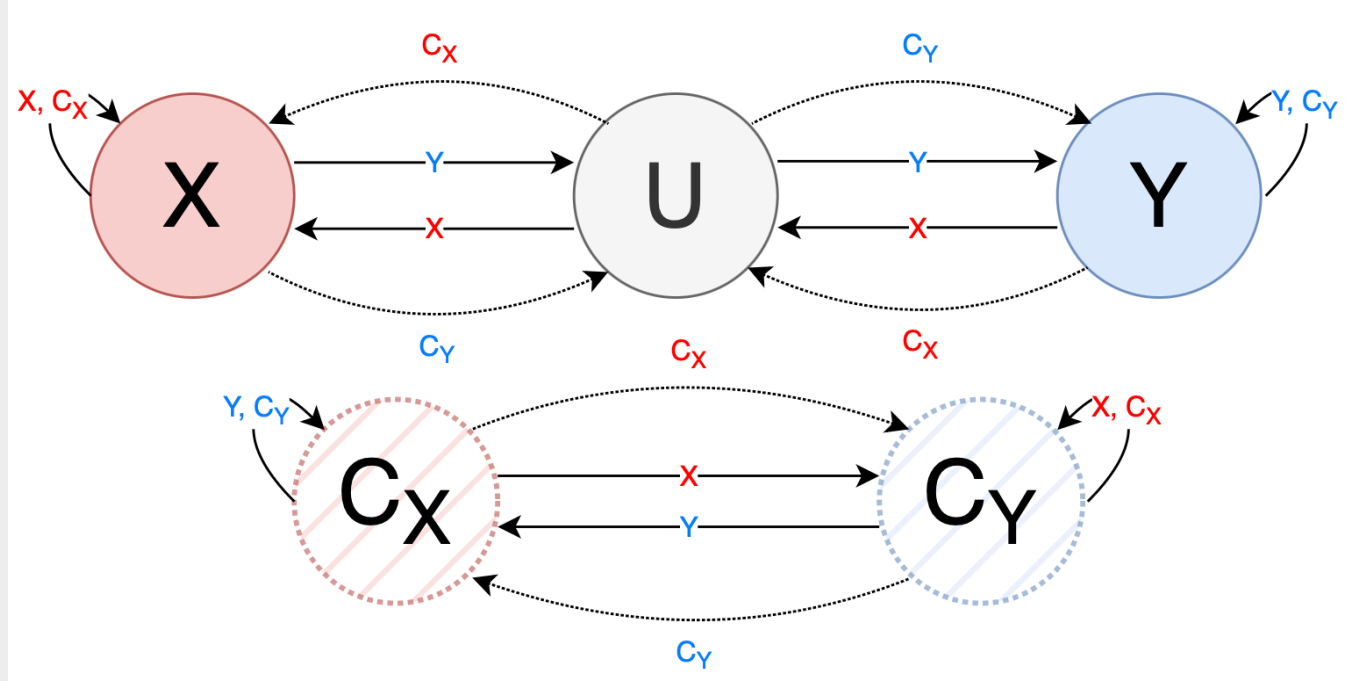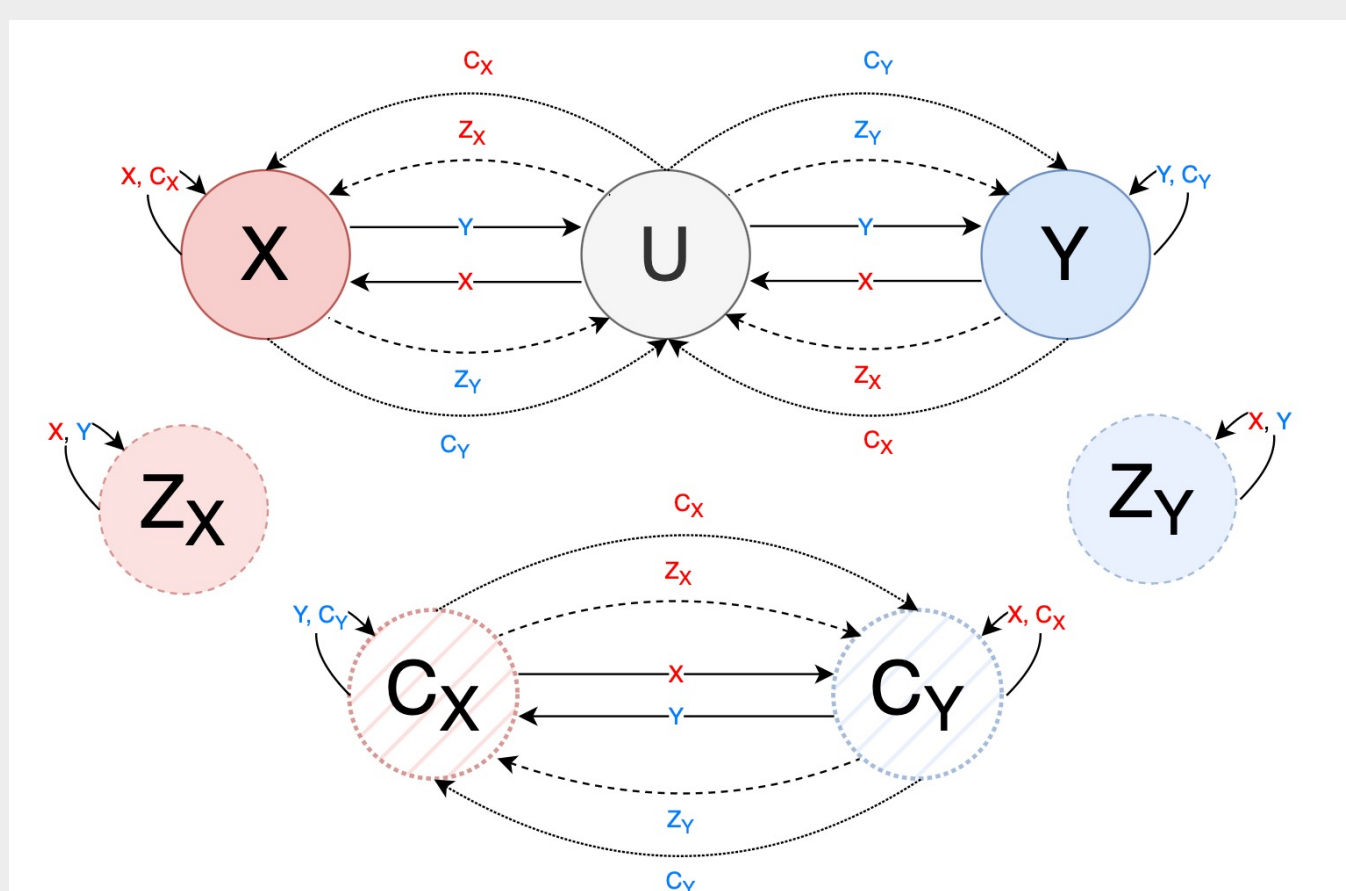

(1a)　　　　(1b)　　　　(1c)

Models

(2a)
(2b)
(2c)
(2d)

Cross-inhibition model

$$X + Y \xrightarrow{q_x} X + U$$
$$X + Y \xrightarrow{q_y} Y + U$$
$$X + U \xrightarrow{q_x} 2X$$
$$Y + U \xrightarrow{q_y} 2Y$$

Zealots

$$Y + Z_X \xrightarrow{q_x} U + Z_X$$
$$U + Z_X \xrightarrow{q_x} X + Z_X$$
$$X + Z_Y \xrightarrow{q_y} U + Z_Y$$
$$U + Z_Y \xrightarrow{q_y} Y + Z_Y$$

Contrarians

$$X + C_Y \xrightarrow{q_y} U + C_Y$$
$$U + C_Y \xrightarrow{q_y} Y + C_Y$$
$$X + C_X \xrightarrow{q_y} X + C_Y$$
$$Y + C_X \xrightarrow{q_x} U + C_X$$
$$U + C_X \xrightarrow{q_x} X + C_X$$
$$Y + C_Y \xrightarrow{q_x} Y + C_X$$
$$C_X + C_X \xrightarrow{q_y} C_X + C_Y$$
$$C_Y + C_Y \xrightarrow{q_x} C_X + C_X$$
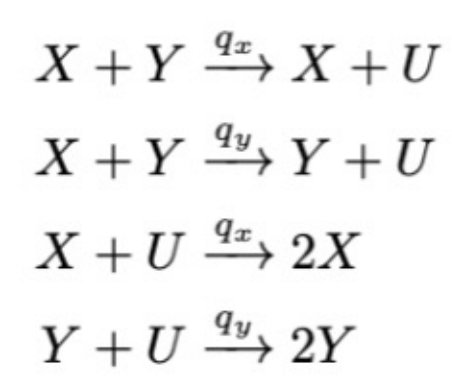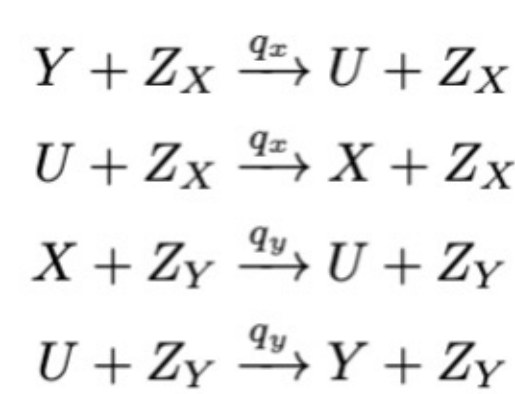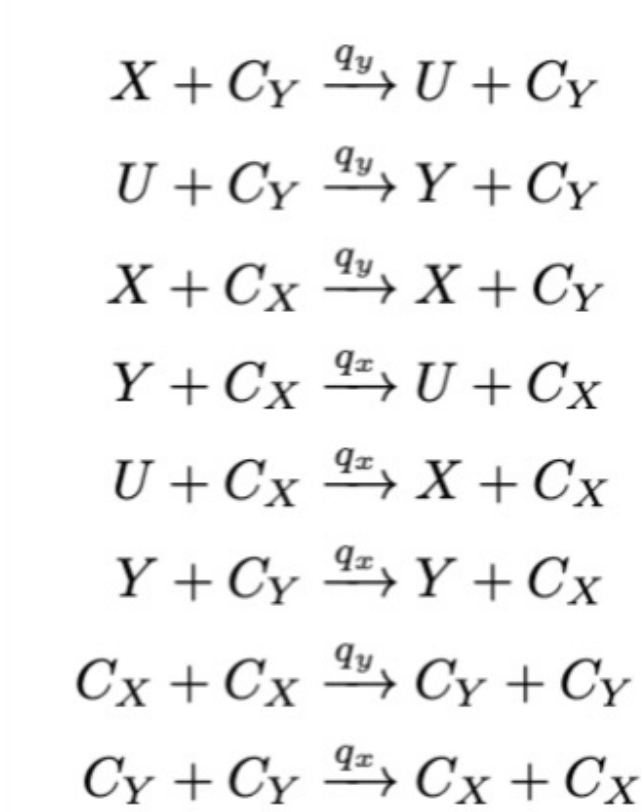
Contrarians and zealots

$$C_X + Z_X \xrightarrow{q_y} C_Y + Z_X$$
$$C_Y + Z_Y \xrightarrow{q_x} C_X + Z_Y$$
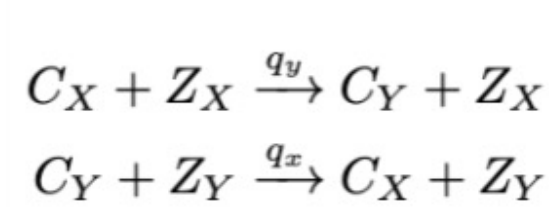
Analysis

- Statistical model checking of properties in Bounded Linear Temporal Logic
- Definition of consensus:
  - **Majority**: more than $m\%$ of population commits to same decision
  - **Distance**: significant difference of at least d between majority & those favouring opposite decision
  - **Reaching time**: consensus is reached within r minutes
  - **Holding time**: group maintains consensus for at least h minutes
  - Baseline: m=50, d=10, r=35, h=40
- Stable consensus (1a) in BLTL:

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq min_m) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$
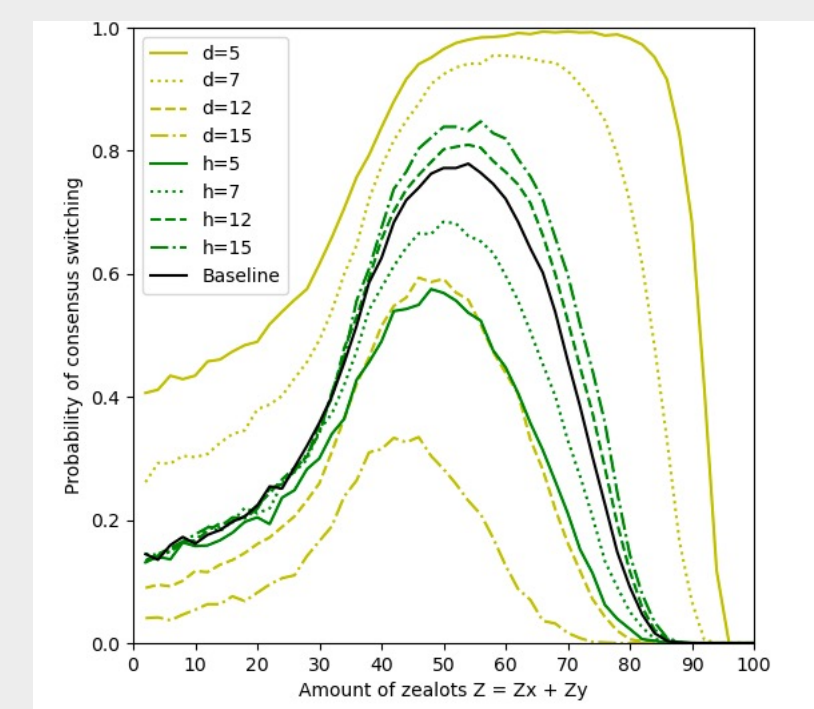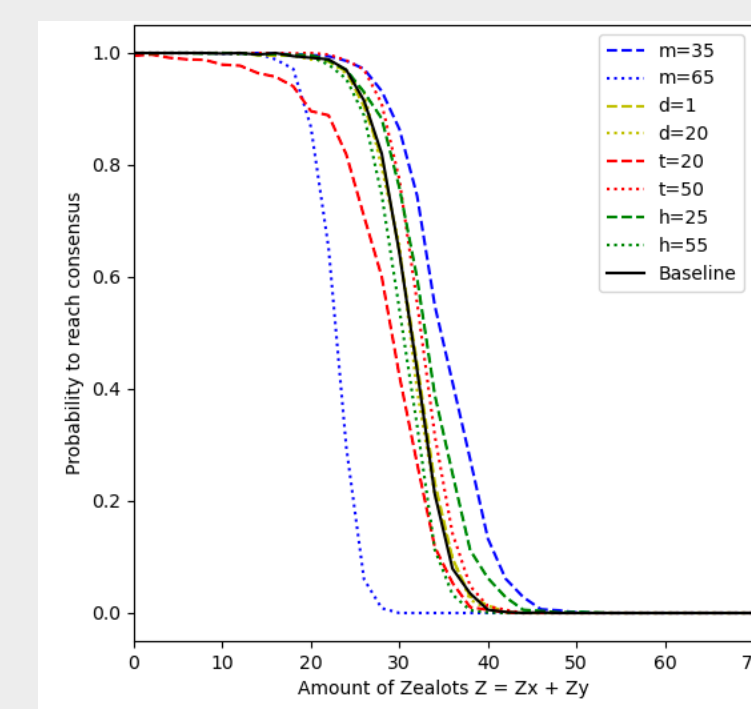
- Switching consensus (1b) in BLTL:

$$F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \wedge (true\ U_{\leq h}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee (((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \wedge (true\ U_{\leq h}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))$$
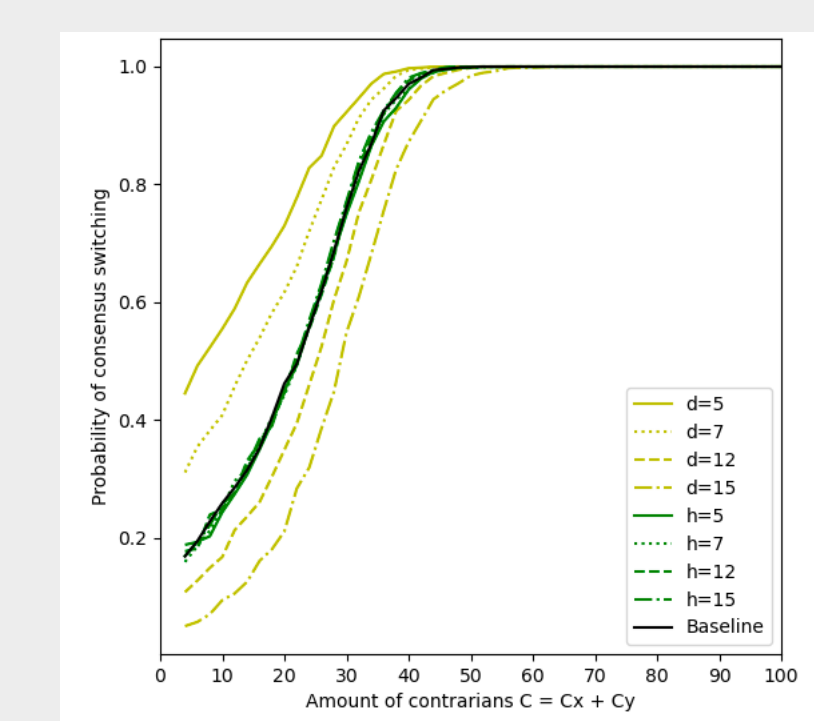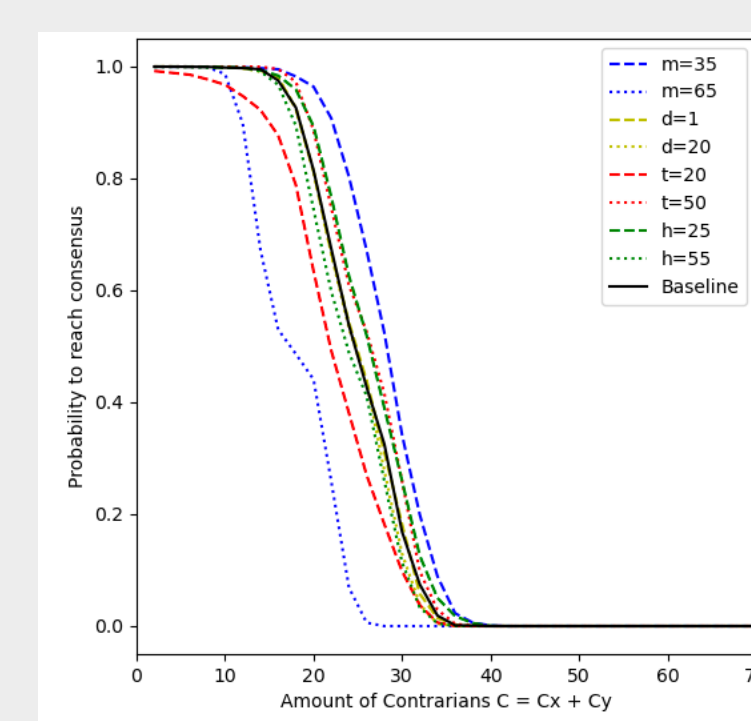
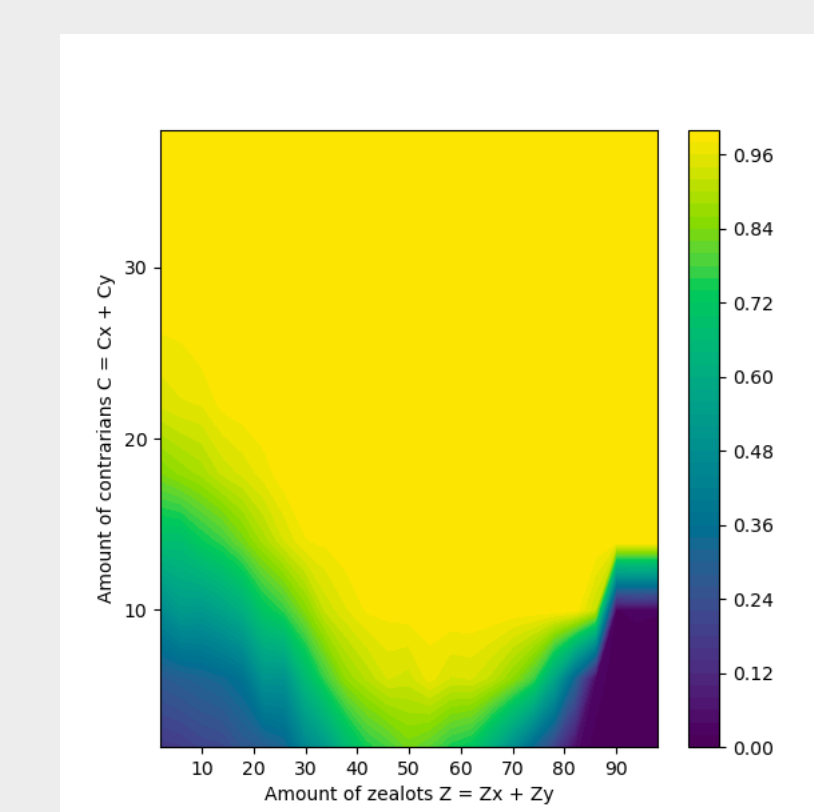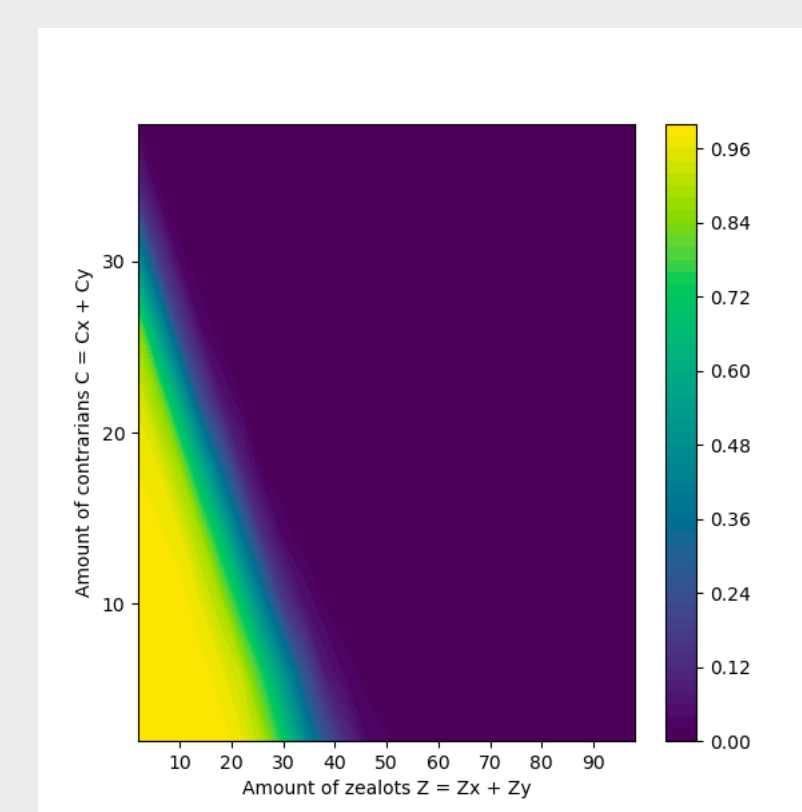- Monte Carlo algorithm to estimate satisfaction probability

Results

- Robustness of consensus in presence of zealots

- Robustness of consensus in presence of contrarians

- Robustness of consensus in presence of both zealots & contrarians

- Expected time to reach and hold majority

#	12	14	16	18	20	24	26	28	60	80
Zealots	t.o.	t.o	t.o.	t.o	t.o.	t.o.	7771	1831	27	0
Contrarians	t.o.	8288	1004	223	94	42	36	32	3.9	1.7
Both	t.o.	t.o.	t.o.	19755	1612	157	n.a.	58.5	9.9	4.1

#	8	12	14	16	18	20	22	24	40
Zealots	t.o.	t.o.	t.o.	t.o.	3033	326	32	183	0.22
Contrarians	25450	14.8	0.6	0.21	0.16	0.142	0.141	0.13	0.12
Both	t.o.	2466	n.a.	4.03	n.a.	0.176	n.a.	0.22	0

- ➤ Disruptive individuals can change opinion dynamics
- ➤ Robust up to certain #zealots/#conrrarians, then phase transition
- ➤ Zealots are less harmful than conrrarians

- Future work: more complex scenarios with more parameters, decision difficulty, number of options, spatial correlation, influence of group size

References

[1] Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)