Centre for the Advanced Study of Collective Behaviour

UNIVERSITÀ DEGLI STUDI DI TRIESTE

MAX-PLANCK-GESELLSCHAFT

Universität Konstanz

Exploring Consensus Robustness In Swarms with Disruptive Individuals

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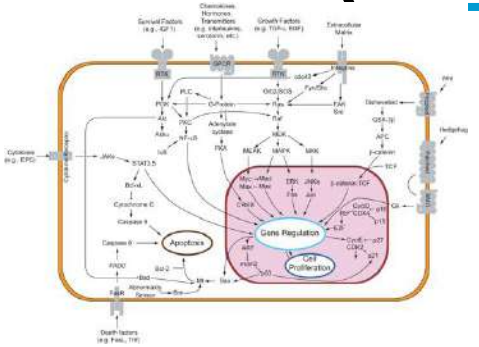
Presentation only

Accepted at ISoLA Conference 2024

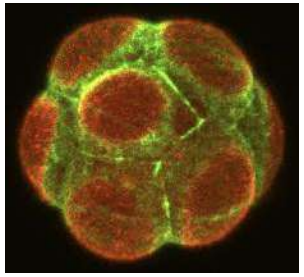
Swarms (collectives)



molecular signaling

social insects

stampede



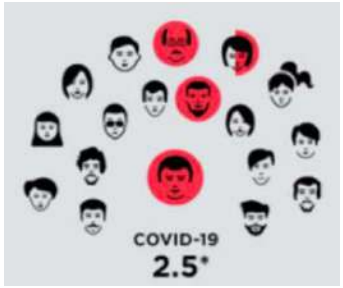
cellular differentiation

coordinated animal groups

traffic jam

groupthink

opinion polarization

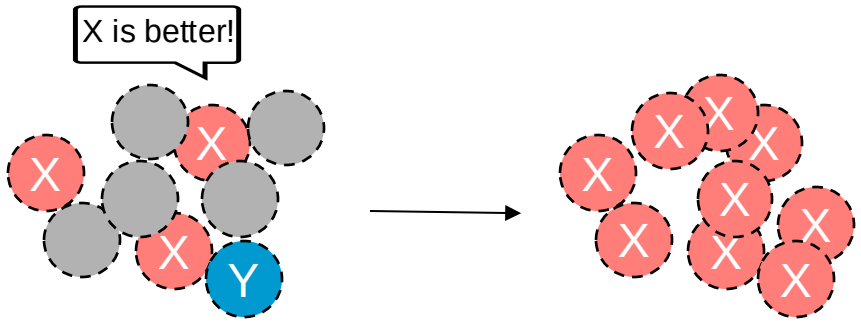


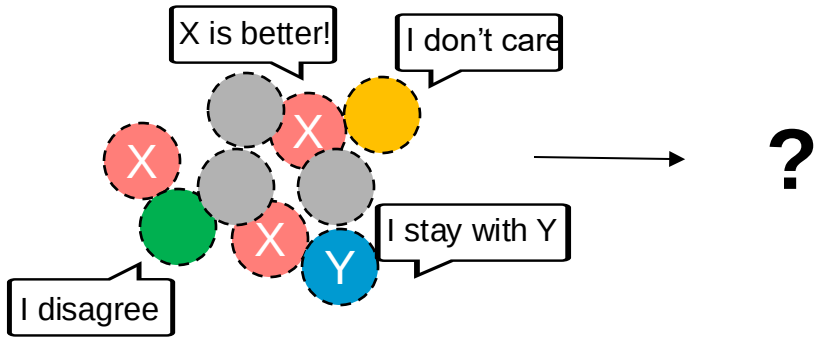
pandemics outbreak

robotic swarms

→ How do swarms agree on decisions?

Collective decision making





- Group needs to collectively decide between 2 or more options
- Individuals interact and convince (infect) peers

→ **Consensus** is ideally achieved with certain speed and accuracy

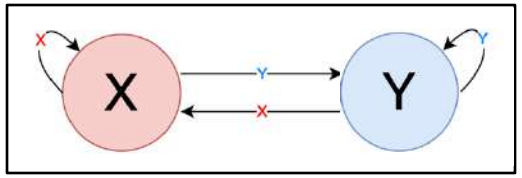
→ What happens in presence of disruptive (asocial) individuals?

Example: Voter model

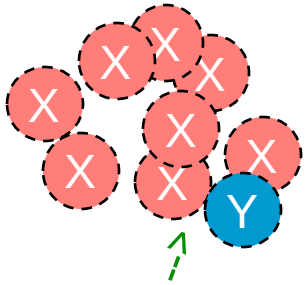
Voter Model

--> $X + Y \xrightarrow{q_y} Y + Y$

$Y + X \xrightarrow{q_x} X + X$



Swarm state evolves as a continuous-time Markov chain



Voter model

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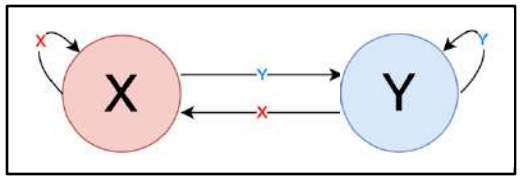

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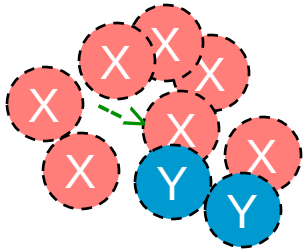

Voter model

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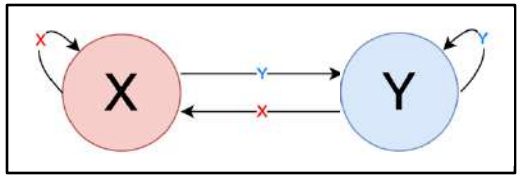

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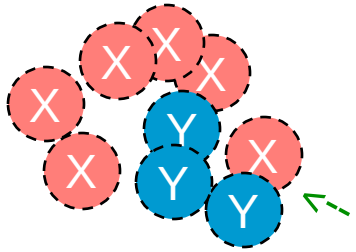

Voter model

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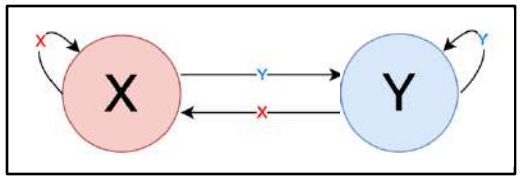

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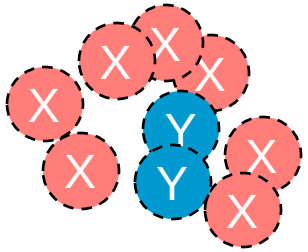

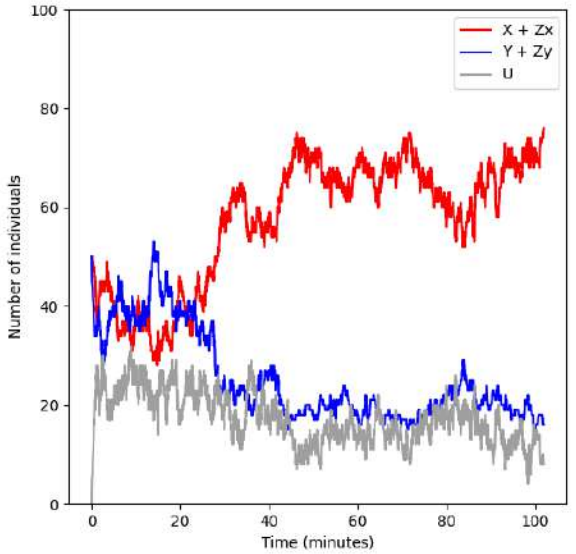

Stable consensus

Voter model with zealots

Voter Model

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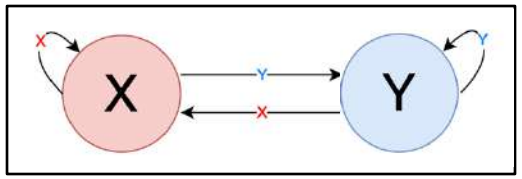
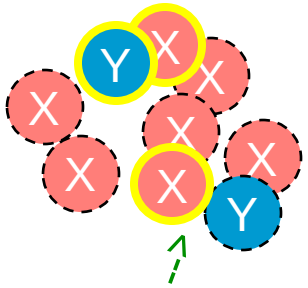


Swarm state with stubborn individuals (**zealots**)



Voter model with zealots

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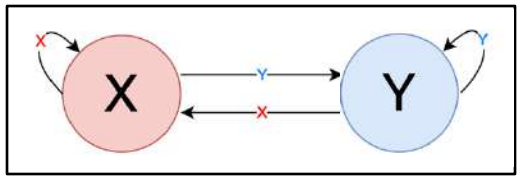

Swarm state with stubborn individuals (zealots)

Voter model with zealots

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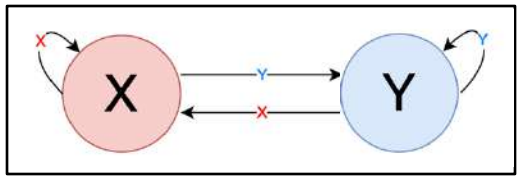

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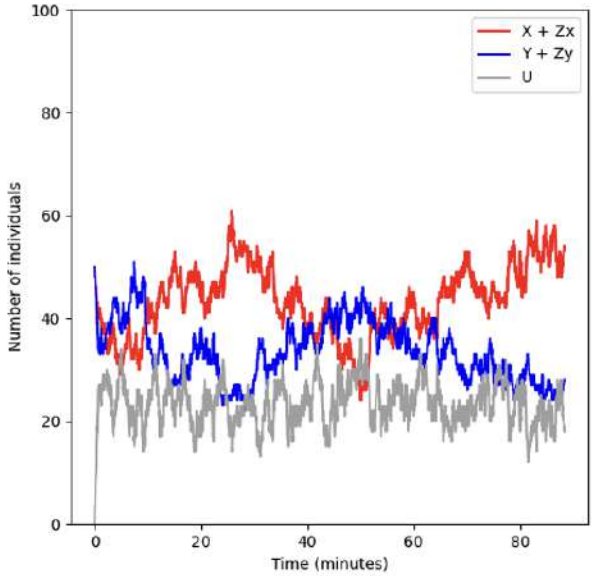

2% zealots
→ permanent indecision

Voter model with contrarians

Voter Model

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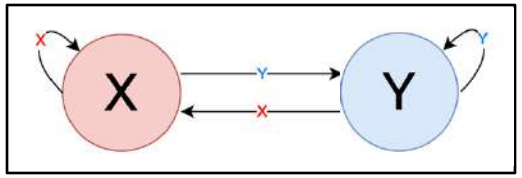
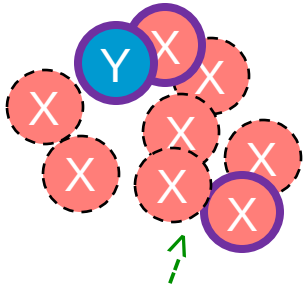


Swarm state with hipster individuals (contrarians)



Voter model with contrarians

Voter Model

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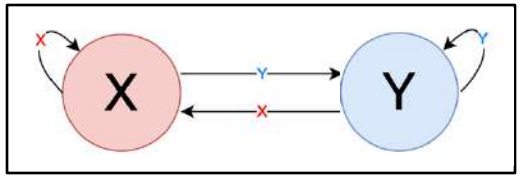
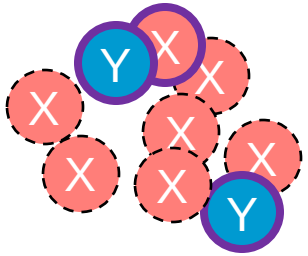


Swarm state with hipster individuals (contrarians)



Voter model with contrarians

Voter Model

--> $X + Y \xrightarrow{q_y} Y + Y$

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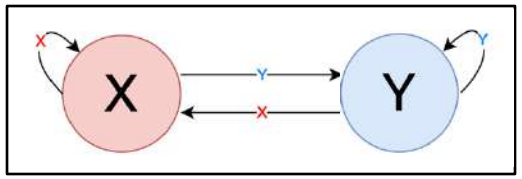

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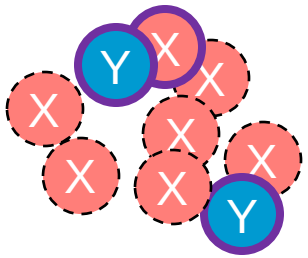

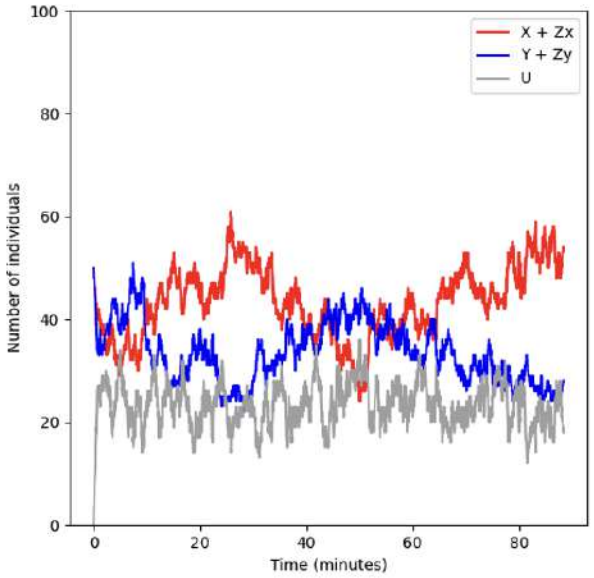

1% contrarians
→ permanent indecision

Cross-inhibition model

Voter Model

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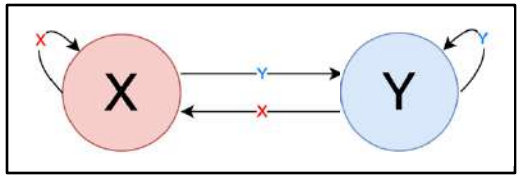

Cross-Inhibition Model

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$X + U \xrightarrow{q_x} 2X$

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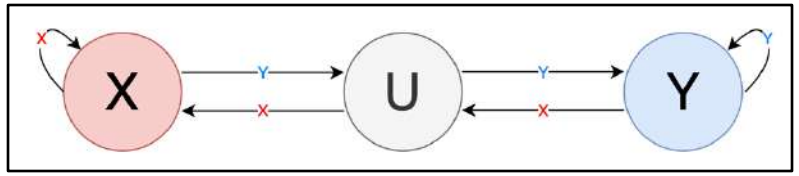
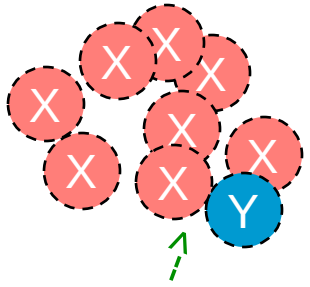


Swarm state



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Voter Model

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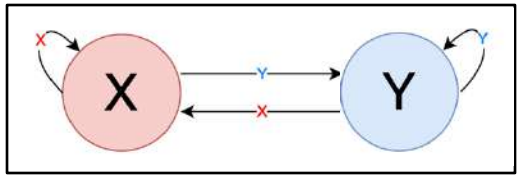

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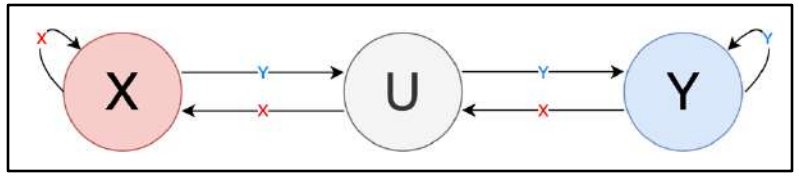

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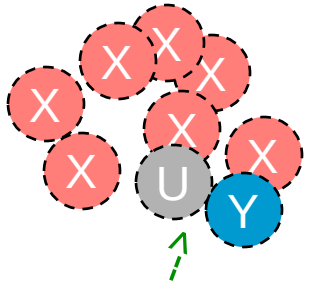

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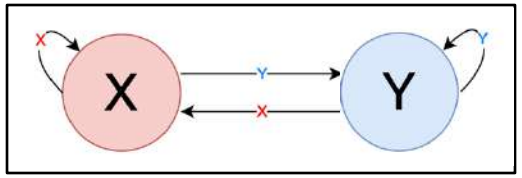

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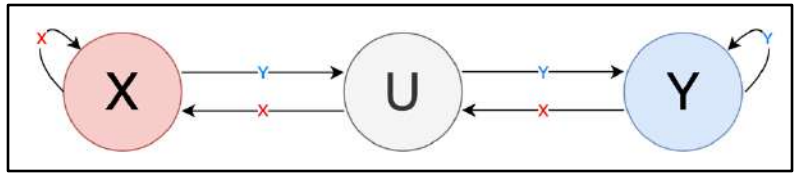

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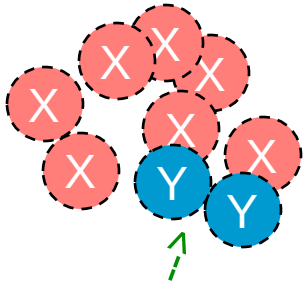

Cross-inhibition model

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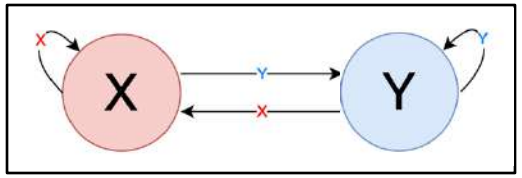

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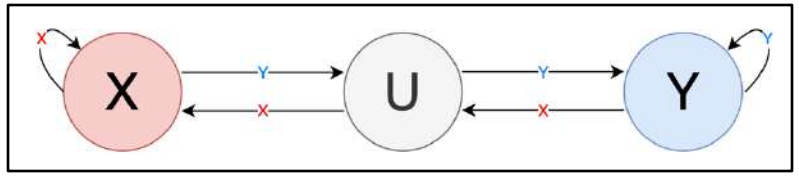

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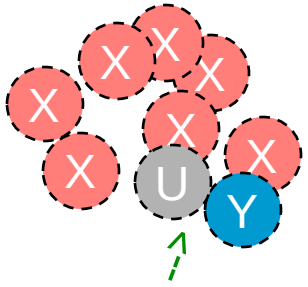

Cross-inhibition model

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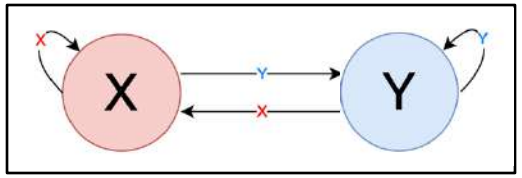

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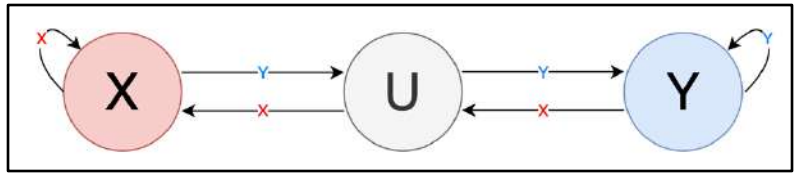
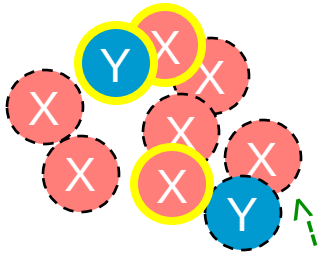


Swarm state with stubborn individuals (zealots)



Cross-inhibition model

Voter Model

$$X + Y \xrightarrow{q_y} Y + Y$$
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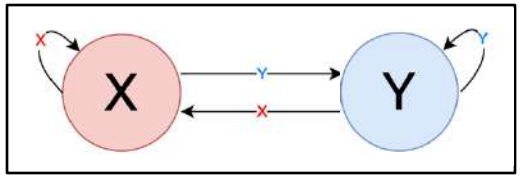

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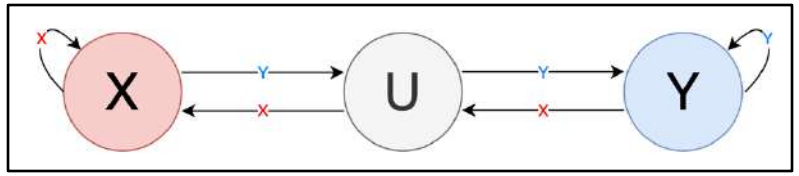

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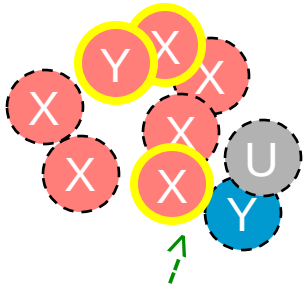

Cross-inhibition model

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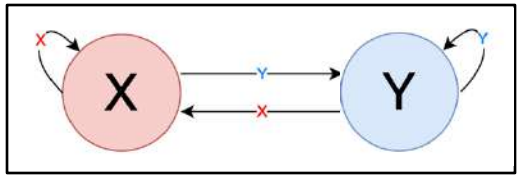

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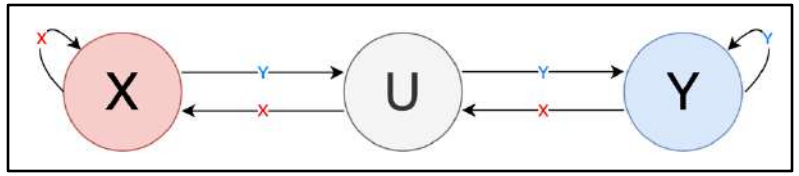

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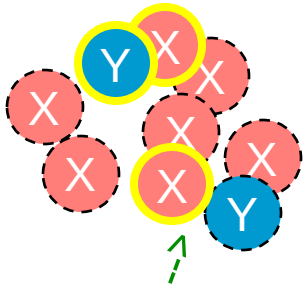

Cross-inhibition model

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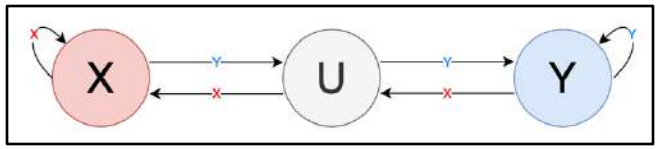
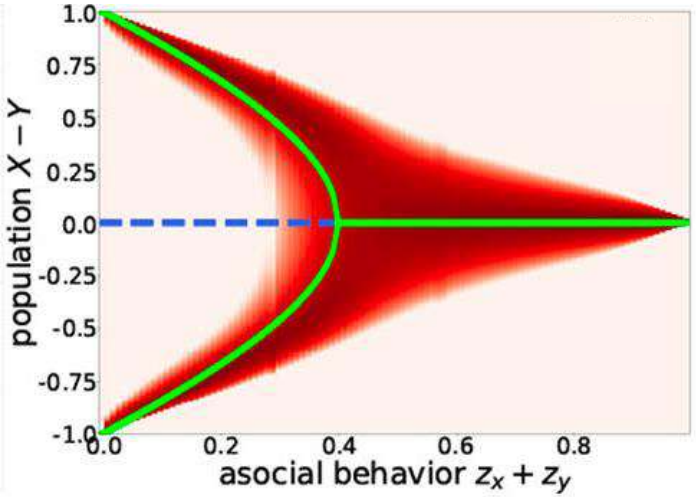
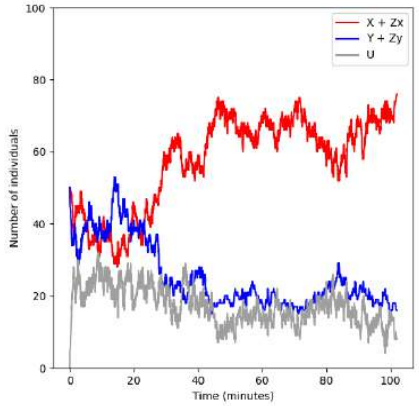


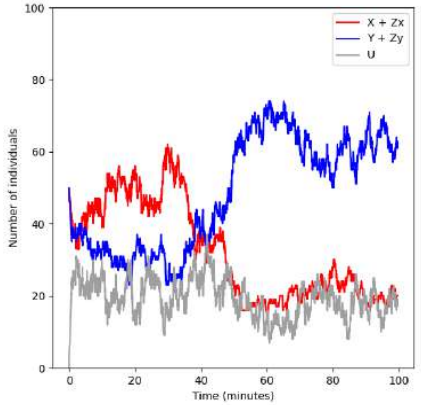


20% zealots



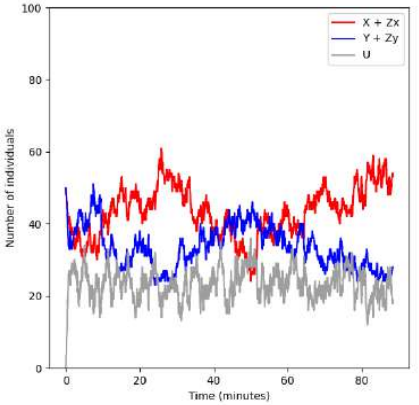
stable consensus

25% zealots



switching consensus

30% zealots



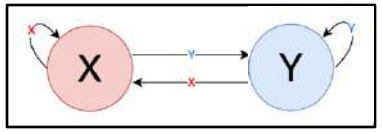
no consensus

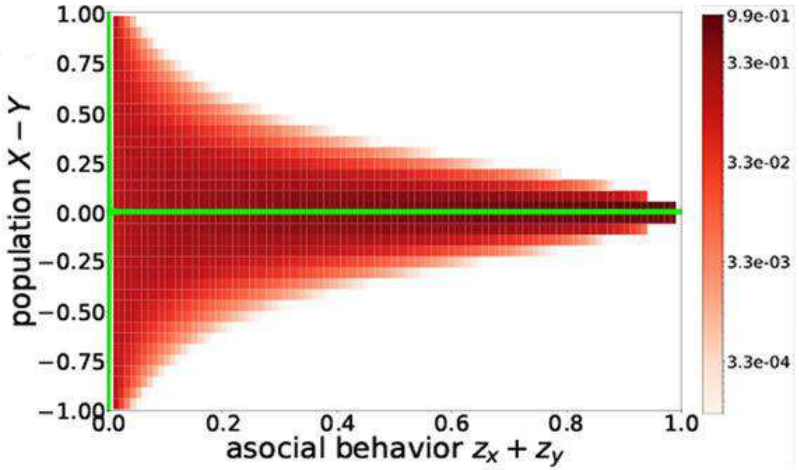
→ the swarm demonstrates resilience against relatively high levels of asocial behaviour

Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

Inspiration

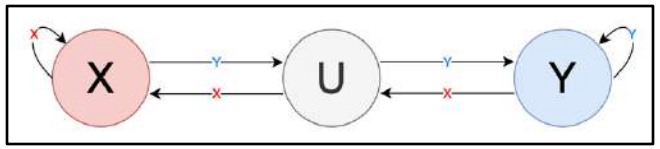
Voter Model

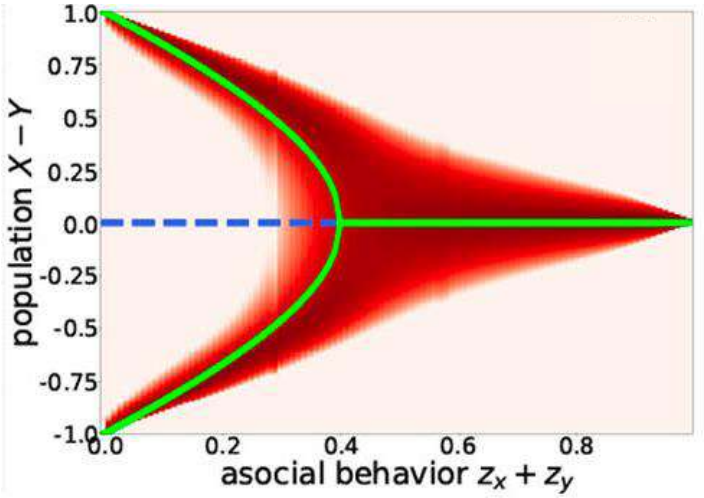




→ in presence of asocial individuals, the swarm gets quickly locked into an indecision state

Cross-Inhibition Model





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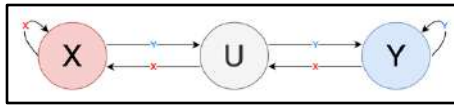
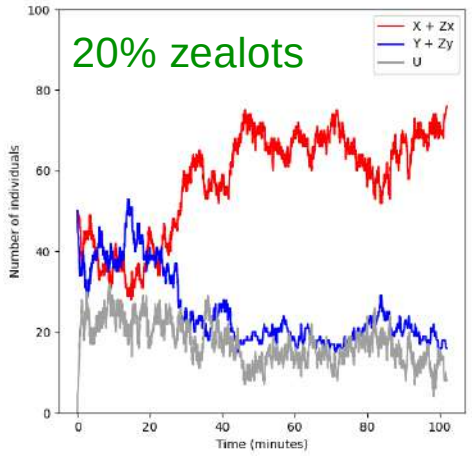
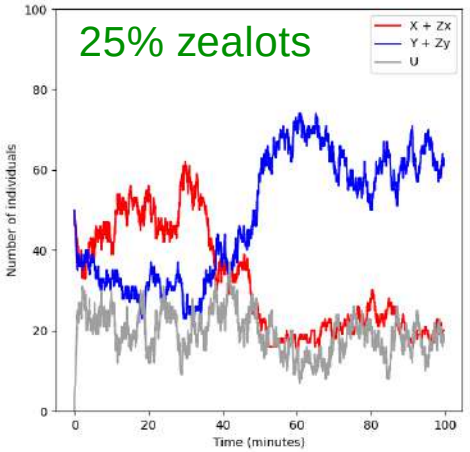


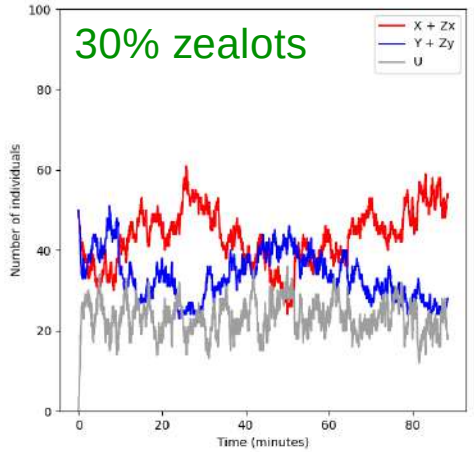
Research Questions



stable consensus



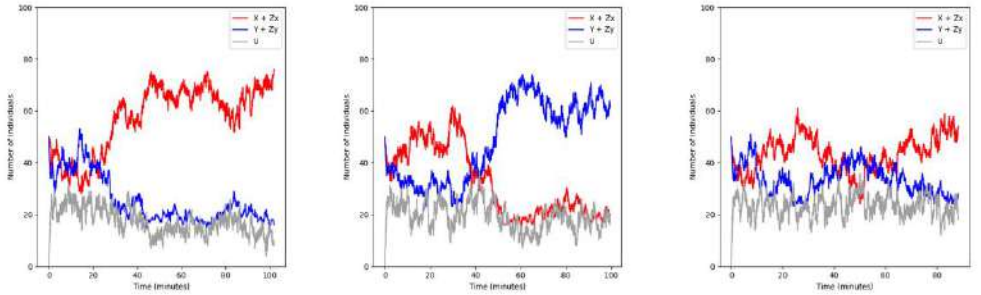
switching consensus



no consensus

- How does the amount of disruptive individuals affect consensus reaching? → **robustness analysis**
- How does the combination of zealots and contrarians affect consensus reaching? → **combined effect**

Approach



Statistical Model Checking of properties in **Bounded Linear Temporal Logic (BLTL)**

STEP 1: Formally describe **stable consensus** and **switching consensus** in BLTL

- Five parameters: majority *m*, distance *d*, reaching time *t*, holding time *h*, switching time *s*

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \ \wedge \ ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee$$
$$((y + Z_y + C_y \geq min_m) \ \wedge \ ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$

$$F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \ \wedge$$
$$(true \ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee$$
$$(((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \ \wedge$$
$$(true \ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))$$

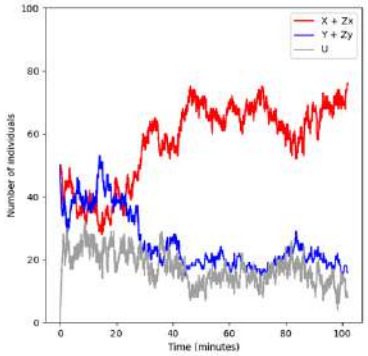
STEP 2: Apply model checking tools (PRISM and PlasmaLab) to explore the relevant scenarios:

- Varying number of zealots and contrarians to explore robustness
- Varying number of both to explore combined effect
- Varying total group size to explore group size effect

Plasma Lab
Statistical Model-Checker

PRISM
model checker

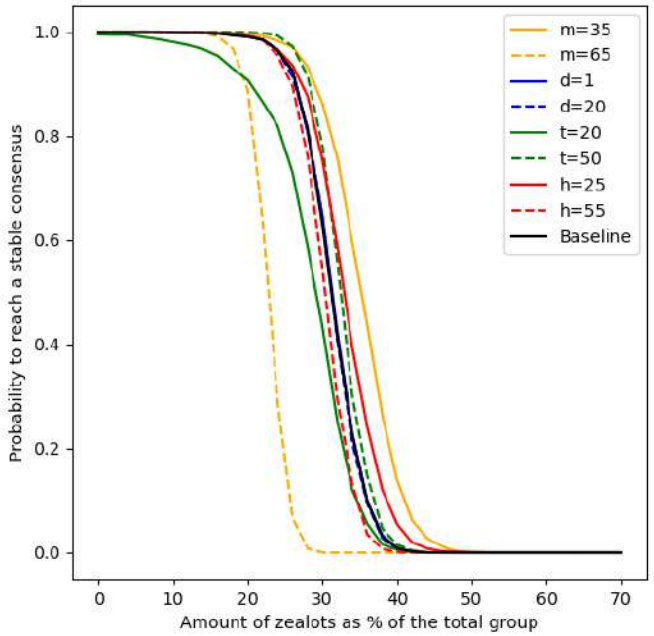
Results



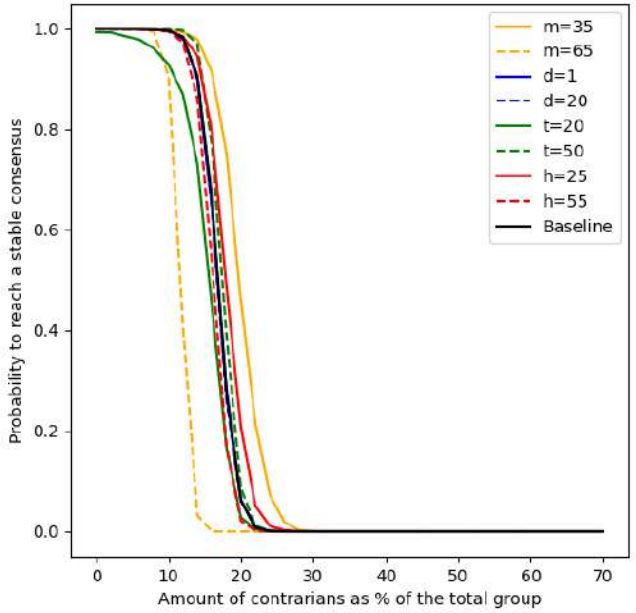
Robustness of reaching a stable consensus under perturbations of number of disruptive individuals

- N = 100 robots, equivalent options X and Y, initially #X=#Y, #U=0, #Zx=#Zy (#Cx=#Cy)
- Baseline: m=50, d=10, t=35, h=40

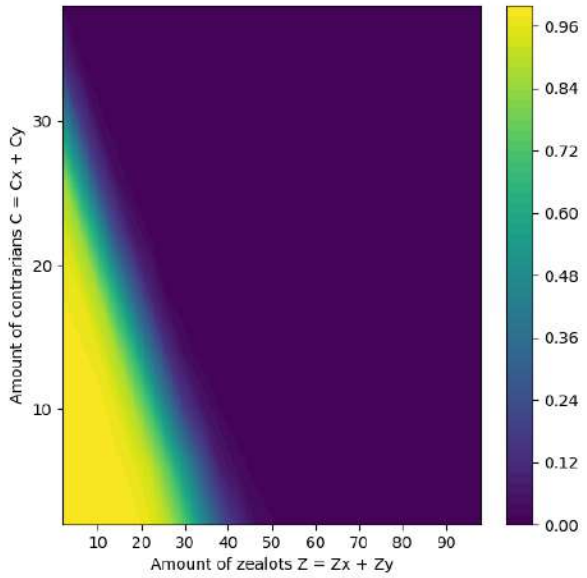
Robustness – Zealots



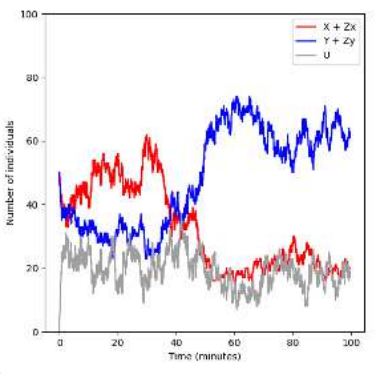
Robustness – Contrarians



Combined effect



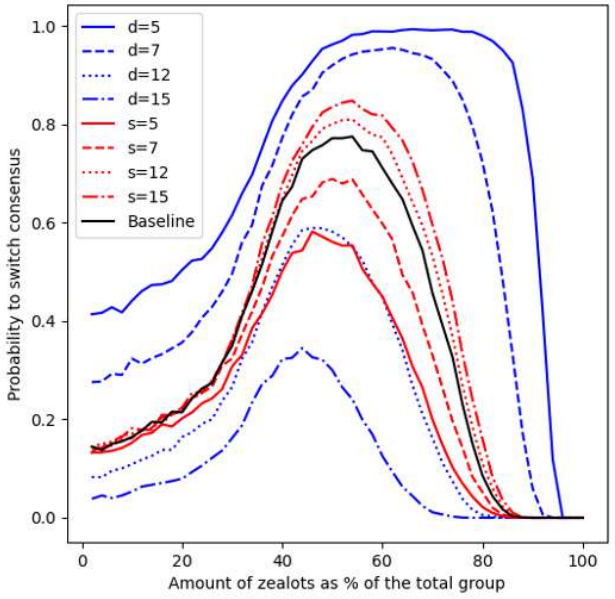
Results



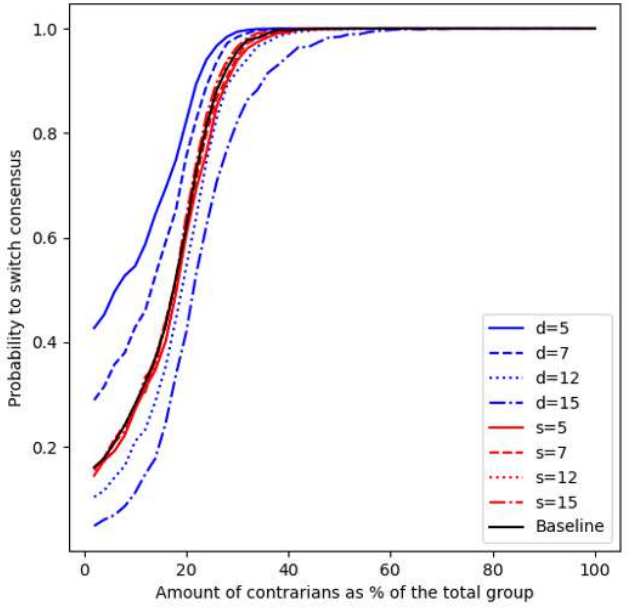
Robustness of switching consensus under perturbations of number of disruptive individuals

- N = 100 robots, equivalent options X and Y, initially #X=#Y, #U=0, #Zx=#Zy (#Cx=#Cy)
- Baseline: d=10, t=35, s=10

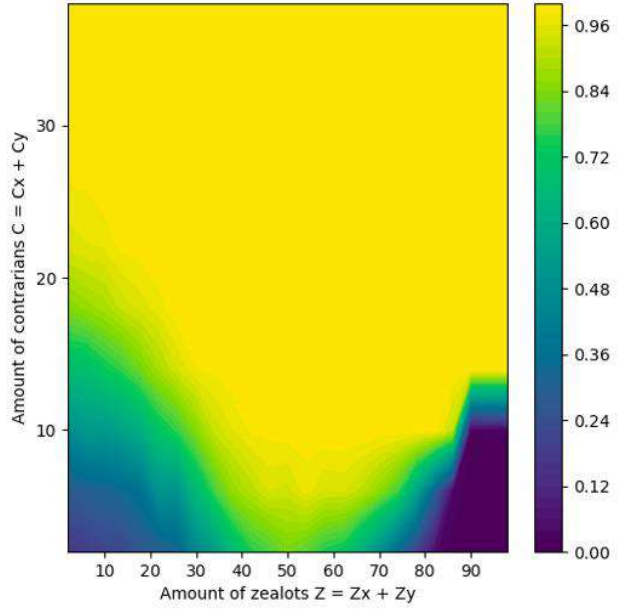
Robustness – Zealots



Robustness – Contrarians



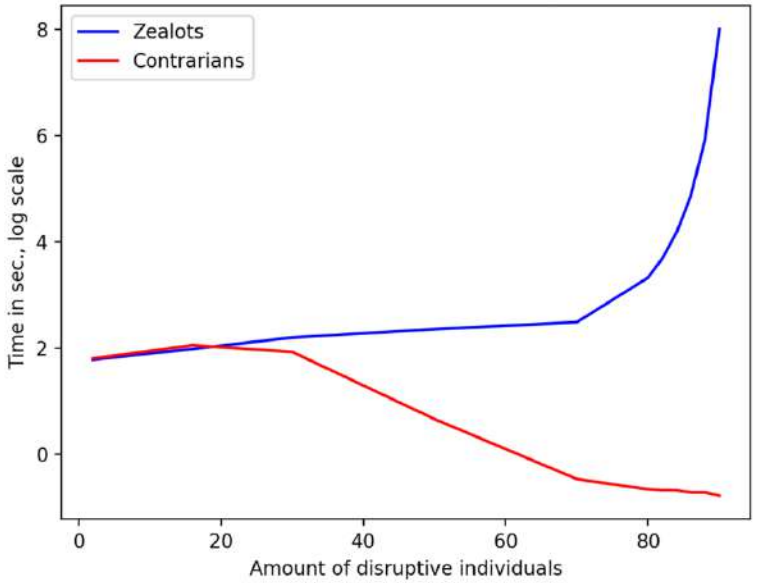
Combined effect



Results

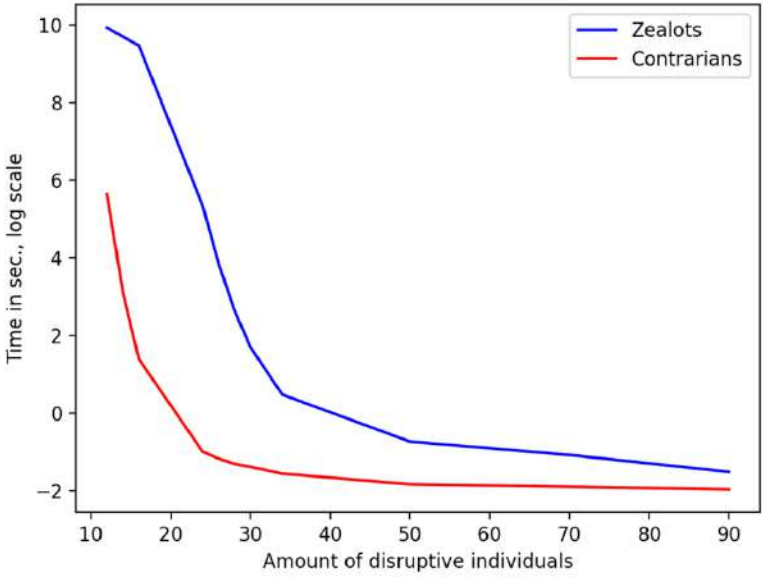
Expected times to reach consensus

#	2	16	30	50	70	80	82	84	86	88	90
Zealots	5.95	7.28	9.02	10.57	12.04	27.82	39.94	64.95	128.85	374.04	2975.68
Contrarians	6.07	7.81	6.89	1.95	0.63	0.52	0.51	0.51	0.49	0.49	0.46



Expected times to hold consensus

#	12	14	16	24	26	28	30	34	50	70	90
Zealots	20686.51	16368.28	13047.85	210.98	47.71	14.13	5.46	1.61	0.48	0.34	0.22
Contrarians	283.57	22.53	4.03	0.37	0.31	0.27	0.25	0.21	0.16	0.15	0.14



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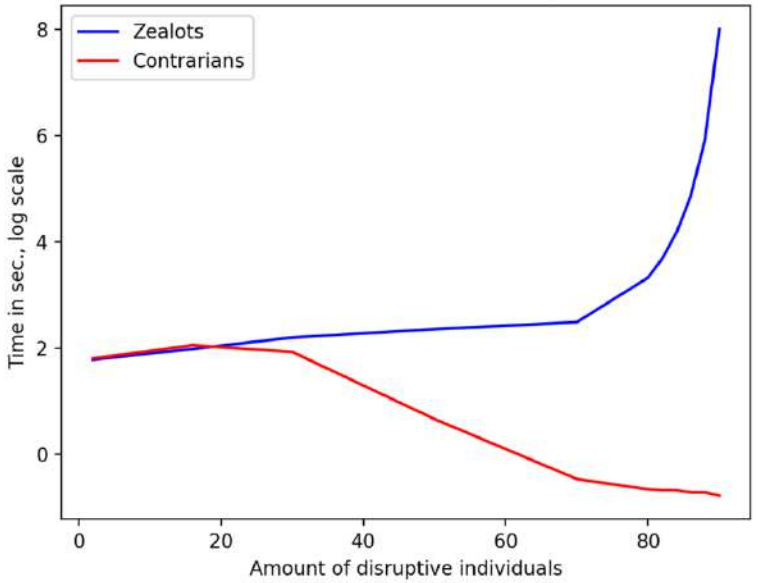

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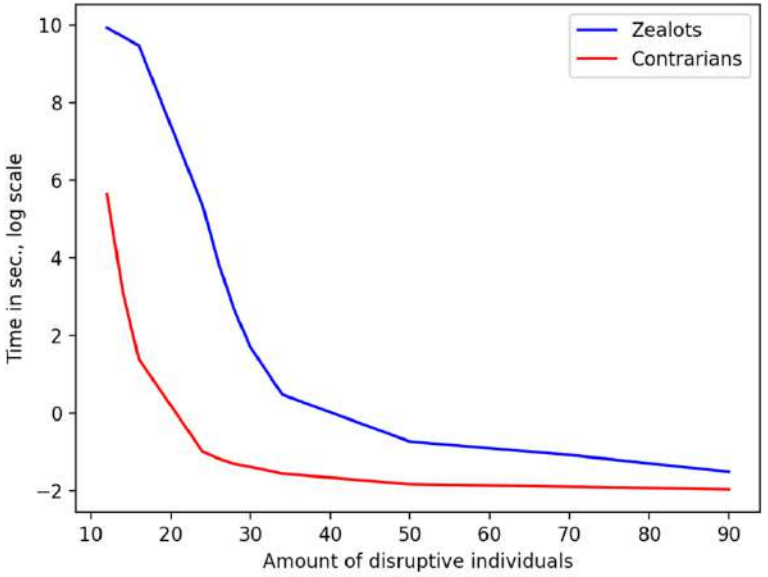


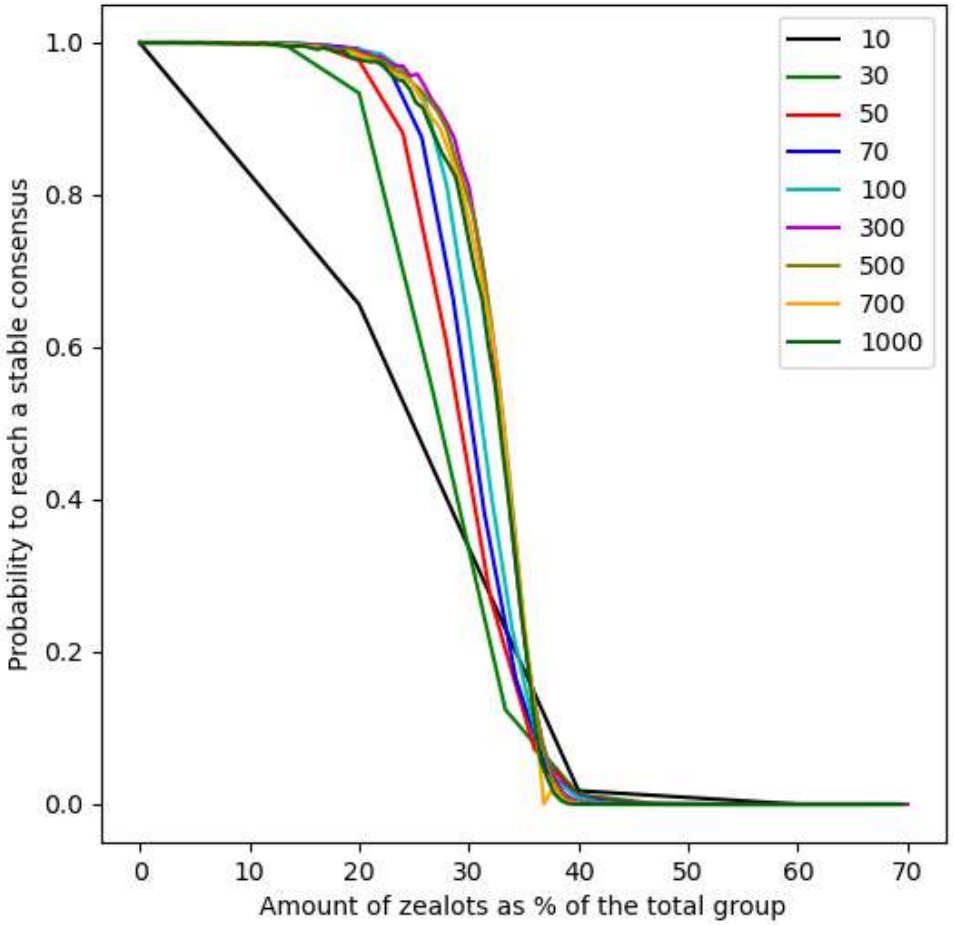
Plasma Lab
Statistical Model-Checker

PRISM
model checker

→ Fully symmetric system… what pushes it to one decision or another?

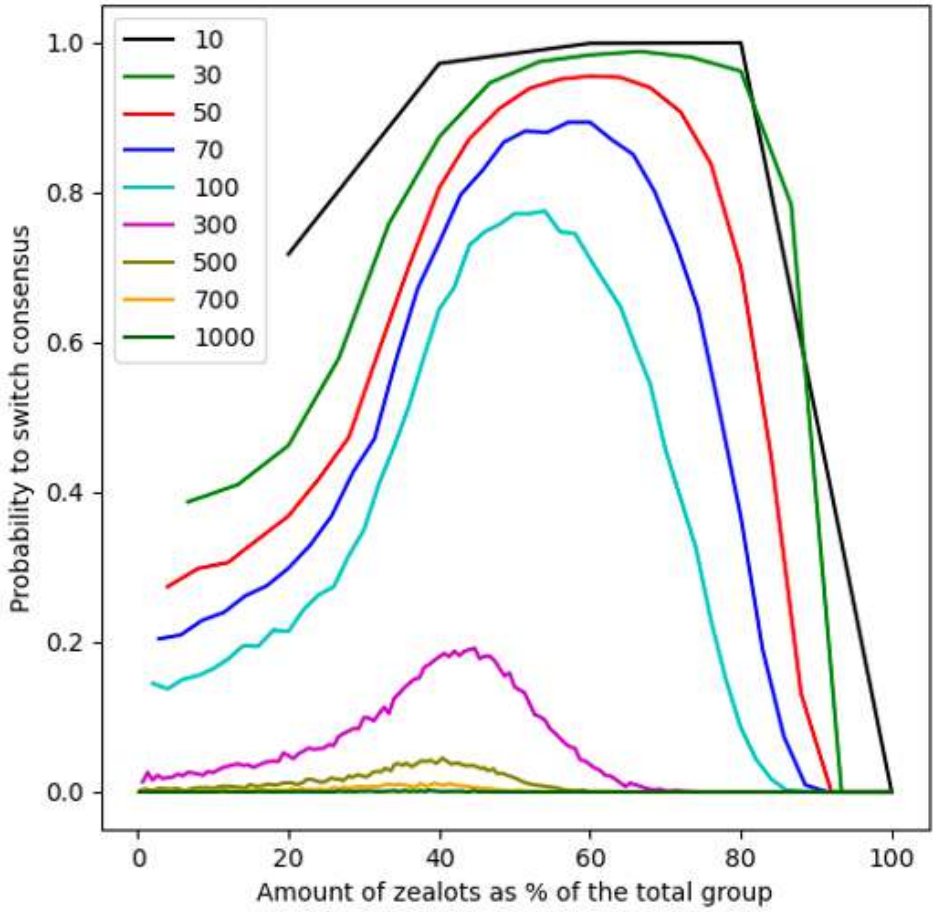
Results: Group size effect

Consensus reaching (zealots)



… robust to group size scaling!

Consensus switching (zealots)



… sensitive to group size scaling

Conclusion and outlook

- A small increase of disruptive individuals can drastically affect consensus dynamics
- Our method with SMC allows to explore consensus beyond mean-field analysis or single simulation

- **Stable consensus**:
  - Cross-inhibition model robust up to certain fraction of zealots/contrarians, then rapid phase transition
  - Zealots are less harmful for reaching consensus than contrarians

- **Switching consensus**:
  - Range of zealots for which such trajectories occur with non-negligible probability, but very rare for high number of zealots
  - Contrarians promote switching dynamics

- Future work
  - Group size effect: characterisation of a class of stochastic systems for which consensus reaching is robust to scaling
  - Asymmetric model – what if only one decision is correct?
  - Control theory: interventions over individuals for a global outcome (e.g. vaccination policy)

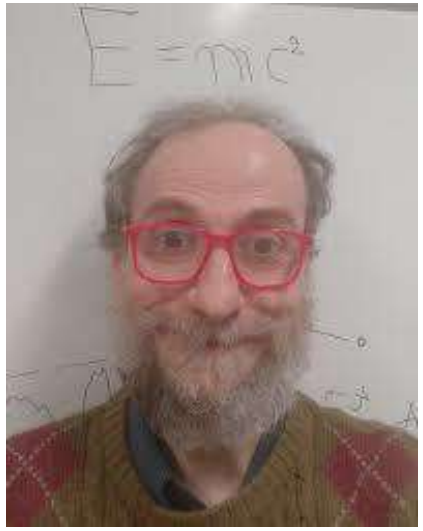
Centre for the Advanced Study of Collective Behaviour

UNIVERSITÀ DEGLI STUDI DI TRIESTE

MAX-PLANCK-GESELLSCHAFT

Universität Konstanz

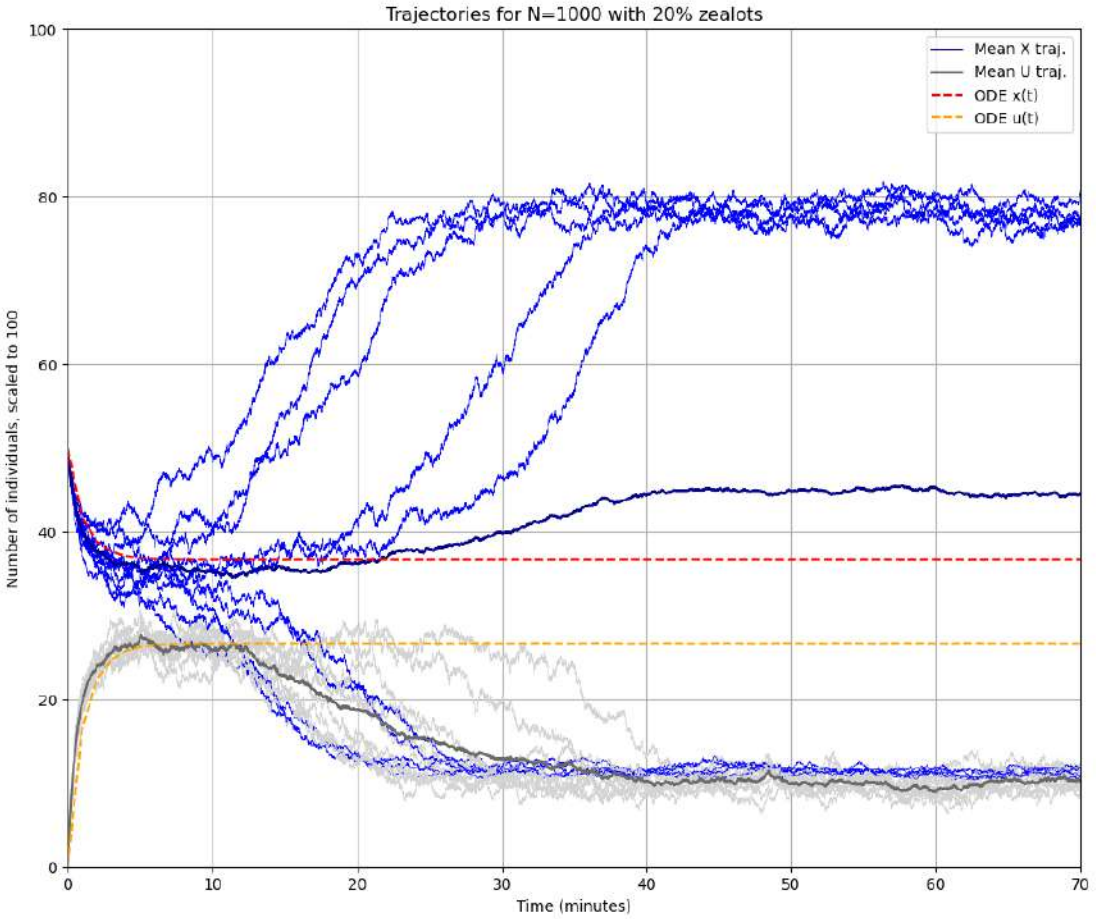
Julia Klein

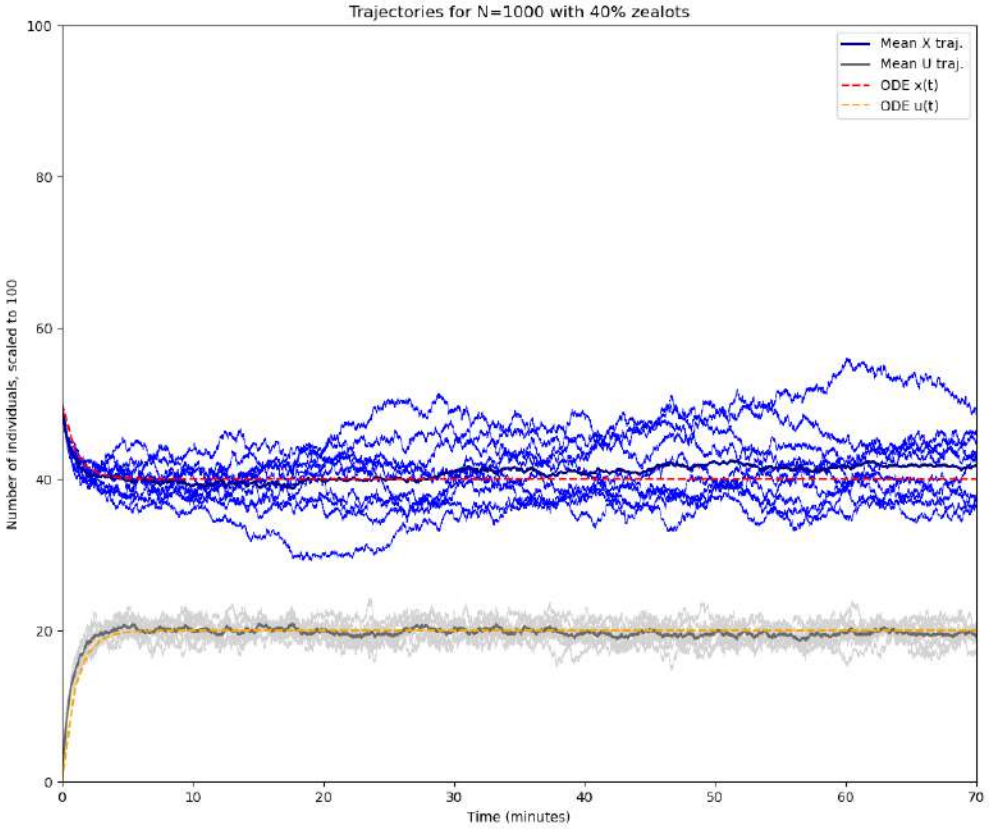


Alberto d'Onofrio

Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani | Sapienza Università di Roma

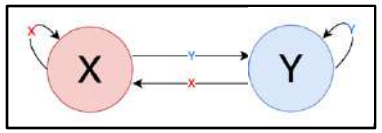
Thank you!

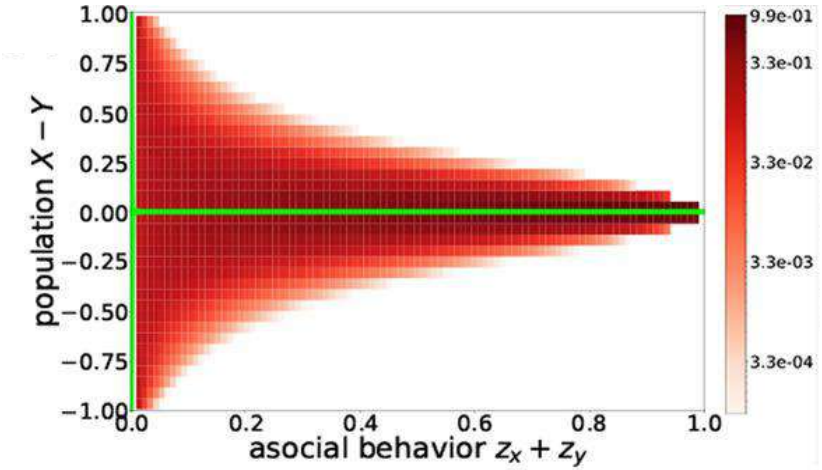

Trajectories for N=1000 with 20% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)
0
20
40
60
80
100
0
10
20
30
40
50
60
70


Trajectories for N=1000 with 40% zealots
Mean X traj.
Mean U traj.
ODE x(t)
ODE u(t)
Number of individuals, scaled to 100
Time (minutes)
0
20
40
60
80
100
0
10
20
30
40
50
60
70

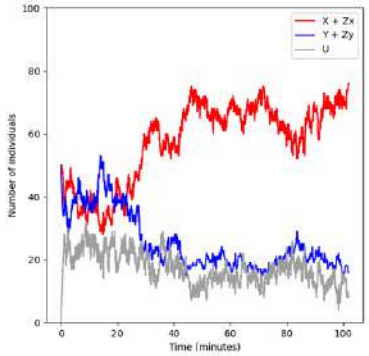
Studied Model of Decision-Making

Voter Model

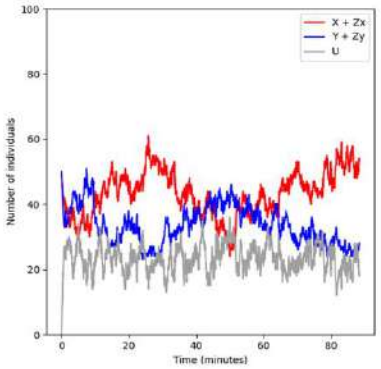




→ in presence of asocial individuals, the swarm gets quickly locked into an indecision state



No zealots
→ quick, stable consensus

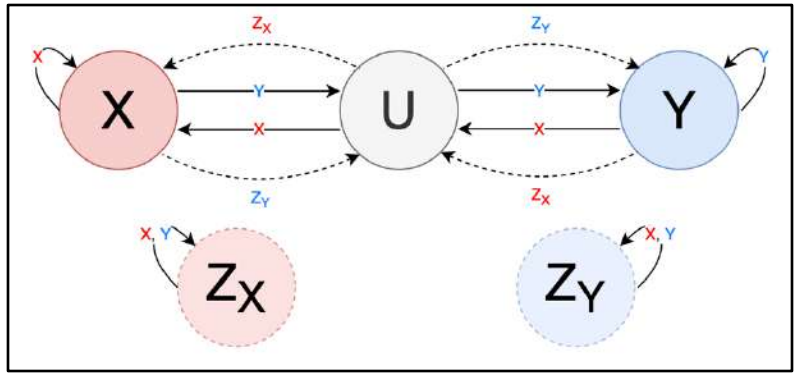


2% zealots
→ permanent indecision

Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

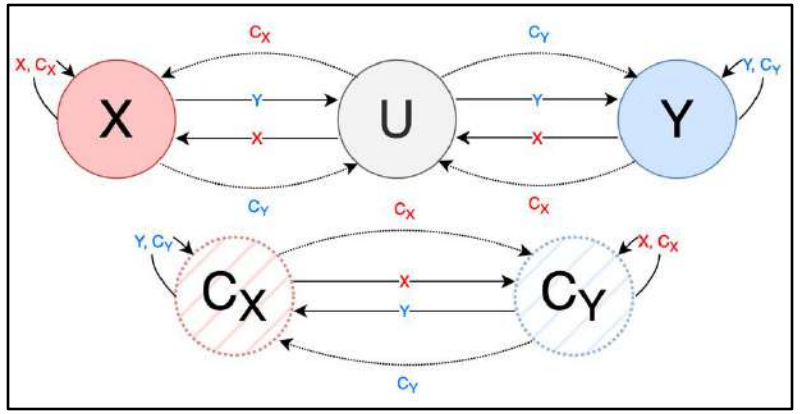
Studied Model with Disruptive Individuals

Cross-Inhibition model with Zealots



- **Zealots**: stubborn individuals which never change their own opinion
- Four additional reactions, where ‘pure’ agents interact with zealots & adjust their own states

Cross-Inhibition model with Contrarians



- **Contrarians**: individuals which counter the opinion of the individual they interact with
- Eight additional reactions, where contrarians influence ‘pure’ individuals & are influenced by others with the same opinion