Exploring Consensus Robustness in Swarms with Disruptive Individuals

Julia Klein[1,2], Alberto d'Onofrio[3], Tatjana Petrov[2,3,4]

[1]University of Konstanz [2]Centre for the Advanced Study of Collective Behaviour, University of Konstanz [3]University of Trieste [4]Max Planck Institute of Animal Behaviour, Radolfzell

julia.klein@uni-konstanz.de

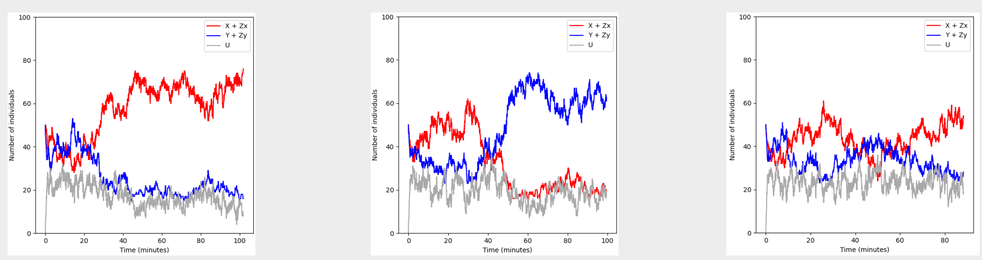
Abstract

- Consensus in collective systems essential for coordinated behaviour
- Strongly opinionated minorities can disrupt opinion dynamics

- **Approach**: investigate robustness of consensus-reaching among stubborn individuals (**zealots**) and **contrarians**
  - Statistical model checking to formally quantify **robustness** under perturbations of the amount of disruptive individuals
  - Investigate robustness landscape for combinations of different disruptive agents
- Goal: guide design and control of swarm robotics systems with focus on resilience to disruptive agents

Background

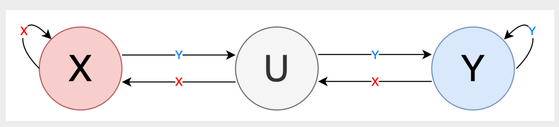
- Underlying model of decision-making: **cross-inhibition model** with 'undecided' state between switching opinion (2a), including
  - **zealots**: never change their own opinion (2b)
  - **contrarians**: counter opinion of individual they interact with (2c)
  - both zealots and contrarians (2d)
- Scenario:
  - 2 equivalent options X and Y
  - Group of N=100 individuals
  - Initial state: equally split between X and Y, varying amount of disruptive individuals
- Observation of 3 different group dynamics:



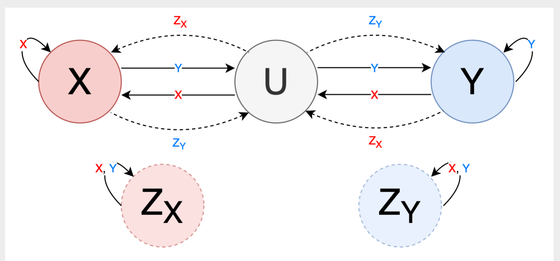
(1a) stable consensus (1b) switching consensus (1c) no consensus

Models

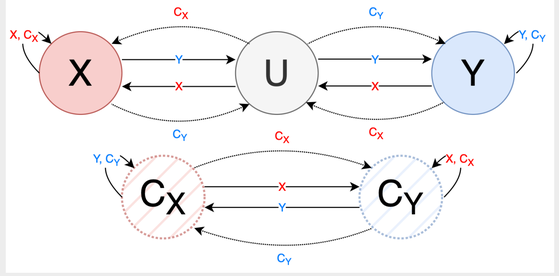
(2a)



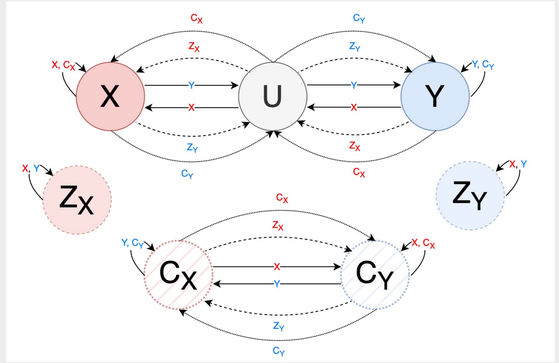
(2b)



(2c)



(2d)



Cross-Inhibition model

$$X + Y \xrightarrow{q_x} X + U$$
$$X + Y \xrightarrow{q_y} Y + U$$
$$X + U \xrightarrow{q_x} 2X$$
$$Y + U \xrightarrow{q_y} 2Y$$

Zealots

$$Y + Z_X \xrightarrow{q_x} U + Z_X$$
$$U + Z_X \xrightarrow{q_x} X + Z_X$$
$$X + Z_Y \xrightarrow{q_y} U + Z_Y$$
$$U + Z_Y \xrightarrow{q_y} Y + Z_Y$$

Contrarians

$$X + C_Y \xrightarrow{q_y} U + C_Y$$
$$U + C_Y \xrightarrow{q_y} Y + C_Y$$
$$X + C_X \xrightarrow{q_y} X + C_Y$$
$$Y + C_X \xrightarrow{q_x} U + C_X$$
$$U + C_X \xrightarrow{q_x} X + C_X$$
$$Y + C_Y \xrightarrow{q_x} Y + C_X$$
$$C_X + C_X \xrightarrow{q_y} C_Y + C_Y$$
$$C_Y + C_Y \xrightarrow{q_x} C_X + C_X$$

Both Zealots & Contrarians

$$C_X + Z_X \xrightarrow{q_y} C_Y + Z_X$$
$$C_Y + Z_Y \xrightarrow{q_x} C_X + Z_Y$$

Analysis

1. Formally describe observed scenarios in **Bounded Linear Temporal Logic**
   - Definition of consensus with five parameters: at least *majority m%* of population commits to same decision, *difference* of at least *d* between both groups, consensus is reached within *reaching time t*, consensus is maintained for at least *holding time h*, switch to other opinion happens within *switching time s*
   - Baseline: m=50, d=10, t=35, h=40, s=10
   - **Stable consensus** (1a) in BLTL:

$$F_{\leq t}(G_{\leq h}(((x + Z_x + C_x \geq min_m) \wedge ((x + Z_x + C_x) - (y + Z_y + C_y) \geq d)) \vee ((y + Z_y + C_y \geq min_m) \wedge ((y + Z_y + C_y) - (x + Z_x + C_x) \geq d)))))$$

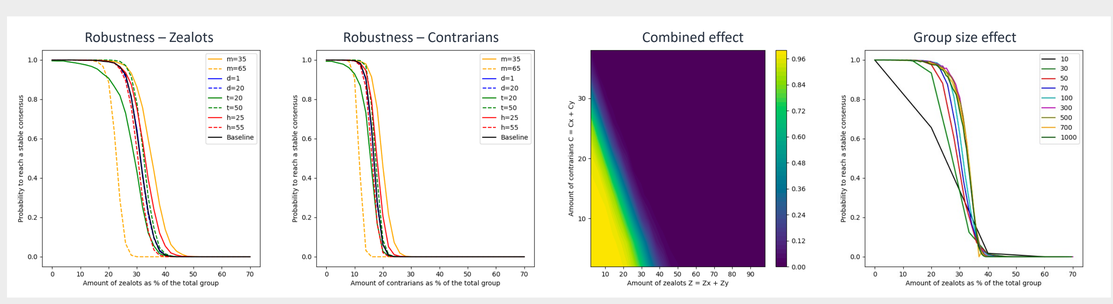
   - **Switching consensus** (1b) in BLTL:

$$F_{\leq t}((((x + Z_x + C_x) - (y + Z_y + C_y) \geq d) \wedge (true\ U_{\leq s}((y + Z_y + C_y) - (x + Z_x + C_x) \geq d))) \vee (((y + Z_y + C_y) - (x + Z_x + C_x) \geq d) \wedge (true\ U_{\leq s}((x + Z_x + C_x) - (y + Z_y + C_y) \geq d))))$$

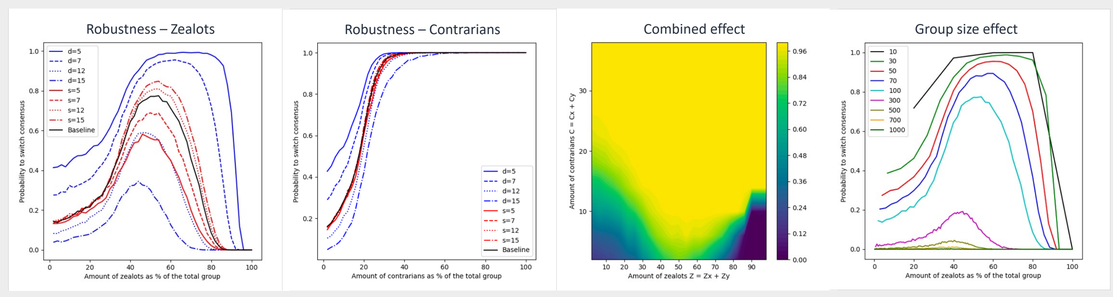
2. **Statistical model checking** to estimate satisfaction probability and explore robustness of scenarios

Results

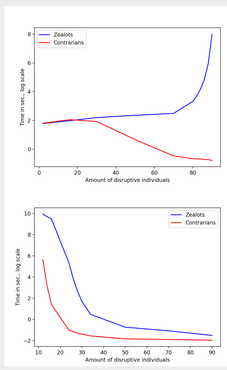
- Robustness of reaching a stable consensus (1a)



- Robustness of switching consensus (1b)



- Expected times



…to reach consensus

#	2	16	30	50	70	80	82	84	86	88	90
Zealots	5.95	7.28	9.02	10.57	12.04	27.82	39.94	64.95	128.85	374.04	2975.68
Contrarians	6.07	7.81	6.89	1.95	0.63	0.52	0.51	0.51	0.49	0.49	0.46

…to hold consensus

#	12	14	16	24	26	28	30	34	50	70	90
Zealots	20686.51	16368.28	13047.85	210.98	47.71	14.13	5.46	1.61	0.48	0.34	0.22
Contrarians	283.57	22.53	4.03	0.37	0.31	0.27	0.25	0.21	0.16	0.15	0.14

Conclusion

- Disruptive individuals can change opinion dynamics
- Our method is more informative
- **Stable consensus**: robust up to certain #zealots/#contrarians, then rapid phase transition; zealots are less harmful for reaching consensus
- **Switching consensus**: only range of zealots for which switching occurs with high probability; contrarians promote switching dynamics

- **Future work**: explore variations of current scenario, asymmetric model (vote for better option), control theory

References

[1] Reina, A., Zakir, R., De Masi, G., Ferrante, E.: Cross-inhibition leads to group consensus despite the presence of strongly opinionated minorities and asocial behaviour. Communications Physics 6(1), 236 (2023)

This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2117 – 422037984.