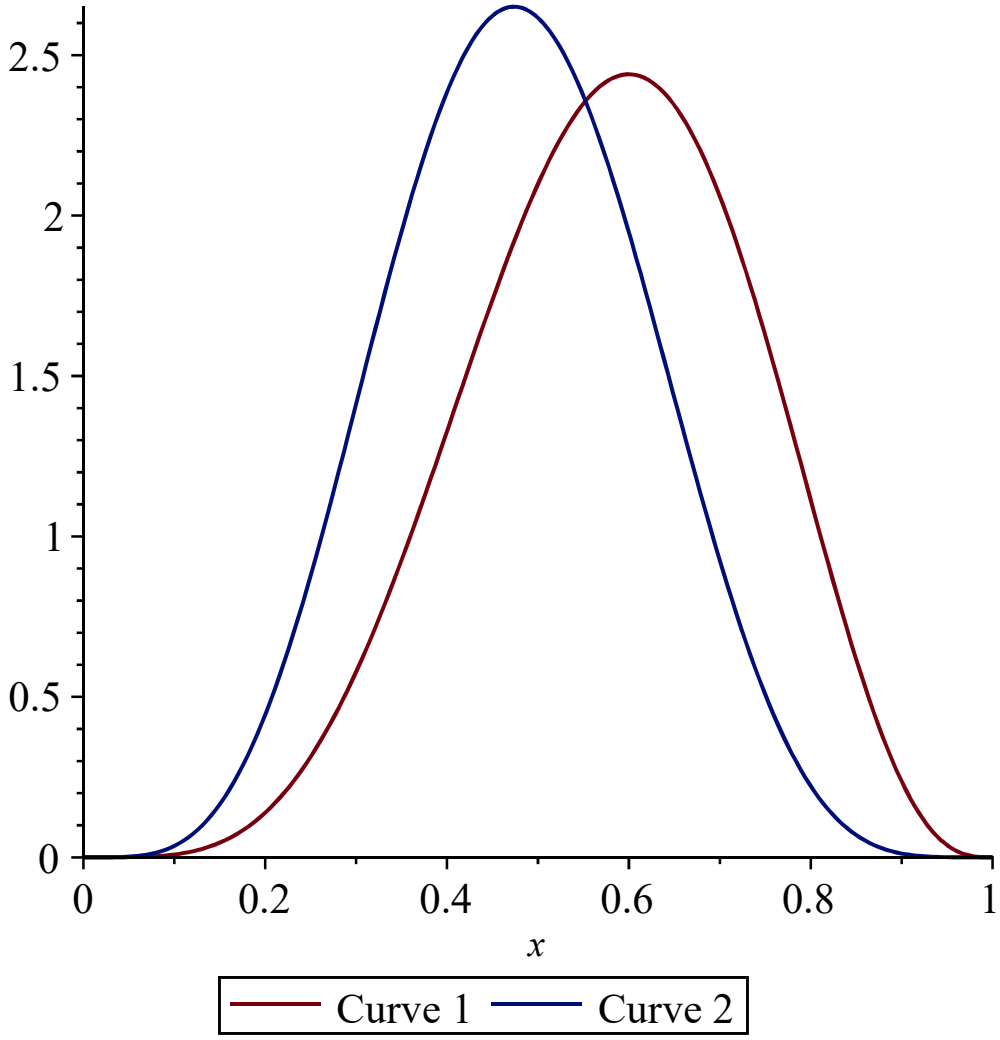
```
> with(plots):
  with(Statistics):
> CA := RandomVariable(BetaDistribution(5.5, 4)):
  CB := RandomVariable(BetaDistribution(5.5, 6)):
> %CA := RandomVariable(UniformDistribution(0, 1)):
  %CB := RandomVariable(UniformDistribution(0, 1)):
> priorA := PDF(CA, pa):
  priorB := PDF(CB, pb):
> plot([PDF(CA, x), PDF(CB, x)], x = 0..1)
```




```
> likelihood := pa/(pa + pb - pa·pb);
```

$$likelihood := \frac{pa}{-pa\,pb + pa + pb} \tag{1}$$

```
> prior := priorA·priorB;
```

$$prior := \tag{2}$$

$$\left(\left\{\begin{array}{ll} 0 & pa < 0 \\ 379.8437500\, pa^{4.5}\, (1 - pa)^3 & pa < 1 \\ 0 & otherwise \end{array}\right\}\right)\left(\left\{\begin{array}{ll} 0 & pb < 0 \\ 1894.470703\, pb^{4.5}\, (-pb + 1)^5 & pb < 1 \\ 0 & otherwise \end{array}\right\}\right)$$

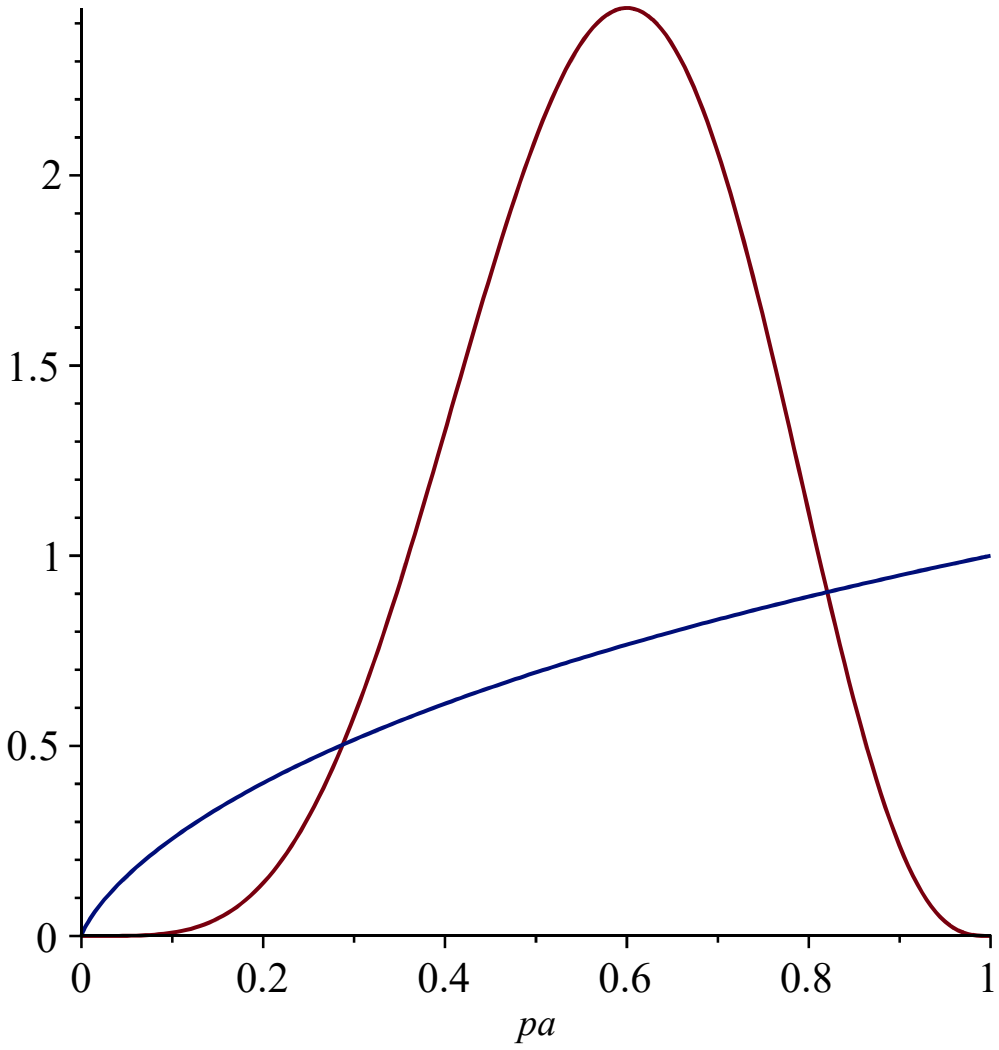
> $prior\_check := \int_0^1 prior \, dpb;$

$likelihood\_check := \int_0^1 likelihood \, dpb;$

$plot([prior\_check, likelihood\_check], pa = 0..1)$

$$prior\_check := \left\{\begin{array}{ll} 0. & pa < 0. \\ -379.8437500\, (pa - 1.)^3\, pa^{9/2} & pa < 1. \\ 0. & 1. \le pa \end{array}\right.$$

$$likelihood\_check := -\frac{pa \ln\left(\frac{1}{pa}\right)}{pa - 1}$$



> $margLikelihood := \int_0^1 \int_0^1 likelihood \cdot prior \, dpb \, dpa;$

$$margLikelihood := 0.7341085410 \quad \textbf{(3)}$$

> $posterior := \frac{likelihood \cdot prior}{margLikelihood};$

$posterior :=$ **(4)**

$$\frac{1}{-pa\,pb + pa + pb}\left(1.362196384\,pa\left(\begin{cases} 0 & pa < 0 \\ 379.8437500\,pa^{4.5}\,(1-pa)^3 & pa < 1 \\ 0 & otherwise \end{cases}\right)\left(\begin{cases} 0 & pb < 0 \\ 1894.470703\,pb^{4.5}\,(-pb+1)^5 & pb < 1 \\ 0 & otherwise \end{cases}\right)\right)$$

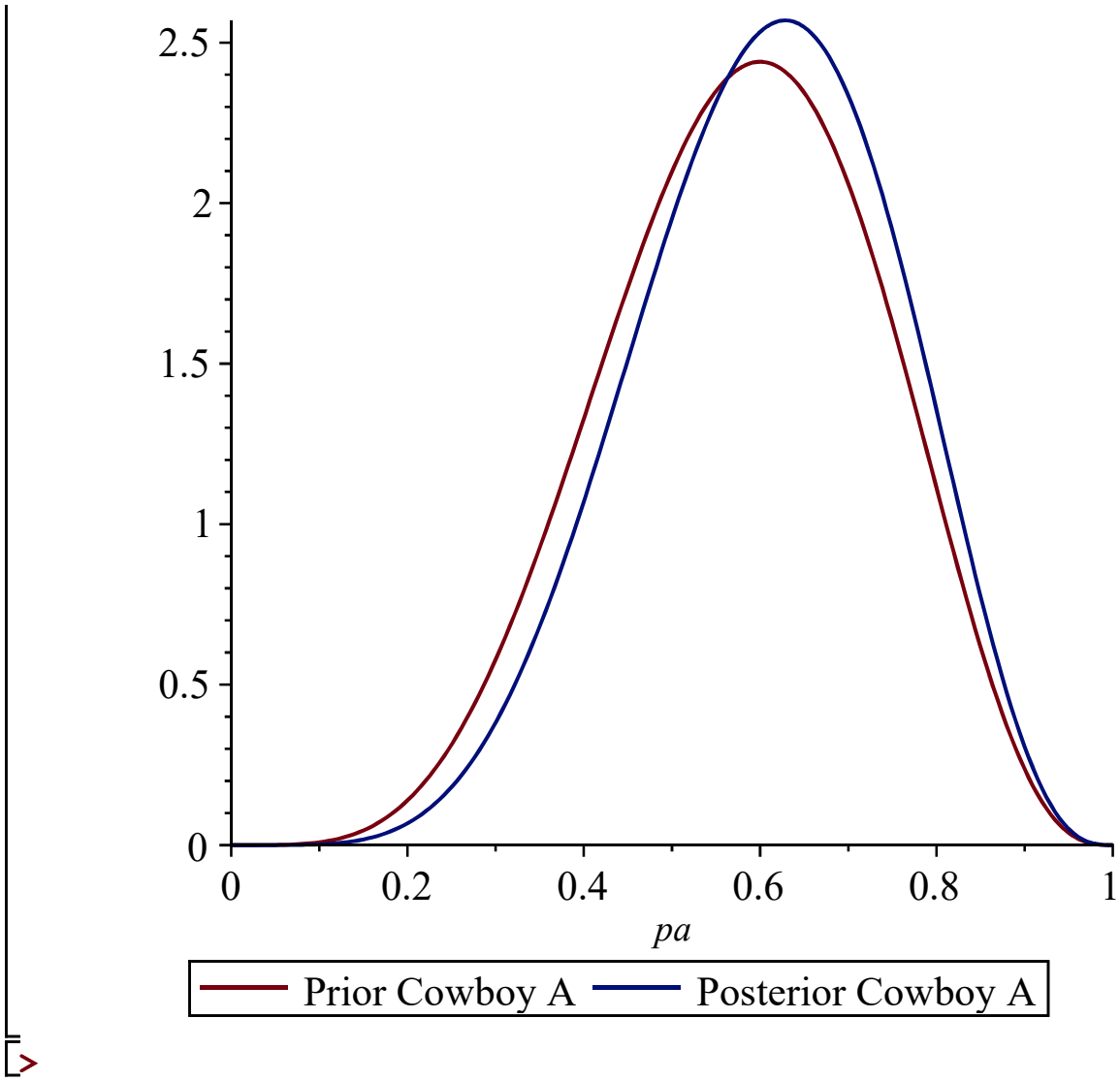
There are some issues with the symbolic integration method by maple -> resort to numerical integration!

```
> marginalPosterior := int(posterior, pb = 0 .. 1);
  marginalPosteriorf := evalf(Int(posterior, pb = 0 .. 1));
```

$$marginalPosterior := \left\{ -\frac{0.1347452559 \left(1.4549535\ 10^{7}\ pa^{21/2} \arctan\left(\frac{\sqrt{1. - 1.\ pa}}{\sqrt{pa}} \right) - 8064.\ pa^{15} \sqrt{1. - 1.} \right.}{}$$

$$marginalPosteriorf := \int_{0.}^{1.} \frac{1}{-pa\ pb + pa + pb} \left(1.362196384\ pa \left(\left\{ \begin{array}{ll} 0. & pa < 0. \\ 379.8437500\ pa^{4.5}\ (1. - 1.\ pa)^{3} & pa < 1. \\ 0. & otherwise \end{array} \right. \right) \left(\left(\left\{ \begin{array}{ll} 0. & pb < 0. \\ 1894.470703\ pb^{4.5}\ (-1.\ pb + 1.)^{5} & pb < 1. \\ 0. & otherwise \end{array} \right. \right) \right) \right) dpb \quad \textbf{(5)}$$

```
> plot([priorA, marginalPosteriorf], pa = 0 .. 1)
```


2.5
2
1.5
1
0.5
0
0
0.2
0.4
0.6
0.8
1
pa
Prior Cowboy A
Posterior Cowboy A