Lab 2 (Week 10)

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Instructions

1. Lab 2 covers materials taught in **week 8 - 10**. It is designed to test and reinforce the concepts you have learned, and in some cases, extend them to more advanced scenarios.
2. **Please make sure you follow the instructions.**
3. Name your file in the following format: **FirstName_LastName_w271_F15_week10_lab2.FileExtension**
4. Note that the lab is a **group exercise**. We encourage that you form a group between *3 - 4 students*. Each group makes 1 submission to the ISVC platform by **the beginning of the live session on Wednesday, November 18. Note that you have 2 weeks to complete the lab instead of 1 week as stated originally**.
5. Submission: Your group will **submit 2 files**:
    1. A report of all of your answers (in pdf of html format if you print them from Rmd or juypter notebook)
    2. An extensively annotated R-scripts, which could an Rmd file. It is very important that you show the steps to arrive at the answers and **highlight in bold** the final answer. Graphs must have a descriptive title, well-labeled x- and y-axis, clear axis tick marks, and legend (where needed). They must be reasonably sized.
6. Late submission will not be given credits. If your group have troubles completing the exam on-time, please let us know as soon as you can, not on the day of submission or the day before.

Question 1: Concepts (15 points)

1.1 (1 points) Define the term *stochastic process*.

1.2 (2 points) Define the term *time series*. What is the difference between a stochastic process and a time series?

1.3 (3 points) In your own words, discuss the mean function, variance function, and measure of dependency structure in the context of time series (week 8 - 10) and compare them with those we studied in classical linear model (week 1 - 7).

1.4 (2 points) Define strict and weak statonarity

1.5 (3 points) Give an example of a time series in real life. Describe the series. Evaluate (not empirical work is needed) whether or not the series can be modeled using the class of autoregressive models?

1.6 (4 points) In your own words, define and describe *partial autocorrelation function (PACF)*. Why is it not enough just to *autocorrelation functon (ACF)* to capture the dependency of a series?

Question 2: (25 points)

Determine if each of the following models is stationary?

2.1 (5 points) $z_t = 0.95z_{t-1} + \omega_t$

2.2 (5 points) $z_t = 0.8z_{t-1} + 0.3z_{t-2} + \omega_t$

2.3 (5 points) $z_t = -0.5z_{t-1} + 0.5z_{t-2} + \omega_t$

2.4 (5 points) $z_t = z_{t-1} + 0.4z_{t-2} + \omega_t$

2.5 (5 points) $z_t = -0.5z_{t-1} - 0.25z_{t-2} + \omega_t$

Question 3: (30 points)

3.1 (1 point) Load the series series1.csv

3.2 (2 points) Describe the basic structure of the data and provide summary statistics of the series

3.3 (4 points) Plot histogram and time-series plot of the series. Describe the patterns exhibited in histograrm and time-series plot. For time series analysis, is it sufficient to use only histogram to describe a series?

3.4 (4 points) Plot the ACF and PACF of the series. Describe the patterns exhibited in the ACF and PACF.

3.5 (2 points) Estimate the series using the *maximum likelihood method* option of *ar()* function.

3.6 (1 points) Report the estimated AR parameters, the order of the model, and standard errors.

3.7 (2 points) Estimate the series using the *Ordinary Least Squre* option of *ar()* function.

3.8 (1 points) Report the estimated AR parameters, the order of the model, and standard errors.

3.9 (2 points) Estimate the series using the *Yule-Walker Equations* option of *ar()* function.

3.10 (1 points) Report the estimated AR parameters, the order of the model, and standard errors.

3.11 (5 points) Are these estimates the same? If so, derive the formular to justify your answer. If not, please explain (but no mathematical derivation is needed.) How does the function *ar()* choose the "best" AR model?

Question 4: (30 points)

4.1 Simulate a time series of lenght 100 for the following model. Name the series x.

$$x_t = \frac{5}{6}x_{t-1} - \frac{1}{6}x_{t-2} + \omega_t$$

4.2 Plot the correlogram and partial correlogram for the simulated series. Comments on the plots.

4.3 Estimate an AR model for this simulated series. Report the estimated AR parameters, standard erros, and the order of the AR model.

4.4 Construct a 95% confidence intervals for the parameter estimates of the estimated model. Do the "true" mode parameters fall within the confidence intervals? Explain the 95% confidence intervals in this context.

4.5 Is the estimated model stationary or non-stationary?

4.6 Plot the correlogram of the residuals of the estimated model. Comment on the plot.