Elements of Sub-Riemannian Geometry and its Applications

Julián Jiménez Cárdenas

Departamento de Matemáticas
Universidad de los Andes

June 01, 2022

Universidad de los Andes

Table of Contents

1. Distributions
    - Distributions and horizontal curves
2. Literature Review
3. Methodology
4. Experiment
5. Conclusion

Vector Bundle

Let E, M be two smooth manifolds. A map $\pi : E \to M$ is said to be a *smooth vector bundle* if it is a submersion, its fibers have the structure of finite dimensional vector spaces, and of every $p \in M$ there is an open neighborhood U of p and a diffeomorphism $\phi : U \times \mathbb{R}^k \to \pi^{-1}(U)$ such that the following diagram commutes

$$\begin{tikzcd} U \times \mathbb{R}^k \arrow[rr, "\phi"] \arrow[dr, "\mathrm{proj}_U"'] & & \pi^{-1}(U) \arrow[dl, "\pi"] \\ & U & \end{tikzcd}$$

and the map $v \mapsto \phi(p, v)$ is a linear isomorphism between $\pi^{-1}(p)$ and $\mathbb{R}^k$.

Remark

References I

Richard Montgomery.
A tour of subriemannian geometries, their geodesics and applications.
Number 91. American Mathematical Soc., 2002.

J.M. Lee.
Introduction to Smooth Manifolds.
Graduate Texts in Mathematics. Springer, 2003.

Frank W Warner.
Foundations of differentiable manifolds and Lie groups, volume 94.
Springer Science & Business Media, 2013.

Glenys Luke and Alexander S Mishchenko.
Vector bundles and their applications, volume 447.
Springer Science & Business Media, 2013.

References II

Allen Hatcher.
Vector bundles and k-theory.
Im Internet unter http://www. math. cornell. edu/˜ hatcher, 2003.

Chris Wendl.
Lecture notes on bundles and connections.
Lecture notes, Humboldt-Universität zu Berlin, 2008.

David W Lyons.
An elementary introduction to the hopf fibration.
Mathematics magazine, 76(2):87–98, 2003.

Chenjia Lin.
Introductory differential topology and an application to the hopf fibration.

References III

Shoshichi Kobayashi and Katsumi Nomizu.
Foundations of differential geometry, volume 1.
New York, London, 1963.

Vladimir Igorevich Arnol'd.
Mathematical methods of classical mechanics, volume 60.
Springer Science & Business Media, 2013.

Hassan Najafi Yunjiang Jiang and Josef Pozny.
Poisson geometry. lectures 1, 2, 3.
"https://webspace.science.uu.nl/~crain101/Poisson/poisson-lectures123.pdf", 2014.
[Online; accessed 29-November-2021].

References IV

Richard W Sharpe.
Differential geometry: Cartan's generalization of Klein's Erlangen program, volume 166.
Springer Science & Business Media, 2000.

John M Lee.
Introduction to Riemannian manifolds.
Springer, 2018.

Barrett O'neill.
Semi-Riemannian geometry with applications to relativity.
Academic press, 1983.

David Bao, S-S Chern, and Zhongmin Shen.
An introduction to Riemann-Finsler geometry, volume 200.
Springer Science & Business Media, 2012.