$$\int e^{ax}\cos(bx)\,dx = \frac{a e^{ax}\cos(bx) + b e^{ax}\,\mathrm{Sen}(bx)}{a^2+b^2} + C$$

Sen(bx)

$$\int_0^{\infty} e^{ax}\cos(bx)\,dx = \lim_{n\to\infty}\int_0^{n} e^{ax}\cos(bx)\,dx = \lim_{n\to\infty}\left(\frac{e^{an}\left(a\cos(bn)+b\,\mathrm{Sen}(bn)\right)}{a^2+b^2} - \frac{a}{a^2+b^2}\right)$$

$$= \left(\lim_{n\to\infty} \frac{e^{an}\left(a\cos(bn)+b\,\mathrm{Sen}(bn)\right)}{a^2+b^2}\right) - \frac{a}{a^2+b^2} \qquad a\in\mathbb{R}.$$

0

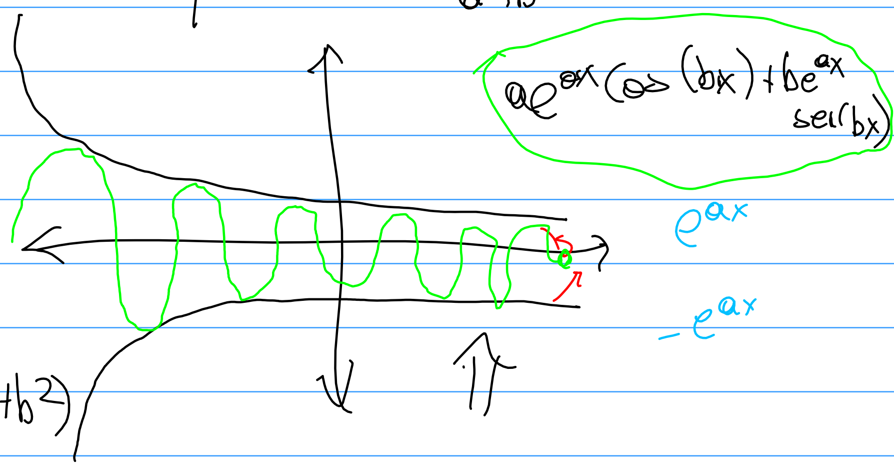
$|\cos(bn)|, |\mathrm{Sen}(bn)| \leq 1.$

- $a<0.$

$$0 \leq \lim_{n\to\infty}\left|\frac{e^{an}\left(a\cos(bn)+b\,\mathrm{Sen}(bn)\right)}{a^2+b^2}\right| \leq \lim_{n\to\infty}\frac{|a|e^{an}+|b|e^{an}}{a^2+b^2} = 0$$

$\parallel$

0 Converge.



$a<0$ converge y su valor es

$-a/(a^2+b^2)$ $\quad b/(a^2+b^2)$

- $a=0.$

$$\lim_{n\to\infty}\left(\frac{b\,\mathrm{Sen}(bn)}{b^2}\right) \text{ diverge para todo } b\in\mathbb{R}. \qquad \int_0^{\infty} dx \text{ diverge}$$

- $a>0$

$$\lim_{n\to\infty}\left(\frac{e^{an}\left(a\cos(bn)+b\,\mathrm{Sen}(bn)\right)}{a^2+b^2}\right)$$

$\parallel$ $f(n)$

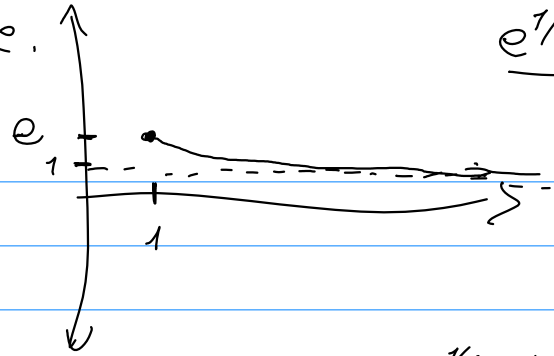
$\exists L\in\mathbb{R}.\ \forall\varepsilon>0\ \exists N\in\mathbb{N}$ t.q. si $n>N$ entonces $|f(n)-L|<\varepsilon$

Negación

$\forall L\in\mathbb{R}\ \exists\varepsilon>0\ \forall N\in\mathbb{N}$ si $n>N$ ocurre que $|f(n)-L|\geq\varepsilon$.

$$\int_1^{\infty} \frac{e^{1/x}}{x^3+1}\,dx \quad // \quad \text{Convergente.}$$

$$\le \int_1^{\infty} \frac{e}{x^3+1}\,dx$$



$e^{1/x} = f(x)$

$\Downarrow$

decreciente

$\lim_{x\to\infty} e^{1/x} = 1.$

$e \ge e^{1/x} \quad \forall x \in [1,\infty).$

$$\le \int_1^{\infty} \frac{e}{x^3}\,dx = -\frac{e}{2x^2}\Big|_1^{\infty} = \lim_{n\to\infty}\left(\frac{-e}{2x^2}\right) - \left(\frac{-e}{2\cdot 1^2}\right) = e/2.$$

$$\lim_{x\to\infty} -\frac{e}{2x^2} = 0.$$

- $$\int t^3\sqrt{1+t^2}\,dt = \int \tan^3\theta\,\sec^3(\theta)\,d\theta = \int(\sec^2\theta-1)\tan\theta\sec^3\theta\,d\theta.$$

$\sec\theta\tan\theta \cdot \sec^2\theta$

u^2

$t = \tan\theta$	$u = \sec\theta$
$dt = \sec^2\theta\,d\theta$	$du = \tan\theta\sec\theta\,d\theta$

$$= \int(u^2-1)\,u^2\,du.$$

- $$\int_1^{\infty} \frac{x}{x^4+1}\,dx \simeq O(1/x^3) \int_1^{\infty}\frac{dx}{x^3} \quad \text{converge}$$

$$\int_1^{\infty} \frac{x}{x^4+1}\,dx \le \int_1^{\infty}\frac{x}{x^4}\,dx = \int_1^{\infty}\frac{dx}{x^3} = \frac{-1}{2x^2}\Big|_1^{\infty} = 1/2.$$

Converge.

- $$\int \frac{x^5}{\sqrt{1+x^2}}\,dx = \int \frac{\tan^5\theta}{\sec\theta}\sec^2\theta\,d\theta = \int \tan^5\sec\theta\,d\theta = \int \tan^4\theta\,\tan\theta\sec\theta\,d\theta$$

$x = \tan\theta$	$= \int(\sec^2\theta-1)^2\tan\theta\sec\theta\,d\theta, = \int(u^2-1)^2\,du$
$dx = \sec^2\theta\,d\theta$	

$u = \sec\theta$

$du = \tan\theta\sec\theta\,d\theta$

$$\int f(x)\,dx = F(x)+C.$$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = \lim_{x\to b}\left(F(x)\right) - \lim_{x\to a}\left(F(x)\right)$$

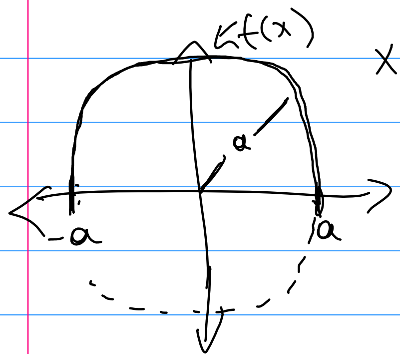
L'Hopital

$$¿\,\infty\cdot 0,\quad 0\cdot\infty,\quad 1^{\infty},\quad \infty^{0}\,?$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\frac{(0)'}{(0)'},\quad \frac{(\infty)'}{(\infty)'},\quad \infty\cdot 0 \quad 0\cdot\infty \qquad \log$$

$$e^{\log(\dots)}.$$



$$x^2+y^2=a^2$$

$$y^2 = a^2-x^2$$

$$y = \sqrt{a^2-x^2} = f(x) \Rightarrow f'(x) = \frac{-2x}{2\sqrt{a^2-x^2}}$$

$$L = \int_{-a}^{a}\sqrt{1+(f'(x))^2}\,dx = \int_{-a}^{a}\sqrt{1+\frac{x^2}{a^2-x^2}}\,dx = \int_{-a}^{a}\frac{\sqrt{a^2\cancel{-x^2}\cancel{+x^2}}}{\sqrt{a^2-x^2}}\,dx = a\int_{-a}^{a}\frac{dx}{\sqrt{a^2-x^2}}$$

$$\left.\begin{array}{l} x = a\,\mathrm{Sen}\,\theta \\ dx = a\cos\theta\,d\theta \end{array}\right| = a\int_{-\pi/2}^{\pi/2}\frac{a\cos\theta}{a\cos\theta}\,d\theta = a\cdot\left(\pi/2+\pi/2\right) = a\pi.$$

$$\int_{-\pi/2}^{\pi/2} d\theta = \theta\Big|_{-\pi/2}^{\pi/2}$$