EP352 , mh
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VSWR and Guided Microwaves

1 Purpose

1. To become familiar with microwave propagation in rectangular waveguides.
2. To become familiar with microwave generation and measurement techniques.

2 Theory

As the speed of electronics increases to the gigahertz region and beyond, conventional circuit analysis starts to break down due to the fact that the signals propagate at a finite speed throughout a circuit. This is due to the intrinsic relationship (Maxwell equations) between voltage and current signals on the one hand, and electromagnetic wave propagation on the other hand. One of the simplest and most direct ways of demonstrating propagation effects is by launching a gigahertz signal into a rectangular metal waveguide, and using a rectifying diode to monitor the voltage at different points along the guide. (How would the voltage vary along the guide if a kilohertz signal generator was connected to the guide?)

To analyze signal propagation along a given conducting channel (here we use a hollow rectangular metal waveguide, but we could equally well consider a coaxial cable or the more-common strip-line geometry), one has to specify the geometry and the material parameters. Usually the metallic components can be considered ideal conductors, and the dielectric components as ideal insulators with real dielectric constants. With this information, Maxwell's equations can be solved for the harmonic electromagnetic modes supported by this system in the frequency range of interest. The solutions for the ideal rectangular metal waveguide are particularly simple, especially if one only considers solutions with electric field vectors perpendicular to the axis of the guide (so called transverse electric modes, or TE modes). With reference to Fig. 1 for the geometry, the solution for the electric field takes the form

$$\begin{aligned}\vec{E}(x,y,z) = &\left[E_x^R \sin\left(\frac{n\pi}{b}z\right)\cos\left(\frac{m\pi}{a}x\right)\hat{x} + E_z^R \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}z\right)\hat{z}\right] e^{-j\left(\omega t - \frac{2\pi y}{\lambda_g}\right)} \\ &+ \left[E_x^L \sin\left(\frac{n\pi}{b}z\right)\cos\left(\frac{m\pi}{a}x\right)\hat{x} + E_z^L \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}z\right)\hat{z}\right] e^{-j\left(\omega t + \frac{2\pi y}{\lambda_g}\right)}\end{aligned} \tag{1}$$

where

$$\left(\frac{1}{\lambda_g}\right)^2 = \left(\frac{1}{\lambda_0}\right)^2 - \left(\frac{n}{2b}\right)^2 - \left(\frac{m}{2a}\right)^2 \tag{2}$$

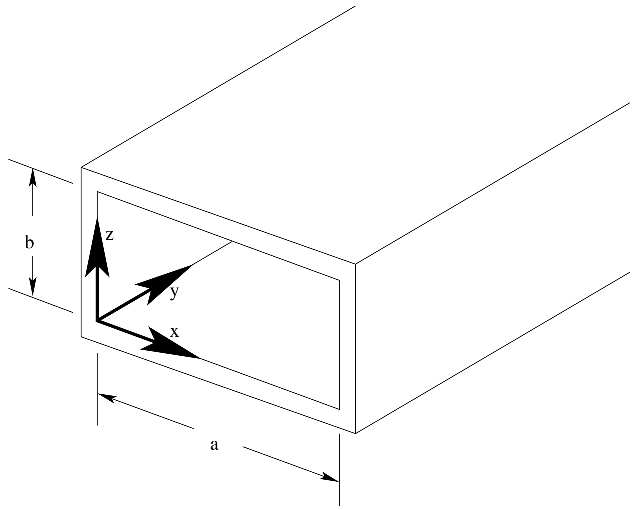

Figure 1: A waveguide.

Here λ_g is the wavelength in the waveguide, $\lambda_0 = c/\nu$ is the wavelength in free space, and R and L refer to right and left propagating waves respectively. The x and z components of the field in each direction are not independent. The spectrum of allowed modes is discrete (due to the similarity of the free space wavelengths and the guide geometry) and they can be labelled by positive integer indices (m and n) that basically specify how many nodes the electric field amplitude exhibits along the two transverse axes (x and z in Fig. 1). One, but not both, of (m, n), can be zero.

From Eq. 2 it is clear that the solution propagates along the guide, unattenuated, if the square of the guide wavelength is real, but is exponentially attenuated, with no oscillatory behaviour along y at all, if the square of the guide wavelength is negative. For each mode m, n, therefore, there is a corresponding maximum wavelength (minimum frequency), beyond which electromagnetic waves will not propagate along the guide. This wavelength is called the cutoff wavelength, and is given by

$$\left(\frac{1}{\lambda_c}\right)^2 = \left(\frac{n}{2b}\right)^2 + \left(\frac{m}{2a}\right)^2 \tag{3}$$

Thus the propagation characteristics at a given frequency are completely determined by the dimensions of the guide, as contained in the expression for the cutoff wavelength.

The right and left propagating field amplitudes are determined by the nature of the microwave source, how it is coupled into the guide, and by what terminates the guide at the end opposite the signal launcher. In the present experiment, since $b < a$, the cutoff frequencies for all modes other than the first mode (TE_{10}) are higher than the frequencies available from the source, hence we only have to be concerned with the ratio $\zeta = E^L/E^R$. The overall magnitude is a function of the source and its coupling into the guide. The ratio ζ is by definition the reflection coefficient of the right propagating wave at the termination of the waveguide. As in conventional electronics, the reflection coefficient ζ is related to the impedance mismatch between (in this case) the guide itself and the load, Z_l:

$$\zeta = \frac{Z_l - Z_g}{Z_l + Z_g} \tag{4}$$

The guide impedance, Z_g can be obtained by solving Maxwell's equations for the magnetic field as well as the electric field, and the result for the TE_{10} mode is $Z_g = 120\pi\lambda_g/\lambda_0$ (Ω). The superposition of left and right propagating waves gives rise to standing waves as illustrated in Fig. 2. You should be able to show that the reflection coefficient ζ, and hence the termination impedance, can be determined by measuring the Voltage Standing Wave Ratio (VSWR)

$$VSWR = \frac{V_{max}}{V_{min}} \tag{5}$$

and the phase of the voltage maxima and/or minima with respect to the end of the guide.

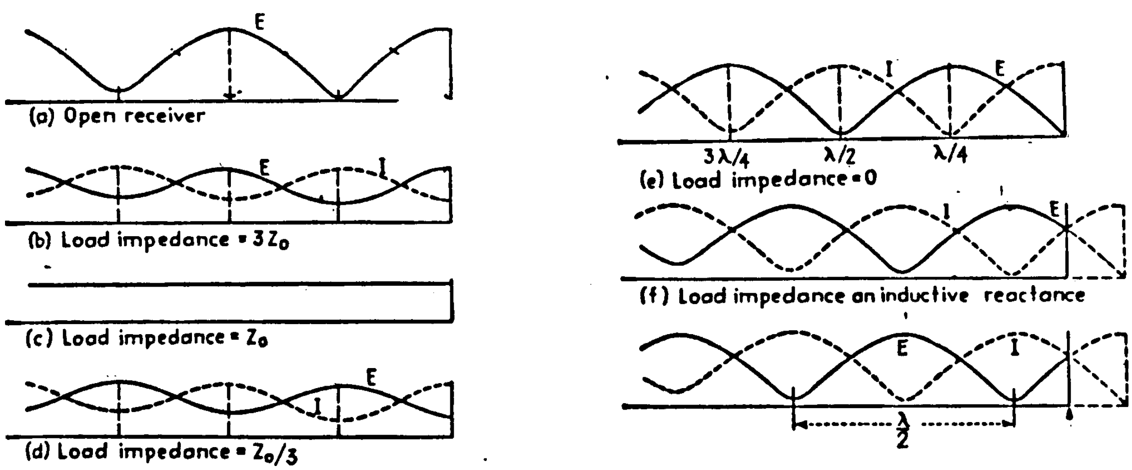

Figure 2: Solid lines show the absolute magnitude of the net electric field with zero baseline. Here Z_0 means the guide impedance Z_g.

The Smith chart is an instrument designed to simplify the conversion from VSWR and phase measurements to the reflection coefficient or load impedance. The theory of the Smith chart is in the Appendix and several books can be found explaining the use of the Smith chart. "Microwave Theory" and "Measurements and Handbook of Microwave Measurements" are both in the lab library.

3 Procedure

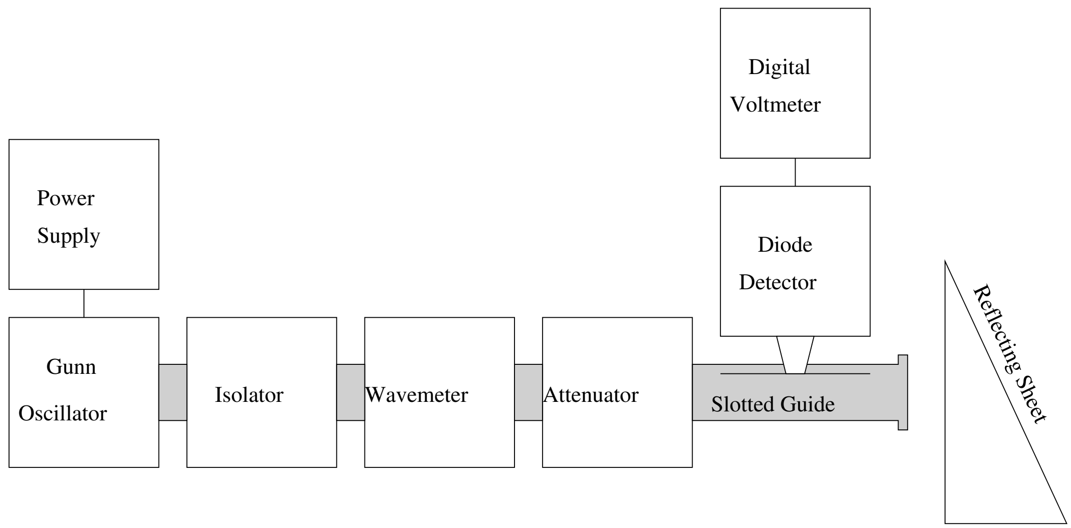

Figure 3: Schematic diagram of the apparatus. Operate in CW mode.

The Gunn oscillator produces microwave power at a given frequency when the applied voltage and the cavity micrometer are adjusted to the values specified on the manufacturer's specification chart. Check you have the correct one for your diode. The slotted guide allows you to sample the field in the waveguide with a small copper probe; the voltage induced in the probe is rectified by a microwave diode in the probe assembly and the resulting DC voltage is observed on the digital voltmeter. The rectified DC voltages observed at low power levels are proportional to the square root of the power level in the guide, and are therefore proportional to the voltage in the guide at the probe wire.

Since the probe is required to sample the field without also interfering with the propagating wave, you may have to adjust the depth of the probe into the waveguide so that you are not loading the line but can still pick up a good signal. To determine whether you must adjust the depth, follow these steps:

1. Turn on the power and attach the silvered plate as the waveguide termination.
2. Move the detector along the guide to obtain a maximal reading.
3. Adjust the tuning plunger on the detector mount to further optimize the output.
4. Move the detector along the guide recording the positions at which successive maxima and minima are observed.
5. The maxima and minima should occur at approximately equal distances from each other along the length of the line. If they are not, then you will have to adjust the depth.

The depth is adjusted by loosening the locking mechanism at the base of the detector mount and raising the entire mount. Once you have found a good depth you can re-tighten the locking mechanism. Adjust the tuning plunger on the detector mount to get the largest voltage output. You can also adjust the wave-meter micrometer and attenuator to get a reasonable signal from the probe. The manual for the detector mount will be located at the bench. You must use the same settings for all parts of both tasks.

4 TASK 1 — Determination of Cut-off Wavelength

1. With the reflecting sheet fixed in place, and no termination attached to the waveguide end, measure λ_g by moving the probe carriage along the guide.

2. Next keep the probe fixed and move the aluminum reflecting sheet to determine λ_0.

3. Deduce a value for λ_c and compare it with the value you obtain by measuring the dimensions of the waveguide cross-section with a pair of calipers.

5 Task 2 — Impedance Measurement

1. Determine the VSWR and the fractional wavelength from the end of the waveguide to the nearest maximum with a short-circuiting termination, using the silver-coated plate. How do these values compare to their expected values?

2. Remove the short circuit, and replace the reflecting sheet used in Part 1 by a sheet of microwave absorbing material inclined at a slight angle to the axis of the waveguide to prevent reflections back to the waveguide system. With nothing attached to the end of the slotted guide, measure the VSWR and deduce the distance of the nearest voltage maximum to the end of the slotted guide. From these measurements, deduce the normalized impedance at the end of the guide using the Smith Chart.

3. Repeat step 2 using a rectangular horn attached to the end of the slotted guide. Measure the distance from the nearest voltage maximum to the input flange of the horn.

4. From the normalized impedances found in steps 2 and 3, deduce the termination impedance for each case by multiplying the normalized impedance by the impedance in the rectangular guide.

5. Which method of termination provides a better match to the guided waves? Why is this so?

6 Appendix — The Smith Transmission Line Chart

Many graphical aids for transmission line computation have been devised. Of these, the most generally useful has been one presented by P.H. Smith, which consists of loci of constant resistance and reactance plotted on a polar diagram in which radius corresponds to magnitude of reflection coefficient, and angle corresponds to phase of reflection coefficient referred to a general point along the line. The chart enables one to find simply how impedances are transformed along the line, or to relate impedance to reflection coefficient or standing wave ratio and positions of voltage minima. By combinations of operations, it enables one to understand the behaviour of complex impedance-matching techniques and to devise new ones.

The normalized impedance, $\frac{Z_i}{Z_0}$, that would be measured at a distance l from the end of the guide is given by

$$\frac{Z_i}{Z_0} = (r + jx) = \frac{1 + \rho(l)}{1 - \rho(l)} \tag{6}$$

where ρ is the complex voltage reflection coefficient ζ, multiplied by a phase factor corresponding to the accumulated phase from the observation point to the end of the guide and back again.

Now if we let $\rho = u + jv$ then

$$r + jx = \frac{1 + (u + jv)}{1 - (u + jv)} \tag{7}$$

which may be separated into real and imaginary parts as

$$r = \frac{1 - (u^2 + v^2)}{(1 - u)^2 + v^2} \tag{8}$$

$$x = \frac{2v}{(1 - u)^2 + v^2} \tag{9}$$

or

$$\left(u - \frac{r}{1 + r}\right)^2 + v^2 = \frac{1}{(1 + r)^2} \tag{10}$$

$$(u - 1)^2 + \left(v - \frac{1}{x}\right)^2 = \frac{1}{x^2} \tag{11}$$

If we then wish to plot the loci of constant resistance r on the ρ plane (u and v serving as rectangular coordinates), Eq. 10 shows that they're circles centred on the u axis at $(r/[1 + r], 0)$ and with radii $1/(1 + r)$. The circles for $r = 0$, $1/2$, 1, and 2 are sketched on Fig. 4. From Eq. 11, the curves of constant x plotted on the ρ plane are also circles, centred at $(1, 1/x)$ and with radii $1/|x|$. Curves for $x = 0$, $\pm 1/2$, ± 1, and ± 2 are sketched on Fig. 4. These circles appear as arcs as they are only drawn until they meet the $r = 0$ boundary.

Any point on a given transmission line will have some impedance with a positive resistive part, and will then correspond to some particular point on the inside of the unit circle of the ρ plane.

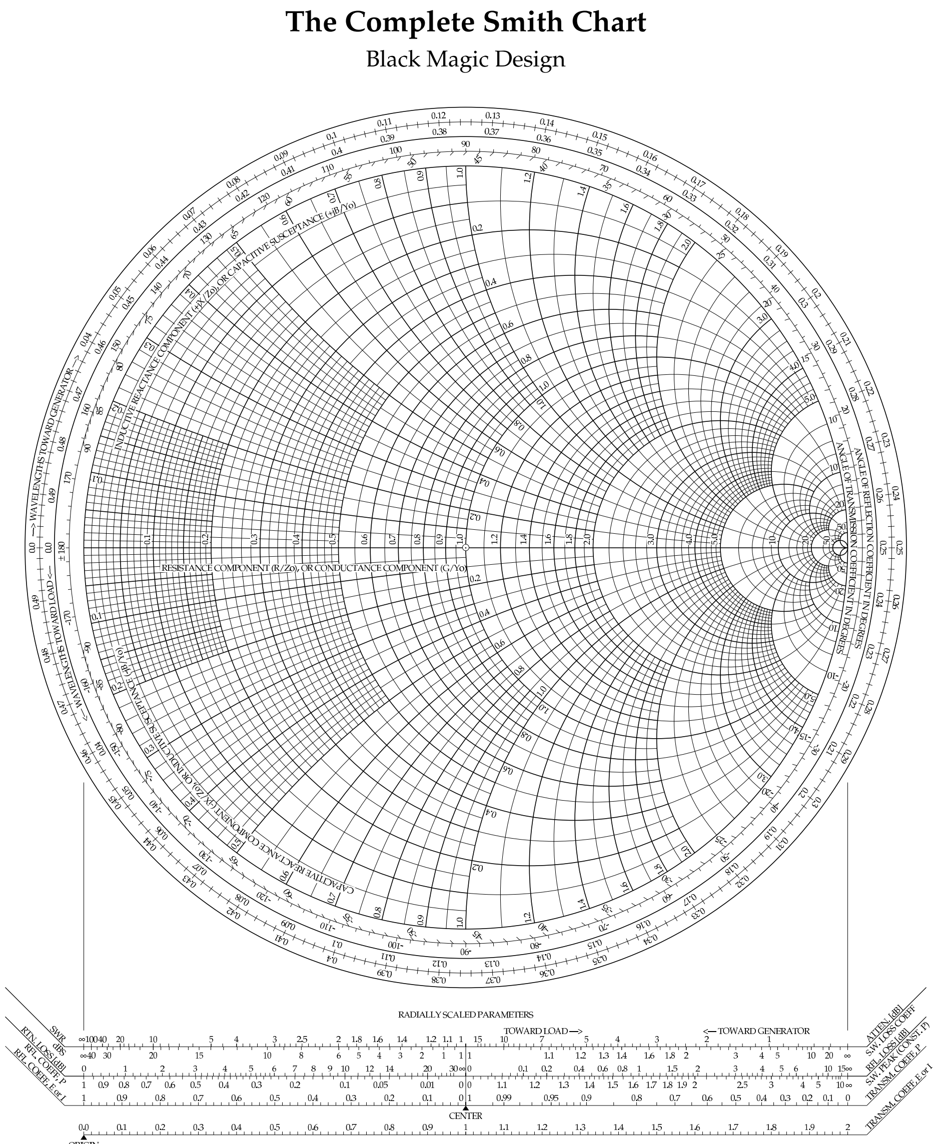

Figure 4: The Smith chart, as stolen from Spread Spectrum Scene, http://www.sss-mag.com/smith.html`http://www.sss-mag.com/smith.html` .