Section III simply contains the source resistor, and we have already shown in Figure 6.6 that

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{III} = \begin{bmatrix} 1 & Z_S \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

If we now cascade these transfer matrices as described by (6.2.9), we find that

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & j25 \\ j0.04 & 0 \end{bmatrix} \begin{bmatrix} 0 & j100 \\ j0.01 & 0 \end{bmatrix} \begin{bmatrix} V_L \\ Y_L V_L \end{bmatrix}$$
$$= \begin{bmatrix} -0.25 & -40 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} V_L \\ Y_L V_L \end{bmatrix}$$

which means that

$$V_S = (-0.25 - 40Y_L)V_L = (-0.25 - j0.4)V_L$$

and

$$I_S = -4Y_L V_L = -j0.04 V_L$$

for $Y_L = 1/Z_L = 0.01j$. The input impedance Z_{in} is related to V_S, I_S, and Z_S by $I_S = V_S/(Z_{\text{in}} + Z_S)$, or

$$Z_{\text{in}} = \frac{V_S}{I_S} - Z_S = \frac{-0.25 - j0.4}{-j0.04} - 10 = -j6.25$$

as was also found in Example 6.1.2. The power supplied by the source is again

$$P = \frac{1}{2}\,\text{Re}\{V_S I_S^*\} = \frac{1}{2}\,\text{Re}\left\{V_S \left(\frac{V_S}{10 - j6.25}\right)^*\right\}$$
$$= 0.036|V_S|^2 \text{ (W)}$$

6.3 GAMMA PLANE AND SMITH CHART ANALYSIS METHODS

Although the methods discussed in Sections 6.1 and 6.2 are adequate to solve for $V(z)$ and $I(z)$ everywhere on a network composed of lumped elements and *TEM* lines, other analysis techniques can also prove helpful. In particular, we shall consider the complex gamma plane $\Gamma(z)$ and its mapping into complex impedance $Z(z)$, which is carried out graphically using a Smith chart. The Smith chart, while not so useful a computational method in this age of computers, still provides physical insight into how impedances transform on a *TEM* line. It is also a useful way to quickly estimate impedances that could be exactly (but more tediously!) computed analytically.

We first recall (5.3.45) and (5.3.49):

$$V(z) = V_+ e^{-jkz} + V_- e^{+jkz} \tag{6.3.1}$$

$$I(z) = Y_0\left[V_+ e^{-jkz} - V_- e^{+jkz}\right] \tag{6.3.2}$$

These equations become

$$V(z) = V_+ e^{-jkz}[1 + \Gamma(z)] \tag{6.3.3}$$

$$I(z) = Y_0 V_+ e^{-jkz}[1 - \Gamma(z)] \tag{6.3.4}$$

where

$$\Gamma(z) \equiv \frac{V_- e^{jkz}}{V_+ e^{-jkz}} = \Gamma_L e^{2jkz} \tag{6.3.5}$$

and $\Gamma_L = \Gamma(z = 0) = V_-/V_+ = (Z_L - Z_0)/(Z_L + Z_0)$ from (6.1.4). It follows that

$$Z(z) = \frac{V(z)}{I(z)} = Z_0\left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)}\right) \tag{6.3.6}$$

where $\Gamma(z)$ and hence $Z(z)$ are periodic functions of position z. Equivalently,

$$\Gamma(z) = \frac{Z_n(z) - 1}{Z_n(z) + 1} \tag{6.3.7}$$

where the *normalized impedance* $Z_n(z)$ is defined as $Z(z)/Z_0$.

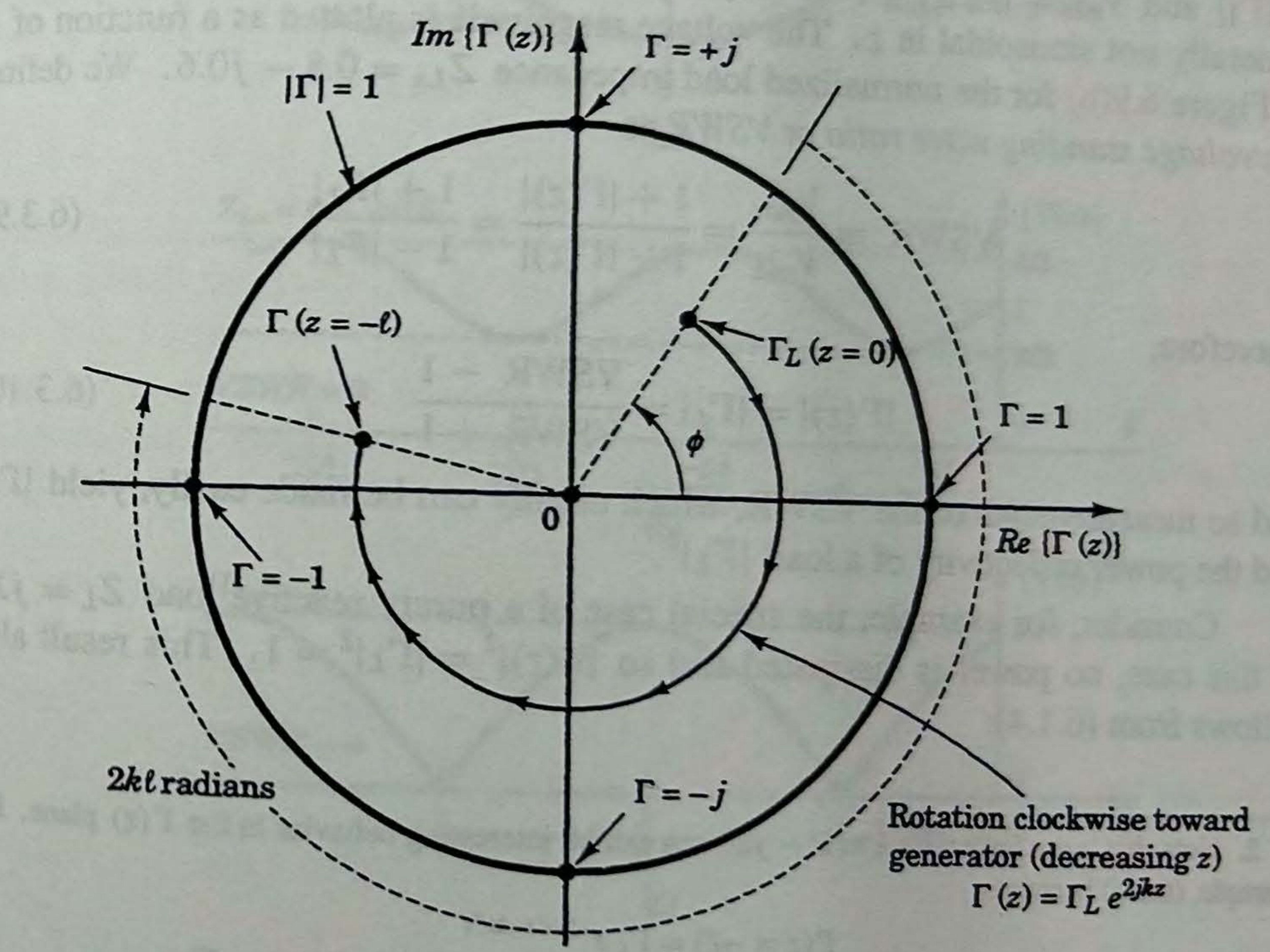


Figure 6.8 Complex gamma plane for $\Gamma(z)$.

Before considering the one-to-one mapping [given by (6.3.6) and (6.3.7)] between $\Gamma(z)$ and $Z_n(z)$ that motivated invention of the Smith chart, we first establish the character of $\Gamma(z)$ and the relationship between $\Gamma(z)$ and the total line voltage $V(z)$. The complex *gamma plane* is illustrated in Figure 6.8, having axes $\text{Re}\{\Gamma\}$ and $\text{Im}\{\Gamma\}$. From (6.3.5), we note immediately that $|\Gamma(z)| = |\Gamma_L|$.[1] If we use (6.3.5) to plot $\Gamma(z)$ in the gamma plane, we find that as we move in the negative

1. In this analysis, we assume that the characteristic impedance Z_0 is a real number so that k is also real.

z direction away from the load Z_L, the angle of $\Gamma(z)$ **decreases** with negative z. Therefore, $\Gamma(z)$ rotates **clockwise** along a circle of radius $|\Gamma_L|$ as z moves away from the load, one complete revolution occurring each time that $2k\Delta z$ equals 2π, or $\Delta z = \lambda/2$.

The voltage magnitude is found from (6.3.3) to be

$$|V(z)| = |V_+||1+\Gamma(z)| \tag{6.3.8}$$

which is represented graphically in Figure 6.9(a), where $1+\Gamma(z)$ is the complex vector connecting the point -1 and $\Gamma(z)$ in the gamma plane. For lossless passive media, we have already mentioned that $|\Gamma(z)| \le 1$, because the reflected power must not exceed the incident power at a junction.[2]

This geometric construction shows how $|V(z)|$ varies between $V_{\max} = |V_+|(1+|\Gamma|)$ and $V_{\min} = |V_+|(1-|\Gamma|)$ with a period $\Delta z = \lambda/2$. The pattern $|V(z)|$ is generally **not** sinusoidal in z. The voltage magnitude is plotted as a function of z in Figure 6.9(b) for the normalized load impedance $Z_{Ln} = 0.8 - j0.6$. We define the *voltage standing wave ratio* or *VSWR* as

$$\text{VSWR} \equiv \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma(z)|}{1-|\Gamma(z)|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} \tag{6.3.9}$$

Therefore,

$$|\Gamma(z)| = |\Gamma_L| = \frac{\text{VSWR}-1}{\text{VSWR}+1} \tag{6.3.10}$$

and so measurements of the VSWR, which usually can be made easily, yield $|\Gamma_L|$ and the power reflectivity of a load $|\Gamma_L|^2$.

Consider, for example, the special case of a purely reactive load $Z_L = jX$. In this case, no power is dissipated and so $|\Gamma(z)|^2 = |\Gamma_L|^2 = 1$. This result also follows from (6.1.4):

---

2. Note that lossy lines with $k = k' - jk''$ can exhibit interesting behavior in the $\Gamma(z)$ plane. For example, (6.3.5) becomes

$$\Gamma(z=-\ell) = \Gamma_L e^{-2jk'\ell - 2k''\ell}$$

and $\Gamma(z=-\ell)$ spirals exponentially inward toward $\Gamma = 0$ as $\ell \to \infty$. Thus the impedance at $z = -\ell$ approaches Z_0 as $\ell \to \infty$. It is also interesting to note that $|\Gamma|$ can exceed unity for a lossy line because $Z_0 = \sqrt{(R+j\omega L)/(G+j\omega C)}$ is in general complex, and Z_0 can have phase angle ϕ where $-\pi/4 < \phi < \pi/4$. Thus, $Z_n = Z_L/Z_0$ can have an angle $\alpha = \beta - \phi$ up to $\pm 3\pi/4$. The angle β of Z_L is limited to $-\pi/2 \le \beta \le \pi/2$ since the real part of Z_L is always positive for passive media. It can easily be seen that

$$|\Gamma| = \frac{|Z_n - 1|}{|Z_n + 1|} = \frac{||Z_n|\, e^{j\alpha} - 1|}{||Z_n|\, e^{j\alpha} + 1|}$$

is largest when $\alpha = \pm 3\pi/4$ and $\text{Re}\{|Z_n|\, e^{j\alpha}\}$ is negative. The maximum possible value of $|\Gamma|$ is $1+\sqrt{2}$. Note than when $|\Gamma| > 1$ on a lossy line, it does not mean that there is power amplification. Here, $|\Gamma| > 1$ is the result of the reactive elements, and power in the forward or reverse directions is no longer simply equal to $|V_+|^2/2Z_0$ or $|V_-|^2/2Z_0$ because Z_0 is complex.

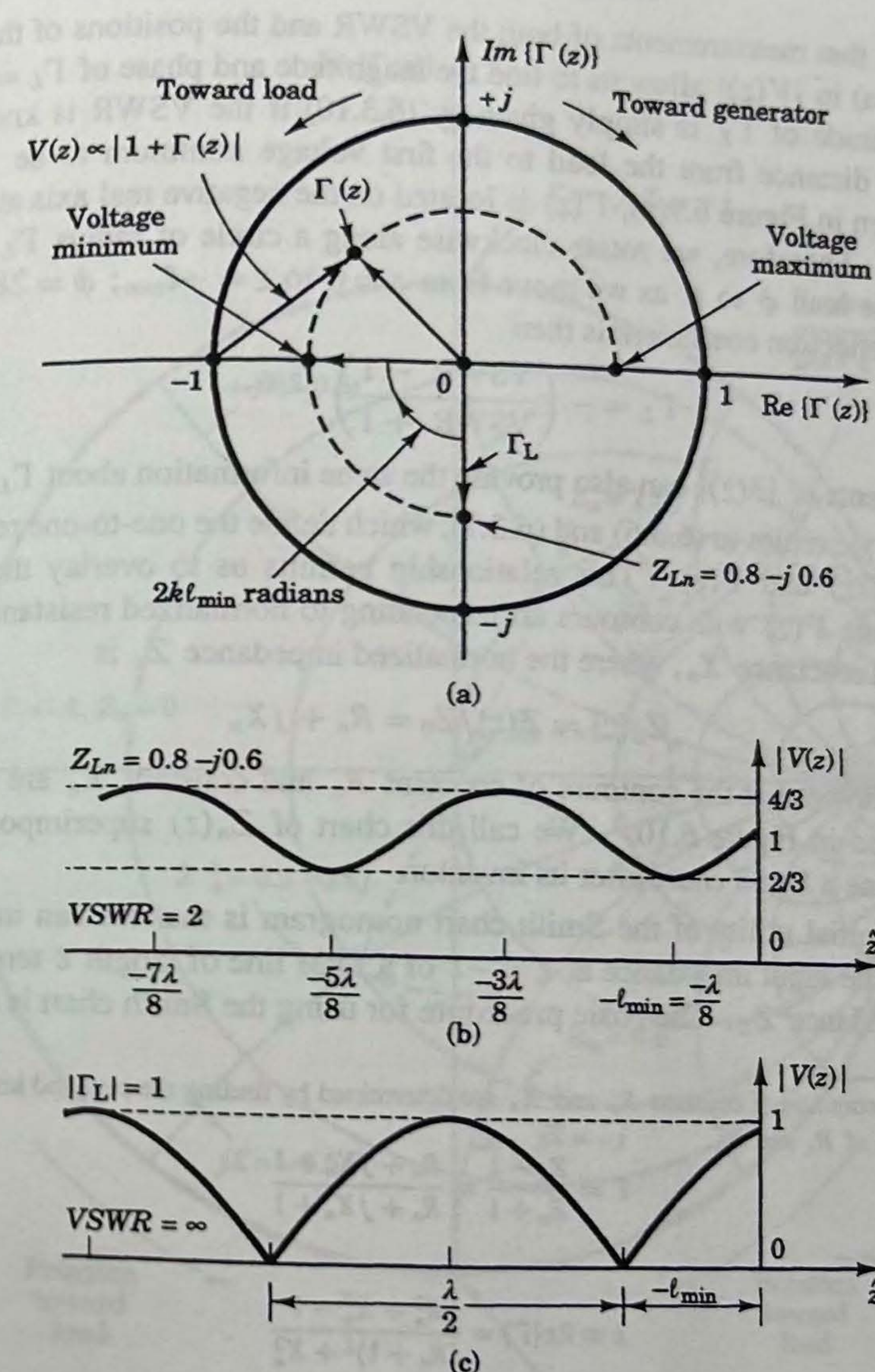


Figure 6.9 Voltage magnitude represented in the gamma plane.

$$|\Gamma_L| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\frac{jX - Z_0}{jX + Z_0}\right| = 1$$

For a purely reactive load, the VSWR defined by (6.3.9) is infinite, and $|V(z)|$ is illustrated in Figure 6.9(c). We can also calculate $|V(z)|$ analytically in this special case by noting that $\Gamma_L = e^{j\phi}$ where ϕ is determined from X and Z_0: $\phi = \tan^{-1}[2XZ_0/(X^2 - Z_0^2)]$. Equation (6.3.8) then yields

$$\begin{aligned} |V(z)| &= |V_+||1 + e^{j\phi} e^{2jkz}| \\ &= 2|V_+||\cos(kz - \phi/2)| \end{aligned}$$

which is exactly what is shown in Figure 6.9(c).

Note that measurements of both the VSWR and the positions of the minima (or maxima) in $|V(z)|$ allow us to find the magnitude and phase of $\Gamma_L = |\Gamma_L|\, e^{j\phi}$. The magnitude of Γ_L is simply given by (6.3.10) if the VSWR is known. We define the distance from the load to the first voltage minimum to be $\ell_{\min}$. As may be seen in Figure 6.9(a), $\Gamma(z)$ is located on the negative real axis at a voltage minimum. Therefore, we rotate clockwise along a circle of radius Γ_L from the angle of the load ϕ to π as we move from $z = 0$ to $z = -\ell_{\min}$; $\phi = 2k\ell_{\min} - \pi$. The load reflection coefficient is then

$$\Gamma_L = -\left(\frac{\text{VSWR} - 1}{\text{VSWR} + 1}\right) e^{2jk\ell_{\min}} \tag{6.3.11}$$

Measurements of $|I(z)|$ can also provide the same information about Γ_L.

Now we return to (6.3.6) and (6.3.7), which define the one-to-one relationship between $Z(z)$ and $\Gamma(z)$. This relationship permits us to overlay the complex gamma plane $\Gamma(z)$ with contours corresponding to normalized resistance R_n and normalized reactance X_n, where the normalized impedance Z_n is

$$Z_n(z) = Z(z)/Z_0 = R_n + jX_n$$

It can be shown that the contours of constant R_n and constant X_n are all circles, as illustrated in Figure 6.10.[3] We call this chart of $Z_n(z)$ superimposed on the gamma plane a *Smith chart* after its inventor.

The initial utility of the Smith chart nomogram is that we can immediately determine the input impedance at $z = -\ell$ of a *TEM* line of length ℓ terminated by a load impedance Z_L. The basic procedure for using the Smith chart is as follows:

---

3. These contours of constant R_n and X_n are determined by finding the real and imaginary parts of Γ in terms of R_n and X_n

$$\Gamma = \frac{Z_n - 1}{Z_n + 1} = \frac{R_n + jX_n - 1}{R_n + jX_n + 1}$$

which means that

$$x \equiv \text{Re}\{\Gamma\} = \frac{R_n^2 + X_n^2 - 1}{(R_n + 1)^2 + X_n^2}$$

and

$$y \equiv \text{Im}\{\Gamma\} = \frac{-2X_n}{(R_n + 1)^2 + X_n^2}$$

If these two equations are solved for x and y as a function of R_n alone, then after some algebra we find that

$$\left(x - \frac{R_n}{R_n + 1}\right)^2 + y^2 = \left(\frac{1}{R_n + 1}\right)^2$$

which is the equation for a circle centered at $(R_n/(R_n + 1), 0)$ of radius $1/(R_n + 1)$. We note that this circle always intersects the point $(1,0)$ in the gamma plane. These are the contours of constant R_n.

Conversely, we may solve for x and y in terms of X_n alone, giving

$$(x - 1)^2 + \left(y + \frac{1}{X_n}\right)^2 = \left(\frac{1}{X_n}\right)^2$$

which is also the equation of a circle centered at $(1, -1/X_n)$ with radius $1/X_n$. The portions of these

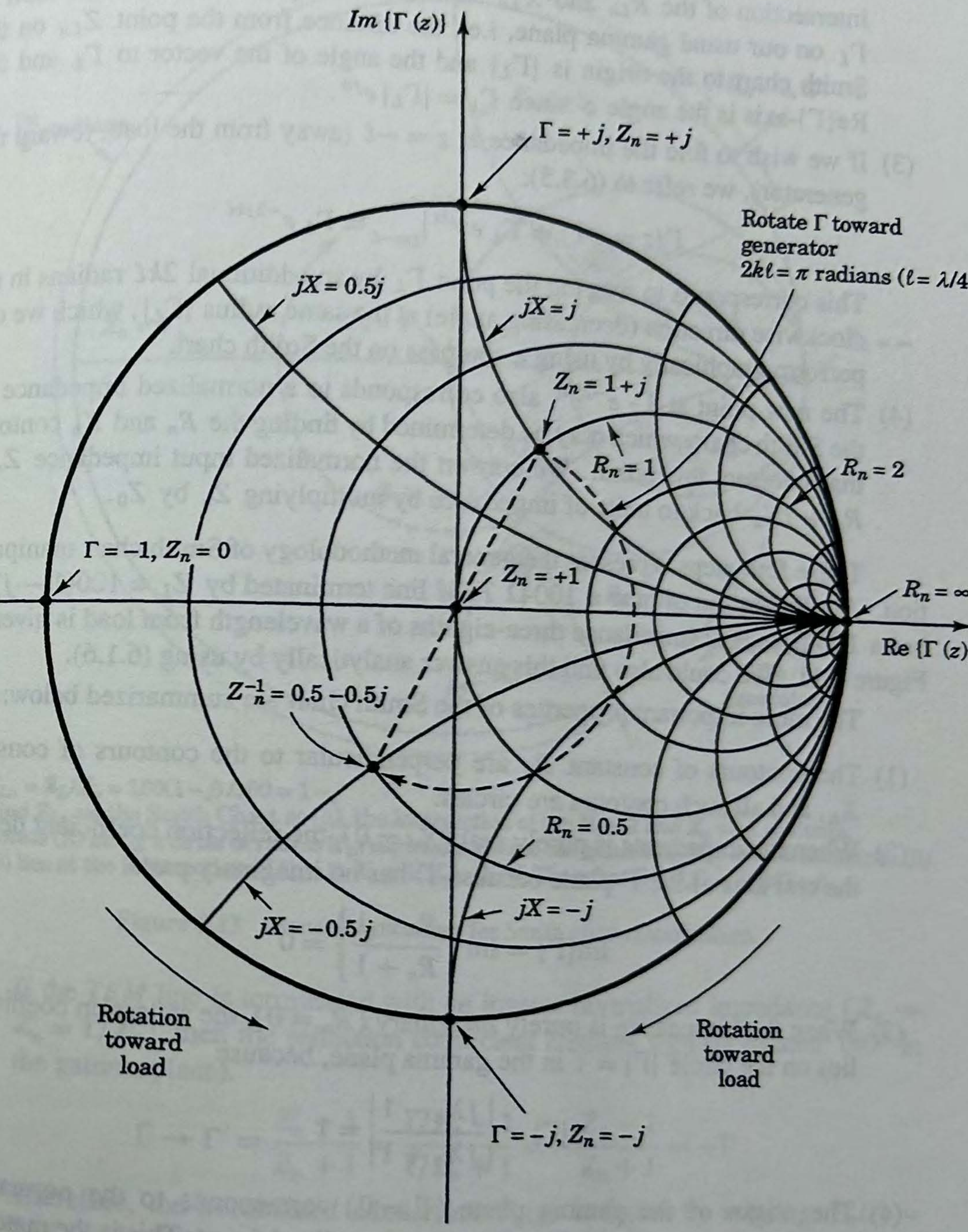


Figure 6.10 Smith chart.

(1) Calculate the normalized load impedance by dividing Z_L by the characteristic impedance of the transmission line: $Z_{Ln} = Z_L/Z_0$.

(2) Find the point $Z_{Ln} = R_{Ln} + jX_{Ln}$ on the Smith chart by looking for the

---

circles that lie within the unit circle on the gamma plane ($|\Gamma| < 1$) are the contours of constant X_n, and these circles also intersect the point $(1, 0)$ in the Γ-plane.

intersection of the R_{Ln} and X_{Ln} contours. This point is also the location of Γ_L on our usual gamma plane, i.e., the distance from the point Z_{Ln} on the Smith chart to the origin is $|\Gamma_L|$ and the angle of the vector to Γ_L and the Re{Γ}-axis is the angle ϕ since $\Gamma_L = |\Gamma_L| e^{j\phi}$.

(3) If we wish to find the impedance at $z = -\ell$ (away from the load, toward the generator), we refer to (6.3.5):

$$\Gamma(z = -\ell) = \Gamma_L e^{2jkz}\big|_{z=-\ell} = \Gamma_L e^{-2jk\ell}$$

This corresponds to rotating the point Γ_L by an additional $2k\ell$ radians in the clockwise direction (decreasing angle) at the same radius $|\Gamma_L|$, which we can perform graphically by using a compass on the Smith chart.

(4) The new point at $\Gamma_L e^{-2jk\ell}$ also corresponds to a normalized impedance on the Smith chart, which may be determined by finding the R_n and X_n contours that intersect this point. We convert the normalized input impedance $Z_n = R_n + jX_n$ back to units of impedance by multiplying Z_n by Z_0.

These four steps represent the general methodology of Smith chart manipulation. An illustration of how a 100-Ω *TEM* line terminated by $Z_L = 100(1 - j)\ \Omega$ has a $100(2 + j)$-Ω impedance three-eighths of a wavelength from load is given in Figure 6.11. We could also find this answer analytically by using (6.1.6).

The more important properties of the Smith chart are summarized below:

(1) The contours of constant R_n are perpendicular to the contours of constant X_n, and all such contours are circles.

(2) When the impedance is purely real ($X_n = 0$), the reflection coefficient lies on the real axis of the Γ-plane because Γ has no imaginary part:

$$\mathrm{Im}\{\Gamma\} = \mathrm{Im}\left\{\frac{R_n - 1}{R_n + 1}\right\} = 0$$

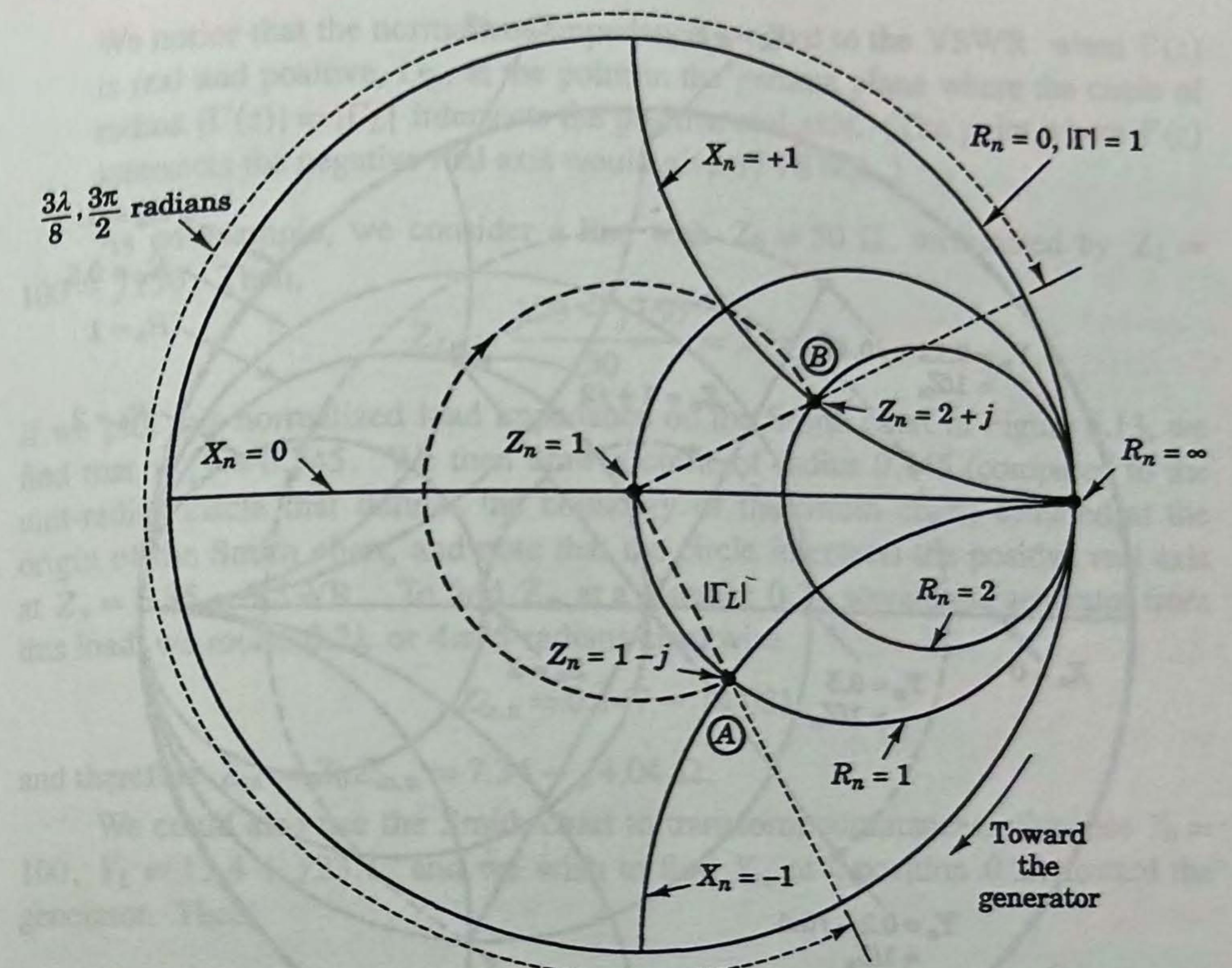
(3) When the impedance is purely imaginary ($R_n = 0$), the reflection coefficient lies on the circle $|\Gamma| = 1$ in the gamma plane, because

$$|\Gamma| = \left|\frac{jX_n - 1}{jX_n + 1}\right| = 1$$

(4) The origin of the gamma plane ($\Gamma = 0$) corresponds to the normalized impedance $Z_n = 1$, which is Z_0 in un-normalized form. This is the matched-load impedance case, where no reflections are observed.

(5) If a lossless line is terminated with the conjugate impedance Z_L^*, then the reflection coefficient must be complex conjugated also:

$$\Gamma^* = \frac{Z_n^* - 1}{Z_n^* + 1}$$



(1) $Z_{Ln} = Z_L/Z_o = 100(1-j)/100 = 1-j$
(2) Find Z_{Ln} on the Smith Chart at (A), the intersection of the $R_n = 1$ and $X_n = -1$ contours,
(3) Rotate (A) along a circle of radius $|\Gamma_L|$ clockwise toward the generator $3\lambda/8$ or $3\pi/2$ radians to (B).
(4) (B) lies at the intersection of $R_n = 2, X_n = 1; Z_n = 2 + j$. Therefore $Z = Z_nZ_o = 100(2 + j)$.

Figure 6.11 General procedure for Smith chart manipulation.

(6) If the *TEM* line is terminated with an inverse normalized impedance ($Z_n \rightarrow Z_n' = 1/Z_n$), then the reflection coefficient changes sign (is rotated 180° in the gamma plane).

$$\Gamma \rightarrow \Gamma' = \frac{Z_n' - 1}{Z_n + 1} = \frac{1/Z_n - 1}{1/Z_n + 1} = -\frac{Z_n - 1}{Z_n + 1} = -\Gamma$$

Therefore, the normalized admittance $Y_n(z)$ is found by rotating $Z_n(z) = 1/Y_n(z)$ by 180° on the Smith chart, as illustrated for three specific cases in Figure 6.12. The Smith chart can thus be used equally well to represent impedances or admittances.

(7) A change Δz in position along the transmission line is represented by rotating Γ by an angle $2k\Delta z$ radians; Γ_L at $z = 0$ is transformed into $\Gamma(z)$ at position z by

$$\Gamma(z) = \Gamma_L e^{2jkz}$$

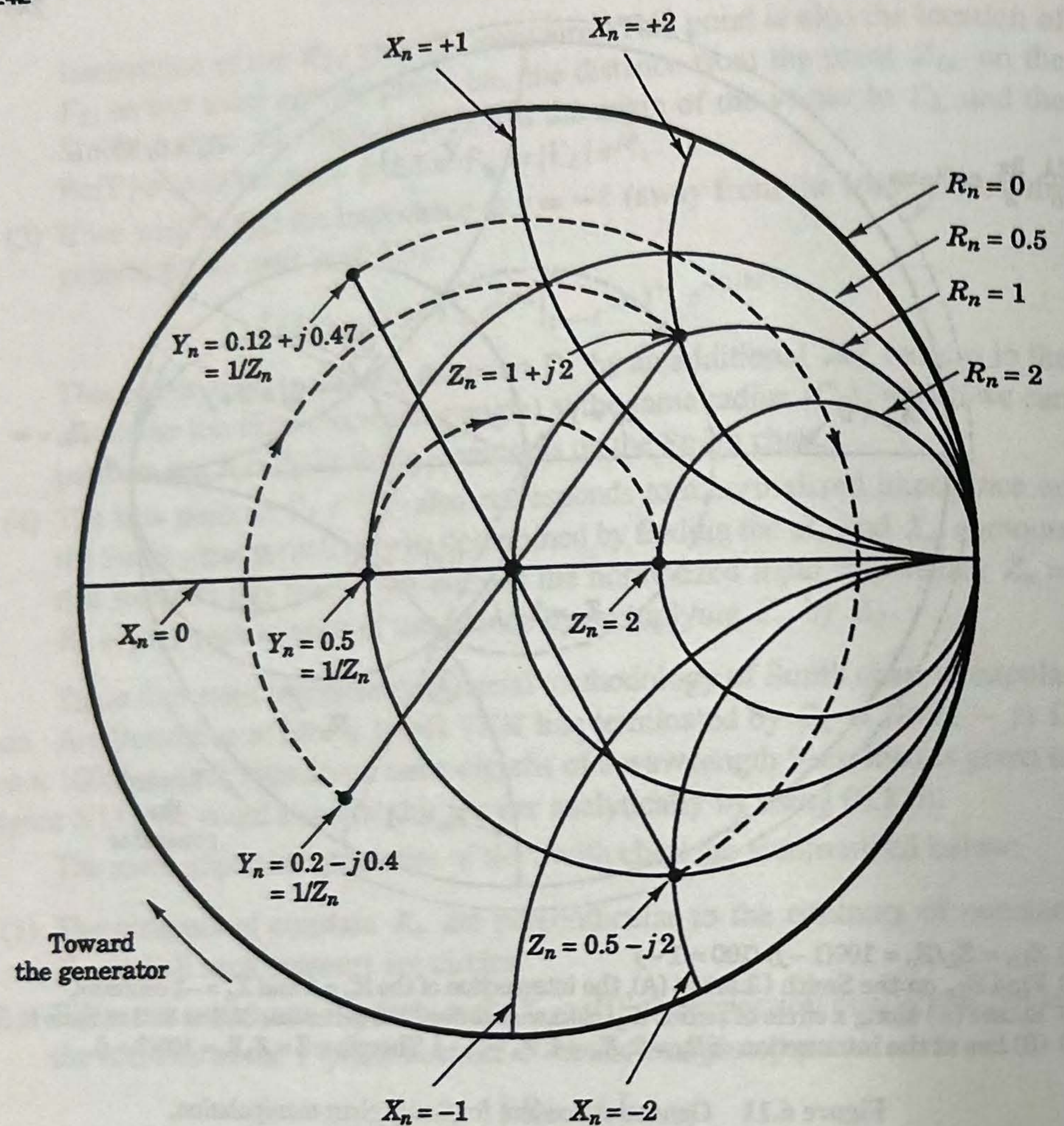


Figure 6.12 Admittances on a Smith chart.

That is, we draw a circle of radius $|\Gamma_L|$ and rotate clockwise from Γ_L to determine the impedance at positions closer to the generator, and counterclockwise to determine the impedance at positions closer to the load, where the load is in the direction of increasing z. As noted earlier, a complete rotation occurs when $2k\Delta z = 2\pi$, or when $\Delta z = \lambda/2$.

(8) The VSWR may be obtained directly from the Smith chart by comparing it with the definition of normalized impedance (6.3.9):

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

versus

$$Z_n(z) = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

We notice that the normalized impedance is equal to the VSWR when $\Gamma(z)$ is real and positive, i.e., at the point in the gamma plane where the circle of radius $|\Gamma(z)| = |\Gamma_L|$ intersects the positive real axis. (The point where $\Gamma(z)$ intersects the negative real axis would give $1/\text{VSWR}$.)

As an example, we consider a line with $Z_0 = 50\ \Omega$, terminated by $Z_L = 100 - j150$. Then,

$$Z_{Ln} = \frac{100 - j150}{50} = 2 - j3$$

If we plot this normalized load impedance on the Smith chart in Figure 6.13, we find that $|\Gamma_L| = 0.745$. We then draw a circle of radius 0.745 (compared to the unit-radius circle that defines the boundary of the Smith chart) centered at the origin of the Smith chart, and note that the circle intersects the positive real axis at $Z_n = 6.85 = \text{VSWR}$. To find Z_{in} at a distance 0.2λ toward the generator from this load, we rotate 0.2λ or $4\pi/5$ radians clockwise

$$Z_{\text{in,n}} = 0.147 - j0.081$$

and therefore $Z_{\text{in}} = Z_0 Z_{\text{in,n}} = 7.34 - j4.04\ \Omega$.

We could also use the Smith chart to transform admittances. Suppose $Y_0 = 100$, $Y_L = 15.4 + j23.1$, and we wish to find Y_{in} at a position 0.2λ toward the generator. Then,

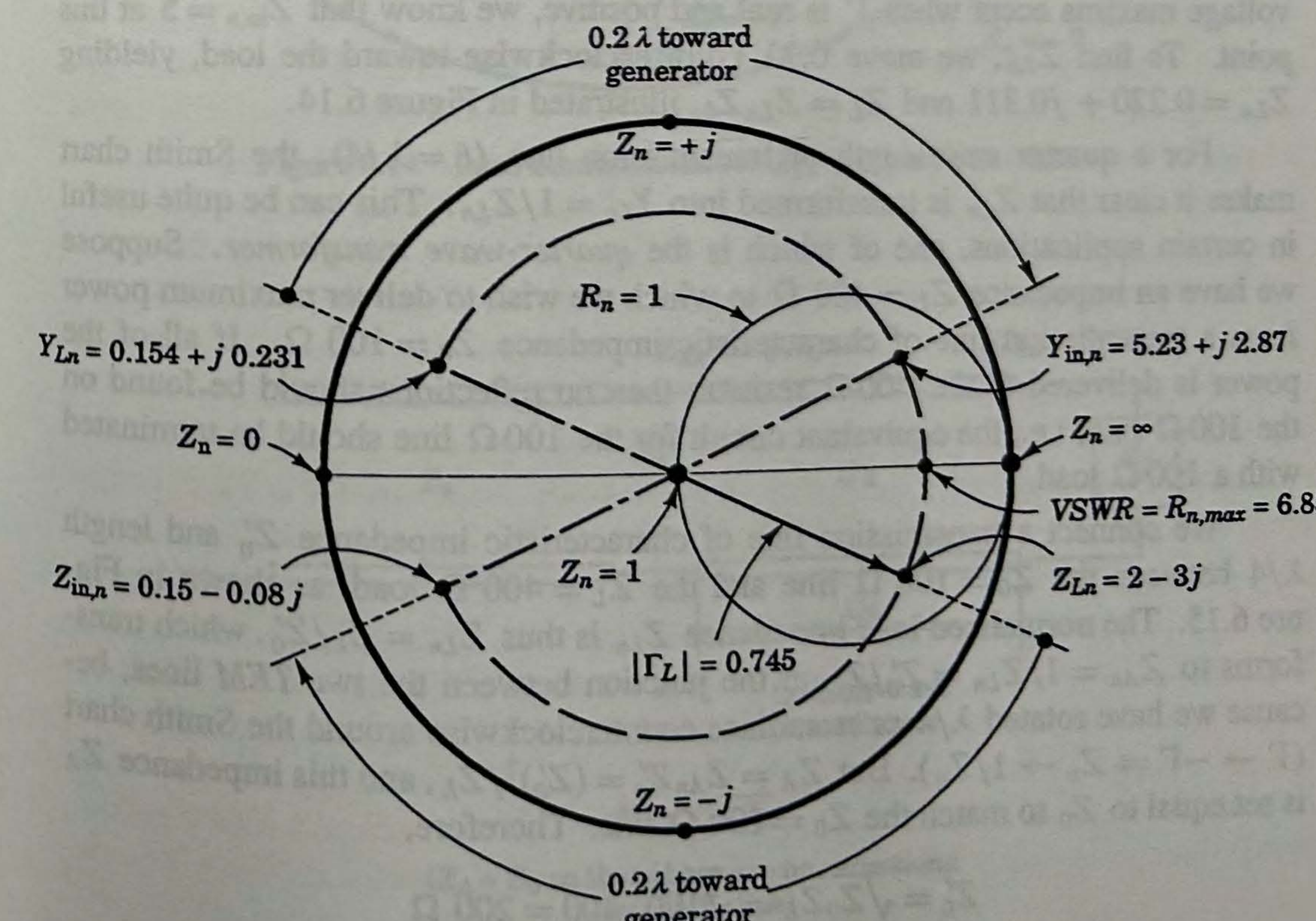


Figure 6.13 Impedance and admittance transformations using a Smith chart.

$$Y_{Ln} = Y_L/Y_0 = 0.154 + j0.231$$

which is plotted in Figure 6.13. The corresponding Z_{Ln} is directly opposite on the chart, as discussed in observation (6), and is given by $Z_{Ln} = 2 - j3$. We rotate this 0.2λ clockwise to find $Z_{\text{in,n}} = 0.147 - j0.081$ as before. The normalized admittance is then directly opposite $Z_{\text{in,n}}$, and is equal to $Y_{\text{in,n}} = 5.23 + j2.87$. The total input admittance is thus

$$Y_{\text{in}} = (5.23 + j2.87)100 = 523 + j287\ \Omega^{-1}$$

But we could have also utilized admittances directly:

(1) $Y_{Ln} = Y_L/Y_0 = (15.4 + j23.1)/100 = 0.154 + j0.231$

(2) Rotate Y_{Ln} clockwise (toward the generator) 0.2λ to get $Y_{\text{in,n}} = 5.23 + j2.87$.

(3) $Y_{\text{in}} = Y_{\text{in,n}}Y_0 = 523 + j287$.

The Smith chart can also facilitate finding Z_L from VSWR measurements. Suppose we measure the voltage maximum to be $\ell_{\max} = 0.2\lambda$ from the unknown load Z_L, and the VSWR is 5. Then we know that Z_{Ln} must lie on the Γ circle intercepting the point $R_n = 5$, $jX_n = 0$, where Γ is real and positive. Since voltage maxima occur when Γ is real and positive, we know that $Z_{\text{in,n}} = 5$ at this point. To find Z_{Ln}, we move 0.2λ **counterclockwise** toward the load, yielding $Z_{Ln} = 0.220 + j0.311$ and $Z_L = Z_{Ln}Z_0$, illustrated in Figure 6.14.

For a quarter wavelength of transmission line ($\ell = \lambda/4$), the Smith chart makes it clear that Z_{Ln} is transformed into $Y_{Ln} = 1/Z_{Ln}$. This can be quite useful in certain applications, one of which is the *quarter-wave transformer*. Suppose we have an impedance $Z_L = 400\ \Omega$ to which we wish to deliver maximum power from a transmission line of characteristic impedance $Z_0 = 100\ \Omega$. If all of the power is delivered to the 400-Ω resistor, then no reflections should be found on the 100-Ω line; i.e., the equivalent circuit for the 100-Ω line should be terminated with a 100-Ω load.

We connect a transmission line of characteristic impedance Z'_0 and length $\lambda/4$ between the $Z_0 = 100\ \Omega$ line and the $Z_L = 400\ \Omega$ load, as shown in Figure 6.15. The normalized load impedance Z_{Ln} is thus $Z_{Ln} = Z_L/Z'_0$, which transforms to $Z_{An} = 1/Z_{Ln} = Z'_0/Z_L$ at the junction between the two *TEM* lines, because we have rotated $\lambda/4$ or π radians counterclockwise around the Smith chart ($\Gamma \to -\Gamma \Rightarrow Z_n \to 1/Z_n$). But $Z_A = Z_{An}Z'_0 = (Z'_0)^2/Z_L$, and this impedance Z_A is set equal to Z_0 to match the $Z_0 = 100\ \Omega$ line. Therefore,

$$Z'_0 = \sqrt{Z_0 Z_L} = \sqrt{100 \cdot 400} = 200\ \Omega$$

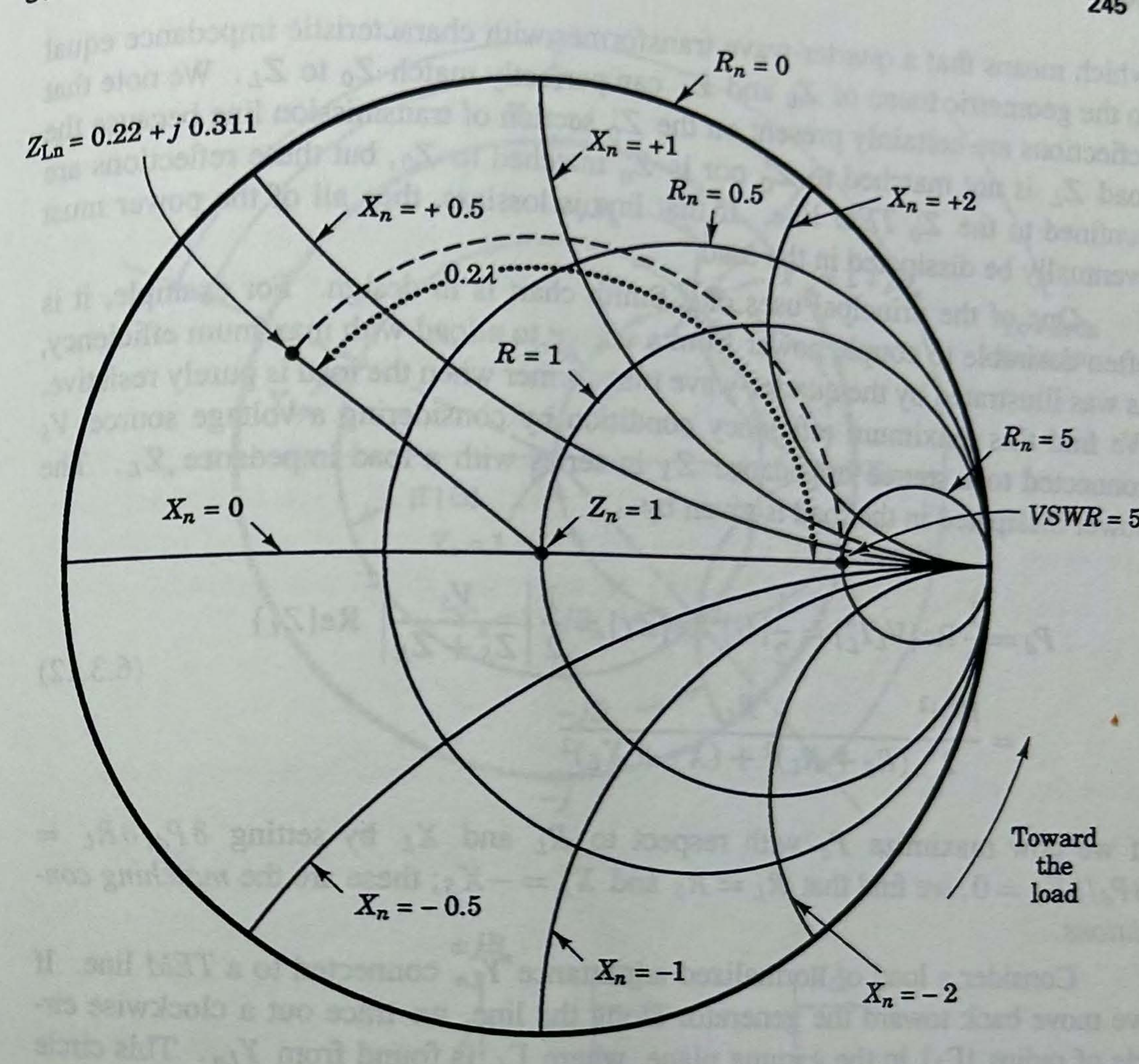


Figure 6.14 Load determination for Z_{Ln} using a Smith chart.

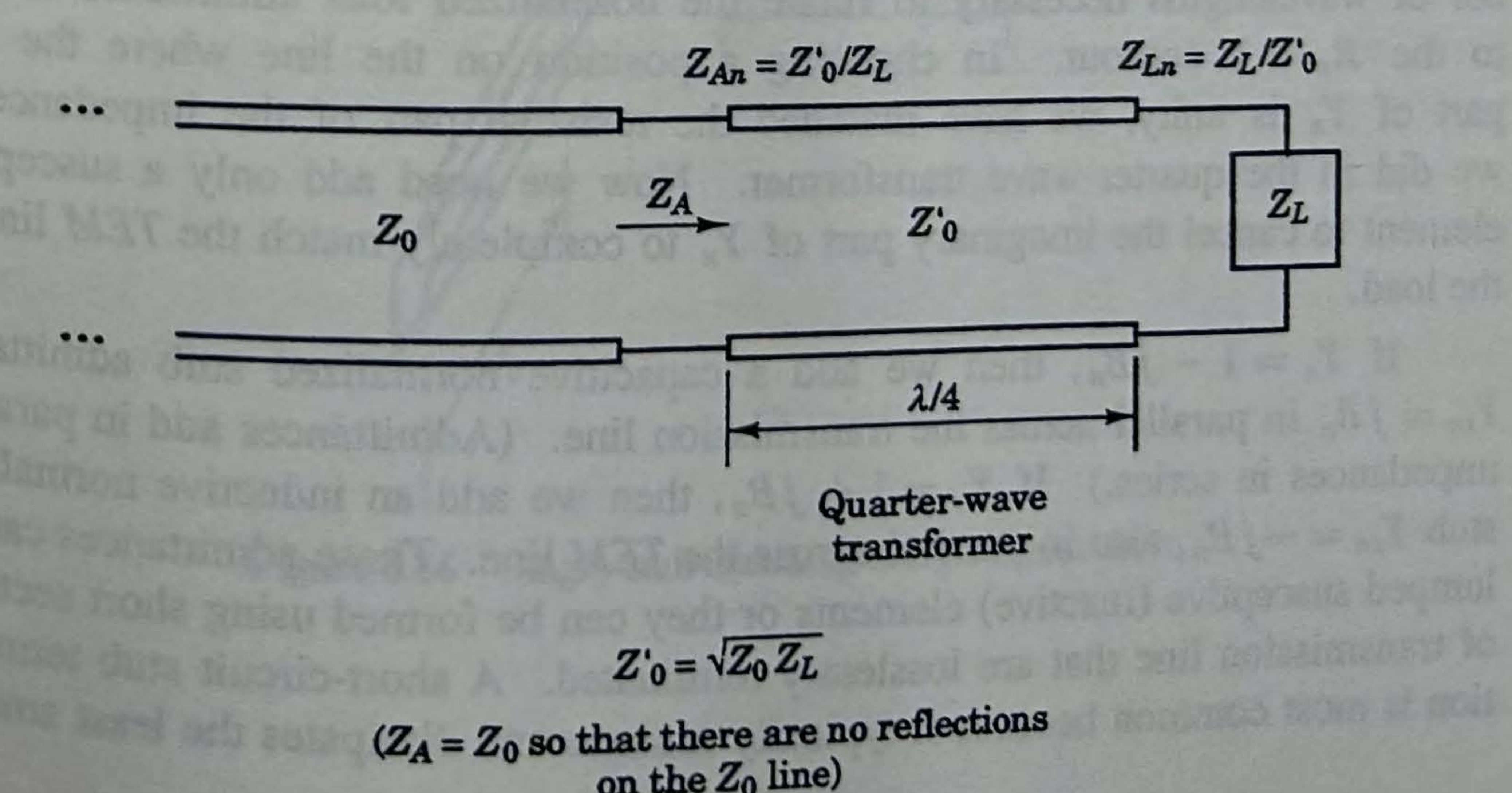


Figure 6.15 Transmission line quarter-wave transformer.

which means that a quarter-wave transformer with characteristic impedance equal to the geometric mean of Z_0 and Z_L can perfectly match Z_0 to Z_L. We note that reflections are certainly present on the Z_0' section of transmission line because the load Z_L is not matched to Z_0' nor is Z_0' matched to Z_0, but these reflections are confined to the Z_0' *TEM* line. If that line is lossless, then all of the power must eventually be dissipated in the load.

One of the principal uses of a Smith chart is in design. For example, it is often desirable to couple power from a source to a load with maximum efficiency, as was illustrated by the quarter-wave transformer when the load is purely resistive. We find this maximum efficiency condition by considering a voltage source V_S connected to a source impedance Z_S in series with a load impedance Z_L. The power dissipated in the load is given by

$$P_d = \frac{1}{2}\mathrm{Re}\{V_L I_L^*\} = \frac{1}{2}|I_L|^2\mathrm{Re}\{Z_L\} = \frac{1}{2}\left|\frac{V_S}{Z_S + Z_L}\right|^2 \mathrm{Re}\{Z_L\}$$

$$= \frac{|V_S|^2}{2}\frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{6.3.12}$$

If we now maximize P_d with respect to R_L and X_L by setting $\partial P_d/\partial R_L = \partial P_d/\partial X_L = 0$, we find that $R_L = R_S$ and $X_L = -X_S$; these are the *matching conditions*.

Consider a load of normalized admittance Y_{Ln} connected to a *TEM* line. If we move back toward the generator along the line, we trace out a clockwise circle of radius $|\Gamma_L|$ in the gamma plane, where Γ_L is found from Y_{Ln}. This circle intersects the contour $R_n = 1$ at two places, as shown in Figure 6.16, so at two positions within a half wavelength of the load, the admittance is $Y_n = 1 \pm jB_n$. The distance between the load and the stub is found by computing the number of wavelengths necessary to rotate the normalized load admittance around to the $R_n = 1$ contour. In choosing a position on the line where the real part of Y_n is unity, we have matched the resistive part of the impedance as we did in the quarter-wave transformer. Now we need add only a susceptive element to cancel the imaginary part of Y_n to completely match the *TEM* line to the load.

If $Y_n = 1 - jB_n$, then we add a capacitive normalized stub admittance $Y_{sn} = jB_n$ in parallel across the transmission line. (Admittances add in parallel, impedances in series.) If $Y_n = 1 + jB_n$, then we add an inductive normalized stub $Y_{sn} = -jB_n$, also in parallel across the *TEM* line. These admittances can be lumped susceptive (reactive) elements or they can be formed using short sections of transmission line that are losslessly terminated. A short-circuit stub termination is most common because it typically radiates and dissipates the least amount

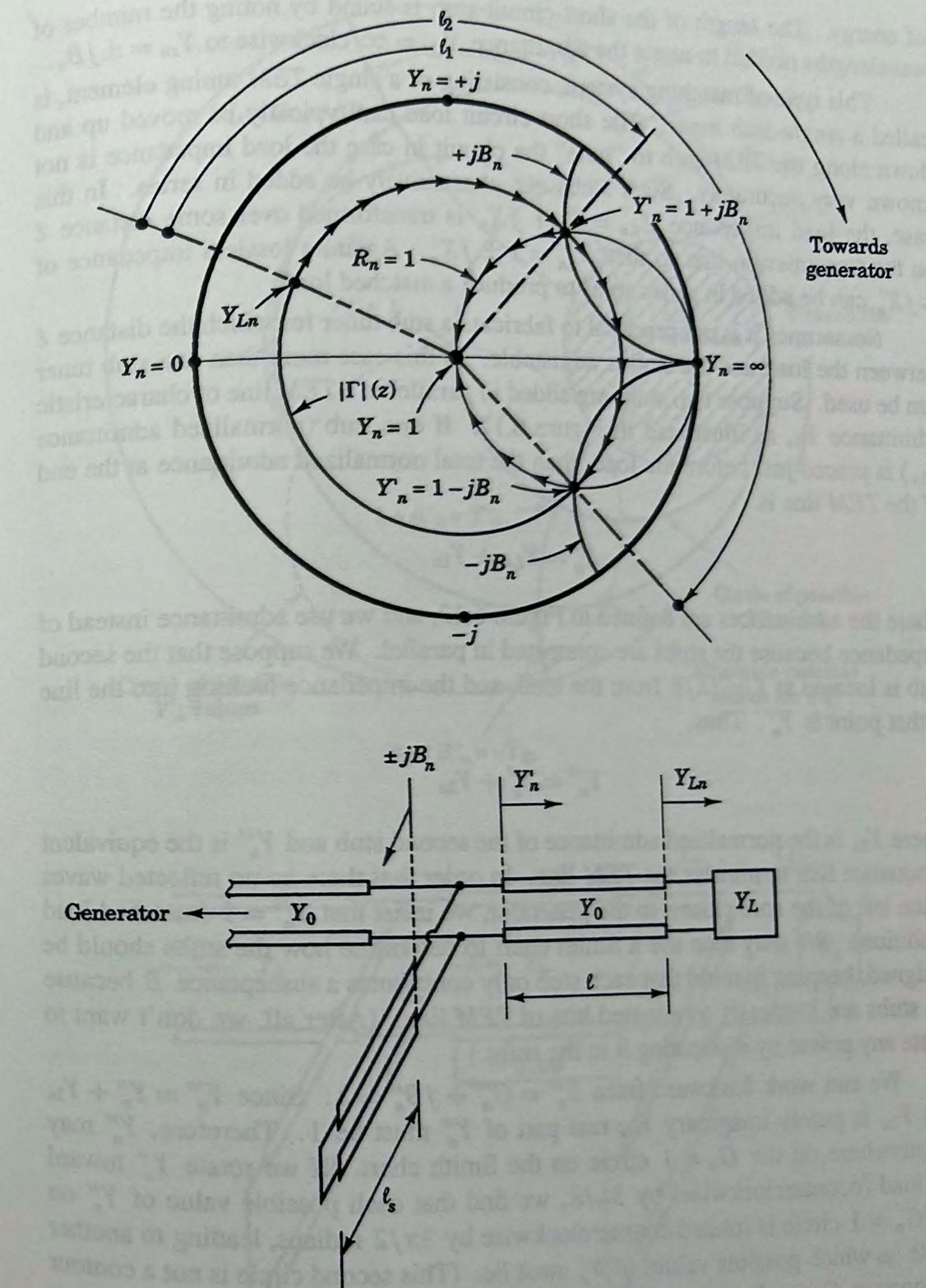


Figure 6.16 Single-stub matching using a Smith chart.