Semismooth Newton Solver for Unilateral Contact

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INTRODUCTION

- The high-cycle fatigue causing the failure of dovetail root joint has been observed in aircraft engines and experiment rigs.
- The Hertz theory for solving the contact pressure is not applicable to complex geometries.
- A robust contact solver based on finite element method is required.

OBJECTIVES

- A general solver based on FEM.
- The discretized problem converge to a unique solution.
- The solutions using the numerical solver converge to the solution of continuous systems.

Figure 1: Dovetail root joint in blade-disk connection

1. UNILATERAL CONTACT

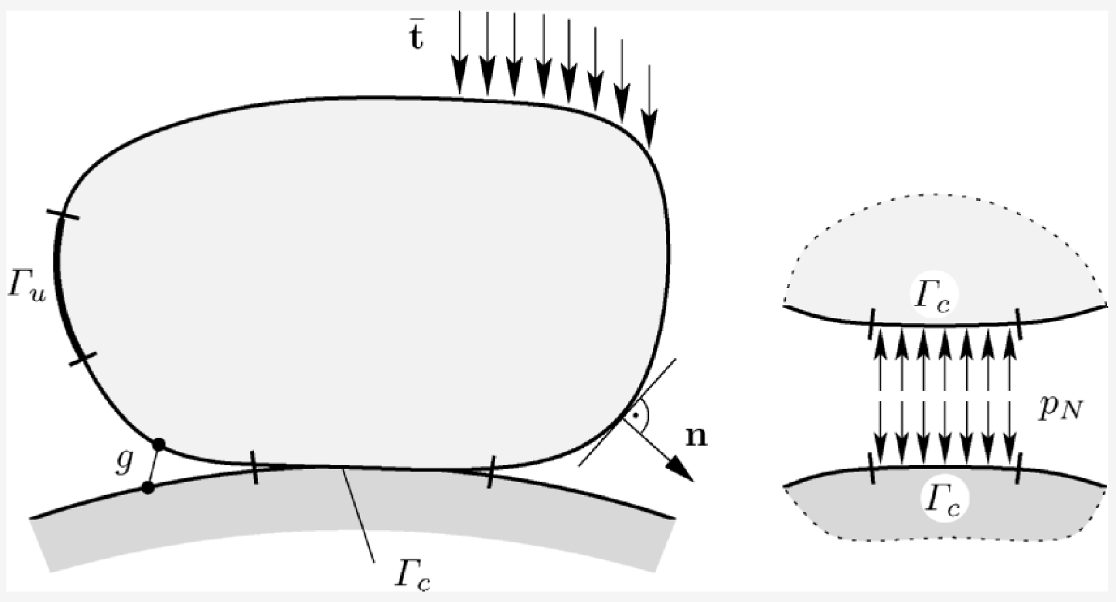

Figure 2: Unilateral contact of an elastic solid

- Γ_c: contact surface
- g: initial gap
- $\mathbf{n}$: normal direction
- p_N: normal contact stress

2. ONE DOF EXAMPLE

- equilibrium equation

$$ku = \lambda + f \tag{1}$$

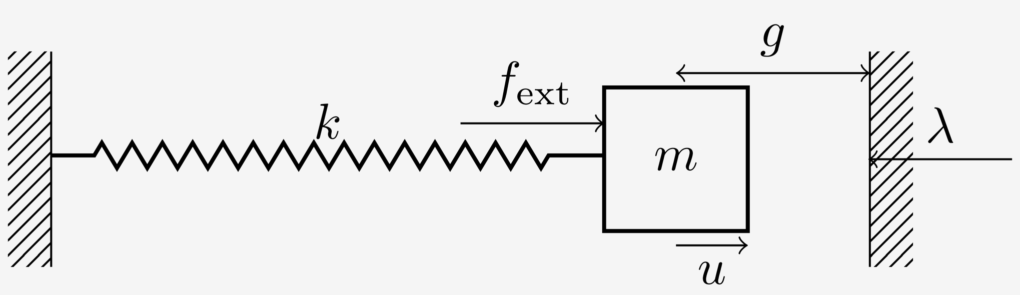

Figure 3: An one degree-of-freedom (DOF) mass spring system, where u is the displacement, λ is the contact force

- unilateral contact condition in KKT form

$$u - g \leq 0, \quad \lambda \leq 0, \quad (u - g)\lambda = 0$$

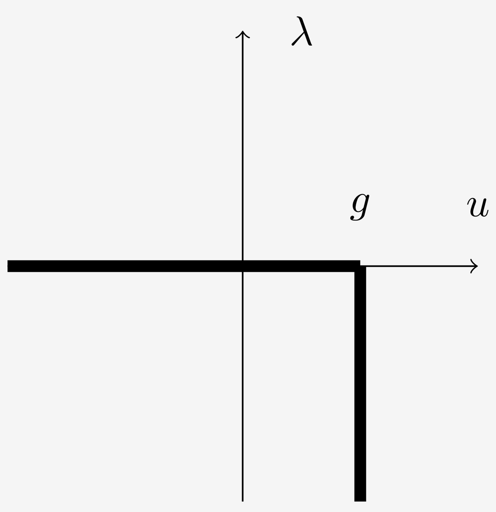

Figure 4: Karush–Kuhn–Tucker condition

- KKT condition reformulation: Lipschitz continuous equation

$$\lambda + \max\{0, c(u - g) - \lambda\} = 0 \tag{2}$$

- Lipschitz continuous equation $\mathbf{f}(\mathbf{x})$ by a Newton-tyep solver – the *semismooth Newton solver*.

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \mathbf{G}^{-1}(\mathbf{x}^k)\mathbf{f}(\mathbf{x}^k) \tag{3}$$

where $\mathbf{G}(x) \in \partial \mathbf{f}(x)$.

3. DOVETAIL ROOT CONTACT

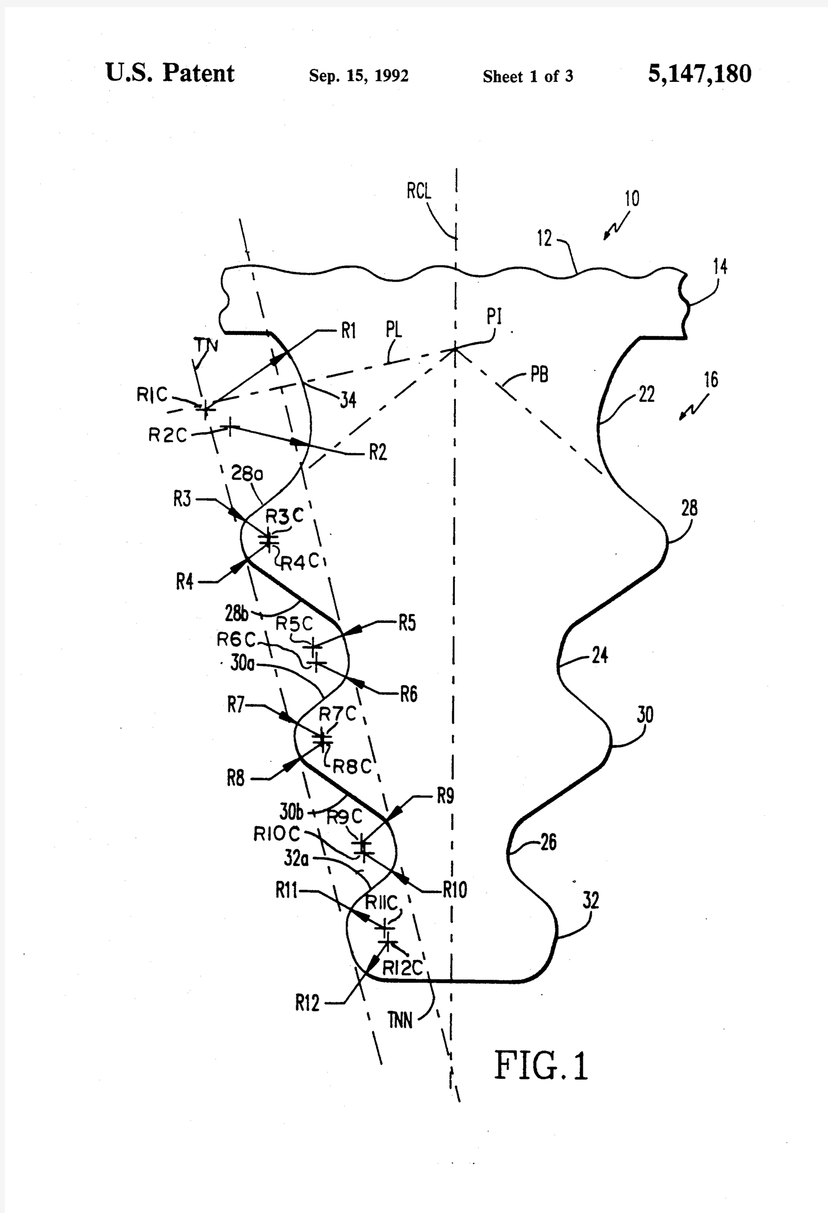

Figure 5: Dovetail root shape

- Assume: no friction, no stick assumption. Allow: detaching and sliding
- Initially: the tooth of the dovetail root fits the shape of the trough nicely.
- Under centrifugal force induced by rotation: the lower border of tooth will detach from the trough.
- The static centrifugal force points upward.

4. SOLVER WORKFLOW

- Draw the shape using CAD.
- Label the contact border Γ_c, the displacement border Γ_u and the force border Γ_σ.

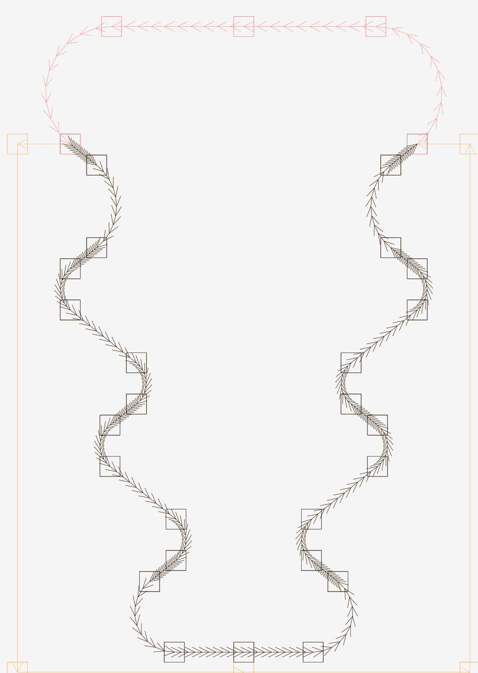

Figure 6: Dovetail root shape. Γ_c (–), Γ_σ (–) and the border of the trough (–)

4. SOLVER WORKFLOW (CONT'D)

- Mesh the elastic body using FreeFem++ (C++ library)

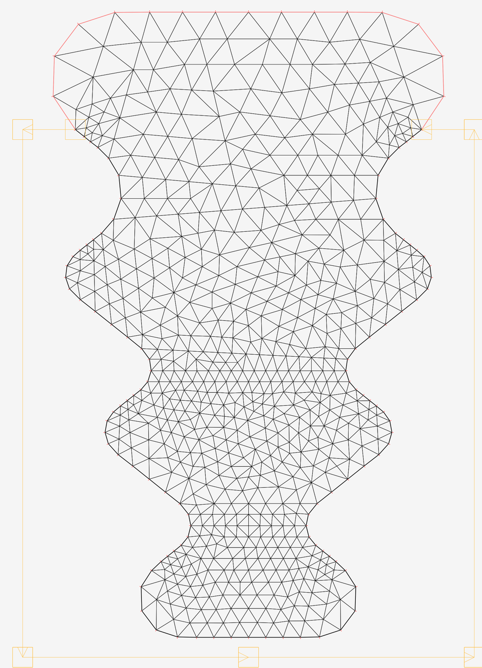

Figure 7: Dovetail root shape.

- Assemble the Stiffness matrix $\mathbf{K}$ and truncate the nodes on Γ_u.
- Compute the normal direction of nodes on Γ_c.
- Map the linear part of the problem to the contact border using rectangle matrix $\mathbf{B}$.

$$\mathbf{K}\mathbf{u} = \mathbf{B}^\top \boldsymbol{\lambda} + \mathbf{f}_{\text{ext}} \tag{4}$$

$$\boldsymbol{\lambda} + \max\{0, c(\mathbf{B}\mathbf{u} - \mathbf{g}) - \boldsymbol{\lambda}\} = 0 \tag{5}$$

- Solve the reduce-order Lipschitz continuous equations (4) and (5) using *semismooth Newton solver*.
- Plug the contact force $\boldsymbol{\lambda}$ back to (4) and calculate the displacement $\mathbf{u}$.
- visualize.

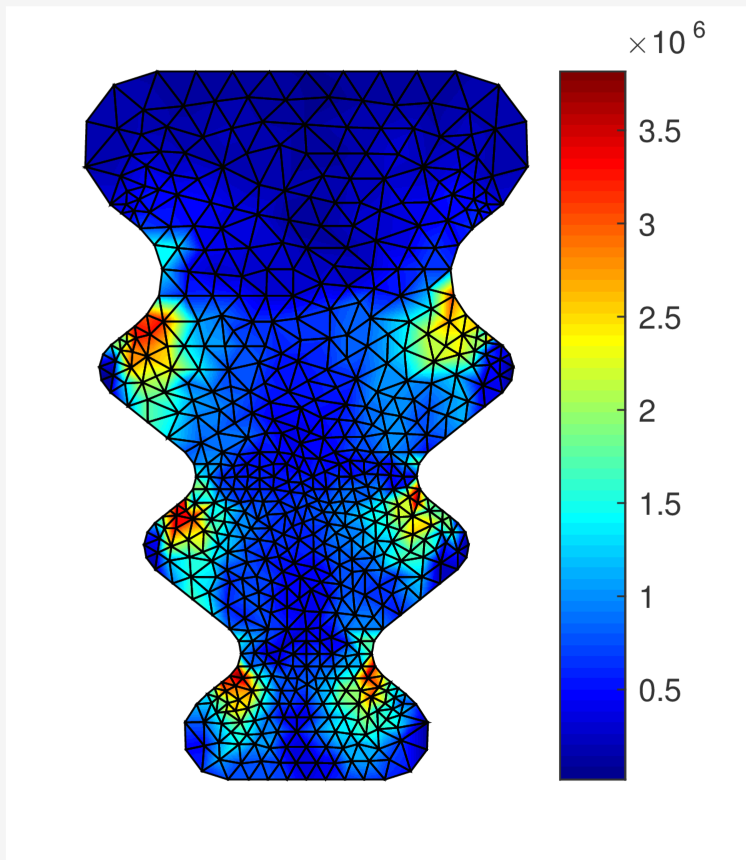

Figure 8: Von Mises stress

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