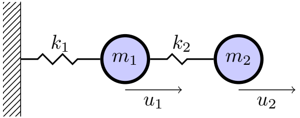
Linear modes of mass-spring oscillators

Governing equation

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{0}, \quad \forall t$$



Assume a harmonic solution

$$\mathbf{u}(t) = \bar{\mathbf{u}}\mathrm{e}^{i\omega t}$$

$\omega_{n1} = 0.618$

$\omega_{n2} = 1.618$

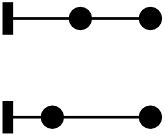


Figure 1: Two modes

Modes predict vibratory *resonances* of periodically forced systems

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{D}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}\cos(\Omega t)$$

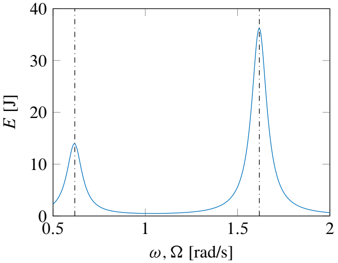


Figure 2: Energy-frequency plot. (-·-·) backbone curves.

Nonlinear modes of oscillators under contact

Equilibrium equation and contact condition

$$m\ddot{u}(t) + ku(t) = \lambda(t)$$
$$\lambda(t) + \max\{0, c(u(t) - g) - \lambda(t)\} = 0$$

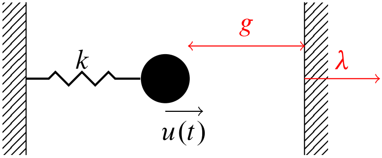


Figure 3: Vibro-impact oscillator

Harmonic Balance Method (HBM) assumes solutions as Fourier series

$$\mathbf{u}(t) = \sum_{m=-M}^{M} \bar{u}_m \mathrm{e}^{im\omega t}, \ \lambda(t) = \sum_{m=-M}^{M} \bar{\lambda}_m \mathrm{e}^{im\omega t}$$

Harmonically forced system. (Watch the right)

$$\ddot{u}(t) + 0.1\dot{u}(t) + u(t) = \lambda(t) + \cos(\Omega t), \ g = 1$$

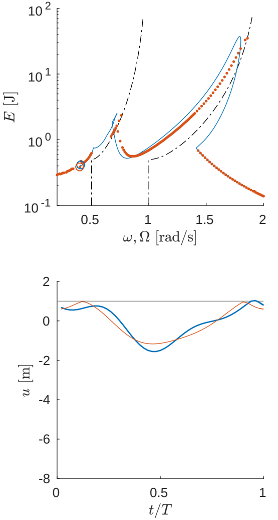


Figure 4: (——) HBM, (——) exact