# Analysis of Jeffcott rotor considering contact mechanism

MECH 501

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December 1, 2015

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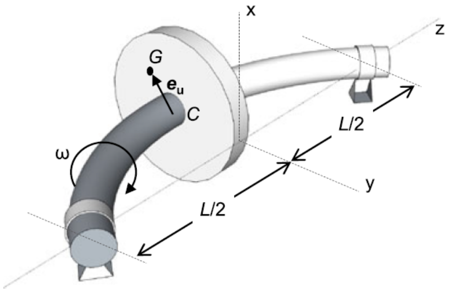
# Problem statement

## Jeffcott Rotor

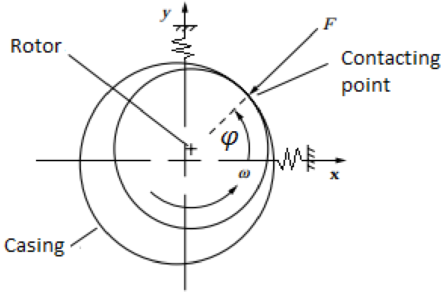
- simplified lumped parameter model
- mathematical idealization



# Problem statement

## Jeffcott Rotor

- Contact Consideration
- Methods to find solution
  - Time Integration method
  - Harmonic Balance method
- Applications



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# Governing Equation

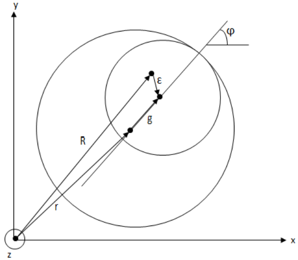
- $$m_r\ddot{r}_r(t) + c_r\dot{r}_r(t) + k_r r_r(t) = \lambda(t) + f_{textext}(t)$$
  $$m_s\ddot{r}_s(t) + c_s\dot{r}_s(t) + k_s r_s(t) = -\lambda(t)$$
  $$\lambda(t) = -k_c[(|r_r(t) - r_s(t)| - g)^+ \angle(r_r - r_s)]$$

  where subscript $r$ means rotor, $s$ means stator.

  $$r(t) = x(t) + iy(t)$$

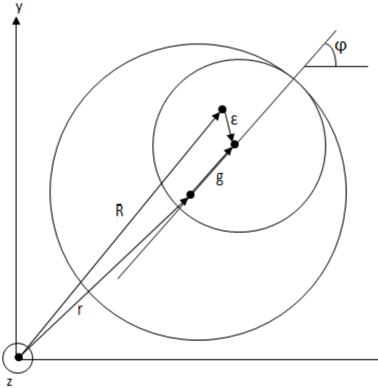
  $$f_{textext}(t) = m_r e_m w^2 \mathrm{e}^{iwt}$$

  $$k_c = 100k_r(1 + i0.1)$$



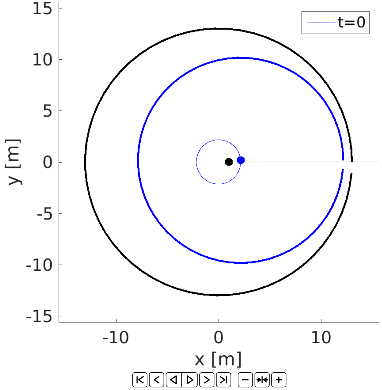
# Solution

- Numerical Solution
- Runge-kutta Method
- ode45
  - State-space form
  - Initial conditions

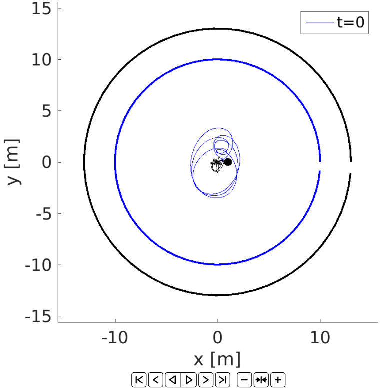


# Results

## periodic solution

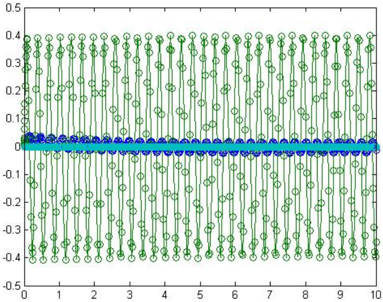


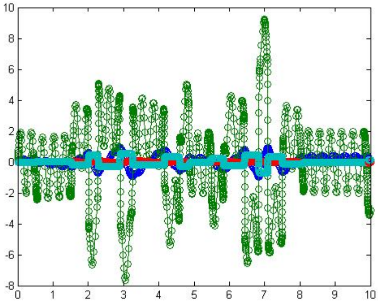

## Non-periodic solution



# Variation in solution

Left: $e = 0.02$, right: $e = 0.1$



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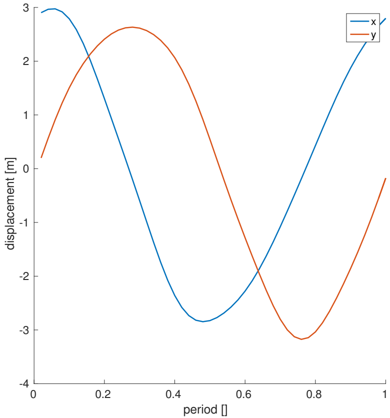
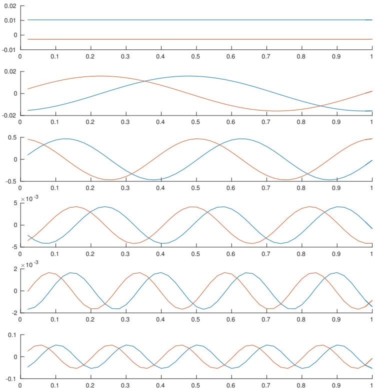
# Harmonic Balance Method basics



# Harmonic Balance Method basics

- for problem

$$\ddot{x}(t) = f(x(t), \dot{x}(t)) \tag{2}$$

- assume an approximate solution $x^h(t) \in C^h$

$$x(t) \approx x^h(t) = \sum_{m=-M}^{M} c_m \mathrm{e}^{imwt} = \mathbf{\Phi}(t)\mathbf{c} \tag{3}$$

where $0 < M < \infty$, $C^h$ is the finite space spanning from the finite base functions $\mathrm{e}^{imwt}$.

- (2) yields

$$\ddot{x}^h(t) = (imw)^2 \sum_{m=-M}^{M} c_m \mathrm{e}^{imwt} = -m^2 w^2 \mathbf{\Phi}(t)\mathbf{v} \tag{4}$$

# Harmonic Balance Method basics

## weak form

- define the test function $v^h(t) \in C^h$ in the same finite space

$$v^h(t) = \sum_{m=-M}^{M} v_m \mathrm{e}^{imwt} = \mathbf{\Phi}(t)\mathbf{v} \tag{5}$$

- Then, the nonlinear ODE (2) is equivalent to the weak form

$$\int_0^{2\pi/w} v^h(t)[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0, \quad \forall v^h(t) \in C^h \tag{6}$$

# Harmonic Balance Method basics

- For $2M+1$ unknown coefficients $c_m$, $m = -M, ..., -1, 0, 1, ..., M$, we have $2M+1$ equations which make solution unique
  -
$$\int_0^{2\pi/w} \mathrm{e}^{-i(-M)wt}[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0$$
$$\ldots$$
$$\int_0^{2\pi/w} \mathrm{e}^{-i(-1)wt}[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0$$
$$\int_0^{2\pi/w} \mathrm{e}^{-i(0)wt}[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0$$
$$\int_0^{2\pi/w} \mathrm{e}^{-i1wt}[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0$$
$$\ldots$$
$$\int_0^{2\pi/w} \mathrm{e}^{-iMwt}[\ddot{x}^h - f(x^h(t), \dot{x}^h(t))]\mathrm{d}t = 0$$

- In summary, we are going to solve a residual function $\mathbf{f} : \mathbb{R}^{2M+1} \to \mathbb{R}^{2M+1}$

$$\mathbf{f}(\mathbf{c}_r, \mathbf{c}_s) = \mathbf{0} \tag{8}$$

# Harmonic Balance Method basics

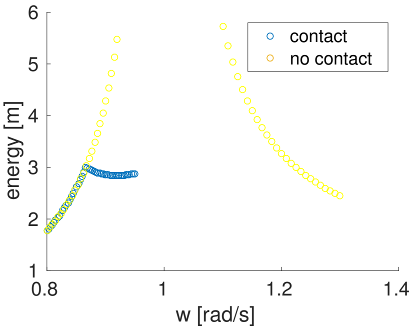
## Iteration algorithm

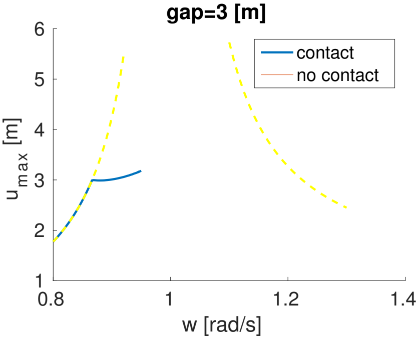
- For a certain $w$, given the initial guess $\mathbf{c}^{(0)}$, We use " time-> frequency -> time -> frequency -> ..." iteration. The one time period $[0, 2\pi/w]$ is divided into $N$ small segments $[t_{n-1}, t_n]$, $n = 1, 2, ..., N$.
  - at iteration step $k$: frequency -> time:
  
    $$r_r(t)^{(k)} = \mathbf{\Phi}(t)\mathbf{c}_r^{(k)}, \quad r_s(t)^{(k)} = \mathbf{\Phi}(t)\mathbf{c}_s^{(k)}$$
  - time domain:
  
    $$\boldsymbol{\lambda}^{(k)}(t_n) = -k_c[(|r_r^{(k)}(t_n) - r_s^{(k)}(t_n)| - g)^+\angle(r_r^{(k)}(t_n) - r_s^{(k)}(t_n))]$$
  - time -> frequency
  
    $$\mathbf{f}(\mathbf{r}_r(t_n, \mathbf{c}_r^{(k)}), \mathbf{r}_s(t_n, \mathbf{c}_s^{(k)}), \boldsymbol{\lambda}(t_n, \mathbf{c}^{(k+1)})) = \mathbf{0}$$
  - if not terminate, go to step 1.

# Results

## Energy - forcing frequency

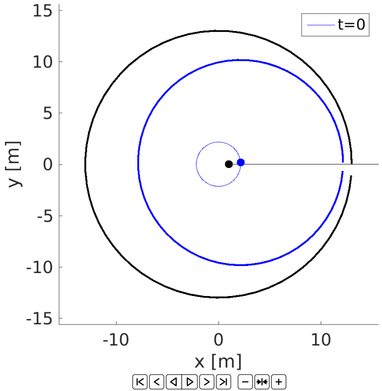
$$energy = \sqrt{c_{-M}^2 + \dots + c_{-1}^2 + c_0^2 + c_1^2 + \dots + c_M^2}$$



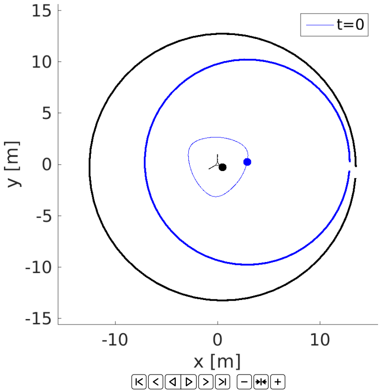
# Results – periodic solution

## No contact




$$r_r(t) = (2.1569 + 0.1128i)\mathrm{e}^{iwt}$$

## With contact




$$r_r(t) = \ldots + (-0.0079 + 0.0011i)\mathrm{e}^{iwt} + (0.005$$

$$r_s(t) = \ldots + (0.0007 - 0.0002i)\mathrm{e}^{iwt} + (-0.002$$