User manual

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Chapter 1

Static contact problem

1.1 Semismooth Newton method

Input: Semismooth function $\mathbf{f}(\mathbf{x}) : \mathbb{R}^N \to \mathbb{R}^N$ and initial guess $\mathbf{x} \in \mathbb{R}^N$; tolerance $\varepsilon > 0$
Init: Initial guess $\mathbf{x} \in \mathbb{R}^N$
while Norm of residual $\|\mathbf{f}(\mathbf{x})\| > \varepsilon$ **do**
    Calculate the residual $\mathbf{f}(\mathbf{x})$
    Select a Jacobian matrix $\mathbf{G}(\mathbf{x})$ in the sub-differential set $\partial \mathbf{f}(\mathbf{x})$
    Update $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{G}(\mathbf{x})^{-1}\mathbf{f}(\mathbf{x})$
end while

Algorithm 1.1: Semismooth Newton method [**?**, **?**].

1.2 One degree-of-freedom contact problem

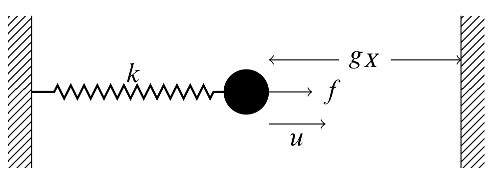

Figure 1.1: One degree-of-freedom contact problem $ku = \lambda + f$ and $u \leq g_X$ where u is the displacement, λ is the contact force, k is the stiffness, f is the external force and g_X is the initial gap. Define the normal gap as $g_N(u) = u - g_X$, denote the penetration as g_N^+

1.3 Semismooth reformulation

$$\begin{cases} r(u, \lambda) = ku - \lambda - f = 0 \\ s(u, \lambda) = \lambda + \max\{0, c(u - g_X) - \lambda\} = 0 \end{cases} \tag{1.1}$$

where positive constant c is used to balance the unit between displacement and force. Function $s(u, \lambda) : \mathbb{R}^2 \to \mathbb{R}$ is semismooth. Solve it with semismooth Newton method.

1.4 Penalty treatment

Regularize the non-differentiable contact constitutive law using a constant penalty coefficient ϵ as is reviewed in [**?**]

$$\lambda(u) = -\epsilon \max\{0, u - g_X\} \tag{1.2}$$

And get the regularized system equation to be solved:

$$r(u) = ku - \lambda(u) - f = 0 \tag{1.3}$$

where positive constant c is used to balance the unit between displacement and force. Function $r(u) : \mathbb{R} \to \mathbb{R}$ is semismooth.

The solution of the regularized system converges to true solution when $\epsilon \to \infty$. It yields the algorithm 1.2

Input: coefficient matrices $\mathbf{K}$, $\mathbf{f}$ and $\mathbf{G}$ (linear or nonlinear w.r.t. $\mathbf{u}$), penalty coefficient ϵ and error tolerances ε and ε_1.
Initialization: $\mathbf{u}$.
while $||\Delta \mathbf{u}|| > \varepsilon$ **do**
  Increase ϵ
  while $||\Delta \mathbf{u}|| > \varepsilon_1$ **do**
    Solve (1.3) using semismooth Newton method in algorithm **??**
  end while
end while

Algorithm 1.2: Solve finite contact problem in penalty form by semismooth Newton method.

1.5 Augmented Lagrange treatment and Uzawa algorithm

In augmented Lagrange method, the contact force is the sum of a constant term and the penalty term with small ϵ as is reviewed in [**?**]

$$\lambda(u) = \bar{\lambda} - \epsilon \max\{0, u - g_X\} \tag{1.4}$$

And get the regularized system equation to be solved:

$$r(u) = ku - \lambda(u) - f = 0 \tag{1.5}$$

where positive constant c is used to balance the unit between displacement and force. Function $r(u) : \mathbb{R} \to \mathbb{R}$ is semismooth.

The augmented term $\bar{p}_N$ is fixed within an iteration step, as to preserve the contact stress. It is updated as

$$\bar{\lambda} = \min\{0, \bar{\lambda} - \epsilon \max\{0, u - g_X\}\} \tag{1.6}$$

It yields a double-loop algorithm. In the inner loop, solve the nonlinear equation with fixed augmented term $\bar{\lambda}$ and fixed penalty coefficient ϵ, in the outer loop, update these two coefficients. An empirical updating law of ϵ given by [**?**]

$$\epsilon \leftarrow 10\epsilon, \qquad ||g_N^{+,(k+1)}|| > \frac{1}{4}||g_N^{+,(k)}|| \tag{1.7}$$

Input: coefficient matrices of the discretized problem (**??**) and error tolerances ε and ε_1.
Initialization: $u, \bar{\lambda}, \epsilon$.
while $||r(u)|| > \varepsilon$ **do**
  while $||r(u)|| > \varepsilon_1$ **do**
    Solve (1.5) using semismooth Newton method in algorithm **??**
  end while
  update $\bar{\lambda}$ according to (1.6)
  update ϵ according to (1.7)
end while

Algorithm 1.3: Penalty method for finite contact problem