Aula 15

3/11/2025

FP3 (10) (d)

$$\begin{vmatrix} 0^+ & 1 & 4 & 5 \\ -1^- & -2 & -4 & 6 \\ 0^+ & 0^- & 1^+ & 0^- \\ 1 & 2 & 7 & 2 \end{vmatrix} = -1 \begin{vmatrix} 0^+ & 1 & 5 \\ 1^- & -2 & 6 \\ 1^+ & 2 & 2 \end{vmatrix} = -\left(-1 \underbrace{\begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix}}_{=2-10=-8} + \underbrace{\begin{vmatrix} 1 & 5 \\ -2 & 6 \end{vmatrix}}_{=6+10=16}\right) = -(-8+16) = -8$$

(3)

$$\begin{vmatrix} \lambda+2^+ & -1 & 3 \\ 2^- & \lambda-1 & 2 \\ 0^+ & 0^- & \lambda+4^+ \end{vmatrix} = 0 \iff (\lambda+4) \begin{vmatrix} \lambda+2 & -1 \\ 2 & \lambda-1 \end{vmatrix} = 0 \iff (\lambda+4)\left((\lambda+2)(\lambda-1)+2\right)=0$$

$$\iff \lambda+4=0 \text{ ou } \underbrace{(\lambda+2)(\lambda-1)+\cancel{2}}_{=\lambda^2-\lambda+2\lambda-\cancel{2}}=0 \iff \lambda=-4 \text{ ou } \lambda^2+\lambda=0 \iff \lambda=-4 \text{ ou } \lambda(\lambda+1)=0$$

$$\iff \lambda=-4 \text{ ou } \lambda=0 \text{ ou } \lambda=-1$$

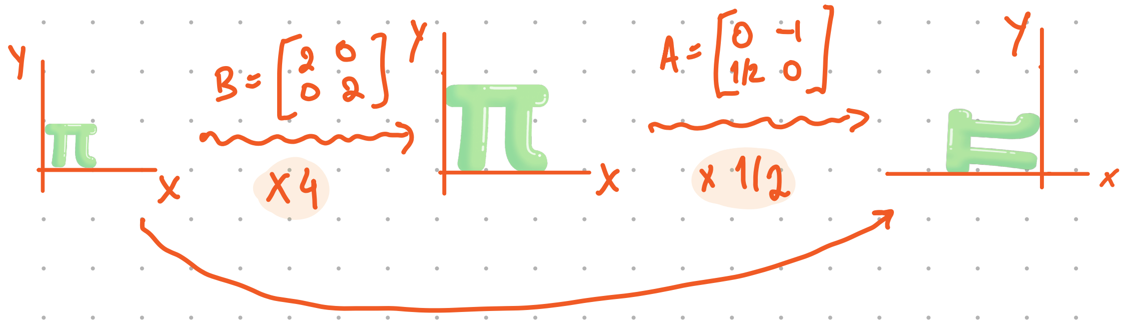
(4) $A_{n\times n}$ tem colunas $C_1, \dots, C_n$

$$\det(cA) = \det(cC_1, cC_2, \dots, cC_n) = c \det(C_1, cC_2, \dots, cC_n)$$
$$= c^2 \det(C_1, C_2, cC_3, \dots, C_n) = \dots = c^n \det(C_1, \dots, C_n) = c^n \det(A).$$

Aplicamos a Prop. 3 do slide 2 sucessivamente.

Propriedades

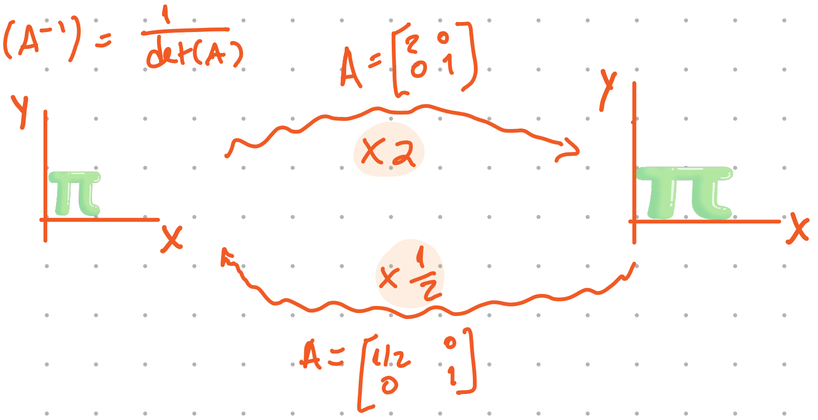
- $\det(AB) = \det(A)\det(B) \; (= \det(BA))$



$$AB = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$$

$$\times 4 \times 1/2 = 2$$

- $\det(A^{-1}) = \dfrac{1}{\det(A)}$



- Se A tem 2 colunas/linhas iguais ou colunas/linhas nulas então $\det(A)=0$.

- $\det(A)=\det(A^T)$

- $A \underset{\substack{L_i=\alpha L_i \\ \alpha\neq 0}}{\sim} B \therefore \det(B)=\alpha\det(A)$

- $A \underset{L_i \leftrightarrow L_j}{\sim} B \therefore \det(B)=-\det(A)$

- $A \underset{L_i=L_i+\beta L_j}{\sim} B \therefore \det(B)=\det(A)$

- $\det(A+B)\neq\det(A)+\det(B)$

Por exemplo,

$$\det\left(\begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\neq\det\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)+\det\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right) \Leftrightarrow 1\neq 0$$

FP3 2

$$\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\4c_1&4c_2&4c_3\end{vmatrix}=4\begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}=4.7=28$$

$$\begin{vmatrix}a_1&a_3&a_2\\b_1&b_3&b_2\\c_1&c_3&c_2\end{vmatrix}=-7$$

$$\begin{vmatrix}a_1-5c_1&a_2-5c_2&a_3-5c_3\\10b_1&10b_2&10b_3\\-4c_1&-4c_2&-4c_3\end{vmatrix}=-4.10\begin{vmatrix}a_1-5c_1&a_2-5c_2&a_3-5c_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}$$

$(L_1=L_1+5L_3)$

$=-4.10.7=-280$

7 $\det(B^{-1})=4$ logo $\det(B)=1/4$

$\det(AB)=24 \Leftrightarrow \det(A)\,1/4=24 \Leftrightarrow \det(A)=96$

6 $A_{4\times4}$, $\det(A)=3$

$$\det(2(A^{-1})^T)=2^4\det((A^{-1})^T)=2^4\det(A^{-1})=\frac{2^4}{\det(A)}=\frac{2^4}{3}=\frac{16}{3}$$

Adjunta

$$\text{adj } A = \begin{bmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \cdots & A_{nn} \end{bmatrix}^T = \begin{bmatrix} A_{11} & \cdots & A_{n1} \\ \vdots & & \vdots \\ A_{1n} & & A_{nn} \end{bmatrix}$$

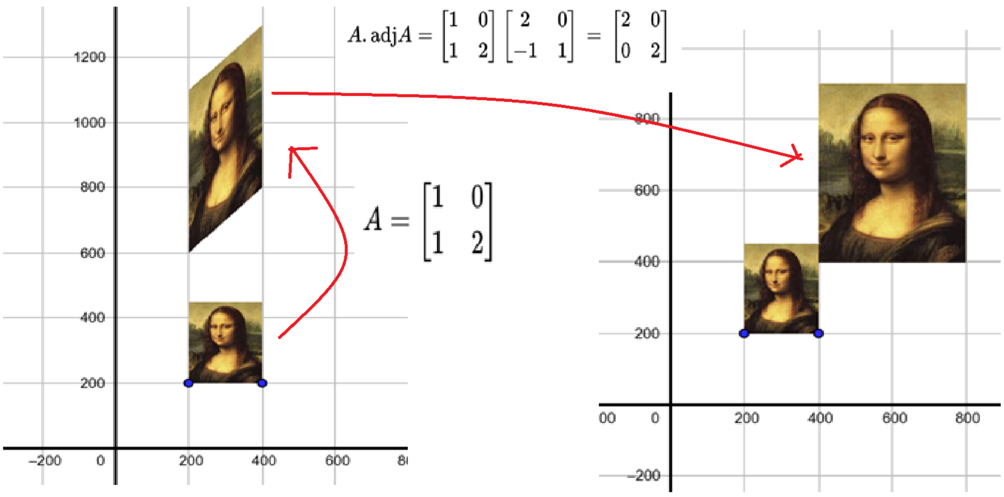
Relembrar: Cofator de a_{ij} é $A_{ij} = (-1)^{i+j}|M_{ij}|$

Exemplo

$$A = \begin{bmatrix} 1^{+} & 0^{-} & 2^{+} \\ 3^{-} & 1^{+} & -1^{-} \\ 0^{+} & 1^{-} & 0^{+} \end{bmatrix} \quad A_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 7 \quad A_{13} = +\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} = 3 \quad A_{22} = +\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 0$$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 0 & -1 \\ -2 & 7 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 0 & 7 \\ 3 & -1 & 1 \end{bmatrix}$$

Teorema: $A \text{ adj } A = \det(A) I_n$



- Se A tem inversa então $\boxed{A^{-1} = \frac{\text{adj } A}{|A|}}$

Intuição: A adjunta de A é quase a inversa de A.

- A é invertível $\Leftrightarrow \det(A) \neq 0$
  $\Leftrightarrow AX = 0$ é poss. det. (Cap. 1.1 slide 37)

(14) $A = \begin{bmatrix} 1^{+} & 1 & 1 \\ 1^{-} & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad A^{-1} = \frac{\text{adj } A}{\det(A)} = \frac{1}{\det(A)} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$

O elemento (1,2) de A^{-1} é $\frac{A_{21}}{\det(A)} = \frac{-\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}{-1} = 0.$

FP3 até ao ex. 22. Recomendo o ex. 5, 8(a), 12, 17.