ReProva Álgebra Linear P1

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1. Considere as bases do $\mathbb{R}$-espaço vetorial $\mathbb{R}^3$, A $= \{(4,2,0),(1,-1,1),(5,3,3)\}$ e B $= \{(1,-2,1),(1,5,2),(1,0,1)\}$. Exiba as matrizes de mudança de base $M_{B\rightarrow\ A}$ e $M_{A\ \rightarrow\ B}$. Escreva também os vetores abaixo nas bases indicadas:

- $M_{B\rightarrow\ A}$

$a_1.(1,-2,1) + a_2.(1,5,2) + a_3.(1,0,1) = (4,2,0)$
$a_1.(1,-2,1) + a_2.(1,5,2) + a_3.(1,0,1) = (1,-1,1)$
$a_1.(1,-2,1) + a_2.(1,5,2) + a_3.(1,0,1) = (5,3,3)$

Resolvendo a equação do vetor (4,2,0)

$$\begin{cases} a_1 + a_2 + a_3 = 4 \\ -2a_1 + 5a_2 = 2 \\ a_1 + 2a_2 + a_3 = 0 \end{cases}$$

$a_1 = 4 - a_2 - a_3$

$4 - a_2 - a_3 + 2a_2 + a_3 = 0$
$4 + a_2 = 0$
$a_2 = -4$

$-\ 2a_1 + 5(-4) = 2$
$-\ 2a_1 - 20 = 2$
$-\ 2a_1 = 2 + 20$
$-\ a_1 = \frac{22}{2}$
$a_1 = -11$

$-\ 11 - 4 + a_3 = 4$
$a_3 = 4 + 11 + 4$
$a_3 = 19$

a_1 = -11; $a2$ = -4; a_3 = 19

Resolvendo o sistema do vetor (1,-1,1)

$$\begin{cases} a_1 + a_2 + a_3 = 1 \\ -2a_1 + 5a_2 = -1 \\ a_1 + 2a_2 + a_3 = 1 \end{cases}$$

$a_1 = 1 - a_2 - a_3$

$1 - a_2 - a_3 + 2a_2 + a_3 = 0$
$1 + a_2 = 1$
$a_2 = 0$

$-2a_1 + 5(0) = -1$
$-2a_1 = -1$
$-a_1 = \frac{-1}{2}$
$a_1 = \frac{1}{2}$

$\frac{1}{2} + 0 + a_3 = 1$
$a_3 = 1 - \frac{1}{2}$
$a_3 = \frac{1}{2}$

$a_1 = \frac{1}{2}$; $a2 = 0$; $a_3 = \frac{1}{2}$

Resolvendo o sistema do vetor (5,3,3)

$$\begin{cases} a_1 + a_2 + a_3 = 5 \\ -2a_1 + 5a_2 = 3 \\ a_1 + 2a_2 + a_3 = 3 \end{cases}$$

$a_1 = 5 - a_2 - a_3$

$5 - a_2 - a_3 + 2a_2 + a_3 = 0$
$5 + a_2 = 3$
$a_2 = 3 - 5$
$a_2 = -2$

$-2a_1 + 5(-2) = 2$
$-2a_1 - 10 = 3$
$-2a_1 = 3 + 10$
$-2a_1 = 13$
$-a_1 = \frac{13}{2}$
$a_1 = \frac{-13}{2}$

$\frac{-13}{2} - 2 + a_3 = 5$
$a_3 = 5 + \frac{13}{2} + 2$
$a_3 = 7 + \frac{13}{2}$
$a_3 = \frac{27}{2}$

$a_1 = \frac{-13}{2}$; $a2$ = -2; $a_3 = \frac{27}{2}$

$$M_B \rightarrow {}_A = \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ -4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix}$$

- $M_A \rightarrow {}_B$

$a_1.(4, 2, 0) + a_2.(1, -1, 1) + a_3.(5, 3, 3) = (1, -2, 1)$

$a_1.(4,2,0) + a_2.(1,-1,1) + a_3.(5,3,3) = (1,5,2)$
$a_1.(4,2,0) + a_2.(1,-1,1) + a_3.(5,3,3) = (1,0,1)$

Resolvendo o sistema do vetor (1,-2,1)

$$\begin{cases} 4a_1 + a_2 + 5a_3 = 1 \\ 2a_1 - a_2 + 3a_3 = -2 \\ a_2 + 3a_3 = 1 \end{cases}$$

$a_2 = 1 - 3a_3$

$4a_1 + 1 - 3a_3 + 5a_1 = 1$
$4a_1 + 1 + 2a_3 = 1$
$4a_1 = -2a_3$
$a_1 = \frac{-2a_3}{2}$

$2\left(\frac{-2a_3}{2}\right) - 1 + 3a_3 + 3a_3 = -2$
$-4\left(\frac{-2a_3}{2}\right) - 1 + 3a_3 + 3a_3 = -2$
$-a_3 - 1 + 6a_3 = -2$
$-a_3 - 1 + 6a_3 = -2$
$-a_3 + 6a_3 = -2 + 1$
$5a_3 = -1$
$a_3 = \frac{-1}{5}$

$a_2 + 3\left(\frac{-1}{5}\right) = 1$
$a_2 - \frac{3}{5} = 1$
$a_2 = \frac{5+3}{5}$
$a_2 = \frac{8}{5}$

$2a_1 - \frac{8}{5} + 3\left(\frac{-1}{5}\right) = -2$
$2a_1 - \frac{8}{5} - \frac{-3}{5} = -2$
$2a_1 - \frac{11}{5} = -2$
$2a_1 = -2 + \frac{11}{5}$
$2a_1 = \frac{-10+11}{5} = \frac{1}{5}$
$a_1 = \frac{\frac{1}{5}}{2}$
$a_1 = \frac{1}{10}$

$a_1 = \frac{1}{10}; a_2 = \frac{8}{5}; a_3 = \frac{-1}{5}$

Resolvendo o sistema do vetor (1,5,2)

$$\begin{cases} 4a_1 + a_2 + 5a_3 = 1 \\ 2a_1 - a_2 + 3a_3 = 5 \\ a_2 + 3a_3 = 2 \end{cases}$$

$a_2 = 2 - 3a_3$

$4a_1 + 2 - 3a_3 + 5a_3 = 1$
$4a_1 + 2a_3 = 1 - 2$

$a_1 = \frac{-2a_3-1}{4}$

$2\left(\frac{-2a_3-1}{4}\right) - 2 + 3a_3 + 3a_3 = 5$
$\left(\frac{-4a_3-2}{4}\right) - 2 + 3a_3 + 3a_3 = 5$
$-a_3 - \frac{2}{4} - 2 + 6a_3 = 5$
$-a_3 - \frac{1}{2} - 2 + 6a_3 = 5$
$-a_3 + 6a_3 = 5 + \frac{1}{2} + 2$
$5a_3 = \frac{10+1+4}{2}$
$5a_3 = \frac{15}{2}$
$a_3 = \frac{\frac{15}{2}}{5}$
$a_3 = \frac{3}{2}$

$a_2 + 3\left(\frac{3}{2}\right) = 2$
$a_2 + \frac{9}{2} = 2$
$a_2 = 2 - \frac{9}{2}$
$a_2 = \frac{4-9}{2}$
$a_2 = \frac{-5}{2}$

$2a_1 - \left(\frac{-5}{2}\right) + 3\left(\frac{3}{2}\right) = 5$
$2a_1 + \frac{5}{2} + \frac{9}{2} = -5$
$2a_1 - \frac{11}{5} = -2$
$2a_1 = 5 - \frac{5}{2} - \frac{9}{2}$
$2a_1 = 5 - \frac{14}{2}$
$2a_1 = \frac{10-14}{2}$
$2a_1 = \frac{-4}{2}$
$2a_1 = -2$
$a_1 = \frac{-2}{2}$
$a_1 = -1$

$a_1 = -1; a_2 = \frac{-5}{2}; a_3 = \frac{3}{2}$

Resolvendo o sistema do vetor (1,0,1)

$$\begin{cases} 4a_1 + a_2 + 5a_3 = 1 \\ 2a_1 - a_2 + 3a_3 = 0 \\ a_2 + 3a_3 = 1 \end{cases}$$

$a_2 = 1 - 3a_3$

$4a_1 + 1 - 3a_3 + 5a_3 = 1$
$4a_1 + 2a_3 = 1 - 1$
$4a_1 + 2a_3 = 0$
$4a_1 = -2a_3$
$a_1 = \frac{-2a_3}{4}$

$2\left(\frac{-2a_3}{4}\right) - 1 + 3a_3 + 3a_3 = 0$
$\frac{-4a_3}{4} - 1 + 6a_3 = 0$
$-a_3 + 6a_3 = 0 + 1$
$5a_3 = 1$
$a_3 = \frac{1}{5}$

$a_2 + 3\left(\frac{1}{5}\right) = 1$
$a_2 + \frac{3}{5} = 1$
$a_2 = 1 - \frac{3}{5}$
$a_2 = \frac{5-3}{5}$
$a_2 = \frac{2}{5}$

$2a_1 - \left(\frac{2}{5}\right) + 3\left(\frac{1}{5}\right) = 0$
$2a_1 - \frac{2}{5} + \frac{3}{5} = 0$
$2a_1 + \frac{1}{5} = 0$
$2a_1 = \frac{-1}{5}$
$a_1 = \frac{\frac{-1}{5}}{2}$
$a_1 = \frac{-1}{10}$

$a_1 = \frac{-1}{10}; a_2 = \frac{2}{5}; a_3 = \frac{1}{5}$

$$M_{A \to B} = \begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5} \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix}$$

- $\mathbf{v}$ = (0,1,2) em B

  $a_1.(1, -2, 1) + a_2.(1, 5, 2) + a_3.(1, 0, 1) = (0, 1, 2)$

  $$\begin{cases} a_1 + a_2 + a_3 = 0 \\ -2a_1 + 5a_2 = 1 \\ a_1 + 2a_2 + a_3 = 2 \end{cases}$$
  $-2a_1 = 1 - 5a_2$
  $-a_1 = \frac{1-5a_2}{2}(-1)$
  $a_1 = \frac{-1+5a_2}{2}$

  $\frac{-1+5a_2}{2} + a_2 + a_3 = 0$
  $\frac{-1+5a_2}{2} + a_2 = -a_3$
  $\frac{-1+5a_2+2a_2}{2} = -a_3$
  $\frac{-1+5a_2+2a_2}{2} = -a_3(-1)$
  $a_3 = \frac{1-7a_2}{2}$

  $\frac{-1+5a_2}{2} + 2a_2 + \frac{1-7a_2}{2} = 2$
  $\frac{-1+5a_2+1-7a_2}{2} + 2a_2 = 2$
  $\frac{-2a_2}{2} + 2a_2 = 2$
  $-a_2 + 2a_2 = 2$
  $a_2 = 2$

  $-2a_1 + 5(2) = 1$
  $-2a_1 + 10 = 1$
  $-2a_1 = 1 - 10$
  $-2a_1 = -9$
  $-a_1 = \frac{-9}{2}(-1)$
  $a_1 = \frac{9}{2}$

  $\frac{9}{2} + 2 + a_3 = 0$

$\frac{9}{2} + \frac{4}{2} + a_3 = 0$
$\frac{13}{2} + a_3 = 0$
$a_3 = \frac{-13}{2}$

$a_1 = \frac{9}{2}; a_2 = 2; a_3 = \frac{-13}{2}$

- $\mathbf{v}$ = (1,3,-1)

$a_1.(4,2,0) + a_2.(1,-1,1) + a_3.(5,3,3) = (1,3,-1)$

$$\begin{cases} 4a_1 + a_2 + 5a_3 = 1 \\ 2a_1 - a_2 + 3a_3 = 3 \\ \quad a_2 + 3a_3 = -1 \end{cases}$$

$a_2 = -1 - 3a_3$
$4a_1 - 1 - 3a_3 + 5a_3 = 1$
$4a_1 - 1 + 2a_3 = 1$
$4a_1 + 2a_3 = 1 + 1$
$4a_1 + 2a_3 = 2$
$4a_1 = -2a_3 + 2$
$a_1 = \frac{-2a_3+2}{4}$

$2\left(\frac{-2a_3+2}{4}\right) + 1 + 3a_3 + 3a_3 = 3$
$\frac{-4a_3+4}{4} + 1 + 3a_3 + 3a_3 = 3$
$-a_3 + 1 + 1 + 6a_3 = 3$
$-a_3 + 6a_3 = 3 - 1 - 1$
$5a_3 = 1$
$a_3 = \frac{1}{5}$

$a_2 + 3\left(\frac{1}{5}\right) = -1$
$a_2 + \left(\frac{3}{5}\right) = -1$
$a_2 = -1 - \frac{3}{5}$
$a_2 = \frac{-5-3}{5}$
$a_2 = \frac{-8}{5}$

$2a_1 - \left(\frac{-8}{5}\right) + 3\left(\frac{1}{5}\right) = 3$
$2a_1 + \frac{8}{5} + \frac{3}{5} = 3$
$2a_1 + \frac{11}{5} = 3$
$2a_1 = 3 - \frac{11}{5}$
$2a_1 = \frac{15-11}{5}$
$2a_1 = \frac{4}{5}$
$a_1 = \frac{\frac{4}{5}}{2}$
$a_1 = \frac{2}{5}$

$a_1 = \frac{2}{5}; a_2 = \frac{-8}{5}; a_3 = \frac{1}{5}$

2. Considere o conjunto S = {(1,1,1,1,1),(2,0,-1,1,3),(3,1,0,2,4),(2,2,5,8,-1),(0,1,0,2,3)}

- S é li ou ld?

$a_1.(1,1,1,1,1) + a_2.(2,0,-1,1,3) + a_3.(3,1,0,2,4) + a_4.(2,2,5,8,-1) + a_5.(0,1,0,2,3)$

$$\begin{cases} a_1 + 2a_2 + 3a_3 + 2a_4 = 0 \\ a_1 + a_3 + 2a_4 + a_5 = 0 \\ a_1 - a_2 + 5a_4 = 0 \\ a_1 + a_2 + 2a_3 + 8a_4 + 2a_5 = 0 \\ a_1 + 3a_2 + 4a_3 - a_4 + 3a_5 = 0 \end{cases}$$

$a_1 = -5a_4 + a_2$

$-5a_4 + a_2 + 2a_a + 3a_3 + 2a_4 = 0$
$-3a_4 + 3a_2 + 3a_3 = 0$
$3a_2 = 3a_4 - 3a_3$
$a_2 = \frac{3a_4 - 3a_3}{3}$
$a_2 = a_4 - a_3$

$-5a_4 + a_2 + a_3 + 2a_4 + a_5 = 0$
$-3a_4 + a_2 + a_3 + a_5 = 0$
$-3a_4 + a_4 - a_3 + a_3 + a_5 = 0$
$-2a_4 + a_5 = 0$
$a_5 = 2a_4$

$-5a_4 + a_2 + a_2 + 2a_3 + 8a_4 + 2(2a_4) = 0$
$-5a_4 + a_4 - a_3 + a_4 - a_3 + 2a_3 + 8a_4 + 4a_4 = 0$
$9a_4 = 0$
$a_4 = \frac{0}{9}$
$a_4 = 0$

$a_1 - a2 + 5a_4 = 0$
$a_1 - a2 + 5(0) = 0$
$a_1 = a2$

$a_1 + 2a_2 + 3a_3 + 2a_4 = 0$
$a_1 + 2a_1 + 3a_3 + 2(0) = 0$
$3a_1 + 3a_3 = 0$
$3a_3 = -3a_1$
$a_3 = \frac{-3a_1}{3}$
$a_3 = -a_1$

$a_1 + 3a_2 + 4a_3 - a_4 + 3a_5 = 0$
$a_1 + 3a_1 - 4a_1 + 0 + 3a_5 = 0$
$3a_5 = 0$
$a_5 = \frac{0}{3}$
$a_5 = 0$

$a_1 = a_2, a_3 = -a_1; a_4 = 0; a_5 = 0$

$1.(1,1,1,1,1) + 1.(2,0,1,1,3) - 1.(3,1,0,2,4) + 0.(2,2,5,8,1) + 0.(0,1,0,2,3) = (0,0,0,0,0)$

$(1,1,1,1,1) + (2,0,1,1,3) + (-3,-1,0,-2,-4) + (0,0,0,0,0) + (0,0,0,0,0) = (0,0,0,0,0)$
$(1+2-3, 1+0-1, 1-1-0, 1+1-2, 1+3-4) = (0,0,0,0,0)$
$(0,0,0,0,0) = (0,0,0,0,0)$

Logo, S é ld, pois temos números $\neq 0$ gerando (0, 0, 0, 0, 0)

- S forma uma base do $\mathbb{R}$-espaço vetorial $\mathbb{R}^5$?

  Não, pois para que um conjunto seja base é necessário que ele seja linearmente independente (li).

3. Considere o conjunto $\mathbb{W}$ = {(x,y,z,w,t,u)—x,y,z,w,t,u $\in \mathbb{R} \land$ x + y + w + z + t + u = 0 $\land$ y - w - z = 0 $\land$ w + t - x = 0} $\subseteq$

$x + y + w + z + t + u = 0$
$y - w - z = 0$
$w + t - x = 0$

$y = w + z$

$x = w + t$

$w + t + w + z + +w + z + t + u = 0$
$3w + 2t + 2z + u = 0$
$u = -3w - 2t - 2z$

$\mathbb{W}$ = {(w + t, w + z, w, z, -3w - 2t -2z) w, z, t $\in \mathbb{R}$ }

- 0 $\in \mathbb{W}$

  w = z = t = 0
  = (0,0,0,0,0,0)

- $\mathbf{u} + \mathbf{v} \in \mathbb{W}$

  $\mathbf{u} = (u_1, u_2, u_3, u_4, u_5, u_6)$
  $\mathbf{v} = (v_1, v_2, v_3, v_4, v_5, v_6)$

  u + v = $(u_1 + v_1, u_2 + v_2, u_3 + v_3, u_4 + v_4, u_5 + v_5, u_6 + v_6) \in \mathbb{W}$
  u + v = $(u_1 + v_1)$. (1,0,0,0,0,0) +
  $(u_2 + v_2)$. (0,1,0,0,0,0) +
  $(u_3 + v_3)$. (0,0,1,0,0,0) +
  $(u_4 + v_4)$. (0,0,0,1,0,0) +
  $(u_5 + v_5)$. (0,0,0,0,1,0) +
  $(u_6 + v_6)$. (0,0,0,0,0,1) =
  $\mathbf{u} + \mathbf{v} \in \mathbb{W}$

- a $\in \mathbb{R}$, $\mathbf{u} \in \mathbb{W} \rightarrow$ a . $\mathbf{u} \in \mathbb{W}$

  $\mathbf{u} = (u_1, u_2, u_3, u_4, u_5, u_6)$

$(a.u_1, a.u_2, a.u_3, u_4, a.u_5, a.u_6) \in \mathbb{W}$

$a.u_1.(1,0,0,0,0,0)+$
$a.u_2.(0,1,0,0,0,0)+$
$a.u_3.(0,0,1,0,0,0)+$
$a.u_4.(0,0,0,1,0,0)+$
$a.u_5.(0,0,0,0,1,0)+$
$a.u_6.(0,0,0,0,0,1)=$

$a.\mathbf{u} \in \mathbb{W}$

Logo, $\mathbb{W}$ é um subespaço vetorial de $\mathbb{R}^6$

- O conjunto $\mathbb{W}$ = {(x,y,z) x,y $\in \mathbb{W} \wedge$ x - z = 1 e y + x = 0} é um subespaço vetorial de $\mathbb{R}^3$?. Esboce graficamente $\mathbb{W}$.

x = 1 + z
y = -x
-x + 1 + z = 0

$\mathbb{W}$ = {(1 + z, -x, z)}

- $0 \in \mathbb{W}$

$x = z = 0$
$(1+0,0,0)$
$(1,0,0)$
$(1,0,0) \neq (0,0,0)$

O conjunto $\in \mathbb{W}$ não passa pela origem, logo, não é subespaço vetorial de $\mathbb{R}^3$.

- Gráfico:

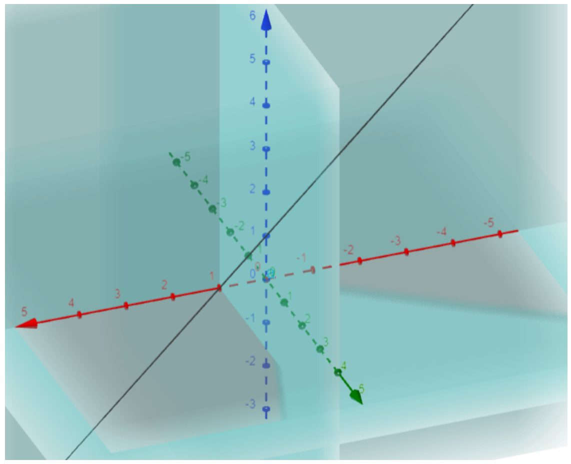

Figure 1:

- Invente seu subespaco vetorial em qualquer R^n com n $\geq$ 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial.

  $\mathbb{W}$ = {(x,y,z) x,y,z $\in \mathbb{R} \wedge$ x + y + z = 0 $\wedge$ y - z = 0} $\subseteq \mathbb{R}^3$

  x + y + z = 0

  y - z = 0
  y = z

  x + y + y = 0
  x + 2y = 0
  x = -2y

  $\mathbb{W}$ = {-2y,y,y}

- 0 $\in \mathbb{W}$

  0 = y
  (-2 . 0,0,0)
  (0,0,0)
  0 $\in \mathbb{W}$

- **u** + **v** $\in \mathbb{W}$

  u = (u_1, u_2, u_3)
  v = (v_1, v_2, v_3)

  u + **v** =$(u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3)$
  u + **v** = $(u_1 + v_1)$. (1,0,0) + $(u_2 + v_2)$. (1,0,0) + $(u_3 + v_3)$. (1,0,0)

$\mathbf{u} + \mathbf{v} \in \mathbb{W}$

- a $\in \mathbb{R}$, $\mathbf{u} \in \mathbb{W} \to$ a . $\mathbf{u} \in \mathbb{W}$

  $\mathbf{u} = (u_1, u_2, u_3)$

  a . $\mathbf{u} = (a.u_1, a.u_2, a.u_3) \in \mathbb{W}$
  $a.u_1.(1, 0, 0, 0, 0, 0) +$
  $a.u_2.(0, 1, 0, 0, 0, 0) +$
  $a.u_3.(0, 0, 1, 0, 0, 0)$

  $a.\mathbf{u} \in \mathbb{W}$

Logo o conjunto $\mathbb{W}$ é um subespaço vetorial de $\mathbb{R}^3$

4. Mostre que o conjunto$\{$(1, 1, 1, 1, 0, 1, 1), (1, 0,1, 1, 1,1, 0), (2, 2, 1, 1,1, 1, 1), (1, 0, 0, 1, 2, 1, 1), (2, 0, 2, 0, 2, 0, 2), (1,1,1,1,1,1, 1), (3, 0, 2, 0, 2,1, 2)$\}$ forma uma base para o $\mathbb{R}$-espaço vetorial $\mathbb{R}^7$

$$\begin{cases} -a_1 + a_2 + 2a_3 + a_4 + 2a_5 + a_6 + 3a_7 = 0 \\ a_1 + 2a_3 - a_6 = 0 \\ a_1 - a_2 + a_3 + 2a_5 - a_6 + 2a7 = 0 \\ a_1 + a_2 + a_3 + a_4 - a_6 = 0 \\ a_2 - a_3 + 2a_4 + 2a_5 - a_6 = 0 \\ a_1 - a_2 + a_3 + a_4 - a_6 - a_7 = 0 \\ a_1 + a_3 + a_4 + 2a_5 + a_6 + 2a7 = 0 \end{cases}$$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

1) $l_1 + l_2 \rightarrow \mathrm{l}_2$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

2) $l_1 + l_3 \rightarrow \mathrm{l}_3$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

3) $l_1 + l_4 \rightarrow \mathrm{l}_4$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

4) $2l_2 - l_4 \rightarrow$ l_4

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 5 & 0 & 2 & 0 & 3 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

5) $5l_3 - 3l_4 \rightarrow$ l_4

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

6) $l_2 - l_5 \rightarrow$ l_5

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 5 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

7) $5l_3 - 3l_5 \rightarrow$ l_5

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 8 & 20 & -3 & 22 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

8) $8l_4 - 5l_5 \rightarrow \mathrm{l}_5$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

9) $l_1 + l_6 \rightarrow \mathrm{l}_6$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

10) $l_3 - l_6 \rightarrow \mathrm{l}_6$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & -1 & 2 & 0 & 3 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

11) $l_4 + 5l_6 \rightarrow \mathrm{l}_6$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 24 & 0 & 31 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

12) $2l_5 - l_6 \rightarrow \mathrm{l}_6$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 \end{bmatrix}$$

13) $l_1 + l_7 \rightarrow \mathrm{l}_7$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 \end{bmatrix}$$

14) $l_2 - l_7 \rightarrow \mathrm{l}_7$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 0 & 1 & -1 & -2 & -2 & -2 \end{bmatrix}$$

15) $l_3 - 3l_7 \rightarrow \mathrm{l}_7$

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 0 & 0 & 4 & 10 & 6 & 11 \end{bmatrix}$$

16) $4l_4 - 5l_7 \rightarrow$ l_7

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 0 & 0 & 0 & 6 & -30 & 19 \end{bmatrix}$$

17) $5l_5 - 2l_7 \rightarrow$ l_7

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 0 & 0 & 0 & 0 & 75 & 0 \end{bmatrix}$$

18) $5l_6 - 2l_7 \rightarrow$ l_7

$$\begin{bmatrix} -1 & 1 & 2 & 1 & 2 & 1 & 3 \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 5 & 14 & 0 & 16 \\ 0 & 0 & 0 & 0 & 12 & 15 & 18 \\ 0 & 0 & 0 & 0 & 0 & 30 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix}$$

Após 18 etapas de escalonamento para isolar o elemento da posição a_{77}, seguiremos pelo método da substituição para verificar de o conjunto é ld ou li:

$25a_7 = 0$

$a_7 = 0$

$30a_6 + 0 = 0$
$30a_6 = 0$
$a_6 = 0$

$12a_5 + 15a_6 + 18a_7 = 0$
$12a_5 + 0 + 0 = 0$
$a_5 = 0$

$5a_4 + 14a_5 + 0 + 16a_7 = 0$
$5a_4 + 0 + 0 + 0 = 0$
$5a_4 = 0$
$a_4 = 0$

$3a_3 + a_4 + 4a_5 + 0 + 5a7 = 0$
$3a_3 + 0 + 0 + 0 + 0 = 0$
$3a_3 = 0$
$a_3 = 0$

$a_2 + 4a_3 + a_4 + 2a_5 + 0 + 3a_7 = 0$
$a_2 + 0 + 0 + 0 + 0 + 0 = 0$
$a_2 = 0$

$-a_1 + a_2 + 2a_3 + a_4 + 2a_5 + a_6 + 3a_7 = 0$
$-a_1 + 0 + 0 + 0 + 0 + 0 + 0 = 0$
$-a_1 = 0$
$a_1 = 0$

$a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0, a_5 = 0, a_6 = 0, a_7 = 0$

Logo, o conjunto proposto é li.

Escreva o vetor (0, 1, 1, 1, 1, 0, 1) nesta base.