# Deliverable 2: X-ray Projection & CT Imaging

BMEN 509/623: Introduction to Biomedical Imaging
Winter 2026

University of Calgary

Due: Sunday, March 2, 2026 at 11:59 PM

## Overview

This deliverable explores the physics and computational methods underlying projection radiography and computed tomography (CT). You will design algorithms for optimizing projection geometry, implement filtered back projection (FBP) reconstruction, and develop artifact detection strategies. These skills are foundational for understanding how clinical X-ray and CT systems produce diagnostic images.

**Estimated Time:** 6 hours

**Learning Objectives:**

- Apply geometric principles to optimize projection radiography parameters
- Implement CT reconstruction using filtered back projection
- Calculate and interpret Hounsfield units from CT data
- Identify and correct common CT artifacts
- Compare projection vs. tomographic imaging for clinical decision-making

## Starter Materials

Download the starter notebook and data files from the course GitHub repository:

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|---|
| **Repository:** github.com/EthanMacDonald/BMEN509-623_Deliverable_2 |
| Click "Code" → "Download ZIP" or clone using Git |

The package includes:

- `README.md` – Assignment overview and quick-start instructions
- `deliverable-02-instructions.pdf` – This document
- `deliverable-02-starter.ipynb` – Jupyter notebook with scaffolding
- `data/` – Data files for CT reconstruction exercises:
- `data/projection_scenarios.csv` – Clinical geometry scenarios

- `data/shepp_logan_phantom.npy` – Standard CT test phantom
- `data/sinogram_180.npy`, `sinogram_90.npy`, `sinogram_45.npy` – Sinograms at different angular sampling
- `data/ct_phantom_clean.npy` – Artifact-free CT phantom image
- `data/ct_phantom_beam_hardening.npy` – CT image with beam hardening
- `data/ct_phantom_ring.npy` – CT image with ring artifact
- `data/hu_calibration.csv` – HU values for calibration materials
- `data/ct_utils.py` – Helper functions for CT operations

# Part 1: Projection Geometry Optimizer (30%)

## Background

In projection radiography, image quality depends critically on geometric factors. The geometric unsharpness ($U_g$) caused by a finite focal spot is:

$$U_g = X_f \cdot \frac{m-1}{m} = X_f \cdot \frac{d_{OID}}{d_{FSD}} \tag{1}$$

where $X_f$ is the focal spot size, $m$ is magnification, $d_{OID}$ is the object-to-image distance, and $d_{FSD}$ is the focus-to-skin distance.

The magnification factor is:

$$m = \frac{d_{FID}}{d_{FID} - d_{OID}} \tag{2}$$

Total unsharpness combines geometric and receptor contributions:

$$U_{total} = \sqrt{\left(\frac{U_r}{m}\right)^2 + U_g^2} \tag{3}$$

The inverse square law governs how mAs must change with distance:

$$mAs_2 = mAs_1 \cdot \left(\frac{d_{FID,2}}{d_{FID,1}}\right)^2 \tag{4}$$

## Your Task

Design an algorithm that optimizes projection geometry for clinical scenarios. Given:

- Focal spot size options (fine: 0.3 mm, broad: 1.0 mm)
- Receptor blur ($U_r$)
- Patient/object thickness and depth within patient
- Maximum tube loading (mAs limit)
- Minimum required spatial resolution (maximum acceptable $U_{total}$)

Your algorithm should recommend:

1. Optimal source-to-image distance (SID/FID)
2. Whether to use fine or broad focal spot
3. Resulting magnification, unsharpness, and required mAs
4. Visualization of the trade-off space

**Deliverables:**

- Working optimization function with clear documentation
- Analysis of at least 3 clinical scenarios from the provided data
- Trade-off visualization (unsharpness vs. mAs vs. SID)
- Discussion of when each focal spot size is preferred

# Part 2: CT Reconstruction Pipeline (30%)

## Background

CT reconstruction recovers a 2D image from its 1D projections (sinogram). The filtered back projection algorithm:

1. Takes the 1D Fourier transform of each projection
2. Applies a ramp filter $|f|$ (or modified versions like Ram-Lak, Shepp-Logan, Hamming)
3. Takes the inverse Fourier transform
4. Back-projects the filtered profiles across the image

The central slice theorem states that the 1D Fourier transform of a projection at angle $\theta$ equals a slice through the 2D Fourier transform of the image at the same angle.

## Your Task

Implement a filtered back projection reconstruction pipeline:

1. **Forward Projection (optional):** Generate a sinogram from the Shepp-Logan phantom
2. **Filtering:** Implement at least two reconstruction filters:
   - Ramp filter: $H(f) = |f|$
   - Hamming-windowed ramp: $H(f) = |f| \cdot (0.54 + 0.46 \cos(\pi f / f_{max}))$
3. **Back Projection:** Implement the back projection operation
4. **Analysis:** Compare reconstruction quality with:
   - Different numbers of projections (180, 90, 45)

- Different filters (sharp vs. smooth)

**Deliverables:**

- Working FBP implementation (you may use FFT libraries, but not pre-built FBP functions)
- Reconstructed images with different parameters
- Quantitative comparison (e.g., RMSE vs. ground truth)
- Analysis of resolution-noise trade-offs with different filters

## Part 3: Hounsfield Unit Analysis & Artifact Investigation (30%)

### Background

Hounsfield units (HU) provide a standardized scale for CT attenuation:

$$HU = 1000 \times \frac{\mu_{tissue} - \mu_{water}}{\mu_{water}} \tag{5}$$

Standard values: air = –1000 HU, water = 0 HU, bone ≈ +1000 HU.
Common CT artifacts include:

- **Beam hardening:** Cupping artifact from polychromatic spectrum
- **Ring artifacts:** Concentric rings from miscalibrated detector elements
- **Partial volume:** Averaging of different tissues within a voxel

### Your Task

1. **HU Calibration:**
   - Measure HU values in the clean phantom for known inserts
   - Calculate linear attenuation coefficients from HU values
   - Assess accuracy against expected values
2. **Artifact Identification:**
   - Design an algorithm to automatically detect beam hardening
   - Design an algorithm to detect ring artifacts
   - Classify artifact severity
3. **Beam Hardening Correction:**
   - Implement a polynomial correction approach
   - Compare corrected vs. uncorrected images
   - Quantify improvement in HU accuracy

**Deliverables:**

- HU measurement and calibration analysis
- Artifact detection algorithms with demonstration
- Beam hardening correction implementation
- Before/after comparison with quantitative metrics

## Part 4: Integration Memo (10%)

Write a technical memo (300–500 words) addressing:

> *"A radiologist asks: For evaluating a patient with suspected lung nodules, when should I order a chest X-ray versus a chest CT? Discuss the trade-offs in terms of projection geometry, dose, contrast resolution, and spatial resolution. Use quantitative reasoning from your analyses in Parts 1–3."*

Your memo should:

- Compare geometric limitations of projection vs. tomographic imaging
- Discuss contrast resolution advantages of CT (low-contrast detectability)
- Address radiation dose considerations
- Provide a reasoned clinical recommendation

## Submission Requirements

Submit via D2L by **Sunday, March 2, 2026 at 11:59 PM**:

1. **Jupyter Notebook** (`.ipynb`): All code, outputs, and analysis
2. **PDF Export** (`.pdf`): Exported version of your notebook

**File naming convention:** `LastName_FirstName_Deliverable2.ipynb` and `.pdf`
**Code Requirements:**

- All code cells must execute without errors
- Include comments explaining your approach
- Use meaningful variable names
- Cite any external resources or code references

## Grading

| Component | Weight | Primary Criteria |
|---|---|---|
| Part 1: Projection Optimizer | 30% | Algorithm design, trade-off analysis |
| Part 2: CT Reconstruction | 30% | FBP implementation, quality comparison |
| Part 3: HU & Artifacts | 30% | Detection algorithms, correction method |
| Part 4: Integration Memo | 10% | Clinical reasoning, synthesis |

Each component is evaluated on a 7-level scale:

| | |
|---|---|
| **Outstanding** | Exceptional work, publication quality |
| **Excellent** | Exceeds expectations, minor improvements possible |
| **Good** | Meets expectations with solid execution |
| **Satisfactory** | Meets basic requirements |
| **Poor** | Below expectations, significant gaps |
| **Very Poor** | Major deficiencies |
| **Incomplete** | Not submitted or fundamentally incomplete |

## Academic Integrity

You may discuss concepts with classmates, but all code and written work must be your own. AI coding assistants may be used for debugging and syntax help, but you must understand and be able to explain all submitted work. Cite any external resources.

## Late Policy

Per the course syllabus, late submissions are penalized as follows:

| Days Late | Penalty |
|---|---|
| 0–24 hours | 25% deduction |
| 24–48 hours | 50% deduction |
| 48–72 hours | 75% deduction |
| More than 72 hours | Not accepted (0%) |

Extensions may be granted for documented extenuating circumstances if requested before the deadline.

## Resources

- IAEA Diagnostic Radiology Physics Handbook, Chapters 5, 6, 7, 11
- Course lecture slides on projection radiography and CT
- NumPy FFT documentation: `https://numpy.org/doc/stable/reference/routines.fft.html`
- SciPy ndimage for image operations: `https://docs.scipy.org/doc/scipy/reference/ndimage.html`

*Questions? Post on the D2L discussion board or attend office hours.*