Anexo da Lista 5 – Análise Fatorial

Exercício 9.26

Example 8.6 (Components from a correlation matrix with a special structure) Geneticists are often concerned with the inheritance of characteristics that can be measured several times during an animal's lifetime. Body weight (in grams) for $n = 150$ female mice were obtained immediately after the birth of their first four litters.[4] The sample mean vector and sample correlation matrix were, respectively,

$$\bar{\mathbf{x}}' = [39.88, 45.08, 48.11, 49.95]$$

and

$$\mathbf{R} = \begin{bmatrix} 1.000 & .7501 & .6329 & .6363 \\ .7501 & 1.000 & .6925 & .7386 \\ .6329 & .6925 & 1.000 & .6625 \\ .6363 & .7386 & .6625 & 1.000 \end{bmatrix}$$

The eigenvalues of this matrix are

$$\hat{\lambda}_1 = 3.085, \quad \hat{\lambda}_2 = .382, \quad \hat{\lambda}_3 = .342, \quad \text{and} \quad \hat{\lambda}_4 = .217$$

We note that the first eigenvalue is nearly equal to $1 + (p - 1)\bar{r} = 1 + (4 - 1)(.6854) = 3.056$, where $\bar{r}$ is the arithmetic average of the off-diagonal elements of $\mathbf{R}$. The remaining eigenvalues are small and about equal, although $\hat{\lambda}_4$ is somewhat smaller than $\hat{\lambda}_2$ and $\hat{\lambda}_3$. Thus, there is some evidence that the corresponding population correlation matrix $\boldsymbol{\rho}$ may be of the "equal-correlation" form of (8-15). This notion is explored further in Example 8.9.

The first principal component

$$\hat{y}_1 = \hat{\mathbf{e}}_1'\mathbf{z} = .49z_1 + .52z_2 + .49z_3 + .50z_4$$

accounts for $100(\hat{\lambda}_1/p)\% = 100(3.058/4)\% = 76\%$ of the total variance. Although the average postbirth weights increase over time, the *variation* in weights is fairly well explained by the first principal component with (nearly) equal coefficients. ■

8.15. The four sample standard deviations for the postbirth weights discussed in Example 8.6 are

$$\sqrt{s_{11}} = 32.9909, \quad \sqrt{s_{22}} = 33.5918, \quad \sqrt{s_{33}} = 36.5534, \quad \text{and} \quad \sqrt{s_{44}} = 37.3517$$

Use these and the correlations given in Example 8.6 to construct the sample covariance matrix $\mathbf{S}$. Perform a principal component analysis using $\mathbf{S}$.

Exercício 9.34

7.26. Measurements of properties of pulp fibers and the paper made from them are contained in Table 7.7 (see also [19] and website: www.prenhall.com/statistics). There are $n = 62$ observations of the pulp fiber characteristics, z_1 = arithmetic fiber length, z_2 = long fiber fraction, z_3 = fine fiber fraction, z_4 = zero span tensile, and the paper properties, y_1 = breaking length, y_2 = elastic modulus, y_3 = stress at failure, y_4 = burst strength.

Table 7.7 Pulp and Paper Properites Data

y_1 BL	y_2 EM	y_3 SF	y_4 BS	z_1 AFL	z_2 LFF	z_3 FFF	z_4 ZST
21.312	7.039	5.326	.932	−.030	35.239	36.991	1.057
21.206	6.979	5.237	.871	.015	35.713	36.851	1.064
20.709	6.779	5.060	.742	.025	39.220	30.586	1.053
19.542	6.601	4.479	.513	.030	39.756	21.072	1.050
20.449	6.795	4.912	.577	−.070	32.991	36.570	1.049
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
16.441	6.315	2.997	−.400	−.605	2.845	84.554	1.008
16.294	6.572	3.017	−.478	−.694	1.515	81.988	.998
20.289	7.719	4.866	.239	−.559	2.054	8.786	1.081
17.163	7.086	3.396	−.236	−.415	3.018	5.855	1.033
20.289	7.437	4.859	.470	−.324	17.639	28.934	1.070

Source: See Lee [19].