Aix*Marseille université
Socialement engagée

NNT/NL : 2020AIXM0001/001ED000

Habilitation à Diriger des Recherches

Présentée à Aix-Marseille Université

le XX janvier 2022 par

Julián Ernesto BAUTISTA

Dark-energy with Spectroscopic Observations of the Universe

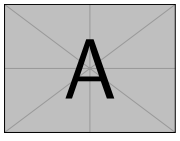
Discipline
Physique et Sciences de la Matière

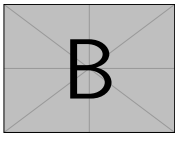
Spécialité
Astrophysique et Cosmologie

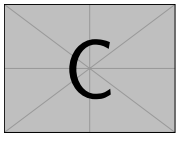
Laboratoire
Centre de Physique des Particules de Marseille

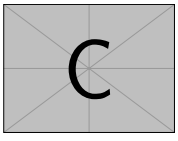
Composition du jury

Prénom NOM Affiliation	Rapporteur·e
Prénom NOM Affiliation	Rapporteur·e
Prénom NOM Affiliation	Examinateur·rice
Prénom NOM Affiliation	Président·e du jury
Prénom NOM Affiliation	Tuteur





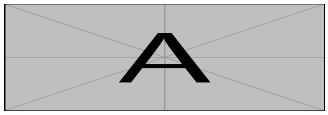




Je soussigné, [Prénom Nom], déclare par la présente que le travail présenté dans ce manuscrit est mon propre travail, réalisé sous la direction scientifique de [Prénom Nom], dans le respect des principes d'honnêteté, d'intégrité et de responsabilité inhérents à la mission de recherche. Les travaux de recherche et la rédaction de ce manuscrit ont été réalisés dans le respect à la fois de la charte nationale de déontologie des métiers de la recherche et de la charte d'Aix-Marseille Université relative à la lutte contre le plagiat.

Ce travail n'a pas été précédemment soumis en France ou à l'étranger dans une version identique ou similaire à un organisme examinateur.

Fait à [ville] le [date]



Résumé

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Mots clés: géométrie algorithmique, complexe planaire et rectangulaire, géodésique, courbure globale non-positive

Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Keywords: computational geometry, planar and rectangular complex, geodesic, global nonpositive curvature

Acknowledgements

Le modèle de thèse AMU n'existerait pas sans la contribution des doctorants. Nous souhaitons remercier tout particulièrement Mickaël Bojados, Flora Cordoleani et Florian Caullery pour leur aide précieuse et la qualité de leurs fichiers sources LaTeX. La mise à jour effectuée en 2018 doit beaucoup à l'excellent travail de Dorian Depriester.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Contents

1 Introduction

Contents

1.1 The history of our Universe and its big open questions

If we gather all knowledge in physics humanity could learn to this day, we can build an interesting story for how our Universe evolved since *very* early times. This great story that can explain most of what we observe in the sky, while being consistent with experimental results here on Earth. However, this story has several weak spots. Either because where we do not have a satisfactory physical explanation for what we observe, or because we simply cannot observe what happens by then (and potentially we never will).

The story in a nutshell goes as follows: the "beginning" of the Universe[1], also known as the *Big Bang* or *inflation*, was an epoch of exponential expansion of space that happened around 13.8 billion years ago. Inflation transformed quantum, microscopic fluctuations of space into macroscopic density fluctuations of the Universe's constituents: quarks, photons,

[1] The actual beginning of the Universe can be seen as a complex phylosophical question or simply as an ill defined physical concept. This discussion is clearly beyond the scope of this work and/or my capabilities. Here, I decide to name as beginning the first epoch of the Universe for which we have a widely known physical theory to describe it.

neutrinos and dark-matter. After inflation, the Universe continues to expand but much slowly. The average temperature of the Universe decreases adiabatiacally. Quarks start to gather to form protons, neutrons and relics of heavier atoms, such as deuterium, tritium, helium, litium and so on. All this during the first few minutes of the Universe's history, in the so called *Big Bang nucleosynthesis*. After 380 thousand years, electrons rebind with protons to form neutral hydrogen, and photons scatter for their last time with them as the Universe becomes more difuse. The photons from this epoch are observable today and are known as the *cosmic microwave background*. The Universe goes through its *dark ages*, where hydrogen is mostly neutral and stars have not yet formed. After several million years, gravity clumps hydrogen into dense clouds that reach high enough temperatures to start thermonuclear reactions in their core. The first stars are formed. Then the first galaxies. These galaxies merge into larger galaxies, galaxies form small virialized groups, large groups, clusters, super clusters. Clusters, filamentary structures and voids compose the so called cosmic web. At around half of the Universe's age, the expansion starts to speed up again, accelerating, as if gravity became repulsive on very large scales. Then, on one of the many billions of galaxies, the *Milky Way*, planet Earth formed around a somewhat isolated star, the Sun, and here we are today, observing the sky, trying to explain the whole Universe.

This story is based on the assumption that general relativity (GR) is the theory of gravity, that the Universe can be modelled as statistically homogeneous and isotropic on large enough scales, and that the Universe is composed of the standard model particles and interactions, with the addition of two exotic components: dark-energy and dark-matter.

The physical origin of dark-energy and dark-matter is a mystery and one of the weak spots of the Universe's story. Another weak spot is the physics of inflation, but observables related to that time are limited. [Any others ?] Solutions for these problems would represent major breakthroughts in physics and large teams of scientists are dedicated to them, either theoretically or experimentally.

My past work has been focused on the observations related to dark-energy. The rest of this chapter will be dedicated to explaining the problem of accelerated expansion, the dark-energy model and other possible solutions, and how do we observe the expansion.

1.2 Evidences for the accelerated expansion of the Universe

Not long after developing his theory of gravity, Einstein wanted to write a model for a static Universe filled with matter. Since gravity is attractive, his solution was to include a new constant term, Λ, in the equations. This will be later known as the *cosmological constant*, one of the simplest models of dark-energy in agreement with a wide variety of cosmological observations.

The first indirect evidences for dark-energy come from measurements of galaxy clustering in the 80's (Maddox et al. 1990; Efstathiou et al. 1990). At the end of the 90's, two independent teams measured the expansion of the Universe using type-Ia supernovae (Riess, Filippenko, et al. 1998; Perlmutter et al. 1999). Their observations could only be explained by an Universe containing around 30 per cent of matter and 70 per cent of dark energy, in the form of a cosmological constant. Few years later, first measurements of the temperature fluctuations in the cosmic microwave background were consistent with an Universe with a flat space (Balbi et al. 2000; de Bernardis et al. 2000), which in combination with measurements of clustering, supernovae and the local expansion rate (Mould et al. 2000), definitely confirmed that the Universe's expansion is accelerating. Age estimates of globular stellar clusters, which

are supposed to be among the oldest astrophysical objects, were also indicating that the Universe had to be older than the age predicted by models of an Universe only filled with matter (Chaboyer 1998). The following decade was enriched by the first measurements of the baryon acoustic oscillations in the distribution of galaxies (Eisenstein, Zehavi, et al. 2005; Cole et al. 2005), re-confirming the accelerated nature of the expansion.

Since the 2010's, we live in the so-called *precision cosmology* era, where several observatories were constructed with the main goal of improving the precision of all these measurements just discussed. Even with the most up-to-date results, with errors of the order of a few percent, there is yet no evidence for a departure from the model of Universe governed by GR, composed by 70 per cent of this mysterious cosmological constant and by 25 per cent by this mysterious dark-matter.

1.3 Model of an expanding Universe

1.3.1 Assumptions and ingredients

In order to define some important cosmological parameters, I will quickly review the basics of the current most accepted model for our Universe in the limit of very large scales, where it can be considered homogenous and isotropic. This is also knwon as the *background* model, since the formation of structures (see next section) can be modeled as small perturbations around it. It is also assumed that gravity is described by general relativity, which interlinks the fabric of space-time and energy densities of the Universe constituents. For these constituents, we assume the presence of *baryons* (protons, electrons, atoms), photons, neutrinos, cold dark-matter and dark-energy.

The expansion of space is usually parametrised by the scale factor $a(t)$, a function of time t that dictates how physical and comoving distances relate to each other. We assume that $a(t_0) = 1$ today ($t_0 = 13.8$ Gyr) and that $a \to 0$ as $t \to 0$. The speed of the expansion and its acceleration are simply derivatives with respect to time, $\dot{a}(t)$ and $\ddot{a}(t)$, respectively. The expansion rate of the Universe is defined as $H(t) = \dot{a}(t)/a(t)$ and takes the Hubble constant value today, $H(t_0) = H_0 = 100h$ km/s/Mpc ~ 70 km/s/Mpc. Since the scale factor is a monotonically increasing function of time (in our simple model), a is commonly used to describe a given cosmic epoch.

One of the consequences of the expansion is that relativistic species, such as photons and massless neutrinos, loose energy or are *redshifted* as they propagate. In astronomy, the redshift z is defined as the relative difference in wavelength between the one emmited by source and the one observed. Thus, in an expanding Universe the scale factor is related to the redshift as $a = 1/(1+z)$. Redshift is also commonly used to describe cosmic epoch or distances. Today corresponds to $z = 0$ while $z \to \infty$ as $t \to 0$.

1.3.2 The expansion rate

The expansion rate can be derived from GR equations and be written as a function of redshift and the densities of each consituent

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_{de}(z) + \Omega_k (1+z)^2\right] \tag{1.1}$$

where $\Omega_x = \rho_x/\rho_{\rm crit}$ is the ratio between the density constituent x today and the critical energy density $\rho_{\rm crit} = 3H_0^2/8\pi G$, which is the density required for a flat space geometry. The

subscripts m, r, de and k stand for non-relativistic species (baryons, cold dark-matter and non-relativistic neutrinos), relativistic species (photons and relativistic neutrinos), dark-energy and curvature, respectively. The curvature "density" parameter is defined as a function of the others, $\Omega_k = (1 - \sum_x \Omega_x)$, and it is zero in a flat space geometry. Non-relativistic species dilute in an expanding universe so their energy density is proportional to $a^{-3} = (1+z)^3$. Relativistic species dilute as well but have an extra factor of a^{-1} due to redshifting. Dark-energy is written as a general function of redshift. In the simplest case of a cosmological constant, $\Omega_{de}(z) = \Omega_\Lambda$, i.e. a constant. The dependency of curvature with a^{-2} simply comes from the field equations.

1.3.3 Distances in cosmology

Distances are non-trivially defined in an expanding, potentially non-flat, Universe. Therefore, different types of distances are more appropriate for certain observables. The *Hubble distance* is related to the expansion rate as

$$D_H(z) = \frac{c}{H(z)}, \tag{1.2}$$

and it is commonly used in measurements of baryon acoustic oscillations in the line-of-sight direction (see Chapters 3 and 4) or to represent the size of a causally connected region of the Universe.

The *comoving distance* to an object at redshift z is written as

$$D_C(z) = \int_0^z \mathrm{d}z'\, D_H(z'). \tag{1.3}$$

This expression yields the distance travelled by a photon from a source towards us but factoring out the scale factor, effectively "removing" the effect of expansion.

The *comoving angular diameter distance* $D_M(z)$ is useful when distances are inferred from angles, which are affected by the curvature of space. An object with comoving size l at a redshift z would be seen with an angular size θ. This allows us to define $D_M(z) = l/\theta(z)$ as

$$D_M(z) = D_C(z)\mathrm{sinc}\left(\sqrt{-\Omega_k}\frac{D_C(z)}{D_H(z=0)}\right), \tag{1.4}$$

where $\mathrm{sinc}(x) = \sin(x)/x$. This distance is used in observations of graviational lensing and in measurements of the baryon acoustic oscillations in the transverse direction to the line-of-sight.

Analogouly, the *luminosity distance* $D_L(z)$ relates the flux f received by an object at redshift z with intrinsic luminosity L. We define then $D_L(z) = \sqrt{L/4\pi f(z)}$ as

$$D_L(z) = D_M(z)(1+z). \tag{1.5}$$

This distance is used in supernova cosmology, where fluxes are used to infer dark-energy properties. Since supernovae fluxes vary by orders of magnitude, it is handy to define the *distance modulus* as the logarithm of the luminosity distance:

$$\mu(z) = 5\log_{10}\frac{D_L(z)}{10\ \mathrm{pc}}. \tag{1.6}$$

This definition is such that it agrees with the definition of magnitude of an object.

In all the distances defined above, the dependency to the cosmological energy densities Ω_x

happens only through $H(z)$ (Eq. 1.1). This is how we can constrain these parameters through distance measurements. However, all cosmological distance measurements are *relative*, which means they are defined to an arbitrary normalisation[2]. It is only by observing the *evolution* of these relative distance measurements as a function of redshift that we can constrain cosmological parameters. Note that in all the distances defined above, the dependency with H_0 only impacts this arbitrary normalisation, so it cannot be measured directly. It is useful to factor out this dependency with H_0, and write distances in units of h^{-1}Mpc (numerically equivalent to set $H_0 = 100$ km/s/Mpc). These are the units used in all works studying the large-scale structures and it is also used thoughout this manuscript.

1.3.4 Dark-energy models

In the Eq. 1.1 for the expansion rate of the Universe, the dark-energy density term is written as a general function of redshift. As mentioned before, the simplest model is to consider a cosmological constant $\Omega_{de}(z) = \Omega_\Lambda$. The cosmological constant can be thought as equivalent to a fluid with relativistic pressure $p_{de} = -\rho_{de}$. This model can be extended by considering a different equation of state $p_{de} = p_{de}(\rho_{de}) = w(z)\rho_{de}$, where $w(z)$ can be a constant or a more general function of redshift, under the condition that $w(z) < -1/3$, needed to obtain an accelerated expansion in a dark-energy dominated Universe. One widely known parametrisation is given by $w(a) = w_0 + w_a(1-a)$ (Chevallier et al. 2001; Linder 2003).

The litterature contains a huge variety of models that attempt to be more physically motivated that those just presented (see Weinberg et al. 2013, section 2.2 for a review). Some of them suggest that maybe general relativity breaks down on large enough scales, causing the expansion to accelerate. They suggest extensions or modifications to Einstein's theory of general relativity, and are commonly known as *modified gravity* models.

1.4 Model of the large-scale structures

The Universe is clearly not homogenous and isotropic. Matter clustered under the influence of gravity, creating the cosmic web, composed of clusters, filaments, empty regions. The model described in the previous section only describes the Universe as a whole, without any inhomogeneities. If we want to describe the evolution of the structures we need to consider an extension to the background model.

Observations of the large-scale structures and the cosmic microwave background (CMB) indicate that the matter density field today is the evolution of tiny density perturbations from the early Universe. Temperature fluctuations in the CMB are of the order of 10^{-5}, while densities today can reach values several orders of magnitude larger than the average density. The most widely accepted theory is that the CMB fluctuations, and today's structures, are originated from the inflation-grown quantum fluctuations, that evolved under the influence of gravity and pressure in the hot dense plasma-like epoch of our Universe.

It is been possible to model the physics of the evolution of these tiny fluctuations using *linear perturbation theory*. The predictions of this theory are an excellent match to observations of the CMB and to the late-time distribution of matter on large scales, i.e., larger than few tens of Mpc. On smaller scales however, gravity becomes highly non-linear and more advanced calculations

[2] Two exceptions are parallax distances which are an absolute measurements based on the known distance between the Earth and the Sun and gravitational wave distances, which amplitudes depend on the distance and on the masses of the progenitors, though the masses affect the wave form as well, breaking the degeneracy.

are required. There are two main approaches to model non-linearities: theoretical calculations going beyond linear terms in perturbation theory or numerical n-body simulations. [Develop?]

The main idea of perturbation theory is to consider that the density of a given species x is given by:

$$\rho_x(\vec{x}, t) = \bar{\rho}_x(t)\,[1 + \delta_x(\vec{x}, t)]\,, \tag{1.7}$$

where the bar indicates average over the whole Universe and $\delta_x \ll 1$ is a small perturbation. The average density $\bar{\rho}_x$ follows the background evolution described in section 1.3. A new set of equations can be derived for the evolution of δ_x for each species depending if x denotes relativitic species (photons, hot baryons, hot neutrinos) or non-relativistic species (cold baryons, cold neutrinos or dark-matter). Only some families of models also consider fluctuations in the dark-energy density.

The evolution of δ_x for a given species is dictated by both Boltzmann and Einstein's GR equations. Boltzmann equations describe the evolution of phase-space distributions of each constituent, considering collisions (pressure) and particle creation and anihilation. GR equations describe how each species influence graviational potentials which in turn make perturbations grow.

Another important ingredient in these equations is the *peculiar velocities* $\vec{v}_x(\vec{x}, t)$ of each species, particularly the non-relativistic ones since relativistic species are assumed to have $v \sim c$. They influence the evolution of perturbations through Boltzmann, Euler and continuity equations. These velocities are observable and are an important probe of the cosmological model and on the strength of gravity itself. They will be a key actor in Chapters 4 and 5.

1.4.1 Statistical description of perturbations

The value of the matter density field at a given position $\vec{x}$ is hardly observable and therefore does not contain much cosmological information, since it is just the evolution of a single random realisation of the initial conditions. In cosmology we are mostly interested by the statistical properties of the density field, or of its perturbations. Particularly, most cosmological information is contained in the first moments of the density field, starting from the two-point function (the one-point function is zero by definition), which is defined as

$$\xi(\vec{x}, \vec{y}) = \langle \delta(\vec{x})\delta(\vec{y}) \rangle \tag{1.8}$$

where $\langle \cdot \rangle$ denotes an ensemble average. Given that we only have one single realisation of the Universe, we assume that averages over space (in an infinite space) are equivalent to ensemble averages.

The assumptions of homogeneity and isotropy of the background can be applied in an statistical sense to the perturbations. Statistical homogeneity makes ξ not to depend on the specific locations $\vec{x}$ and $\vec{y}$ but only on their separation $\vec{r} = \vec{y} - \vec{x}$. Statistical isotropic makes ξ no longer depend on the orientation of $\vec{r}$, only on its absolute value $r = |\vec{r}|$. Therefore, the two-point correlation function simplifies to a function of r:

$$\xi(r) = \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle. \tag{1.9}$$

The correlation function $\xi(r)$ is a powerful observable which can be compared to predictions from a given cosmological model.

1.4.2 Configuration and Fourier space

The time evolution of linear density perturbations $\delta(\vec{x})$ is dictated by a set of second-order differential equations, containing derivatives with respect to time and space. It is thus convient to apply Fourier transforms to these equations, where derivatives with respect to space become simple products. Separations $\vec{r}$ in the so-called *Configuration space* become wavevectors $\vec{k}$ in *Fourier space*. The density perturbations in Fourier space are defined by

$$\tilde{\delta}(\vec{k}) = \int \mathrm{d}^3 x \, \mathrm{e}^{-i\vec{k}\cdot\vec{x}} \delta(\vec{x}), \tag{1.10}$$

and its inverse is

$$\delta(\vec{x}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 k \, \mathrm{e}^{+i\vec{k}\cdot\vec{x}} \tilde{\delta}(\vec{k}). \tag{1.11}$$

As previously, we are interested in the two (or more) point functions of the field. In Fourier space, the two-point correlation function is known as the *power spectrum* and is defined as

$$\left\langle \tilde{\delta}(\vec{k}) \tilde{\delta}(\vec{k})' \right\rangle = (2\pi)^3 P(k) \delta^3_{\mathrm{Dirac}}(\vec{k} - \vec{k}'), \tag{1.12}$$

where $\delta^3_{\mathrm{Dirac}}(\vec{x})$ is a three-dimensional Dirac distribution. The assumption of spatial homogeneity is guaranteed by the properties of Fourier space, while the assumption of isotropy makes the power spectrum to be only a function of the absolute value of the wavevector $k = |\vec{k}|$.

When considering the evolution of perturbations in Fourier space to the linear level, each mode $\tilde{\delta}(\vec{k})$ evolves independently of the others. We often say that there is no *mode coupling* in linear theory.

The power spectrum $P(k)$ is an observable, as powerful as the correlation function, to constrain cosmological models. While in theory these two contain exactly the same compressed statistical information, in pratice the analyses of real data use different finite ranges of scales which are affected differently by systematic effects. Therefore, analyses of real data in configuration and Fourier space complement each other. Such analyses are presented in Chapter 4.

1.4.3 Cosmological dependency of the power spectrum

The power spectrum $P(k)$, or the correlation function $\xi(r)$, of density perturbations are an excellent probe of the cosmological model. It encodes information accumulated of the whole Universe's history, since the Big Bang until today.

At the end of inflation, we think that perturbations were roughly Gaussian (as are quantum fluctuations of the vacuum), having a nearly scale independent power spectrum defined by

$$P_0(k) = A_s k^{n_s - 1}, \tag{1.13}$$

where $A_s \sim 2 \times 10^{-9}$ is the amplitude of scalar perturbations and $n_s \sim 0.96$ is the scalar spectral index. The term scalar refers to standard density (or gravitational potential) perturbations, while tensorial perturbations (of the space-time metric) refer to primordial gravitational waves. Vectorial perturbations decay and rapidly become negligible in most common cosmological models.

After inflation, the majority of the energetic budget of Universe was held by photons and neutrinos. Since their energy density decays as a^{-4}, their contribution quickly drops. This radiation-dominated era lasted until $z \sim 5000$, or $t \sim 20$ kyr, when the energy density of matter

became the dominant source. Matter density decays at a slower rate, proportional to a^{-3}. Dark-energy became dominant at $z \sim 0.5$, or $t \sim 9$ Gyr, since its energy density is constant (or close to constant). The actual duration of each era depends on relative values of the energy densities, parametrised by Ω_x (see 1.3).

Add figure Matter perturbations grew at different rates during these different eras. In radiation-dominated era, large-scale modes (large values of r or small values of k) grew while small-scale ones oscillated, due to battle between gravity and radiative pressure. These are known as *baryon acoustic oscillations*. These oscillations are imprinted in the power spectrum of the photon-baryon fluid, at the time of the CMB, and in the matter power spectrum at later times. The radiative pressure prohibited the growth of small-scale perturbations, so the matter power spectrum is damped, which results in this hill-like shape. In the matter-dominated era, the radiative pressure becomes negligible and all modes grow equally (neglecting non-linear growth). When dark-energy is dominant, the expansion accelerates and growth of structures slows down slightly.

Well after recombination, matter (baryons + dark) density perturbations obey the following differential equation, if we assume GR and linear theory:

$$\ddot{\delta}(\vec{x},t) + 2H(z)\dot{\delta}(\vec{x},t) - \frac{3}{2}\Omega_m H_0^2 (1+z)^3 \delta(\vec{x},t) = 0. \tag{1.14}$$

If we assume that $\delta(\vec{x},t) = \delta(\vec{x},t_0)G(t)/G(t_0)$, we can factor out the spatial dependency and only solve for the time-dependent term $G(t)$, known as the *growth factor*. Its logarithmic derivative with respect to the scale factor $a(t)$ is called *growth rate* of structures:

$$f(a) \equiv \frac{\mathrm{d}\,\ln G(a)}{\mathrm{d}\,\ln a}, \tag{1.15}$$

which is a key observable for which the predicted value depends on the cosmological parameters and on the assumed theory of gravity (e.g., GR in Eq. 1.14).

In linear theory, peculiar velocities are simply related to density perturbations via the continuity equation

$$\nabla \cdot \vec{v}(\vec{x},a) \equiv \theta(\vec{x},a) = -a^2 H(a) \frac{\mathrm{d}\delta(\vec{x},a)}{\mathrm{d}a} = -aH(a)f(a)\delta(\vec{x},a) \tag{1.16}$$

where θ is the velocity divergence, a convenient scalar field describing velocities. In Fourier space, the velocity field is simply proportional to the density. The velocity divergence power spectrum $P_{\theta\theta}$ is an important ingredient for models of redshift-space distortions, as we discuss in the following section.

1.4.4 The amplitude of the power spectrum

The matter power spectrum $P(k)$ represents the variance of density perturbations at a given wavenumber k. The total variance of the density field is therefore the sum of all contributions over all available three dimensional modes. This is equivalent to the correlation function at zero separation:

$$\sigma^2 = \xi(r=0) = \frac{1}{(2\pi)^3}\int \mathrm{d}^3k\, P(k) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^2 P(k), \tag{1.17}$$

which diverges if we use the linear prediction for the matter power spectrum. This is because the linear power spectrum is the evolution of a nearly scale invariant power spectrum, equivalent to white noise, so the variance simply keeps increasing as we consider larger ranges of scales. Therefore, it is convenient to smooth the linear density field using a three-dimensional top-hat filter with radius R. The variance of the smoothed field is then

$$\sigma_R^2 = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}k\, k^2 P(k) W_R^2(k), \tag{1.18}$$

where $W_R(k) = 3[\sin(kR) - kR\cos(kR)]/(kR)^3$ is the Fourier transform of the top-hat filter.

In studies of large-scales structures, it is common to use σ_R as a parameter defining the amplitude of the linear power spectrum. As discussed before, this amplitude depends on the values of A_s (Eq. 1.13) and h (which determines the age of the Universe and how long structures could have grown). Therefore σ_R is degenerate with A_s and h though is more representative of the amplitude of $P(k)$ at later times. Other energy density parameters Ω_x also affect this amplitude, but these also change the overall shape and are not degenerate with σ_R.

The variance σ_R is a decreasing function of R. Historically, the chosen values for R give variances near unity, which correspond to the regime where linear theory should break. Recently, the value of $R = 8h^{-1}$Mpc has been used in several analysis, though it has been argued that chosing $R = 12$ Mpc (without the h dependency) is a better choice to break degeneracies (Sánchez 2020).

In measurements of redshift-space distortions and peculiar velocities, there is a degeneracy between the value of f and the amplitude of the power spectrum, σ_8. Therefore, measurements can only constrain the combination $f\sigma_8$.

1.5 Cosmological probes of expansion

In this section, I present the basics of the cosmological observables that allow us to learn about dark-energy. Some of these are mostly sensitive to the background evolution presented in section 1.3, while others depend on the matter perturbations and their statistical properties as discussed in section 1.4.

1.5.1 Direct measurements of H_0

The Hubble constant H_0 is probably one of the first cosmological parameters to be estimated by the one giving its name (Hubble 1929). Today, there are a few techniques to estimate H_0 which yield roughly independent results. See Riess 2020 for a quick review of the latest results.

The most traditional method is known as the *distance ladder*. The idea is to measure the distance-redshift relationship of objects well in the Hubble flow. The distance to the closest objects in the Solar neighbourhood can be estimated with the parallax method. The Gaia satellite has the current largest catalogue of parallax measurements, of more than a billion stars. Parallax is one of the few direct distance estimating methods. They can be used to measure the intrinsic luminosity of some objects thought to be standard candles, such as Cepheid stars, RR Lyrae, and the largest red giant stars. Distances to farther objects can be estimated using these candles. They can also be used to calibrate other brighter standard candles, such as type-Ia supernovae, which can be observed at the largest distances. Riess, Casertano, et al. 2021 contains the latest measurements using Gaia parallaxes, Cepheids and

type-Ia supernovae to determine $H_0 = 73.0 \pm 1.4$ km/s/Mpc. Freedman et al. 2019 is latest measurement of distances using the tip of the red giant branch.

The distance to the galaxy NGC 4258 could be determined thanks to the presence of a water maser orbiting the center of this galaxy. The proper motions of several clouds orbiting close to the central massive black hole could be measured both in radial and angular directions, strongly constraining the dynamics of the system (Herrnstein et al. 1999). The most recent measurement yields $D = 7.576 \pm 0.082$ (stat.) ± 0.076 (sys.) Mpc (Reid et al. 2019), which can be used as an alternative to Cepheids to anchor the distance ladder and provide an estimate to H_0.

Another alternative method is based on the time-delay of the signals emitted from a quasar behind a strong gravitational lens. The lens creates multiple images of the same background quasar, but the light paths have slightly different lengths. Since quasars are variable objects, the same variability is observed with a delay of several days between the different images. By modelling the distribution of matter between the quasar and us, it is possible to convert these time delays into an estimate of H_0. The "H_0 Lenses in COSMOGRAIL's Wellspring", or H0LiCOW, collaboration produced the latest comprehensive measurement of H_0 using strongly-lensed quasars (Wong et al. 2019). An alternative measurement has been performed using strong lenses from the Sloan (SLACS). They are compared to H0LiCOW in Birrer et al. 2020.

More recently, gravitational waves from a merger of two black holes were observed for the first time by the LIGO collaboration (T. L. S. Collaboration and the Virgo Collaboration 2016). Not long after, a merger of two neutron stars was observed by both LIGO and Virgo, but this time an electromagnetic counterpart was also detected (T. L. S. Collaboration and T. V. Collaboration 2017), pointing to the galaxy where the event occurred. Given the well predicted shape of the wave form and its dependency on the masses of the neutron stars, it was possible to estimate the distance to the host galaxy. Combining with a measurement of its redshift, a single event could provide an estimate of H_0 (Abbott et al. 2017). This opened the field of cosmology using standard sirens, i.e., gravitational wave sources.

The cosmic microwave background can yield an estimate of H_0 but it is strongly degenerate with other unknown parameters, such as curvature or dark-energy densities. Some of these degeneracies are reduced when considering the effect of gravitational lensing of the CMB or when combining with other probes of late times. Therefore, precise measurements of H_0 from the CMB alone are only possible when considering more restrictive models, such as a flat space with a cosmological constant.

1.5.2 Type-Ia supernovae

As mentioned earlier, type-Ia supernovae (SNIa) can be used as standard candles for absolute distance measurements, if they are anchored by another method. The SNIa can also be used alone to produce relative distance measurements and constrain dark-energy. In this case, no anchor is required and the only assumption is that the SNIa intrinsic brightness does not evolve with redshift.

The main observable are the fluxes of SNIa as a function of time in different photometric bands, known as light-curves. Spectroscopic follow up observations of the explosion can confirm the type of the supernova based on the features present in their spectra. Another key ingredient is the redshift of the host galaxy of the SNIa, which is commonly measured with spectroscopy as well.

Spectro-photometric models of SNIa are used to fit the observed light-curves, yielding their apparent magnitudes at peak luminosity in a given photometric band. These models account for correlations between color and duration of light curves and the peak magnitude, reducing the intrinsic scatter in these magnitudes from 40% to roughly 15%. It is essential to obtain accurate and precise measurements of SNIa fluxes in order to obtain the best cosmological constraints. Large sets of realistic simulations of the data are required in order to correct for selection effects. The final product of the analysis is a set of distance moduli (Eq. 1.6) and their host-galaxy redshift, which can then be compared to models of expansion of the Universe.

Distance moduli depend on dark-energy through the integral of $H^{-1}(z)$. In order to obtain good constraints on dark-energy properties, it is important to have a large redshift coverage, at least covering the transition between matter-dominated to dark-energy-dominated eras, so $0 < z < 1$. The advantage of SNIa for dark-energy studies is that they can span these redshifts with a high sampling rate, which helps in the study of the expansion-rate. One of the inconveniences is that SNIa are complex and poorly understood astrophysical events, the intrinsic scatter in luminosity cannot be reduced to better than 12%, potentially limiting the gains constraining power from future experiments.

The latest comprehensive study of SNIa combines data from more than a dozen projects into a single sample, the Pantheon sample, from which dark-energy constraints were derived (D. M. Scolnic et al. 2018). The Dark Energy Survey also measured a more recent sample of few hundreds of SNe (Brout, Sako, et al. 2019; Brout, D. Scolnic, et al. 2019; Kessler et al. 2019), but not all of them have spectroscopic confirmation that they are of type Ia. Their constraints on dark-energy are not yet competitive compared to the Panthon sample. The Zwicky Transient Facility (ZTF, Graham et al. 2019) is currently observing the Northern sky on a search for transient events and is expected to discover around 5000 spectroscopically confirmed SNIa at low redshifts ($0 < z < 0.12$) until 2023. After 2023, the Rubin Observatory Legacy Survey of Space and Time (LSST) will take over in the Southern hemisphere and will discover more than 300 thousand SNIa up to $z < 0.5$.

While ZTF and Rubin's samples of SNIa will not decrease significantly the errors on dark-energy parameters, they cover a redshift range where other powerful probes, such as BAO or weak-lensing, lack of statistical power. Furthermore, at lower redshifts when dark-energy is dominant, SNIa are complementary to redshift-space distortions (RSD) when it comes to testing the validity of General Relativity or constraining altenate models of gravity, as solutions for dark-energy.

Not only SNIa are a great probe of the expansion history, they can also provide peculiar velocities of their host galaxies via their inferred distances. These peculiar velocities and their statistical properties can complement RSD analysis when measuring the growth-rate of structures (Eq. 1.15). Estimates of $H(z)$ and $f(z)$ can help break degeneracies between simple dark-energy models and more involved models of gravity (Kim et al. 2020; Graziani et al. 2020). Chapter 5 is dedicated to this topic, to which I plan to dedicate the future years of my research.

1.5.3 Big Bang nucleosynthesis

In the post-inflation Universe, when temperatures are below the equivalent of 100 MeV, perturbations in the matter and radiation fields are quite small and most of the physics is dictated by the interactions between protons, electrons, neutrons, and photons, as described by the Boltzmann equations. As the Universe cools down and rarefies, protons and neutrons start forming atoms of deuterium, tritium, helium and heavier elements. This process is known as the Big Bang nucleosynthesis (BBN).

The relative amount of each of the formed elements depends on the expansion rate of the Universe at that time as well as the physical density of baryons $\omega_b = \Omega_b h^2$ and radiation $\omega_r = \Omega_r h^2$. Given that the energies involved are within reach of particle accelerators, these reactions can be studied with great detail on Earth, allowing us to build accurate models of the BBN (see Pitrou et al. 2021 for the latest calculations and references therein).

Observations of the primordial abundances can be compared to the predictions by BBN models. Abundances of deuterium, helium, and others can be estimated from spectroscopic observations of HII regions in metal-poor galaxies or from absorption lines of the intergalactic medium in quasar spectra. The most up-to-date measurements of the primordial helium-4 abundance yields $Y_p = 0.2453 \pm 0.0034$ (Aver et al. 2021), while the deuterium one is D/H = $(2.527 \pm 0.030) \times 10^{-5}$ (Pitrou et al. 2021). While observations are consistent with BBN models for deuterium and helium, the abundance of lithum-7 exhibits a factor 3 discrepancy, which is a huge problem in BBN but quite often neglected.

The temperature fluctuations in the cosmic microwave background (CMB) are also very sensitive to the physical baryon density ω_b. Historically, the values obtained from CMB have been in good agreement with BBN measurements, showing that baryons make up to around 16% of the total matter content of the Universe, or $\omega_b = (2.195 \pm 0.022) \times 10^{-2}$. The agreement between two quite independent probes is one of the great successes of the current cosmological model, thought they also enforce the need for a dark-matter component.

1.5.4 Baryon acoustic oscillations

Baryon acoustic oscillations (BAO) is the name given to the propagation of sound waves in the primordial plasma (baryons and photons), prior to recombination. Because of the high pressure on small scales at those times, each initial density perturbation had a spherical density wave around them propagating outwards at the speed of sound in that medium. The speed of sound in the plasma is given by

$$c_s(z) \equiv \sqrt{\frac{1}{3[1+R(z)]}} \tag{1.19}$$

where $R(z) = 3\rho_b/4\rho_\gamma$ is the baryon-to-photon ratio. The propagation of sound waves occurred until the temperatures and densities dropped to values such that baryons no longer felt the pressure from photons, known as the *drag epoch*, which is close in time to the recombination (but technically not the same eopch). This process left a slight overdense shell around each initial perturbation with a radius given by

$$r_{\rm drag} \equiv r_s(z_{\rm drag}) = \int_\infty^{z_{\rm drag}} {\rm d}z' \frac{c_s(z')}{H(z')} \tag{1.20}$$

which is known as the *sound horizon at drag epoch*, or the BAO scale. Today, r_d has a physical size of about 147 Mpc, much larger than any collapsed structure in the Universe. As one can see from Eqs. 1.19 and 1.20, $r_{\rm drag}$ mainly depends on ω_b and ω_c assuming the CMB gives a precise estimate of ω_γ. The dependency of $r_{\rm drag}$ is mainly through ω_b, which is useful when constraining dark energy models for instance.

After recombination, the sound horizon scale only increases in size due to the expansion, or equivalently, its comoving size remains unchanged. Therefore, the BAO scale is a great *standard ruler* to study the expansion rate of the Universe. In practice, the BAO scale is observed statistically in the two-point function of the matter density field, so it is often classified as

an statistical standard ruler. In configuration space, the correlation function $\xi(r)$ presents a small peak at separations corresponding to the BAO scale r_d, while in Fourier space the power spectrum $P(k)$ contains an oscillatory pattern as a function of scale with a period proportional to the BAO scale. In Chapters 3 and 4, I present my past work in the measurement of the BAO scale using Lyα forests and galaxies, respectively, as tracers of the matter density field.

Given that a galaxy survey is made of angular positions and redshifts, which are true observables, the BAO peak is effectively measured as an angle $\Delta\theta_{\rm BAO}$ or as a difference in redshift $\Delta z_{\rm BAO}$. These can be modelled as ratios of distances to the BAO scale r_d as

$$\Delta\theta_{\rm BAO}(z_{\rm eff}) = \frac{D_M(z_{\rm eff})}{r_d} \tag{1.21}$$

$$\Delta z_{\rm BAO}(z_{\rm eff}) = \frac{D_H(z_{\rm eff})}{r_d} \tag{1.22}$$

where $z_{\rm eff}$ is the effective redshift of the galaxy survey, D_M is the comoving angular diameter distance (Eq. 1.4), and D_H is the Hubble distance (Eq. 1.2).

There are two ways BAO can be used to constrain cosmological models, depending if we assume that $r_{\rm drag}$ is known or not. If $r_{\rm drag}$ is known, i.e. given by Eq. 1.20 using some value for ω_b (given by the CMB or BBN for instance), then BAO measurements are converted to absolute distance measurements which depend only on $H(z)$. Therefore, BAO can constrain H_0, curvature and dark energy. If $r_{\rm drag}$ is supposed to be unknown but still a standard ruler, then BAO constraints the ratio of $E(z) = H(z)/H_0$ to the combination $H_0 r_{\rm drag}$ which is now degenerate. This is similar to type Ia supernovae, where their absolute magnitude is degenerate with H_0. By combining several BAO measurements at different effective redshifts, BAO is a powerful probe of dark energy and curvature.

Current BAO measurements span effective redshifts from 0.1 to 2.3. The Sloan Digital Sky Survey (SDSS, Eisenstein, Weinberg, et al. 2011; Blanton et al. 2017) has measured more than 2 million redshifts spectroscopically in the past twenty years, producing the largest maps to date of the distribution of matter in the Universe. In addition to SDSS, surveys such as FastSound (Okumura et al. 2016), Vipers (Pezzotta et al. 2017), 6 degree field galaxy survey (6dFGS, Beutler et al. 2012) and WiggleZ (Parkinson et al. 2012) also produced BAO measurements using galaxies, though the volumes probed and the number of galaxies is inferior to SDSS. The latest cosmological constraints from BAO are described in Alam, Aubert, et al. 2021.

1.5.5 The cosmic microwave background

The cosmic microwave background (CMB) is one of the richest cosmological probes of all. The photons we receive today last scattered on baryons at a redshift of about 1100, corresponding to roughly 380 000 years after the Big Bang. It is the oldest information that we can measure from the Universe today.

The average temperature $T_{\rm CMB} = 2.72548 \pm 0.00057$ K (Mather et al. 1994; Fixsen 2009) of these photons, measured the last time by the COBE satellite, tell us on how much energy density from radiation there is in the Universe. From the black body Bose-Einstein distribution of photons, we have

$$\rho_\gamma = \frac{\pi^2 k^4}{15\hbar^3 c^3} T_{\rm CMB}^4 \tag{1.23}$$

The fluctuations around this average temperature, ΔT, and the polarisation of these photons trace the structures back at the recombination epoch. Given its early-times nature, the

temperature and polarisation fields are extremely well described by Gaussian statistics. Most of the information is therefore contained in the two point functions of these fields. Given that these fields are two dimensional, functions of the position angle in the sky (θ, ϕ), it is convenient to decompose these fields into a basis of spherical harmonic functions $Y_{\ell m}(\theta, \phi)$, which amplitudes are denoted $a_{\ell m}$. The angular power spectrum is simply the variance of these amplitudes: $C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle$, which in linear theory is simply a function of ℓ and independent of m. The temperature and both polarisation modes (E and B) can be decomposed into spherical harmonics, so we can estimate all cross power spectra, e.g., $C_\ell^{TE} = \langle a^T_{\ell m} a^{E*}_{\ell m} \rangle$ is the cross temperature and E-mode polarisation power spectrum.

The temperature and polarisation auto and cross power spectra are exquisitely well modelled by linear perturbation theory. In the most basic ΛCDM model, the power spectrum is a function of only six parameters: the angular scale of baryon acoustic oscillations θ_*, the physical density of baryons ω_b and dark-matter ω_c, the primordial power spectrum amplitude A_s and slope n_s, and the optical depth τ. The fact that such a model with so few free parameters can describe so well the observations is one of the greatest achievements in modern physics. A great description of the CMB physics from first principles can be found in Dodelson et al. 2020. Several codes are available to compute models for the CMB such as CAMB[3] (Lewis et al. 2000) and CLASS[4] (Lesgourgues 2011).

Explain how the power spectrum depends on the parameters

Latest measurements: Planck satellite (Planck Collaboration et al. 2020), Atacama Cosmology Telescope (ACT, Aiola et al. 2020), South Pole Telescope (SPT, **dutcherMeasurementsEModePolarization2021**; Aylor et al. 2017; Balkenhol et al. 2021), BICEP/Keck (BICEP/Keck et al. 2021) for B-modes.

1.5.6 Redshift-space distortions

Matter is not static in the Universe. Due to gravity, matter flows from underdense regions towards overdense ones. These velocities are commonly called *peculiar velocities*. The radial component of peculiar velocities alters the observed redshift of an object, due to the Doppler effect. Therefore, the total observed redshift $z_{\rm obs}$ is a combination of its cosmological redshift $z_{\rm cos}$, due to the expansion of the Universe, and the peculiar redshift $z_{\rm pec}$, due to the Doppler effect:

$$1 + z_{\rm obs} = (1 + z_{\rm cos})(1 + z_{\rm pec}) \tag{1.24}$$

where

$$z_{\rm pec} = \sqrt{\frac{1 + \frac{\vec{v}\cdot\hat{n}}{c}}{1 - \frac{\vec{v}\cdot\hat{n}}{c}}} - 1 \approx \frac{\vec{v}\cdot\hat{n}}{c} \tag{1.25}$$

The first equality is the definition of the relativistic Doppler effect, $\vec{v}$ is the velocity and $\hat{n}$ is the radial unitary vector. The right-hand side is the non-relativistic approximation, which is accurate for typical velocities of matter flows in our Universe.

When converting observed redshifts into comoving distances, peculiar velocities introduce a small error such that

$$s(z_{\rm obs}) \approx r(z_{cos}) + \frac{(1 + z_{\rm cos})}{H(z_{\rm cos})} \vec{v}\cdot\hat{n}, \tag{1.26}$$

where s is the *redshift-space* distance and r is the *real-space* distance. The errors caused by peculiar velocities distort the observed distribution of matter/galaxies since we cannot in

[3] https://camb.info/
[4] https://lesgourg.github.io/class_public/class.html

general decouple the velocities and the Hubble flow. This effect is observable and named *redshift-space distortions* (RSD). RSD are a powerful probe of the dynamics of the matter field, i.e., on how velocities are related to the densities, and how they contribute to the growth of structures over time. More generally, observations of RSD can be used to test the validity of general relativity since it is a probe of the strength of gravity.

The density contrast of matter is modified when observed in redshift-space relative to real-space, given the redshift-space distortions. Mass conservation implies that

$$[1+\delta_s(\vec{s})]\mathrm{d}^3 s = [1+\delta(\vec{x})]\mathrm{d}^3 x, \tag{1.27}$$

such that in Fourier space we obtain, in the linear regime (Kaiser 1987)

$$\delta_s(\vec{k}) = (1+f\mu^2)\delta(\vec{k}) \tag{1.28}$$

where f is the growth-rate of structures defined in Eq. 1.15 and $\mu = \vec{k}\cdot\hat{n}/k$ or the cosine of the angle between the wavevector and the line of sight. The power spectrum in redshift-space assuming linear perturbations is simply

$$P_s(k,\mu) = (1+f\mu^2)^2 P_m^{\mathrm{lin}}(k) \tag{1.29}$$

This means that the redshift-space matter power spectrum has radial modes enhanced by a factor of $(1+f)^2$ (where $f \sim 1$) relative to modes transverse to the line of sight. This enhancement is observable with galaxy surveys. If the normalisation of the linear matter power spectrum is parametrised by σ_8 (Eq. 1.18), then the actual measured quantity is the product $f\sigma_8$. Transposed to a given effective redshift z_{eff} of a galaxy survey, we need to scale σ_8, which is usually defined at $z=0$, using the growth factor $D(z)$, such that the measured quantity is $f(z)\sigma_8 D(z)$.

The observable $f(z)\sigma_8 D(z)$ is mostly sensitive to the amount of dark matter ω_m, which drives the growth of structures, and $H(z)$, which damps the growth (see Eq. 1.14). Therefore, it can be used to constrain dark energy models. More generally, measurements of $f(z)\sigma_8 D(z)$ can be used to test the the validity of general relativity on large scales, which is the commonly assumed theory of gravity.

The latest most relevant measurements were performed using data from the Sloan Digital Sky Survey (SDSS), including

- the SDSS Main Galaxy Sample Howlett et al. 2015,
- the Baryon Oscillation Spectroscopic Survey (BOSS, Alam, Ata, et al. 2017),
- the extended BOSS luminous red galaxy sample (eBOSS LRG, Bautista et al. 2020; Gil-Marín et al. 2020),
- the eBOSS emission line galaxy sample (eBOSS ELG, Tamone et al. 2020; de Mattia et al. 2021),
- the eBOSS quasar sample (eBOSS QSO, Hou et al. 2021; Neveux et al. 2020),

In chapter 4 I present my contributions to the measurement of the growth-rate of structures using the eBOSS LRG sample (Bautista et al. 2020). The cosmological implications of SDSS growth-rate measurements are described in Alam, Aubert, et al. 2021.

1.5.7 Weak gravitational lensing

Photons follow space-time geodesics. Space-time is distorted in the presence of a source of gravitational potential, which is typically in the form of a mass concentration. Therefore, the matter distribution in the Universe bends photon trajectories from distant sources. This phenomenon receives the name of *gravitational lensing*, since the theory describing light propagation on a gravitational field is analogous to classical optics. Gravitational lensing is a rich cosmological probe since the distorion of photon trajectories depends on both the baryonic and dark matter. From lensing measurements we can learn about the total matter distribution in an expanding Universe.

In the case of an extended source of photons, e.g., a galaxy, light from different angular positions within the source take slighly different paths towards the observer. If this extended source is placed in the background of a mass concentration - the lens - each path will suffer a slightly different bending, which depends on the impact parameter of each photon relative to the center of this lens. Also, due to conservation of surface brightness (is this the correct name?), the total flux is also increased. The final result is an image shifted, distorted and brighter.

Lensing measurements use shifts, distortions and increase in flux to determine properties of the lenses. These measurements are hard since we do not have access to the original unlensed position, shape and flux of a given galaxy. One remarkable exception is lensing in its strong regime. When the ratio between impact parameter of the source and mass of the lens is below some threshold - the Einstein radius - multiple images of the same source are created. Strong lensing allows us to estimate the mass and density profile of a given lens. Moreover, if the source is variable in time, this variability is slightly delayed between each of the multiple images, which allows us to constrain the expansion of the Universe. As mentioned in section 1.5.1, strongly lensed quasars (variable sources) are used to constrain H_0, under the assumption that we can properly model the lens density profile.

However, statistical measurements of lensing, such as correlations with overdensities, can be performed by making assumptions on the statistical properties of the unlensed sample of galaxies.

weak regime we measure shapes and correlate themselves (shear-shear) or with the galaxy field (galaxy-shear)

Sensitive to: the amplitude of the shear power spectrum, $S_8 = \sigma_8\sqrt{\Omega_m/0.3}$.

Latest measurements: Kilo Degree Survey (KiDS, Heymans et al. 2021), Dark Energy Survey Year 3 (DES-Y3, D. Collaboration et al. 2021), Hyper Supreme Cam first year (HSC, Hikage et al. 2019)

2 Observing the Universe with spectroscopy

Contents

In the last two decades, spectroscopy became one of the most important methods to survey the Universe.

Base this chapter on the Sloan Digital Sky Survey III and IV.

2.1 Selecting the objects to observe

2.2 Pointing fibres to the sky

2.3 From electrons to redshifts

2.4 From redshifts to cosmology

3 The high-redshift Universe and its forests

Contents

The high-redshift Universe ($2 < z < 4$) could be explored, until now, thanks to the Lyman-α forests.

3.1 Forests as a tracer of neutral hydrogen

3.2 Statistics of the absorption

3.3 Baryon acoustic oscillations in the forests

3.4 Weak-lensing of forests

3.5 Impact of redshift errors

4 The mid-redshift Universe and its galaxies

Contents

At redshifts between 0 and 2, galaxies can be used as tracers of the matter distribution.

4.1 Galaxies as a tracer of the matter field

4.2 Baryon acoustic oscillations with galaxies

4.3 Redshift-space distortions

4.4 Cross-correlation with radio surveys

5 The low-redshift Universe and its velocities

Contents

At low redshifts, below 0.1, peculiar velocities can be directly measured and their statistical properties can provide more information about a gravity on large scales.

5.1 Measuring peculiar velocities

5.2 Combining velocities with densities

5.3 Testing general relativity with velocities

Conclusion

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Bibliography

[Abb+17] B. P. Abbott et al. “A Gravitational-Wave Standard Siren Measurement of the Hubble Constant”. In: *Nature* 551.7678 (7678 2017), pp. 85–88. DOI: 10.1038/nature24471 (cit. on p. 17).

[Aio+20] Simone Aiola et al. “The Atacama Cosmology Telescope: DR4 Maps and Cosmological Parameters”. In: *Journal of Cosmology and Astroparticle Physics* 2020.12 (30, 2020), pp. 047–047. DOI: 10.1088/1475-7516/2020/12/047 (cit. on p. 21).

[Ala+17] Shadab Alam, Metin Ata, Stephen Bailey, Florian Beutler, Dmitry Bizyaev, Jonathan A. Blazek, Adam S. Bolton, Joel R. Brownstein, Angela Burden, Chia-Hsun Chuang, Johan Comparat, Antonio J. Cuesta, Kyle S. Dawson, Daniel J. Eisenstein, Stephanie Escoffier, Héctor Gil-Marín, Jan Niklas Grieb, Nick Hand, Shirley Ho, Karen Kinemuchi, David Kirkby, Francisco Kitaura, Elena Malanushenko, Viktor Malanushenko, Claudia Maraston, Cameron K. McBride, Robert C. Nichol, Matthew D. Olmstead, Daniel Oravetz, Nikhil Padmanabhan, Nathalie Palanque-Delabrouille, Kaike Pan, Marcos Pellejero-Ibanez, Will J. Percival, Patrick Petitjean, Francisco Prada, Adrian M. Price-Whelan, Beth A. Reid, Sergio A. Rodríguez-Torres, Natalie A. Roe, Ashley J. Ross, Nicholas P. Ross, Graziano Rossi, Jose Alberto Rubiño-Martín, Shun Saito, Salvador Salazar-Albornoz, Lado Samushia, Ariel G. Sánchez, Siddharth Satpathy, David J. Schlegel, Donald P. Schneider, Claudia G. Scóccola, Hee-Jong Seo, Erin S. Sheldon, Audrey Simmons, Anže Slosar, Michael A. Strauss, Molly E. C. Swanson, Daniel Thomas, Jeremy L. Tinker, Rita Tojeiro, Mariana Vargas Magaña, Jose Alberto Vazquez, Licia Verde, David A. Wake, Yuting Wang, David H. Weinberg, Martin White, W. Michael Wood-Vasey, Christophe Yèche, Idit Zehavi, Zhongxu Zhai, and Gong-Bo Zhao. “The Clustering of Galaxies in the Completed SDSS-III Baryon Oscillation Spectroscopic Survey: Cosmological Analysis of the DR12 Galaxy Sample”. In: *Monthly Notices of the Royal Astronomical Society* 470 (1, 2017), pp. 2617–2652. DOI: 10.1093/mnras/stx721 (cit. on p. 22).

[Ala+21] Shadab Alam, Marie Aubert, Santiago Avila, Christophe Balland, Julian E. Bautista, Matthew A. Bershady, Dmitry Bizyaev, Michael R. Blanton, Adam S. Bolton, Jo Bovy, Jonathan Brinkmann, Joel R. Brownstein, Etienne Burtin, Solène Chabanier, Michael J. Chapman, Peter Doohyun Choi, Chia-Hsun Chuang, Johan Comparat, Marie-Claude Cousinou, Andrei Cuceu, Kyle S. Dawson, Sylvain de la Torre, Arnaud de Mattia, Victoria de Sainte Agathe, Hélion du Mas des Bourboux, Stephanie Escoffier, Thomas Etourneau, James Farr, Andreu Font-Ribera, Peter M. Frinchaboy, Sebastien Fromenteau, Héctor Gil-Marín, Jean-Marc Le Goff, Alma X. Gonzalez-Morales, Violeta Gonzalez-Perez, Kathleen Grabowski, Julien Guy, Adam J. Hawken, Jiamin Hou, Hui Kong, James Parker, Mark Klaene, Jean-Paul Kneib, Sicheng Lin, Daniel Long, Brad W. Lyke, Axel de la Macorra, Paul Martini, Karen Masters, Faizan G. Mohammad, Jeongin Moon, Eva-Maria Mueller, Andrea Muñoz-Gutiérrez, Adam D. Myers, Seshadri Nadathur, Richard Neveux, Jeffrey A. Newman, Pasquier Noterdaeme, Audrey Oravetz, Daniel Oravetz, Nathalie

Palanque-Delabrouille, Kaike Pan, Romain Paviot, Will J. Percival, Ignasi Pérez-Ràfols, Patrick Petitjean, Matthew M. Pieri, Abhishek Prakash, Anand Raichoor, Corentin Ravoux, Mehdi Rezaie, James Rich, Ashley J. Ross, Graziano Rossi, Rossana Ruggeri, Vanina Ruhlmann-Kleider, Ariel G. Sánchez, F. Javier Sánchez, José R. Sánchez-Gallego, Conor Sayres, Donald P. Schneider, Hee-Jong Seo, Arman Shafieloo, Anže Slosar, Alex Smith, Julianna Stermer, Amelie Tamone, Jeremy L. Tinker, Rita Tojeiro, Mariana Vargas-Magaña, Andrei Variu, Yuting Wang, Benjamin A. Weaver, Anne-Marie Weijmans, Christophe Yèche, Pauline Zarrouk, Cheng Zhao, Gong-Bo Zhao, and Zheng Zheng. "Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Cosmological Implications from Two Decades of Spectroscopic Surveys at the Apache Point Observatory". In: *Physical Review D* 103.8 (28, 2021), p. 083533. DOI: 10.1103/PhysRevD.103.083533 (cit. on pp. 20, 22).

[Ave+21] Erik Aver, Danielle A. Berg, Keith A. Olive, Richard W. Pogge, John J. Salzer, and Evan D. Skillman. "Improving Helium Abundance Determinations with Leo P as a Case Study". In: *Journal of Cosmology and Astroparticle Physics* 2021.03 (1, 2021), p. 027. DOI: 10.1088/1475-7516/2021/03/027 (cit. on p. 19).

[Ayl+17] K. Aylor, Z. Hou, L. Knox, K. T. Story, B. A. Benson, L. E. Bleem, J. E. Carlstrom, C. L. Chang, H.-M. Cho, R. Chown, T. M. Crawford, A. T. Crites, T. de Haan, M. A. Dobbs, W. B. Everett, E. M. George, N. W. Halverson, N. L. Harrington, G. P. Holder, W. L. Holzapfel, J. D. Hrubes, R. Keisler, A. T. Lee, E. M. Leitch, D. Luong-Van, D. P. Marrone, J. J. McMahon, S. S. Meyer, M. Millea, L. M. Mocanu, J. J. Mohr, T. Natoli, Y. Omori, S. Padin, C. Pryke, C. L. Reichardt, J. E. Ruhl, J. T. Sayre, K. K. Scaffer, E. Shirokoff, Z. Staniszewski, A. A. Stark, K. Vanderlinde, J. D. Vieira, and R. Williamson. "A Comparison of Cosmological Parameters Determined from CMB Temperature Power Spectra from the South Pole Telescope and the Planck Satellite". In: *The Astrophysical Journal* 850.1 (21, 2017), p. 101. DOI: 10.3847/1538-4357/aa947b (cit. on p. 21).

[Bal+00] A. Balbi, P. Ade, J. Bock, J. Borrill, A. Boscaleri, P. De Bernardis, P. G. Ferreira, S. Hanany, V. Hristov, A. H. Jaffe, A. T. Lee, S. Oh, E. Pascale, B. Rabii, P. L. Richards, G. F. Smoot, R. Stompor, C. D. Winant, and J. H. P. Wu. "Constraints on Cosmological Parameters from MAXIMA-1". In: *The Astrophysical Journal* 545 (1, 2000), pp. L1–L4. DOI: 10.1086/317323 (cit. on p. 9).

[Bal+21] L. Balkenhol et al. "Constraints on ΛCDM Extensions from the SPT-3G 2018 EE and TE Power Spectra". In: *Physical Review D* 104.8 (4, 2021), p. 083509. DOI: 10.1103/PhysRevD.104.083509 (cit. on p. 21).

[Bau+20] Julian E Bautista, Romain Paviot, Mariana Vargas Magaña, Sylvain de la Torre, Sebastien Fromenteau, Hector Gil-Marín, Ashley J Ross, Etienne Burtin, Kyle S Dawson, Jiamin Hou, Jean-Paul Kneib, Arnaud de Mattia, Will J Percival, Graziano Rossi, Rita Tojeiro, Cheng Zhao, Gong-Bo Zhao, Shadab Alam, Joel Brownstein, Michael J Chapman, Peter D Choi, Chia-Hsun Chuang, Stéphanie Escoffier, Axel de la Macorra, Hélion du Mas des Bourboux, Faizan G Mohammad, Jeongin Moon, Eva-Maria Müller, Seshadri Nadathur, Jeffrey A Newman, Donald Schneider, Hee-Jong Seo, and Yuting Wang. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Measurement of the BAO and Growth Rate of Structure of the Luminous Red Galaxy Sample from the Anisotropic Correlation

Function between Redshifts 0.6 and 1". In: *Monthly Notices of the Royal Astronomical Society* 500.1 (3, 2020), pp. 736–762. DOI: 10.1093/mnras/staa2800 (cit. on p. 22).

[Beu+12] Florian Beutler, Chris Blake, Matthew Colless, D. Heath Jones, Lister Staveley-Smith, Gregory B. Poole, Lachlan Campbell, Quentin Parker, Will Saunders, and Fred Watson. "The 6dF Galaxy Survey: z \approx 0 Measurement of the Growth Rate and Sigma_8". In: *Monthly Notices of the Royal Astronomical Society* 423.4 (11, 2012), pp. 3430–3444. DOI: 10.1111/j.1365-2966.2012.21136.x (cit. on p. 20).

[BIC+21] BICEP/Keck et al. "A Demonstration of Improved Constraints on Primordial Gravitational Waves with Delensing". In: *Physical Review D* 103.2 (26, 2021), p. 022004. DOI: 10.1103/PhysRevD.103.022004 (cit. on p. 21).

[Bir+20] S. Birrer, A. J. Shajib, A. Galan, M. Millon, T. Treu, A. Agnello, M. Auger, G. C.-F. Chen, L. Christensen, T. Collett, F. Courbin, C. D. Fassnacht, L. V. E. Koopmans, P. J. Marshall, J.-W. Park, C. E. Rusu, D. Sluse, C. Spiniello, S. H. Suyu, S. Wagner-Carena, K. C. Wong, M. Barnabè, A. S. Bolton, O. Czoske, X. Ding, J. A. Frieman, and L. Van de Vyvere. "TDCOSMO - IV. Hierarchical Time-Delay Cosmography – Joint Inference of the Hubble Constant and Galaxy Density Profiles". In: *Astronomy & Astrophysics* 643 (1, 2020), A165. DOI: 10.1051/0004-6361/202038861 (cit. on p. 17).

[Bla+17] Michael R. Blanton et al. "Sloan Digital Sky Survey IV: Mapping the Milky Way, Nearby Galaxies, and the Distant Universe". In: *The Astronomical Journal* 154.1 (2017), p. 28. DOI: 10.3847/1538-3881/aa7567 (cit. on p. 20).

[Bro+19a] D. Brout, M. Sako, D. Scolnic, R. Kessler, C. B. D'Andrea, T. M. Davis, S. R. Hinton, A. G. Kim, J. Lasker, E. Macaulay, A. Möller, R. C. Nichol, M. Smith, M. Sullivan, R. C. Wolf, S. Allam, B. A. Bassett, P. Brown, F. J. Castander, M. Childress, R. J. Foley, L. Galbany, K. Herner, E. Kasai, M. March, E. Morganson, P. Nugent, Y.-C. Pan, R. C. Thomas, B. E. Tucker, W. Wester, T. M. C. Abbott, J. Annis, S. Avila, E. Bertin, D. Brooks, D. L. Burke, A. Carnero Rosell, M. Carrasco Kind, J. Carretero, M. Crocce, C. E. Cunha, L. N. da Costa, C. Davis, J. De Vicente, S. Desai, H. T. Diehl, P. Doel, T. F. Eifler, B. Flaugher, P. Fosalba, J. Frieman, J. García-Bellido, E. Gaztanaga, D. W. Gerdes, D. A. Goldstein, D. Gruen, R. A. Gruendl, J. Gschwend, G. Gutierrez, W. G. Hartley, D. L. Hollowood, K. Honscheid, D. J. James, K. Kuehn, N. Kuropatkin, O. Lahav, T. S. Li, M. Lima, J. L. Marshall, P. Martini, R. Miquel, B. Nord, A. A. Plazas, A. Roodman, E. S. Rykoff, E. Sanchez, V. Scarpine, R. Schindler, M. Schubnell, S. Serrano, I. Sevilla-Noarbe, M. Soares-Santos, F. Sobreira, E. Suchyta, M. E. C. Swanson, G. Tarle, D. Thomas, D. L. Tucker, A. R. Walker, B. Yanny, Y. Zhang, and DES COLLABORATION. "First Cosmology Results Using Type Ia Supernovae from the Dark Energy Survey: Photometric Pipeline and Light-curve Data Release". In: *The Astrophysical Journal* 874 (1, 2019), p. 106. DOI: 10.3847/1538-4357/ab06c1 (cit. on p. 18).

[Bro+19b] D. Brout, D. Scolnic, et al. "First Cosmology Results Using SNe Ia from the Dark Energy Survey: Analysis, Systematic Uncertainties, and Validation". In: *The Astrophysical Journal* 874.2 (2019), p. 150. DOI: 10.3847/1538-4357/ab08a0 (cit. on p. 18).

[Cha98] B. Chaboyer. "The Age of the Universe." In: *Physics Reports* 307 (1, 1998), pp. 23–30. DOI: 10.1016/S0370-1573(98)00054-4 (cit. on p. 10).

[CP01] Michel Chevallier and David Polarski. "Accelerating Universes with Scaling Dark Matter". In: *International Journal of Modern Physics D* 10 (1, 2001), pp. 213–223. DOI: 10.1142/S0218271801000822 (cit. on p. 12).

[Col+05] Shaun Cole, Will J. Percival, John A. Peacock, Peder Norberg, Carlton M. Baugh, Carlos S. Frenk, Ivan Baldry, Joss Bland-Hawthorn, Terry Bridges, Russell Cannon, Matthew Colless, Chris Collins, Warrick Couch, Nicholas J. G. Cross, Gavin Dalton, Vincent R. Eke, Roberto De Propris, Simon P. Driver, George Efstathiou, Richard S. Ellis, Karl Glazebrook, Carole Jackson, Adrian Jenkins, Ofer Lahav, Ian Lewis, Stuart Lumsden, Steve Maddox, Darren Madgwick, Bruce A. Peterson, Will Sutherland, and Keith Taylor. "The 2dF Galaxy Redshift Survey: Power-Spectrum Analysis of the Final Data Set and Cosmological Implications". In: *Monthly Notices of the Royal Astronomical Society* 362 (1, 2005), pp. 505–534. DOI: 10.1111/j.1365-2966.2005.09318.x (cit. on p. 10).

[Col+21] DES Collaboration et al. *Dark Energy Survey Year 3 Results: Cosmological Constraints from Galaxy Clustering and Weak Lensing*. 27, 2021. URL: http://arxiv.org/abs/2105.13549 (visited on 06/17/2021) (cit. on p. 23).

[CC17] The LIGO Scientific Collaboration and The Virgo Collaboration. "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral". In: *Physical Review Letters* 119.16 (16, 2017), p. 161101. DOI: 10.1103/PhysRevLett.119.161101 (cit. on p. 17).

[CtV16] The LIGO Scientific Collaboration and the Virgo Collaboration. "Observation of Gravitational Waves from a Binary Black Hole Merger". In: *Physical Review Letters* 116.6 (11, 2016), p. 061102. DOI: 10.1103/PhysRevLett.116.061102 (cit. on p. 17).

[dBer+00] P. de Bernardis, P. A. R. Ade, J. J. Bock, J. R. Bond, J. Borrill, A. Boscaleri, K. Coble, B. P. Crill, G. De Gasperis, P. C. Farese, P. G. Ferreira, K. Ganga, M. Giacometti, E. Hivon, V. V. Hristov, A. Iacoangeli, A. H. Jaffe, A. E. Lange, L. Martinis, S. Masi, P. V. Mason, P. D. Mauskopf, A. Melchiorri, L. Miglio, T. Montroy, C. B. Netterfield, E. Pascale, F. Piacentini, D. Pogosyan, S. Prunet, S. Rao, G. Romeo, J. E. Ruhl, F. Scaramuzzi, D. Sforna, and N. Vittorio. "A Flat Universe from High-Resolution Maps of the Cosmic Microwave Background Radiation". In: *Nature* 404 (1, 2000), pp. 955–959. DOI: 10.1038/35010035 (cit. on p. 9).

[dMat+21] Arnaud de Mattia, Vanina Ruhlmann-Kleider, Anand Raichoor, Ashley J. Ross, Amélie Tamone, Cheng Zhao, Shadab Alam, Santiago Avila, Etienne Burtin, Julian Bautista, Florian Beutler, Jonathan Brinkmann, Joel R. Brownstein, Michael J. Chapman, Chia-Hsun Chuang, Johan Comparat, Hélion du Mas des Bourboux, Kyle S. Dawson, Axel de la Macorra, Héctor Gil-Marín, Violeta Gonzalez-Perez, Claudio Gorgoni, Jiamin Hou, Hui Kong, Sicheng Lin, Seshadri Nadathur, Jeffrey A. Newman, Eva-Maria Mueller, Will J. Percival, Mehdi Rezaie, Graziano Rossi, Donald P. Schneider, Prabhakar Tiwari, M. Vivek, Yuting Wang, and Gong-Bo Zhao. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Measurement of the BAO and Growth Rate of Structure of the Emission Line Galaxy Sample from the Anisotropic Power Spectrum between Redshift 0.6 and

1.1". In: *Monthly Notices of the Royal Astronomical Society* 501 (1, 2021), pp. 5616–5645. DOI: 10.1093/mnras/staa3891 (cit. on p. 22).

[DS20] Scott Dodelson and Fabian Schmidt. *Modern Cosmology - 2nd Edition*. 2nd ed. Elsevier, 18, 2020 (cit. on p. 21).

[ESM90] G. Efstathiou, W. J. Sutherland, and S. J. Maddox. "The Cosmological Constant and Cold Dark Matter". In: *Nature* 348 (1, 1990), pp. 705–707. DOI: 10.1038/348705a0 (cit. on p. 9).

[Eis+11] Daniel J. Eisenstein, David H. Weinberg, et al. "SDSS-III: Massive Spectroscopic Surveys of the Distant Universe, the Milky Way, and Extra-Solar Planetary Systems". In: *The Astronomical Journal* 142 (1, 2011), p. 72. DOI: 10.1088/0004-6256/142/3/72; (cit. on p. 20).

[Eis+05] Daniel J. Eisenstein, Idit Zehavi, David W. Hogg, Roman Scoccimarro, Michael R. Blanton, Robert C. Nichol, Ryan Scranton, Hee-Jong Seo, Max Tegmark, Zheng Zheng, Scott F. Anderson, Jim Annis, Neta Bahcall, Jon Brinkmann, Scott Burles, Francisco J. Castander, Andrew Connolly, Istvan Csabai, Mamoru Doi, Masataka Fukugita, Joshua A. Frieman, Karl Glazebrook, James E. Gunn, John S. Hendry, Gregory Hennessy, Zeljko Ivezić, Stephen Kent, Gillian R. Knapp, Huan Lin, Yeong-Shang Loh, Robert H. Lupton, Bruce Margon, Timothy A. McKay, Avery Meiksin, Jeffery A. Munn, Adrian Pope, Michael W. Richmond, David Schlegel, Donald P. Schneider, Kazuhiro Shimasaku, Christopher Stoughton, Michael A. Strauss, Mark SubbaRao, Alexander S. Szalay, István Szapudi, Douglas L. Tucker, Brian Yanny, and Donald G. York. "Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies". In: *The Astrophysical Journal* 633.2 (2005), p. 560. DOI: 10.1086/466512 (cit. on p. 10).

[Fix09] D. J. Fixsen. "The Temperature of the Cosmic Microwave Background". In: *The Astrophysical Journal* 707 (1, 2009), pp. 916–920. DOI: 10.1088/0004-637X/707/2/916 (cit. on p. 20).

[Fre+19] Wendy L. Freedman, Barry F. Madore, Dylan Hatt, Taylor J. Hoyt, In-Sung Jang, Rachael L. Beaton, Christopher R. Burns, Myung Gyoon Lee, Andrew J. Monson, Jillian R. Neeley, Mark M. Phillips, Jeffrey A. Rich, and Mark Seibert. "The Carnegie-Chicago Hubble Program. VIII. An Independent Determination of the Hubble Constant Based on the Tip of the Red Giant Branch". In: *The Astrophysical Journal* 882.1 (29, 2019), p. 34. DOI: 10.3847/1538-4357/ab2f73 (cit. on p. 17).

[Gil+20] Héctor Gil-Marín, Julián E. Bautista, Romain Paviot, Mariana Vargas-Magaña, Sylvain de la Torre, Sebastien Fromenteau, Shadab Alam, Santiago Ávila, Etienne Burtin, Chia-Hsun Chuang, Kyle S. Dawson, Jiamin Hou, Arnaud de Mattia, Faizan G. Mohammad, Eva-Maria Müller, Seshadri Nadathur, Richard Neveux, Will J. Percival, Anand Raichoor, Mehdi Rezaie, Ashley J. Ross, Graziano Rossi, Vanina Ruhlmann-Kleider, Alex Smith, Amélie Tamone, Jeremy L. Tinker, Rita Tojeiro, Yuting Wang, Gong-Bo Zhao, Cheng Zhao, Jonathan Brinkmann, Joel R. Brownstein, Peter D. Choi, Stephanie Escoffier, Axel de la Macorra, Jeongin Moon, Jeffrey A. Newman, Donald P. Schneider, Hee-Jong Seo, and Mariappan Vivek. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Measurement of the BAO and Growth Rate of Structure of the Luminous Red Galaxy Sample from the Anisotropic Power Spectrum between Redshifts 0.6 and 1.0". In:

Monthly Notices of the Royal Astronomical Society 498 (1, 2020), pp. 2492–2531. DOI: 10.1093/mnras/staa2455 (cit. on p. 22).

[Gra+19] Matthew J. Graham et al. "The Zwicky Transient Facility: Science Objectives". In: *Publications of the Astronomical Society of the Pacific* 131 (1, 2019), p. 078001. DOI: 10.1088/1538-3873/ab006c (cit. on p. 18).

[Gra+20] R. Graziani, M. Rigault, N. Regnault, Ph Gris, A. Möller, P. Antilogus, P. Astier, M. Betoule, S. Bongard, M. Briday, J. Cohen-Tanugi, Y. Copin, H. M. Courtois, D. Fouchez, E. Gangler, D. Guinet, A. J. Hawken, Y.-L. Kim, P.-F. Léget, J. Neveu, P. Ntelis, Ph Rosnet, and E. Nuss. *Peculiar Velocity Cosmology with Type Ia Supernovae*. 24, 2020. URL: http://arxiv.org/abs/2001.09095 (visited on 10/15/2020) (cit. on p. 18).

[Her+99] J. R. Herrnstein, J. M. Moran, L. J. Greenhill, P. J. Diamond, M. Inoue, N. Nakai, M. Miyoshi, C. Henkel, and A. Riess. "A Geometric Distance to the Galaxy NGC4258 from Orbital Motions in a Nuclear Gas Disk". In: *Nature* 400.6744 (6744 1999), pp. 539–541. DOI: 10.1038/22972 (cit. on p. 17).

[Hey+21] Catherine Heymans, Tilman Tröster, Marika Asgari, Chris Blake, Hendrik Hildebrandt, Benjamin Joachimi, Konrad Kuijken, Chieh-An Lin, Ariel G. Sánchez, Jan Luca van den Busch, Angus H. Wright, Alexandra Amon, Maciej Bilicki, Jelte de Jong, Martin Crocce, Andrej Dvornik, Thomas Erben, Maria Cristina Fortuna, Fedor Getman, Benjamin Giblin, Karl Glazebrook, Henk Hoekstra, Shahab Joudaki, Arun Kannawadi, Fabian Köhlinger, Chris Lidman, Lance Miller, Nicola R. Napolitano, David Parkinson, Peter Schneider, HuanYuan Shan, Edwin Valentijn, Gijs Verdoes Kleijn, and Christian Wolf. "KiDS-1000 Cosmology: Multi-probe Weak Gravitational Lensing and Spectroscopic Galaxy Clustering Constraints". In: *Astronomy & Astrophysics* 646 (2021), A140. DOI: 10.1051/0004-6361/202039063 (cit. on p. 23).

[Hik+19] Chiaki Hikage, Masamune Oguri, Takashi Hamana, Surhud More, Rachel Mandelbaum, Masahiro Takada, Fabian Köhlinger, Hironao Miyatake, Atsushi J. Nishizawa, Hiroaki Aihara, Robert Armstrong, James Bosch, Jean Coupon, Anne Ducout, Paul Ho, Bau-Ching Hsieh, Yutaka Komiyama, François Lanusse, Alexie Leauthaud, Robert H. Lupton, Elinor Medezinski, Sogo Mineo, Shoken Miyama, Satoshi Miyazaki, Ryoma Murata, Hitoshi Murayama, Masato Shirasaki, Cristóbal Sifón, Melanie Simet, Joshua Speagle, David N. Spergel, Michael A. Strauss, Naoshi Sugiyama, Masayuki Tanaka, Yousuke Utsumi, Shiang-Yu Wang, and Yoshihiko Yamada. "Cosmology from Cosmic Shear Power Spectra with Subaru Hyper Suprime-Cam First-Year Data". In: *Publications of the Astronomical Society of Japan* 71 (1, 2019), p. 43. DOI: 10.1093/pasj/psz010 (cit. on p. 23).

[Hou+21] Jiamin Hou, Ariel G. Sánchez, Ashley J. Ross, Alex Smith, Richard Neveux, Julian Bautista, Etienne Burtin, Cheng Zhao, Román Scoccimarro, Kyle S. Dawson, Arnaud de Mattia, Axel de la Macorra, Hélion du Mas des Bourboux, Daniel J. Eisenstein, Héctor Gil-Marín, Brad W. Lyke, Faizan G. Mohammad, Eva-Maria Mueller, Will J. Percival, Graziano Rossi, Mariana Vargas Magaña, Pauline Zarrouk, Gong-Bo Zhao, Jonathan Brinkmann, Joel R. Brownstein, Chia-Hsun Chuang, Adam D. Myers, Jeffrey A. Newman, Donald P. Schneider, and M. Vivek. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: BAO and RSD Measurements from Anisotropic Clustering Analysis of the Quasar Sample

in Configuration Space between Redshift 0.8 and 2.2". In: *Monthly Notices of the Royal Astronomical Society* 500 (1, 2021), pp. 1201–1221. DOI: 10.1093/mnras/staa3234 (cit. on p. 22).

[How+15] Cullan Howlett, Ashley J. Ross, Lado Samushia, Will J. Percival, and Marc Manera. "The Clustering of the SDSS Main Galaxy Sample - II. Mock Galaxy Catalogues and a Measurement of the Growth of Structure from Redshift Space Distortions at z = 0.15". In: *Monthly Notices of the Royal Astronomical Society* 449 (1, 2015), pp. 848–866. DOI: 10.1093/mnras/stu2693 (cit. on p. 22).

[Hub29] Edwin Hubble. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae". In: *Proceedings of the National Academy of Sciences* 15.3 (15, 1929), pp. 168–173. DOI: 10.1073/pnas.15.3.168 (cit. on p. 16).

[Kai87] Nick Kaiser. "Clustering in Real Space and in Redshift Space". In: *Monthly Notices of the Royal Astronomical Society* 227 (1, 1987), pp. 1–21. DOI: 10.1093/mnras/227.1.1 (cit. on p. 22).

[Kes+19] R Kessler et al. "First Cosmology Results Using Type Ia Supernova from the Dark Energy Survey: Simulations to Correct Supernova Distance Biases". In: *Monthly Notices of the Royal Astronomical Society* 485.1 (1, 2019), pp. 1171–1187. DOI: 10.1093/mnras/stz463 (cit. on p. 18).

[KL20] Alex G. Kim and Eric V. Linder. "Complementarity of Peculiar Velocity Surveys and Redshift Space Distortions for Testing Gravity". In: *Physical Review D* 101 (1, 2020), p. 023516. DOI: 10.1103/PhysRevD.101.023516 (cit. on p. 18).

[Les11] Julien Lesgourgues. "The Cosmic Linear Anisotropy Solving System (CLASS) I: Overview". In: (14, 2011) (cit. on p. 21).

[LCL00] Antony Lewis, Anthony Challinor, and Anthony Lasenby. "Efficient Computation of Cosmic Microwave Background Anisotropies in Closed Friedmann-Robertson-Walker Models". In: *The Astrophysical Journal* 538 (1, 2000), pp. 473–476. DOI: 10.1086/309179 (cit. on p. 21).

[Lin03] Eric V. Linder. "Exploring the Expansion History of the Universe". In: *Physical Review Letters* 90 (1, 2003), p. 091301. DOI: 10.1103/PhysRevLett.90.091301 (cit. on p. 12).

[Mad+90] S. J. Maddox, G. Efstathiou, W. J. Sutherland, and J. Loveday. "Galaxy Correlations on Large Scales." In: *Monthly Notices of the Royal Astronomical Society* 242 (1, 1990), p. 43. DOI: 10.1093/mnras/242.1.43P (cit. on p. 9).

[Mat+94] J. C. Mather, E. S. Cheng, D. A. Cottingham, R. E. Eplee Jr., D. J. Fixsen, T. Hewagama, R. B. Isaacman, K. A. Jensen, S. S. Meyer, P. D. Noerdlinger, S. M. Read, L. P. Rosen, R. A. Shafer, E. L. Wright, C. L. Bennett, N. W. Boggess, M. G. Hauser, T. Kelsall, S. H. Moseley Jr., R. F. Silverberg, G. F. Smoot, R. Weiss, and D. T. Wilkinson. "Measurement of the Cosmic Microwave Background Spectrum by the COBE FIRAS Instrument". In: *The Astrophysical Journal* 420 (1, 1994), pp. 439–444. DOI: 10.1086/173574 (cit. on p. 20).

[Mou+00] Jeremy R. Mould, John P. Huchra, Wendy L. Freedman, Robert C. Kennicutt Jr., Laura Ferrarese, Holland C. Ford, Brad K. Gibson, John A. Graham, Shaun M. G. Hughes, Garth D. Illingworth, Daniel D. Kelson, Lucas M. Macri, Barry F. Madore, Shoko Sakai, Kim M. Sebo, Nancy A. Silbermann, and Peter B. Stetson. "The Hubble Space Telescope Key Project on the Extragalactic Distance Scale. XXVIII. Combining the Constraints on the Hubble Constant". In: *The Astrophysical Journal* 529 (1, 2000), pp. 786–794. DOI: 10.1086/308304 (cit. on p. 9).

[Nev+20] Richard Neveux, Etienne Burtin, Arnaud de Mattia, Alex Smith, Ashley J. Ross, Jiamin Hou, Julian Bautista, Jonathan Brinkmann, Chia-Hsun Chuang, Kyle S. Dawson, Héctor Gil-Marín, Brad W. Lyke, Axel de la Macorra, Hélion du Mas des Bourboux, Faizan G. Mohammad, Eva-Maria Müller, Adam D. Myers, Jeffrey A. Newman, Will J. Percival, Graziano Rossi, Donald Schneider, M. Vivek, Pauline Zarrouk, Cheng Zhao, and Gong-Bo Zhao. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: BAO and RSD Measurements from the Anisotropic Power Spectrum of the Quasar Sample between Redshift 0.8 and 2.2". In: *Monthly Notices of the Royal Astronomical Society* 499 (1, 2020), pp. 210–229. DOI: 10.1093/mnras/staa2780 (cit. on p. 22).

[Oku+16] Teppei Okumura, Chiaki Hikage, Tomonori Totani, Motonari Tonegawa, Hiroyuki Okada, Karl Glazebrook, Chris Blake, Pedro G. Ferreira, Surhud More, Atsushi Taruya, Shinji Tsujikawa, Masayuki Akiyama, Gavin Dalton, Tomotsugu Goto, Takashi Ishikawa, Fumihide Iwamuro, Takahiko Matsubara, Takahiro Nishimichi, Kouji Ohta, Ikkoh Shimizu, Ryuichi Takahashi, Naruhisa Takato, Naoyuki Tamura, Kiyoto Yabe, and Naoki Yoshida. "The Subaru FMOS Galaxy Redshift Survey (FastSound). IV. New Constraint on Gravity Theory from Redshift Space Distortions at z \sim 1.4". In: *Publications of the Astronomical Society of Japan* 68.3 (1, 2016). DOI: 10.1093/pasj/psw029 (cit. on p. 20).

[Par+12] David Parkinson, Signe Riemer-Sørensen, Chris Blake, Gregory B. Poole, Tamara M. Davis, Sarah Brough, Matthew Colless, Carlos Contreras, Warrick Couch, Scott Croom, Darren Croton, Michael J. Drinkwater, Karl Forster, David Gilbank, Mike Gladders, Karl Glazebrook, Ben Jelliffe, Russell J. Jurek, I.-hui Li, Barry Madore, D. Christopher Martin, Kevin Pimbblet, Michael Pracy, Rob Sharp, Emily Wisnioski, David Woods, Ted K. Wyder, and H. K. C. Yee. "The WiggleZ Dark Energy Survey: Final Data Release and Cosmological Results". In: *Physical Review D* 86.10 (2012), p. 103518. DOI: 10.1103/PhysRevD.86.103518 (cit. on p. 20).

[Per+99] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R. S. Ellis, M. Irwin, R. G. McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, T. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, W. J. Couch, and Supernova Cosmology Project. "Measurements of Omega and Lambda from 42 High-Redshift Supernovae". In: *The Astrophysical Journal* 517 (1, 1999), pp. 565–586. DOI: 10.1086/307221 (cit. on p. 9).

[Pez+17] A. Pezzotta, S. de la Torre, J. Bel, B. R. Granett, L. Guzzo, J. A. Peacock, B. Garilli, M. Scodeggio, M. Bolzonella, U. Abbas, C. Adami, D. Bottini, A. Cappi, O. Cucciati, I. Davidzon, P. Franzetti, A. Fritz, A. Iovino, J. Krywult, V. Le Brun, O. Le Fèvre, D. Maccagni, K. Małek, F. Marulli, M. Polletta, A. Pollo, L. a. M. Tasca, R. Tojeiro,

D. Vergani, A. Zanichelli, S. Arnouts, E. Branchini, J. Coupon, G. De Lucia, J. Koda, O. Ilbert, F. Mohammad, T. Moutard, and L. Moscardini. “The VIMOS Public Extragalactic Redshift Survey (VIPERS). The Growth of Structure at 0.5 < z < 1.2 from Redshift-Space Distortions in the Clustering of the PDR-2 Final Sample”. In: *Astronomy and Astrophysics* 604 (2017), A33. DOI: 10.1051/0004-6361/201630295 (cit. on p. 20).

[Pit+21] Cyril Pitrou, Alain Coc, Jean-Philippe Uzan, and Elisabeth Vangioni. “A New Tension in the Cosmological Model from Primordial Deuterium?” In: *Monthly Notices of the Royal Astronomical Society* 502.2 (10, 2021), pp. 2474–2481. DOI: 10.1093/mnras/stab135 (cit. on p. 19).

[Pla+20] Planck Collaboration et al. “Planck 2018 Results. VI. Cosmological Parameters”. In: *Astronomy and Astrophysics* 641 (1, 2020), A6. DOI: 10.1051/0004-6361/201833910 (cit. on p. 21).

[RPR19] M. J. Reid, D. W. Pesce, and A. G. Riess. “An Improved Distance to NGC 4258 and Its Implications for the Hubble Constant”. In: *The Astrophysical Journal* 886.2 (2019), p. L27. DOI: 10.3847/2041-8213/ab552d (cit. on p. 17).

[Rie20] Adam G. Riess. “The Expansion of the Universe Is Faster than Expected”. In: *Nature Reviews Physics* 2.1 (2020), pp. 10–12. DOI: 10.1038/s42254-019-0137-0 (cit. on p. 16).

[Rie+21] Adam G. Riess, Stefano Casertano, Wenlong Yuan, J. Bradley Bowers, Lucas Macri, Joel C. Zinn, and Dan Scolnic. “Cosmic Distances Calibrated to 1% Precision with Gaia EDR3 Parallaxes and Hubble Space Telescope Photometry of 75 Milky Way Cepheids Confirm Tension with ${upLambda”. In: 908.1 (2021), p. L6. DOI: 10.3847/2041-8213/abdbaf (cit. on p. 16).

[Rie+98] Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiatti, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, B. Leibundgut, M. M. Phillips, David Reiss, Brian P. Schmidt, Robert A. Schommer, R. Chris Smith, J. Spyromilio, Christopher Stubbs, Nicholas B. Suntzeff, and John Tonry. “Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant”. In: *The Astronomical Journal* 116 (1, 1998), pp. 1009–1038. DOI: 10.1086/300499 (cit. on p. 9).

[Sán20] Ariel G. Sánchez. “Arguments against Using $h\hat{}-1}$ Mpc Units in Observational Cosmology”. In: *Physical Review D* 102.12 (2, 2020), p. 123511. DOI: 10.1103/PhysRevD.102.123511 (cit. on p. 16).

[Sco+18] D. M. Scolnic, D. O. Jones, A. Rest, Y. C. Pan, R. Chornock, R. J. Foley, M. E. Huber, R. Kessler, G. Narayan, A. G. Riess, S. Rodney, E. Berger, D. J. Brout, P. J. Challis, M. Drout, D. Finkbeiner, R. Lunnan, R. P. Kirshner, N. E. Sanders, E. Schlafly, S. Smartt, C. W. Stubbs, J. Tonry, W. M. Wood-Vasey, M. Foley, J. Hand, E. Johnson, W. S. Burgett, K. C. Chambers, P. W. Draper, K. W. Hodapp, N. Kaiser, R. P. Kudritzki, E. A. Magnier, N. Metcalfe, F. Bresolin, E. Gall, R. Kotak, M. McCrum, and K. W. Smith. “The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample”. In: *The Astrophysical Journal* 859 (1, 2018), p. 101. DOI: 10.3847/1538-4357/aab9bb (cit. on p. 18).

[Tam+20] Amélie Tamone, Anand Raichoor, Cheng Zhao, Arnaud de Mattia, Claudio Gorgoni, Etienne Burtin, Vanina Ruhlmann-Kleider, Ashley J. Ross, Shadab Alam, Will J. Percival, Santiago Avila, Michael J. Chapman, Chia-Hsun Chuang, Johan Comparat, Kyle S. Dawson, Sylvain de la Torre, Hélion du Mas des Bourboux, Stephanie Escoffier, Violeta Gonzalez-Perez, Jiamin Hou, Jean-Paul Kneib, Faizan G. Mohammad, Eva-Maria Mueller, Romain Paviot, Graziano Rossi, Donald P. Schneider, Yuting Wang, and Gong-Bo Zhao. "The Completed SDSS-IV Extended Baryon Oscillation Spectroscopic Survey: Growth Rate of Structure Measurement from Anisotropic Clustering Analysis in Configuration Space between Redshift 0.6 and 1.1 for the Emission Line Galaxy Sample". In: *Monthly Notices of the Royal Astronomical Society* (1, 2020). DOI: 10.1093/mnras/staa3050 (cit. on p. 22).

[Wei+13] David H. Weinberg, Michael J. Mortonson, Daniel J. Eisenstein, Christopher Hirata, Adam G. Riess, and Eduardo Rozo. "Observational Probes of Cosmic Acceleration". In: *Physics Reports* 530.2 (10, 2013), pp. 87–255. DOI: 10.1016/j.physrep.2013.05.001 (cit. on p. 12).

[Won+19] Kenneth C. Wong, Sherry H. Suyu, Geoff C.-F. Chen, Cristian E. Rusu, Martin Millon, Dominique Sluse, Vivien Bonvin, Christopher D. Fassnacht, Stefan Taubenberger, Matthew W. Auger, Simon Birrer, James H. H. Chan, Frederic Courbin, Stefan Hilbert, Olga Tihhonova, Tommaso Treu, Adriano Agnello, Xuheng Ding, Inh Jee, Eiichiro Komatsu, Anowar J. Shajib, Alessandro Sonnenfeld, Roger D. Blandford, Léon V. E. Koopmans, Philip J. Marshall, and Georges Meylan. "H0LiCOW - XIII. A 2.4 per Cent Measurement of H0 from Lensed Quasars: 5.3sigma Tension between Early- and Late-Universe Probes". In: *Monthly Notices of the Royal Astronomical Society* 498 (1, 2019), pp. 1420–1439. DOI: 10.1093/mnras/stz3094 (cit. on p. 17).