43075-01 Shape modelling and analysis

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Exercise sheet 5: Building a GP Model

Introduction: 28. March 2023

Discussion: 4. April 2023

Introduction

The goal of this exercise is to build a Gaussian process model for the femur. For this first model we only use one example femur. The shape variability is defined by defining a Gaussian process, whose mean and covariance function are defined analytically. The derived model should be consistent with the prior information we have from exercise sheet 1 about the femur length and width. The model will later in the project be used to establish correspondence to the other training examples.

1. Building a GP Model

Write Scalismo code to build a Gaussian process model of the femur. As a reference, use the reference femur from the previous exercise. Use the code in tutorial 7 as a basis for your developments:

- https://scalismo.org/docs/Tutorials/tutorial07

Think about the following points and substantiate your answers with suitable experiments.

- What is the influence of the scale and smoothness parameters?
- What are reasonable parameters for the scale parameter s and the smoothness σ?
- How does the smoothness of a kernel (i.e. the parameter σ of the Gaussian kernel) influence the number of basis functions that are needed to approximate a given Gaussian process? Can you explain this?
- What are the units of these parameters (mm, mm^2, ...)?
- Would it make sense to combine kernels with different scale and smoothness? What would this achieve?
- If you answered yes to the previous question: Should the scale be large for the smooth kernels and small for the less smooth ones, or is it vice versa? Why?
- Think about other combination of kernels that could make sense for modelling femur shapes?
- How would you define a kernel that has more variance in the direction, which corresponds to the length of the femur?

2. Prior predictive checks

In order to understand if our model is useful for explaining femur shapes, we perform prior predictive checks. This means, we generate samples from the model and use them to assess if the samples have the characteristics that we expect from a femur model. From exercise sheet 3, we have already quite some information about the distribution of width and length of the femur. Generate samples from the model, compute the mean and variance of the mean and variance and compare them to the values you computed on the example femur bones in exercise sheet 3. Furthermore, create a scatterplot of length vs. width and compare it to the one you obtained in exercise sheet 3.

If the samples from your model are very different in length and width than the data you observed, change your model.

Note that at this stage it is normal that the samples show deformations that are anatomically implausible. The reason is that we have only incorporated smoothness assumptions in our model, and not used actual example data to learn the covariance function. It is, important, however, that the modelled shape variation should be able to represent all anatomically correct femur shapes. Think about why this is the case

3 Theory

Work through the theory parts of week 5 of the online course

- https://shapemodelling.cs.unibas.ch/ssm-course/week5/

You can add questions and topics that you would like to discuss in class to the Etherpad on Adam:

- https://adam.unibas.ch/goto_adam_xpdl_1553025.html

Note, you don't have to work through the practical parts of the online course. We will work through it together in class.