POKER & GAME THEORY

JULIAN CANALES

OVERVIEW OF PRESENTATION

What are we going to talk about?

INTRODUCTION TO POKER	What is poker and how is it played?
RELATIONSHIP WITH MATHEMATICS	What does poker have to do with mathematics?
UMBRELLAS OF POKER PLAYERS	How is this math actually used in poker?
DEEP DIVE ON THE UMBRELLAS	How can I learn this for myself?
KHUN POKER DEMO	Can we use computational power to play well?
THANKS & QUESTIONS	Questions?

01

WHAT'S POKER?

TEXAS HOLD'EM

THE INGREDIENTS

DECK

2-10 PLAYERS

POKER DEALER

CHIPS

GAME FORMAT


MENU
8
Q
7
8
A
4
4044
4044
2044
2044
2044
3044
4044
5044
DALLAS
FOLD
FOLD & SKIP
CALL
RAISE
POT $ 26352
$ 7000

POKER HANDS VALUE

WHAT ACTIONS CAN I DO?

WHEN IS THERE ACTION??

02 ♥

MATH??

MATH INBOUND

PROBABILITY

- Hand probability?
- What probability of winning do I need to bet some amount?
- What probability do I have of drawing a card I need given info?

MATH STATS

- Given player data, can we estimate true win rate?
- If an unknown player steps onto the table, what skill level are they most likely at?

GAME THEORY

- What is the optimal way to play this game to maximize expected return?
- What if others aren't playing optimally?

03 ♥

UMBRELLAS OF POKER

WHAT ARE THEY?

EXPLOITIVE	GAME THEORY OPTIMAL
Main goal is to maximization expectation against opponents' hand Calculates what the best sequence of actions are for each player… if another play makes a mistake, take advantage of it immediately Assumes that other players are not playing optimally and are indeed exploitable	Typically most used when it is assumed other players are more skilled or unexploitable Main goal is to make your own actions unexploitable so this umbrella typically has lower profits There is no regard to what other players' actions or tells are

04

DEEP DIVE OF UMBRELLAS

EXPLOITIVE PLAY

1. **Made Hand**
2. **Pot Odds**
3. Drawn Hand
4. Implied Pot Odds
5. Hand Reading
6. Creating probability distributions on players

MADE HANDS & POT ODDS

The simplest way to maximize EV

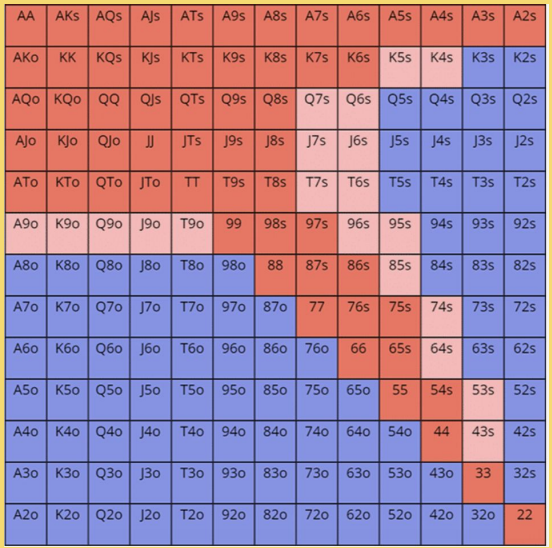
Pot Odds - ratio between size of the pot if you were to call and the size of the bet you are facing

QUICK POT ODDS EXAMPLE (PRE-FLOP)

- Playing 3 player $5/$10 Texas Hold'Em
- Third player raises to $30.
- Small blind folds and action goes to you (Big blind)... you have 7 Diamond, 5 Diamond
- Calculate pot odds
  - Size of final pot?
    - $5 (SB) + $10 (BB) + $30 (Raise) + $20 (Call) = $65
  - Divide size of call (bet you are facing) by pot to find pot odds
    - $20 / $65 = 0.308
- Pot odds tells us what percentage of the time we need to win to place the facing bet

Pot Odds - ratio between size of the pot if you were to call and the size of the bet you are facing

BUT HOW DO I KNOW THE PERCENTAGE OF TIMES MY HAND WOULD WIN?



AA	AKs	AQs	AJs	ATs	A9s	A8s	A7s	A6s	A5s	A4s	A3s	A2s
AKo	KK	KQs	KJs	KTs	K9s	K8s	K7s	K6s	K5s	K4s	K3s	K2s
AQo	KQo	QQ	QJs	QTs	Q9s	Q8s	Q7s	Q6s	Q5s	Q4s	Q3s	Q2s
AJo	KJo	QJo	JJ	JTs	J9s	J8s	J7s	J6s	J5s	J4s	J3s	J2s
ATo	KTo	QTo	JTo	TT	T9s	T8s	T7s	T6s	T5s	T4s	T3s	T2s
A9o	K9o	Q9o	J9o	T9o	99	98s	97s	96s	95s	94s	93s	92s
A8o	K8o	Q8o	J8o	T8o	98o	88	87s	86s	85s	84s	83s	82s
A7o	K7o	Q7o	J7o	T7o	97o	87o	77	76s	75s	74s	73s	72s
A6o	K6o	Q6o	J6o	T6o	96o	86o	76o	66	65s	64s	63s	62s
A5o	K5o	Q5o	J5o	T5o	95o	85o	75o	65o	55	54s	53s	52s
A4o	K4o	Q4o	J4o	T4o	94o	84o	74o	64o	54o	44	43s	42s
A3o	K3o	Q3o	J3o	T3o	93o	83o	73o	63o	53o	43o	33	32s
A2o	K2o	Q2o	J2o	T2o	92o	82o	72o	62o	52o	42o	32o	22

In order to find a hand's equity, you need to calculate the probability your hand wins against another player's hand. But what's the opponent's hand? This is where something called a pre-flop chart comes into the picture. This chart tells us what action to do depending on the pre-flop cards dealt. Red is raise, pink is optional raise, blue is fold. Since the opponent raised, we can assume they have a card combination in the red (and if they don't they made a bad play anyways). Then you have to calculate the probability your hand beats their hand's probability distribution.

37%

Is your hand's equity... that's more than 30%....CALL!

DRAWN HAND & IMPLIED ODDS & HAND READING

Drawn hands and implied odds are slightly more advanced tactics that incorporate bayesian probability but are relatively the same idea... which is maximize EV using probability of winning.

Hand reading usually involves using something similar to the chart shown earlier as it gives insight on what hand opponents are playing as. If they are misplaying a hand, it will be revealed in showdown and you can exploit that

There are three types of poor poker players that are easy to exploit:

1. Maniacs - loose and aggressive
2. Rocks - tight and passive
3. Calling stations - call on everything

Takes too long to cover here

GAME THEORY OPTIMAL (GTO) PLAY

1. Strategic game
2. Payoff matrix & function
3. Nash-equilibrium
4. Bayesian games
5. Mixed-Nash equilibrium
6. Poker???

HOW CAN WE SIMULATE POKER?

START SMALL AND BUILD UP?

STRATEGIC GAMES

PRELIMINARY DEFINITION

Actions

Players

Preferences

PAYOFF FUNCTION & MATRIX

PRELIMINARY DEFINITION

A payoff function is a way to assign ordering to our preferences. So our greatest preference has the highest ordering and vice versa.

A payoff matrix is a method of visualizing combinations of the decisions players make and the corresponding payoffs for each

Ex. Prisoner's Dilemma

- A game of snitching or not snitching to the cops

$$u_1(Fink, Quiet) > u_1(Quiet, Quiet) > u_1(Fink, Fink) > u_1(Quiet, Fink).$$

		Suspect 2	
		Quiet	*Fink*
Suspect 1	*Quiet*	2, 2	0, 3
	Fink	3, 0	1, 1

PURE NASH STRATEGY & EQUILIBRIUM

PRELIMINARY DEFINITION

A nash equilibrium is where there exists an action profile for each player that will give them no advantage from switching

In our example, the Prisoner's Dilemma, our nash equilibrium is both players choose fink (aka snitch).

Ex. Prisoner's Dilemma

- A game of snitching or not snitching to the cops

$$u_1(Fink, Quiet) > u_1(Quiet, Quiet) > u_1(Fink, Fink) > u_1(Quiet, Fink).$$

		Suspect 2	
		Quiet	*Fink*
Suspect 1	*Quiet*	2, 2	0, 3
	Fink	3, 0	1, 1

BUT POKER IS STILL MORE COMPLEX...

BAYESIAN GAME

PRELIMINARY DEFINITION

Loosely speaking, a bayesian game is a strategic game where each player has incomplete information about the environment and must still take action. Some relevant info

- Possible routes game can go are called **game states**
- Players' action gives others **signals** on their knowledge of the game state
- These signals and states are consistent with one another (players are playing correctly) and players are labeled as their signal's type

We may define a Nash equilibrium of a Bayesian game to be a Nash equilibrium of a strategic game in which each player type is considered one independent player in the Bayesian game.

MIXED-NASH STRATEGY & EQUILIBRIUM

PRELIMINARY DEFINITION

In contrast to pure nash strategy and equilibrium, where there is a definite action profile that will be found and the equilibrium set to, a mixed nash strategy is a game where the nash equilibrium lives in a probability distribution

HOW DOES THIS LOOK LIKE IN ACTION AND HOW DOES IT RELATE TO POKER?

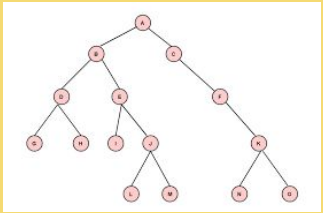
05 ♥

KUHN POKER DEMO

WHAT IS KHUN POKER?

COUNTERFACTUAL REGRET MINIMIZATION TO FIND MIXED-NASH EQUILIBRIUM

1. Make decisions that will minimize regret
2. In order to solve this, we need computational power that will go through many possible trajectories of actions. This should reinforce our decisions and converge to the equilibrium
3. Nodes simulate chance and edges simulate decisions/ actions. Terminal nodes simulate the end of the game and can measure the reward, if another trajectory gave a better reward, update our decision probabilities
4. This would blow up for real poker, so we do it for kuhn poker! If you wanted to do this for real poker you would need many abstractions and plenty computational power



I IMPLEMENTED THE ALGORITHM AND GOT THESE RESULTS

```
Average game value: -0.05615804729589458
  1: [0.7294477322765173, 0.2705522677234826]
 1b: [0.9999985012035335, 1.4987964664374506E-6]
 1p: [0.667935980183777, 0.332064019816223]
1pb: [0.9999989740245846, 1.025975415456584E-6]
  2: [0.9999627967223111, 3.7203277688802016E-5]
 2b: [0.6663126280288155, 0.3336873719711845]
 2p: [0.9999679747961645, 3.2025203835418467E-5]
2pb: [0.3926467409295936, 0.6073532590704063]
  3: [0.18014492494104994, 0.8198550750589501]
 3b: [1.500024000384006E-6, 0.9999984999759997]
 3p: [1.500024000384006E-6, 0.9999984999759997]
3pb: [4.1736360234077406E-6, 0.9999958263639765]
```

```
Average game value: -0.05695550245432511
  1: [0.8039246712603392, 0.19607532873966088]
 1b: [0.9999984982685036, 1.5017314964153668E-6]
 1p: [0.6672413769766591, 0.33275862302334086]
1pb: [0.9999990680264522, 9.319735477247937E-7]
  2: [0.9999739315853066, 2.606841469343382E-5]
 2b: [0.6656856606715507, 0.33431433932844934]
 2p: [0.9999685054485574, 3.149455144260042E-5]
2pb: [0.46337962078621664, 0.5366203792137834]
  3: [0.39299253868320483, 0.6070074613167952]
 3b: [1.4985314391895943E-6, 0.9999985014685608]
 3p: [1.4985314391895943E-6, 0.9999985014685608]
3pb: [1.9092369679614093E-6, 0.999998090763032]
```

```
Average game value: -0.055765266649995746
  1: [0.7818665884546412, 0.21813341154535887]
 1b: [0.999998499944498, 1.5000555020535759E-6]
 1p: [0.6667471753613403, 0.3332528246386598]
1pb: [0.9999990405920482, 9.594079517765871E-7]
  2: [0.9999518079149039, 4.8192085096224814E-5]
 2b: [0.6640099790145666, 0.33599002098543346]
 2p: [0.9999921224634333, 7.877536566774501E-6]
2pb: [0.44786912583897565, 0.5521308741610244]
  3: [0.34702641075404245, 0.6529735892459576]
 3b: [1.4994616932521224E-6, 0.9999985005383067]
 3p: [2.9989233865042448E-6, 0.9999970010766135]
3pb: [2.1630833691668903E-6, 0.9999978369166308]
```

LOOKS LIKE MIXED NASH

06

THANKS...
QUESTIONS?