Estimating the conditional variance by local linear regression

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Aircraft Data

We are using *Aircraft data*, from the R library `sm`. These data record six characteristics of aircraft designs which appeared during the twentieth century.

```
Yr:     year of first manufacture
Period: a code to indicate one of three broad time periods
Power:  total engine power (kW)
Span:   wing span (m)
Length: length (m)
Weight: maximum take-off weight (kg)
Speed:  maximum speed (km/h)
Range:  range (km)
```

We transform data taken logs (except `Yr` and `Period`): `lgPower`, ..., `lgRange`.

Go to `R` and charge the library `sm`:

```
library(sm)
```

Now upload the data:

```
data(aircraft)
help(aircraft)
attach(aircraft)
lgPower <- log(Power)
lgSpan <- log(Span)
lgLength <- log(Length)
lgWeight <- log(Weight)
lgSpeed <- log(Speed)
lgRange <- log(Range)
```

Estimating the conditional variance

Consider the heteroscedastic regression model

$$Y = m(x) + \sigma(x)\varepsilon = m(x) + \epsilon,$$

where $\mathbb{E}(\varepsilon) = 0$, $\mathrm{Var}(\varepsilon) = 1$ and $\sigma^2(x)$ is an unknown function that gives the conditional variance of Y given that the explanatory variable is equal to x.

Let us define $Z = \log((Y - m(x))^2) = \log \epsilon^2$ and $\delta = \log \varepsilon^2$. Then

$$Z = \log \sigma^2(x) + \delta,$$

and $\delta = \log \varepsilon^2$ is a random variable with expected value close to 0 (observe that $\mathbb{E}(\log \varepsilon^2)$ is close to $\log \mathbb{E}(\varepsilon^2) = \log \mathrm{Var}(\varepsilon) = \log 1 = 0$, at least when $\mathrm{Var}(\varepsilon^2)$ is small) taking the role of *noise* in the regression of Z against x (that is, Z is the response variable and x is the predicting variable).

Given that the values of ϵ_i^2 are not observable, a way to estimate the function $\sigma^2(x)$ is as follows:

1. Fit a nonparametric regression to data (x_i, y_i) and save the estimated values $\hat{m}(x_i)$.

2. Transform the estimated residuals $\hat{\epsilon}_i = y_i - \hat{m}(x_i)$:

$$z_i = \log \epsilon_i^2 = \log((y_i - \hat{m}(x_i))^2).$$

3. Fit a nonparametric regression to data (x_i, z_i) and call the estimated function $\hat{q}(x)$. Observe that $\hat{q}(x)$ is an estimate of $\log \sigma^2(x)$.

4. Estimate $\sigma^2(x)$ by

$$\hat{\sigma}^2(x) = e^{\hat{q}(x)}.$$

Apply this procedure to estimate the conditional variance of `lgWeigth` (variable Y) given `Yr` (variable x). Draw a graphic of $\hat{\epsilon}_i^2$ against x_i and superimpose the estimated function $\hat{\sigma}^2(x)$. Lastly draw the function $\hat{m}(x)$ and superimpose the bands $\hat{m}(x) \pm 1{,}96\hat{\sigma}(x)$.

Attention: Do the work twice:

- First, use the function `loc.pol.reg` that you can find in ATENEA and choose all the bandwidth values you need by leave-one-out cross-validation (you have not to program it again! Just look for the right function in the `*.Rmd` files you can find in ATENEA)

- Second, use the function `sm.regression` from library `sm` and choose all the bandwidth values you need by *direct plug-in* (use the function `dpill` from the library `KernSmooth`).