Problem Set 1

FIT 4012 – Advanced Topics in Computational Science

12 September 2015 – Prepared by Julian García

The **learning objectives** of this assignment are:

- Analyse results from an agent-based model using the Montecarlo approach.
- Approximate results from agent-based models using Markov chain theory.
- Apply game-theoretical thinking to modelling problems.

A. The simplified Schelling model

In the simplified Schelling model agents live on a cycle of finite size n. Agents can be of two types, say 0 and 1. There are no empty positions, thus, a cycle of size n also implies n agents.[1] In this simplified version there are no thresholds. Instead, an agent is "happy" if at least one of her neighbours is of the same type.

Time is discrete, so $t = 1, 2, 3, \ldots$ The dynamics go as follows: At each time-step, two individuals (residing in different slots) are matched to potentially trade places. This *matching* procedure is randomly uniform. Each encounter may result in the agents trading places or retaining their position. Agents will agree to trade places if at least one of the two benefits and none of the two is worse off after the swap.

[1] This is a big difference with the standard Schelling model in which vacant spots are a fundamental feature.

1. Specify explicitly the transition Matrix of the MC for $n = 4$. Explain how the transition probabilities are computed and how the states are labeled.

2. Show the canonical form of the Markov chain for $n = 4$. Make sure to specify clearly how states are re-labeled or re-ordered if necessary.

3. Using an agent-based model and montecarlo simulations show how the absorption time varies with n.

4. Numerically approximate the absorption times for $n = 4$ and $n = 5$ and show that they agree with the montecarlo simulations.[2]

[2] For the numerical calculations in the case of n=5, it may not be necessary to specify a full transition matrix.

B. Mistakes

We now turn to a model in which agent may swap places "by mistake". This means with a probability ϵ they will fail to swap places when they intend to, or will fail to stay put when they should. This small change results in a new chain that is ergodic.[3]

[3] For numerical and simulation results assume a small ϵ

1. Specify the full transition matrix for $n = 4$, compute the stationary distribution numerically and show that it is in agreement with Montecarlo simulations. What can you conclude from this model?

2. Repeat this analysis in the case where agents do not live on a cycle, but on a simple linear structure; i.e., the agents on both ends only have one neighbour.

C. Discussion

Discuss reasonable extensions of this model that would allow for richer, and perhaps more realistic dynamics, while keeping tractability at hand.

D. A routing game

50 agents are tasked with routing one packet each through a network from vertex S to vertex T. There is no central control, so each agent is autonomous and strives to minimize total latency for each packet sent through. The structure of the network is presented in Figure one. The edges $S - A$, and $B - T$ have a latency equal to the number of packets going through the edges. Edges $S - B$ and $A - T$ have constant latency. For example, a single packet going through a route $S - A - T$ has latency 51. If 20 agents are using that very same route, each will experience a latency of 70, and so on.

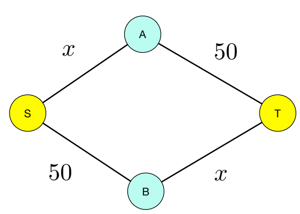

Figure 1: A network

1. Use a game theoretical argument to predict what each agent should do. Compute the average latency experienced by each agent for your prediction.

2. A new high-sped technology is introduced that allows for very low-latency traffic. This technology is expensive, so an edge is afforded and the network is modified as specified in Figure 2. How does your prediction change, and what is the gain experienced by each agent in this new configuration?

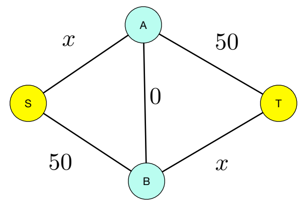

Figure 2: Network after new technology is introduced

Notes:

- Section A and is worth 30 marks, Section B is worth 40 marks, Section C is worth 10 marks. Section D is worth 20 marks.
- Provide pseudocode and properly documented source code (with build instructions, if necessary) for any computer programs you may have used to solve this assignment.
- The assignment is due by September 13, 2015. Please submit your assignment via moodle in PDF format, and include a zip file with your source code and/or executable files.
- This problem set is worth 30% of your final mark.