Quicksort

Julián García

Sorting

<u>Input:</u>

A sequence of *n* elements

$$\langle a_1, a_2, \ldots, a_n \rangle$$

<u>Output:</u>

A permutation of the input sequence

$$\langle a'_1, a'_2, \ldots, a'_n \rangle$$

do it efficiently.

such that $a'_1 \leq a'_2 \leq \ldots \leq a'_n$

Quicksort

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- Divide and conquer algorithm.

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- Recursive

Quicksort

- <u>Divide and conquer</u> algorithm.
- <u>Recursive</u>
- Sorts <u>in place</u> (no extra arrays!).

Quicksort

- Divide and conquer algorithm.
- Recursive
- Sorts in place (no extra arrays!).
- Popular: Top-10 algorithms 20th century (SIAM).

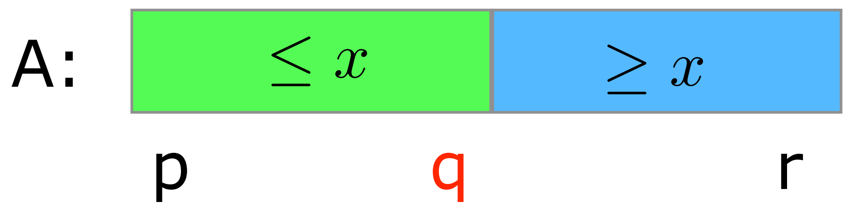
Quicksort strategy

<u>Input:</u>

Quicksort strategy

Input:

Divide:



(partition step)

Quicksort strategy

<u>Input:</u>

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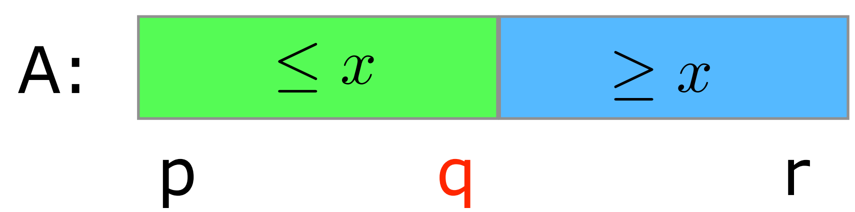
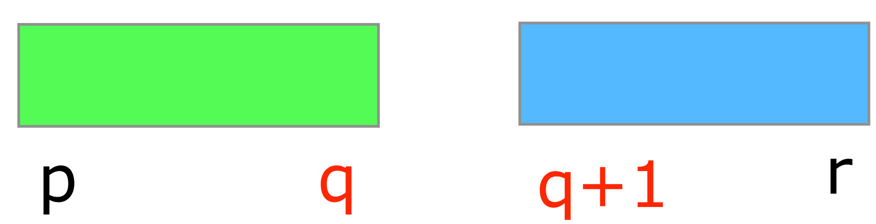


(partition step)

<u>Conquer:</u> Recursively solve two smaller problems.



Quicksort strategy

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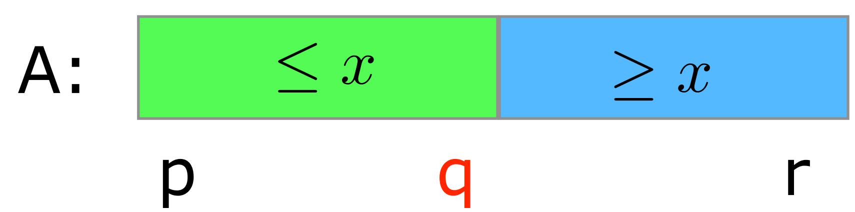

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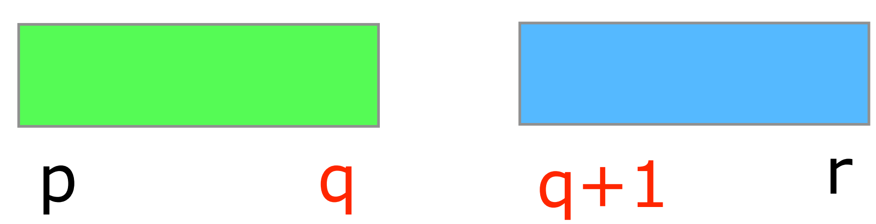
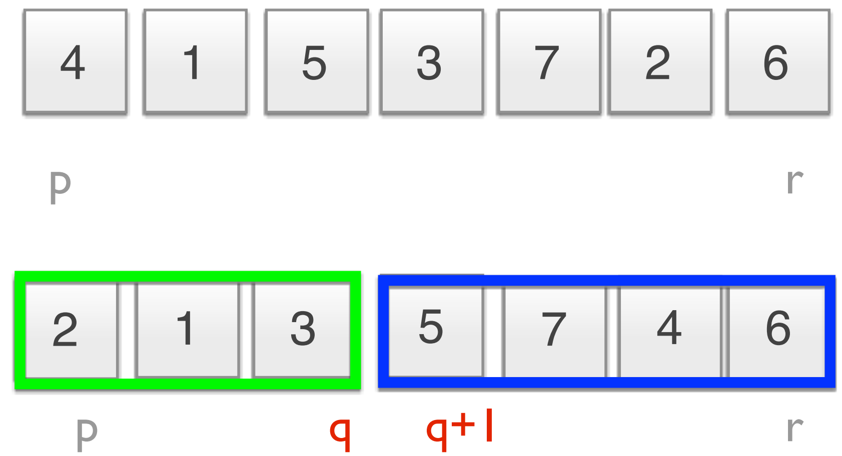
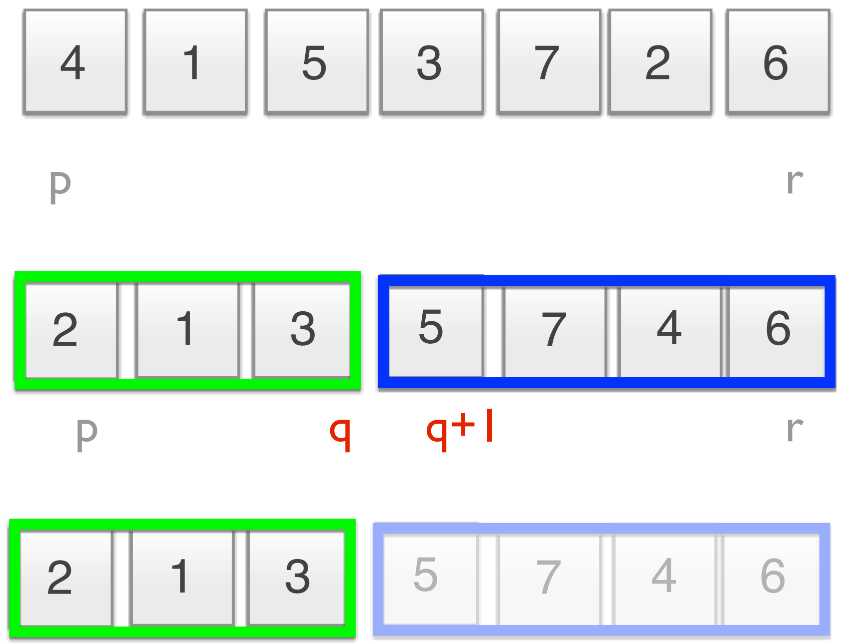


<u>Combine:</u> In-place! so we are done.

4	1	5	3	7	2	6
p						r



(partition step)



(partition step)

sort left

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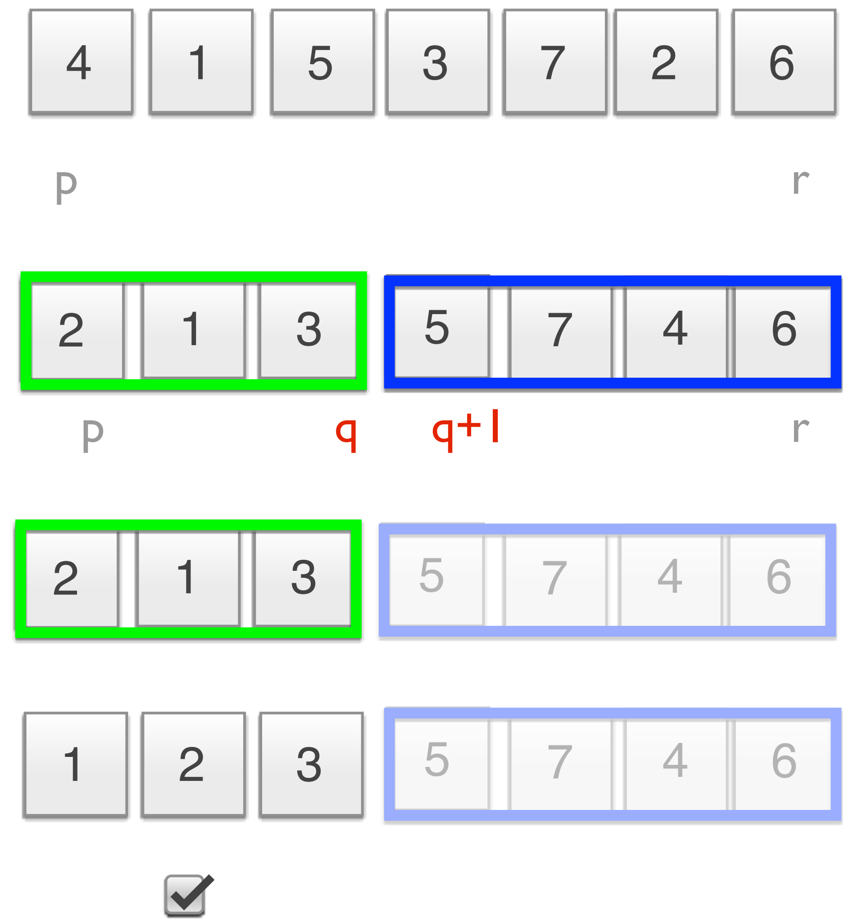
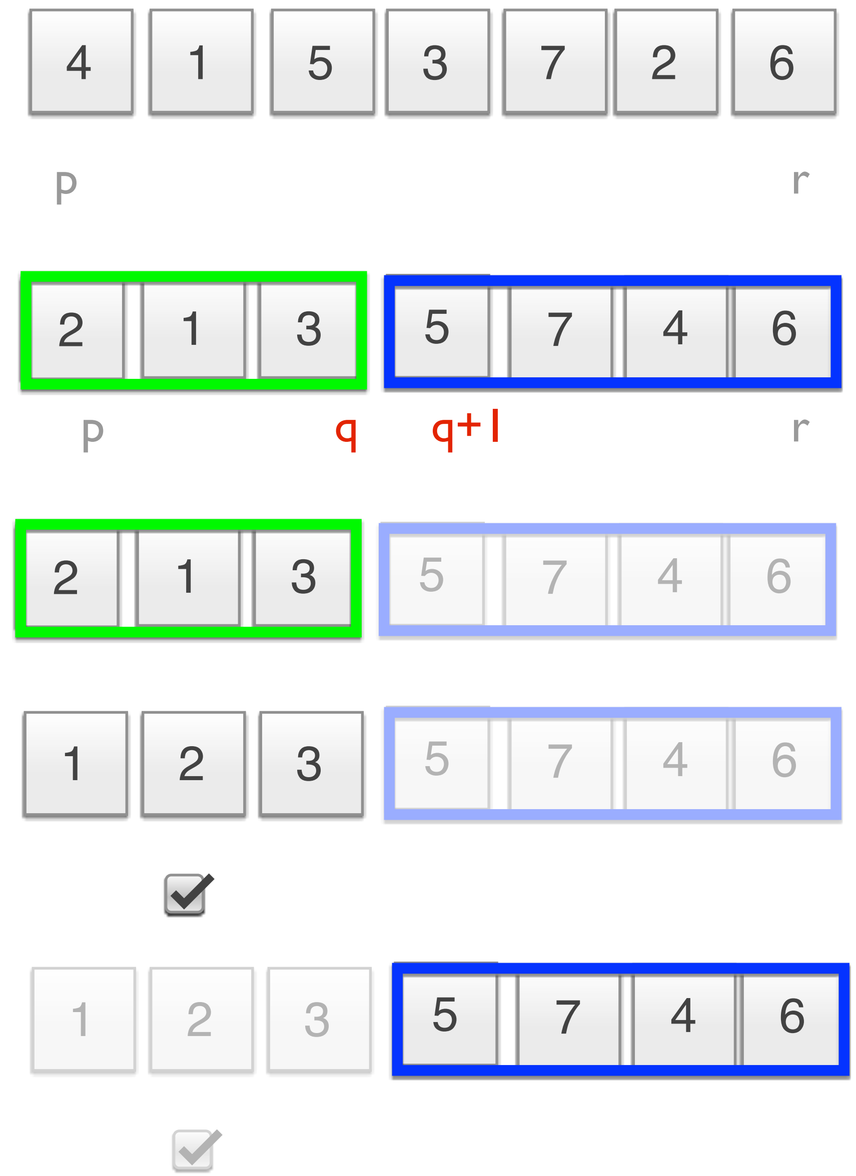
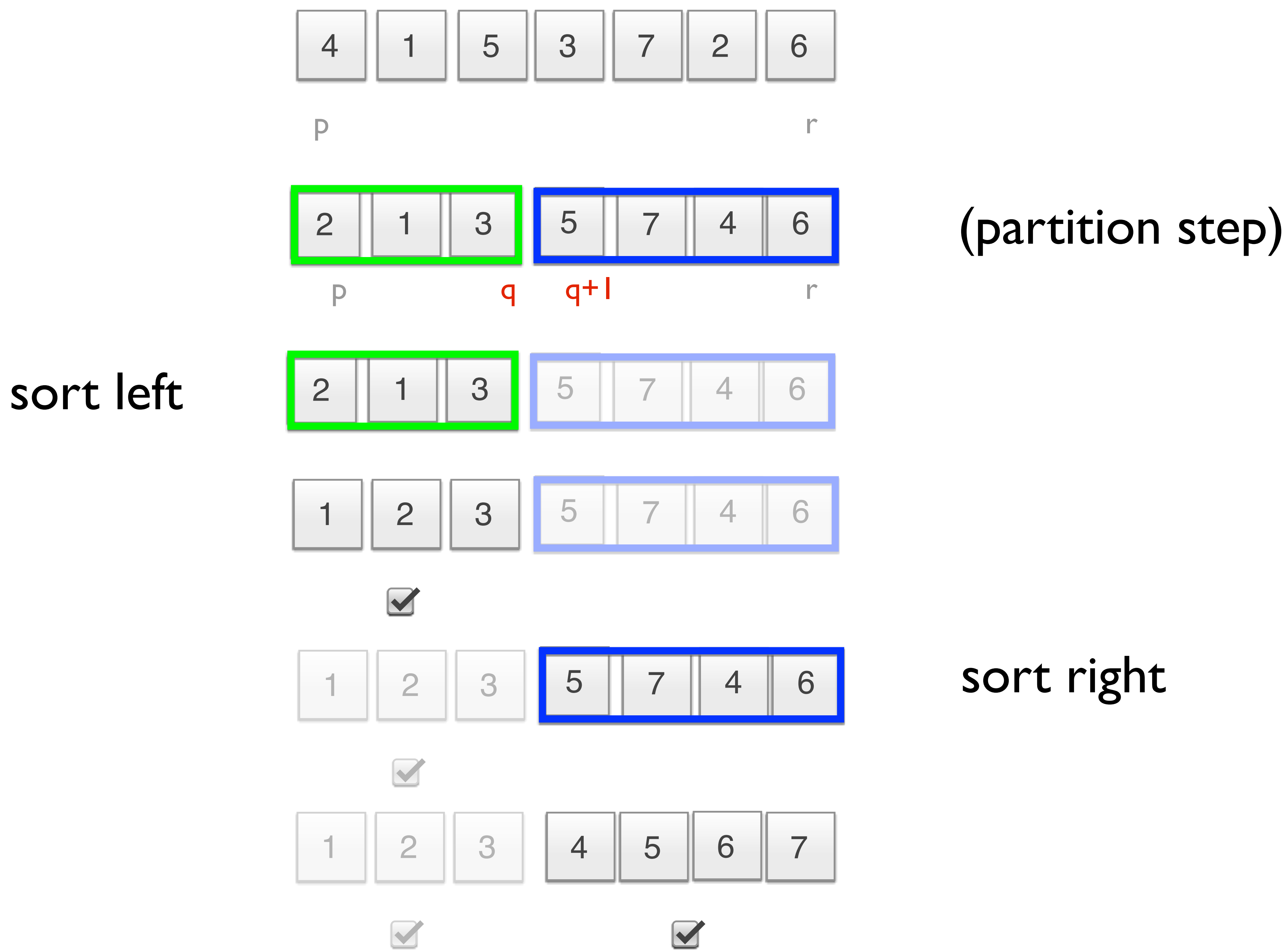


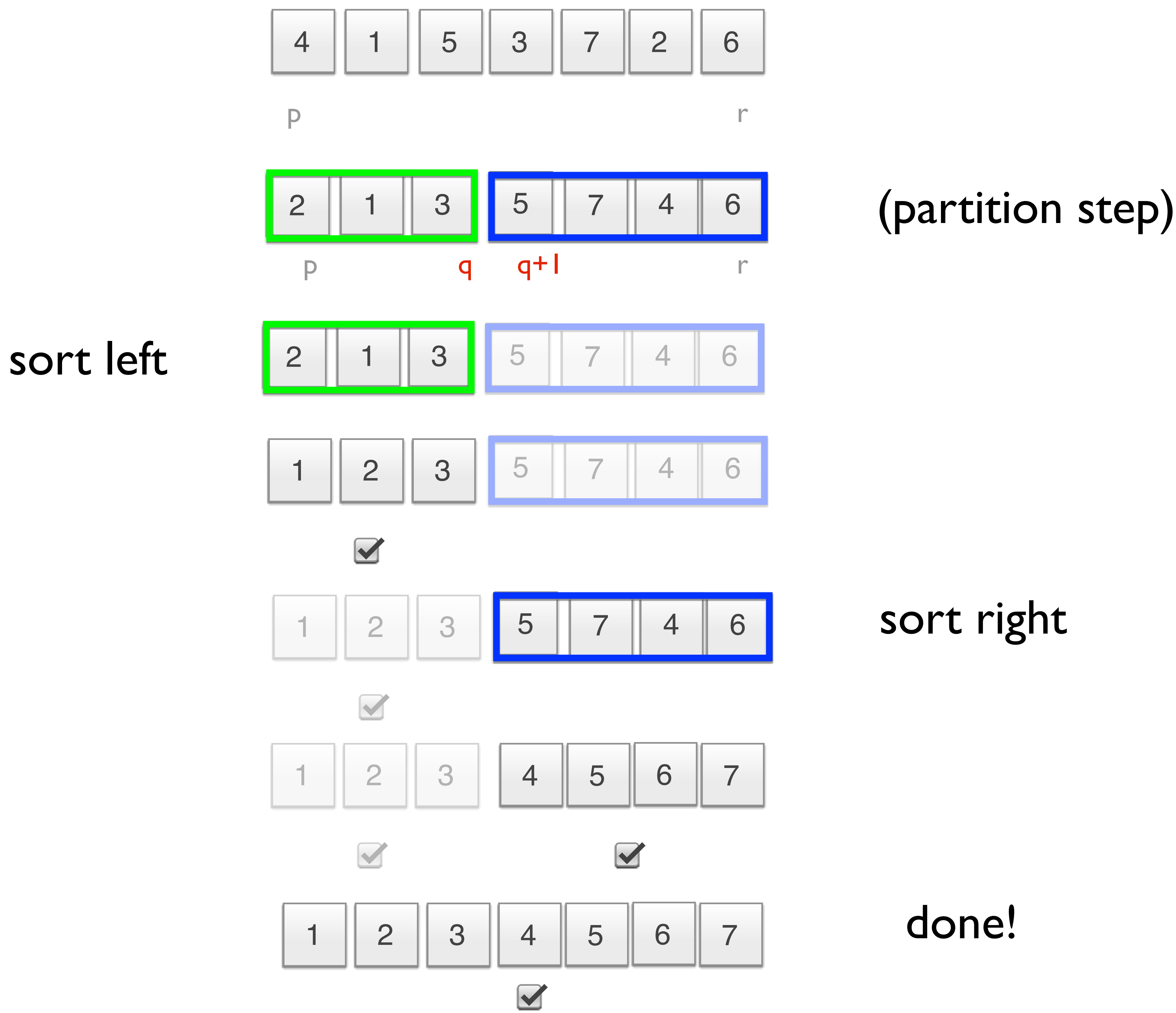


(partition step)

sort left

sort right

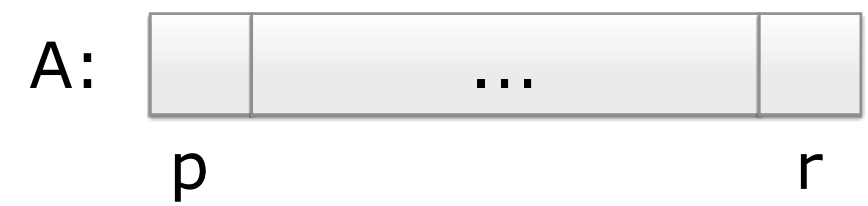

4 1 5 3 7 2 6
p r
2 1 3 5 7 4 6
(partition step)
p q q+1 r
sort left
2 1 3 5 7 4 6
1 2 3 5 7 4 6
1 2 3 5 7 4 6
sort right
1 2 3 4 5 6 7


4 1 5 3 7 2 6
p r
2 1 3 5 7 4 6
(partition step)
p q q+1 r
sort left
2 1 3 5 7 4 6
1 2 3 5 7 4 6
1 2 3 5 7 4 6
sort right
1 2 3 4 5 6 7
1 2 3 4 5 6 7
done!

```
QUICKSORT(A, p, r)
if p < r
        q = PARTITION(A, p, r)
        QUICKSORT(A, p, q)
        QUICKSORT(A, q + 1, r)
```

QUICKSORT(A, p, r)

if $p < r$
  $q = \text{PARTITION}(A, p, r)$
  QUICKSORT(A, p, q)
  QUICKSORT$(A, q + 1, r)$



Is it non-trivial?
(exit door)

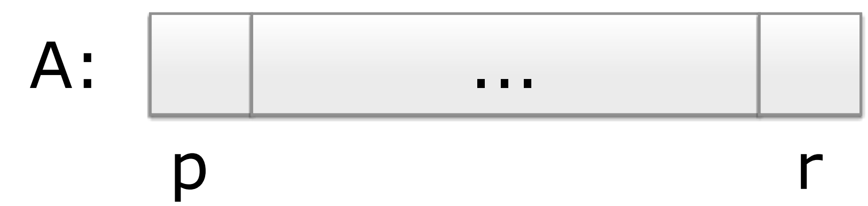
$\textsc{Quicksort}(A, p, r)$

if $p < r$

$\quad q = \textsc{Partition}(A, p, r)$

$\quad \textsc{Quicksort}(A, p, q)$

$\quad \textsc{Quicksort}(A, q + 1, r)$



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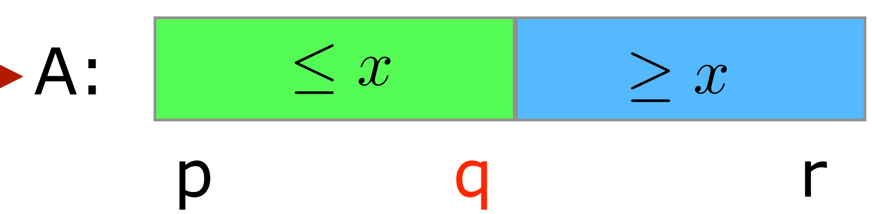

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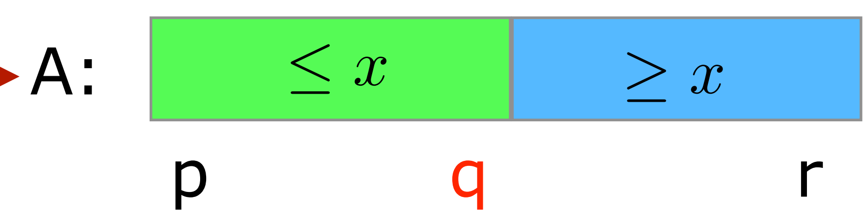

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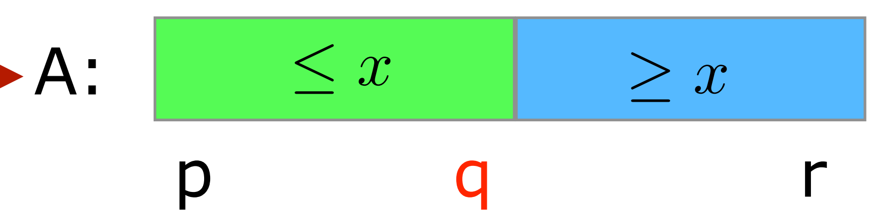
    q = Partition(A, p, r)
    Quicksort(A, p, q)
    Quicksort(A, q + 1, r)
```



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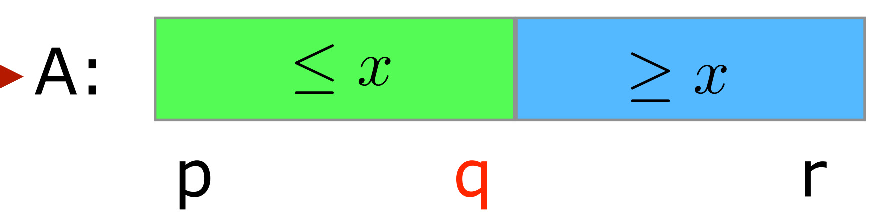

partition really does all the work!

$\textsc{Partition}(A, p, r)$

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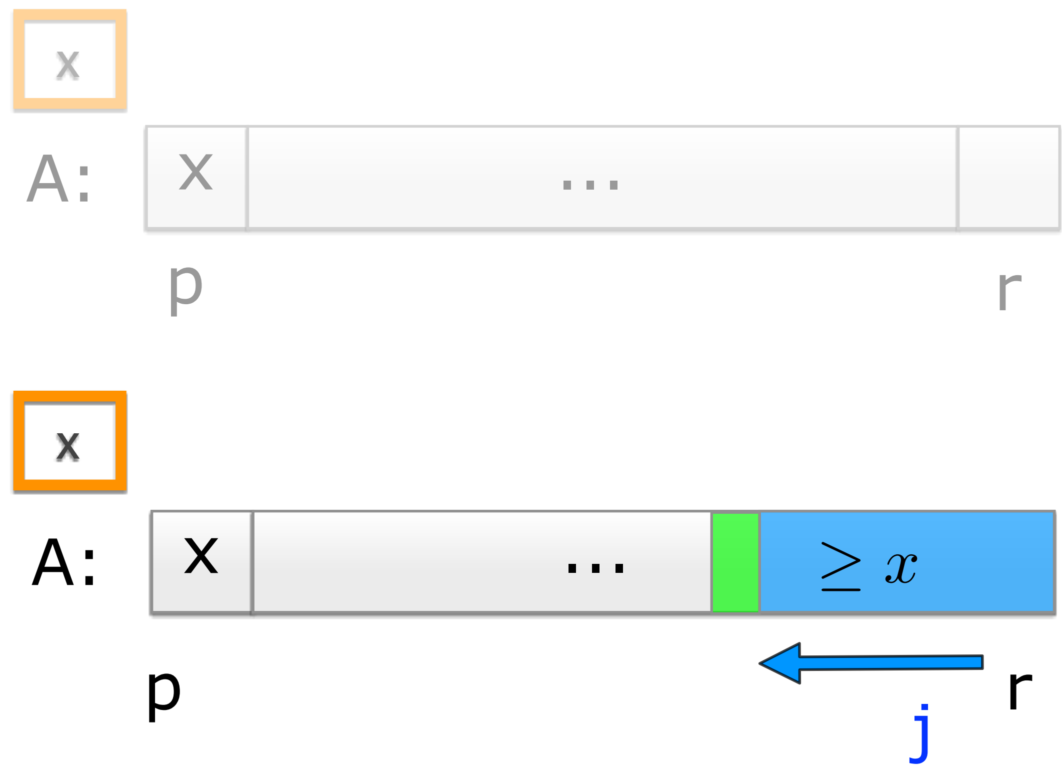
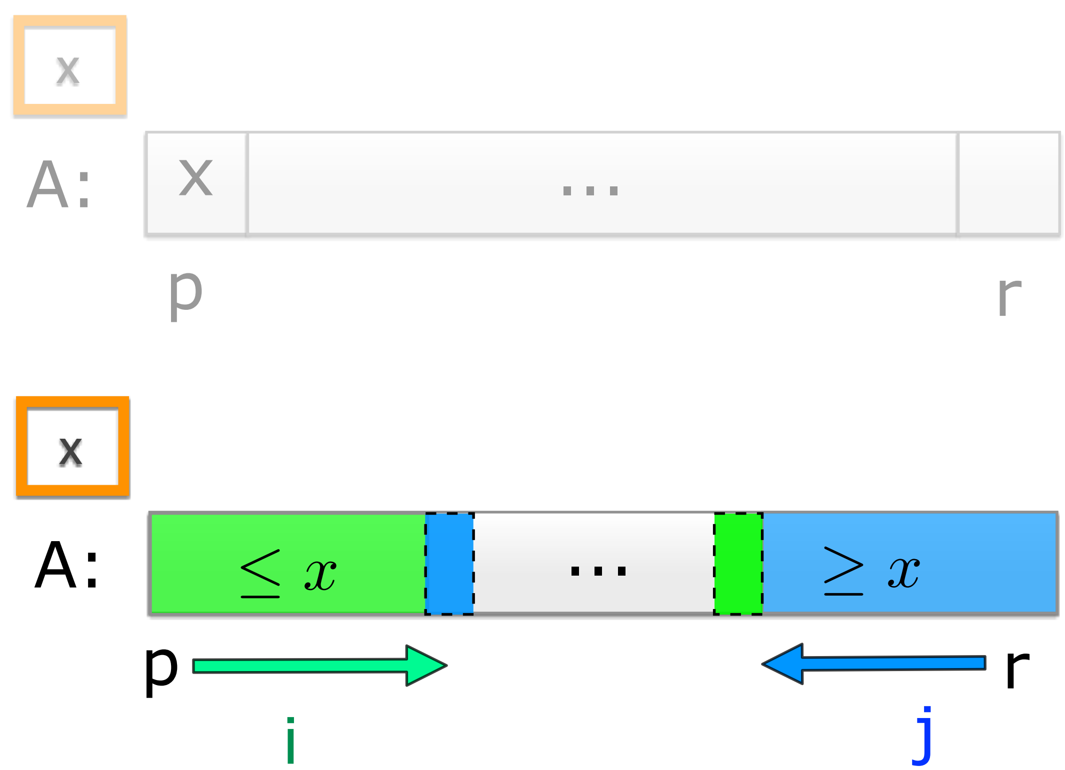
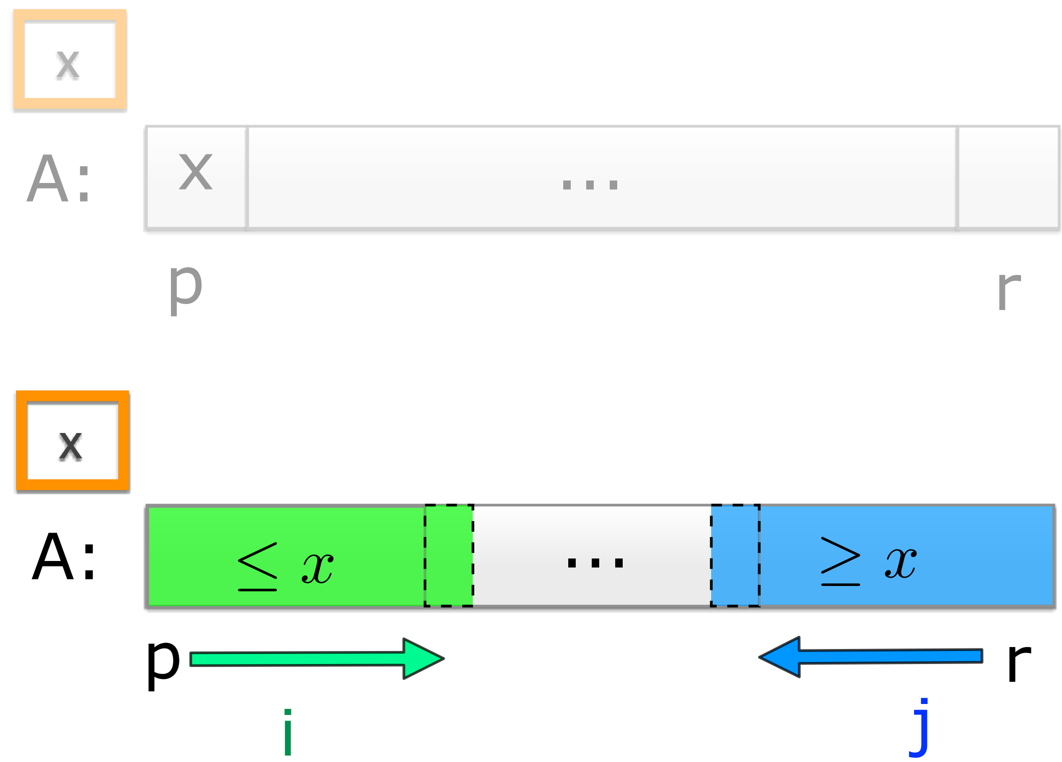


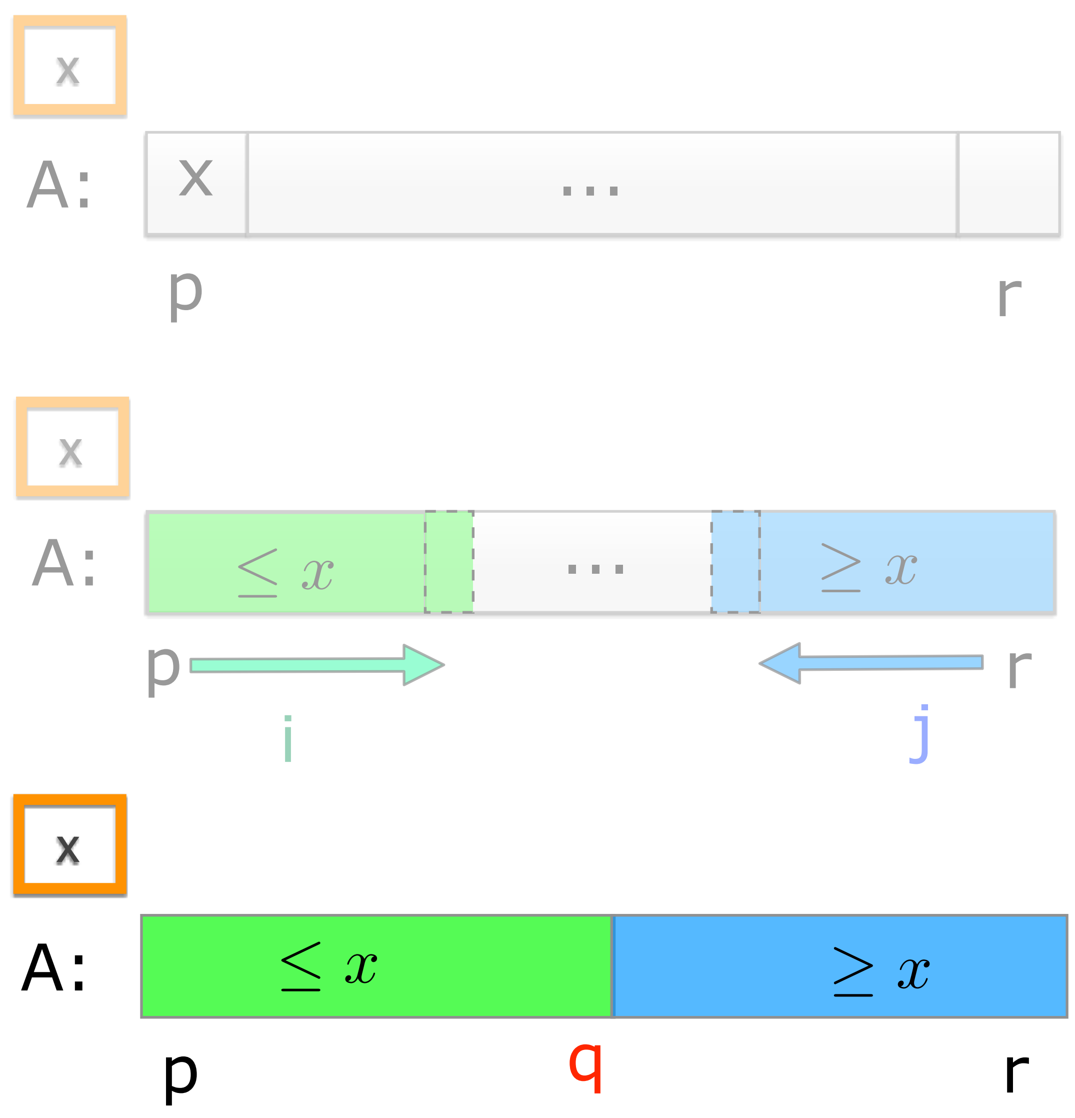
$\textsc{Partition}(A, p, r)$



$\textsc{Partition}(A, p, r)$

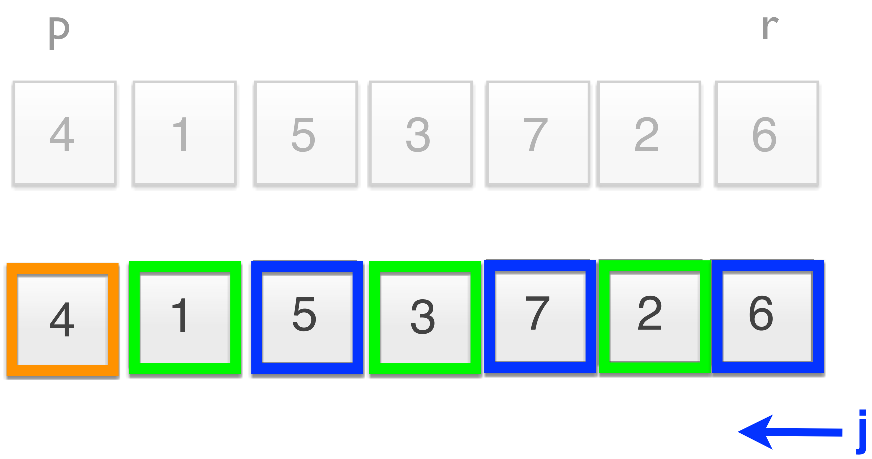


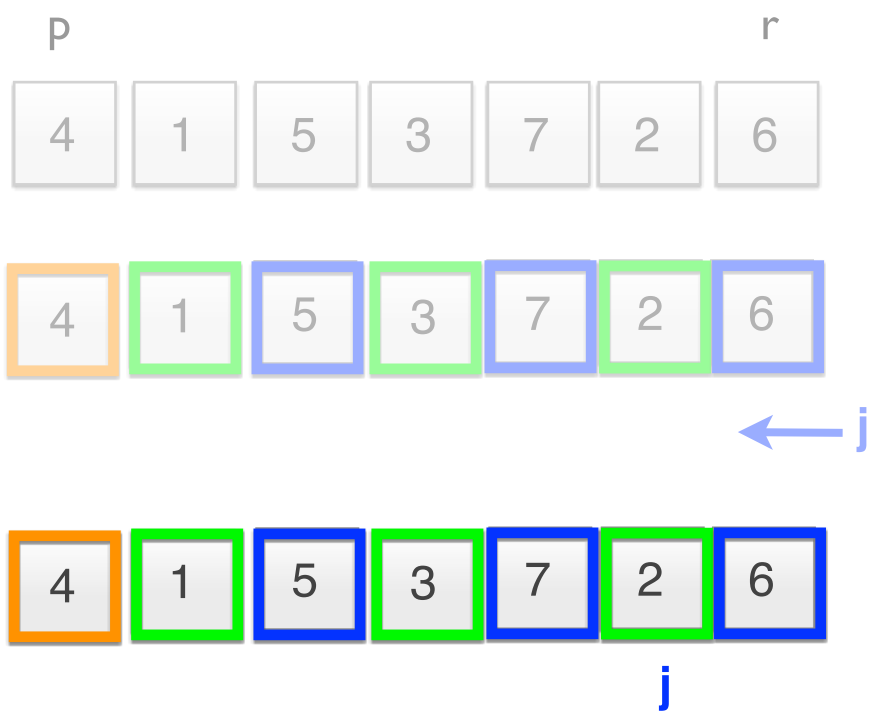
$\textsc{Partition}(A, p, r)$

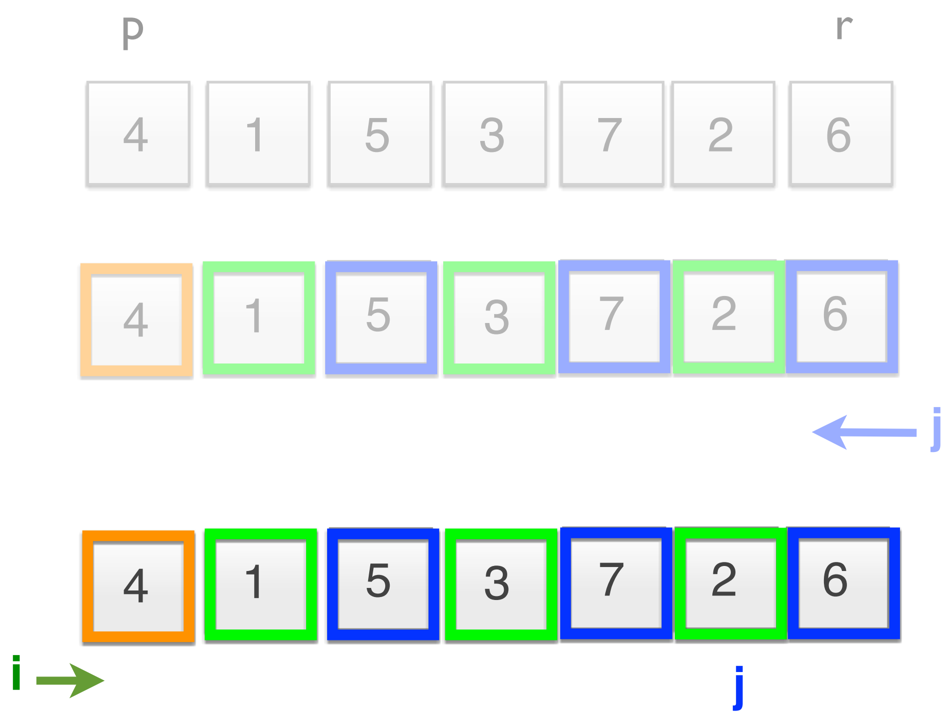


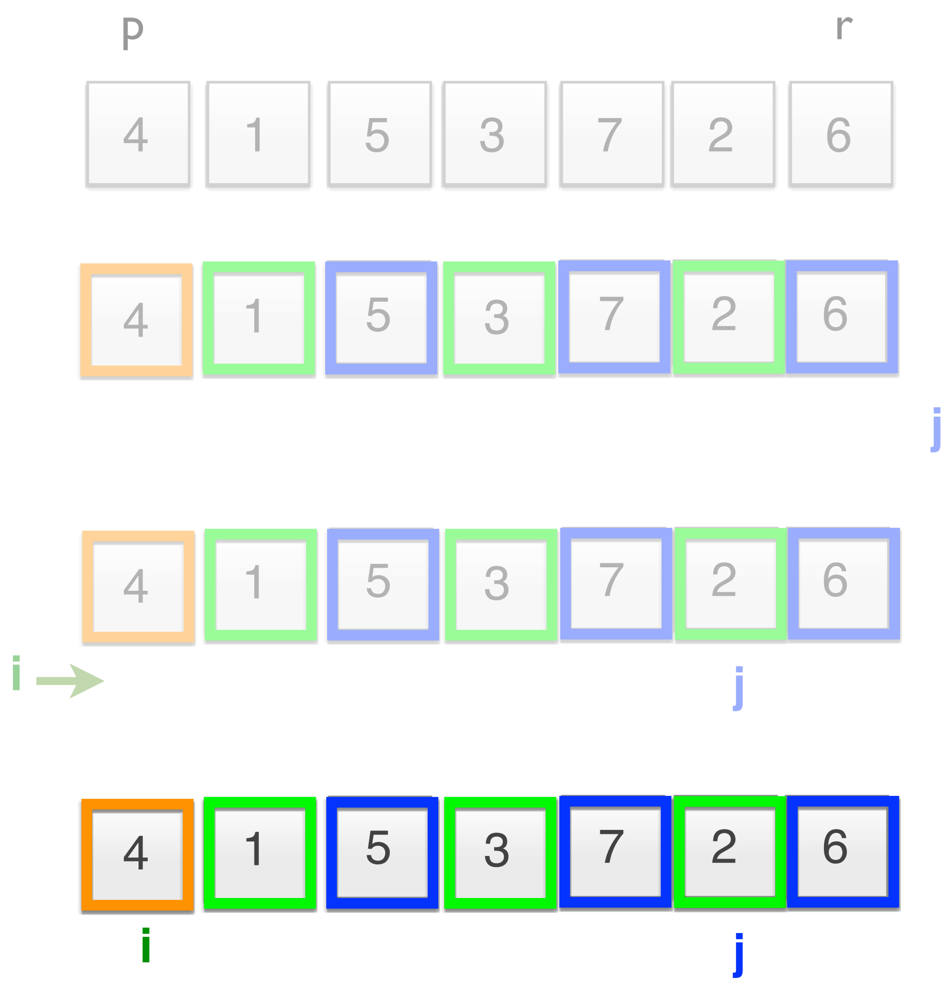

p
r
4 1 5 3 7 2 6

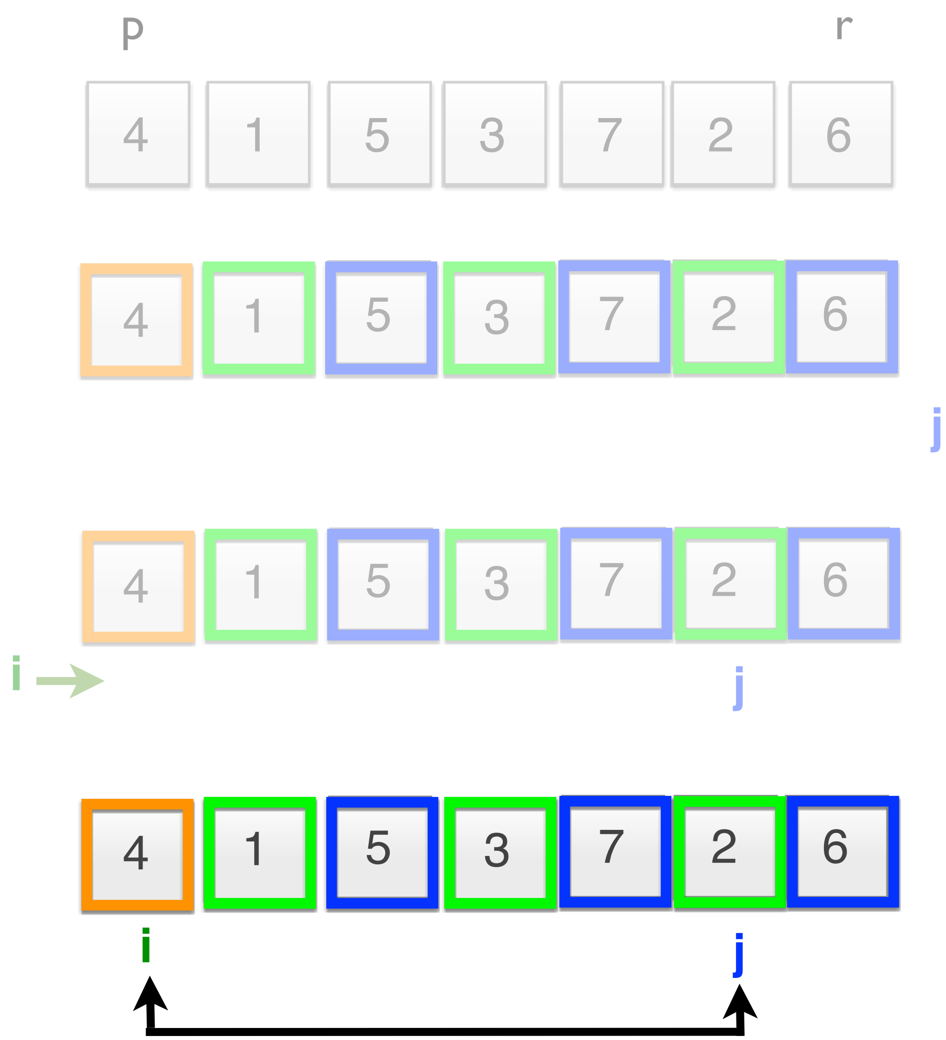

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6

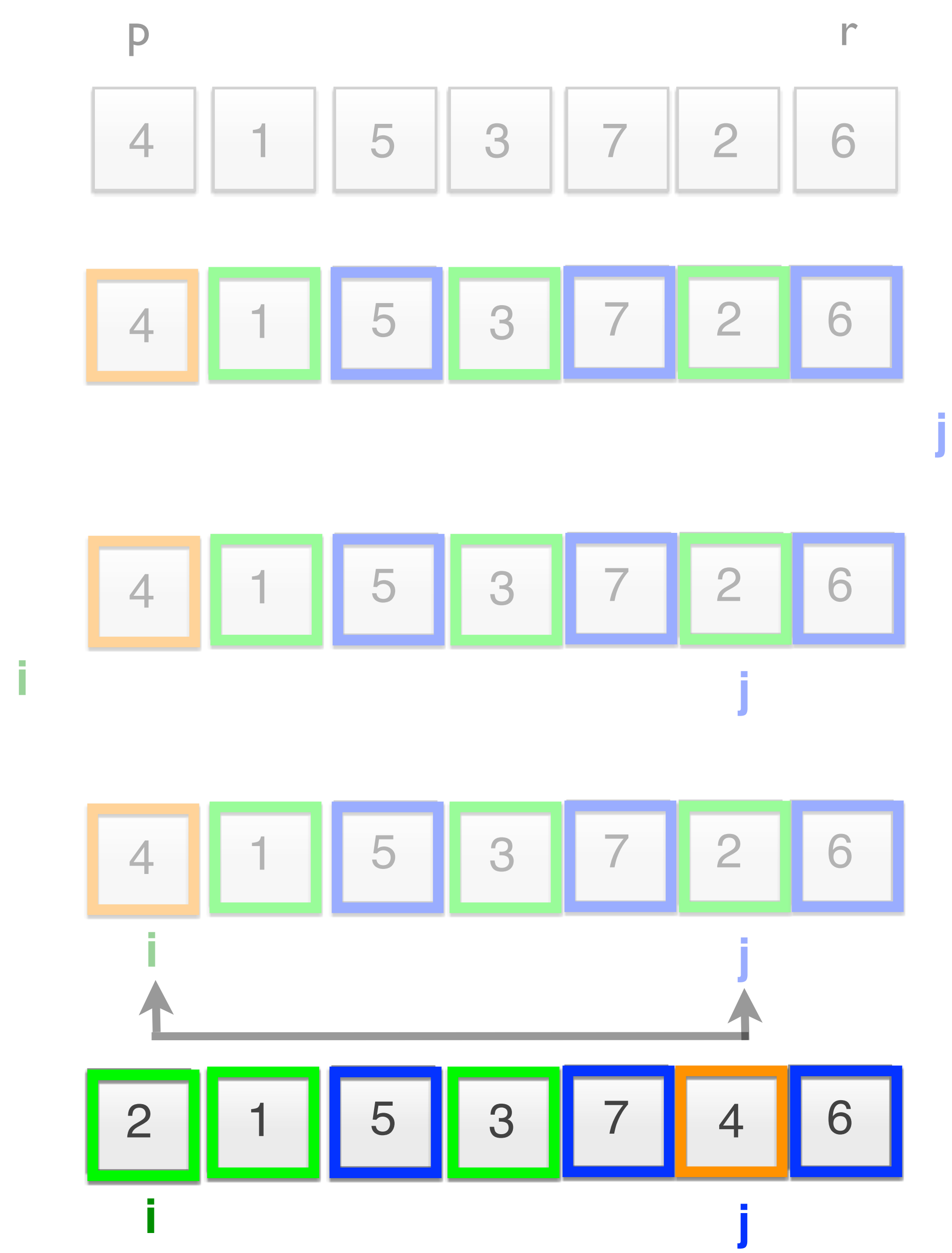

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j

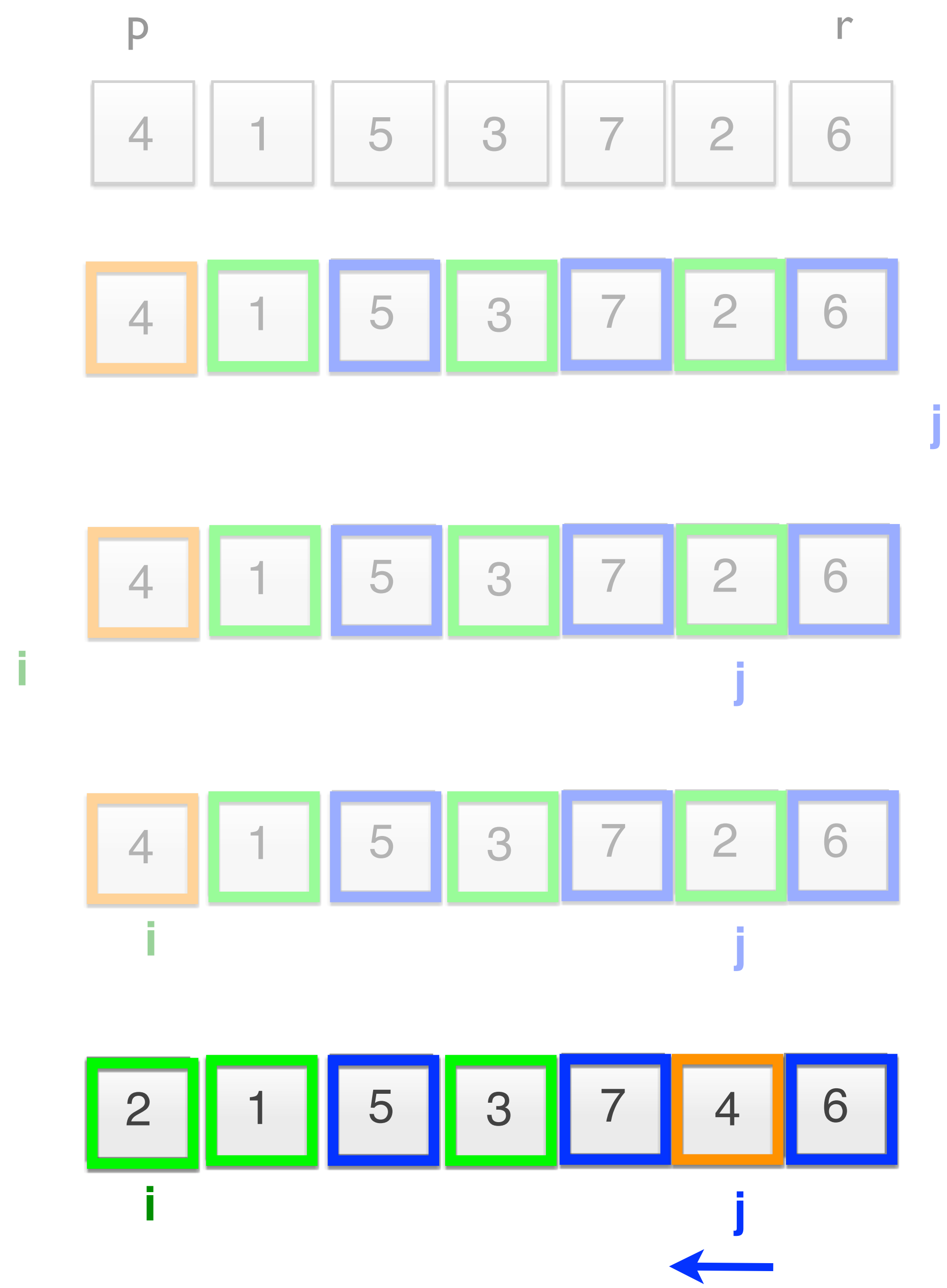

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
j


p
r
4
1
5
3
7
2
6
4
1
5
3
7
2
6
j
4
1
5
3
7
2
6
i
j

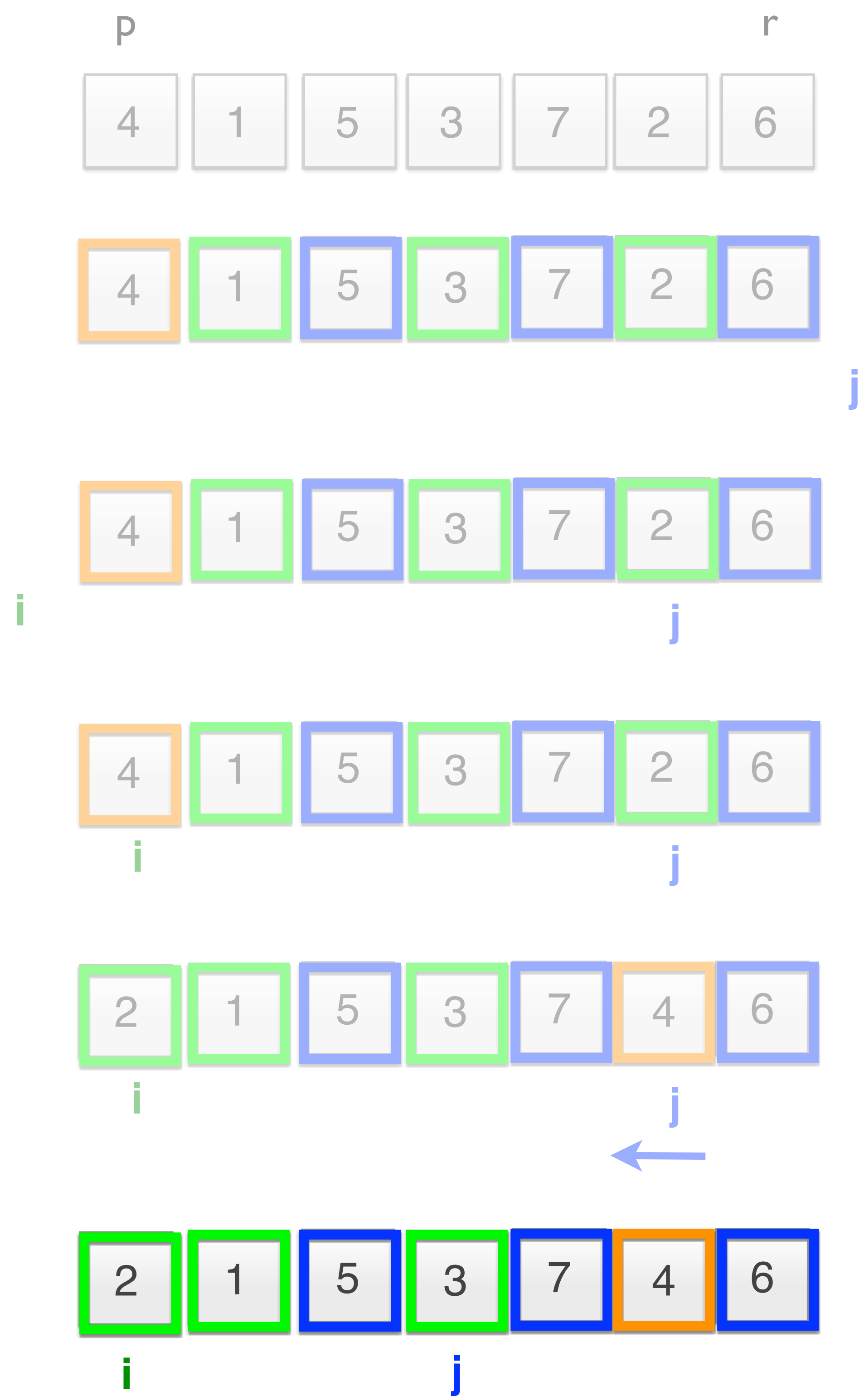

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j


p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j

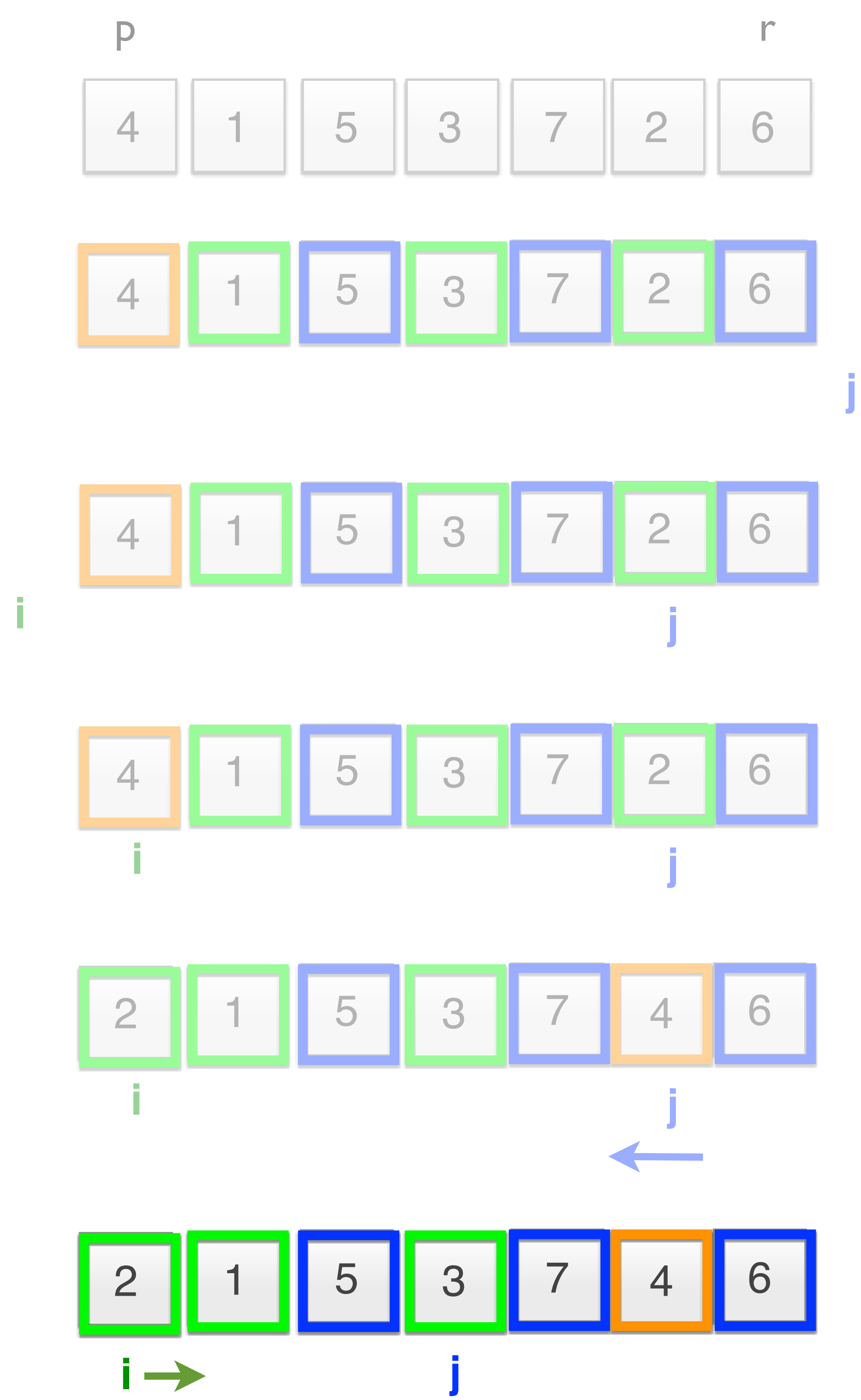

p
r
4 1 5 3 7 2 6
j
i
2 1 5 3 7 4 6

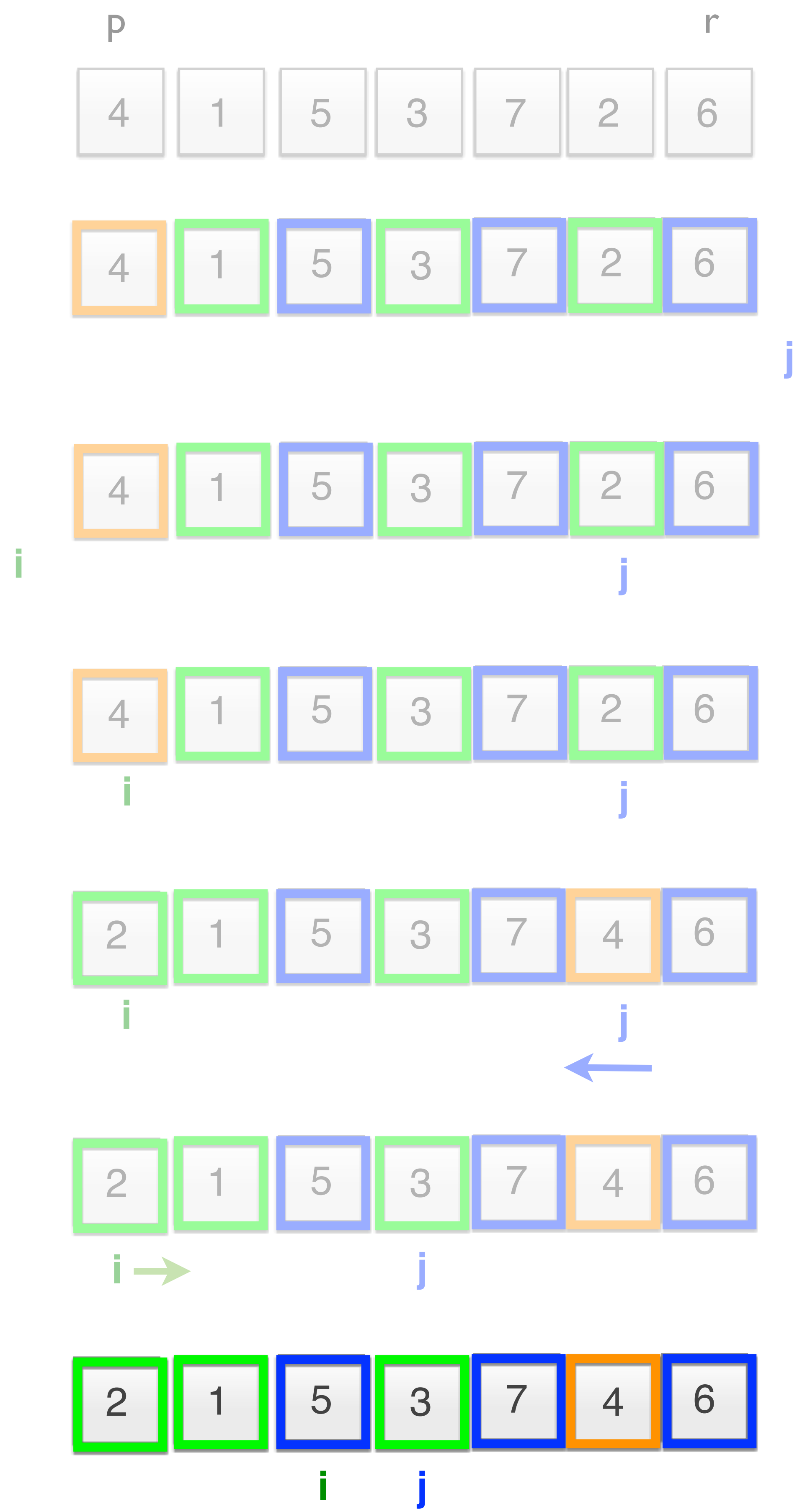


p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j

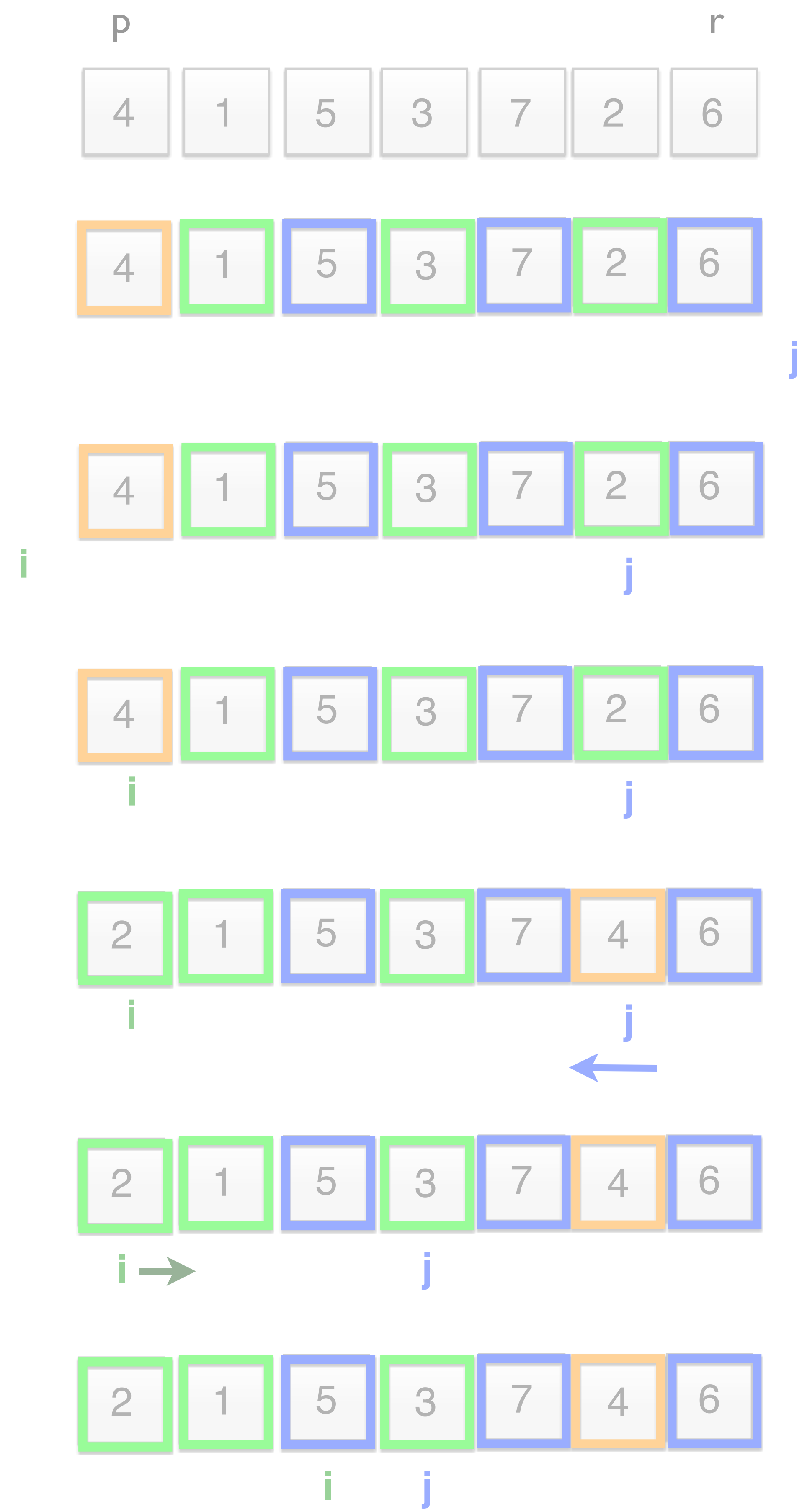
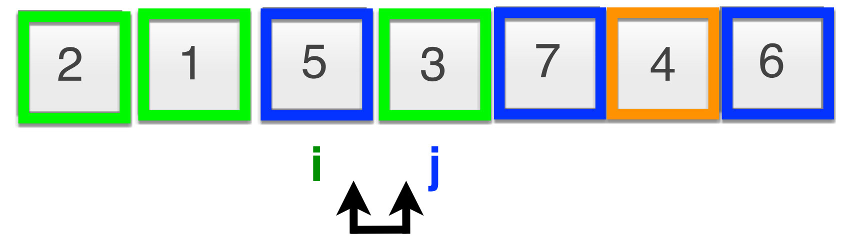


p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j


p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j



p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j


p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j

2 1 5 3 7 4 6
i
j

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 3 5 7 4 6
i
j

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
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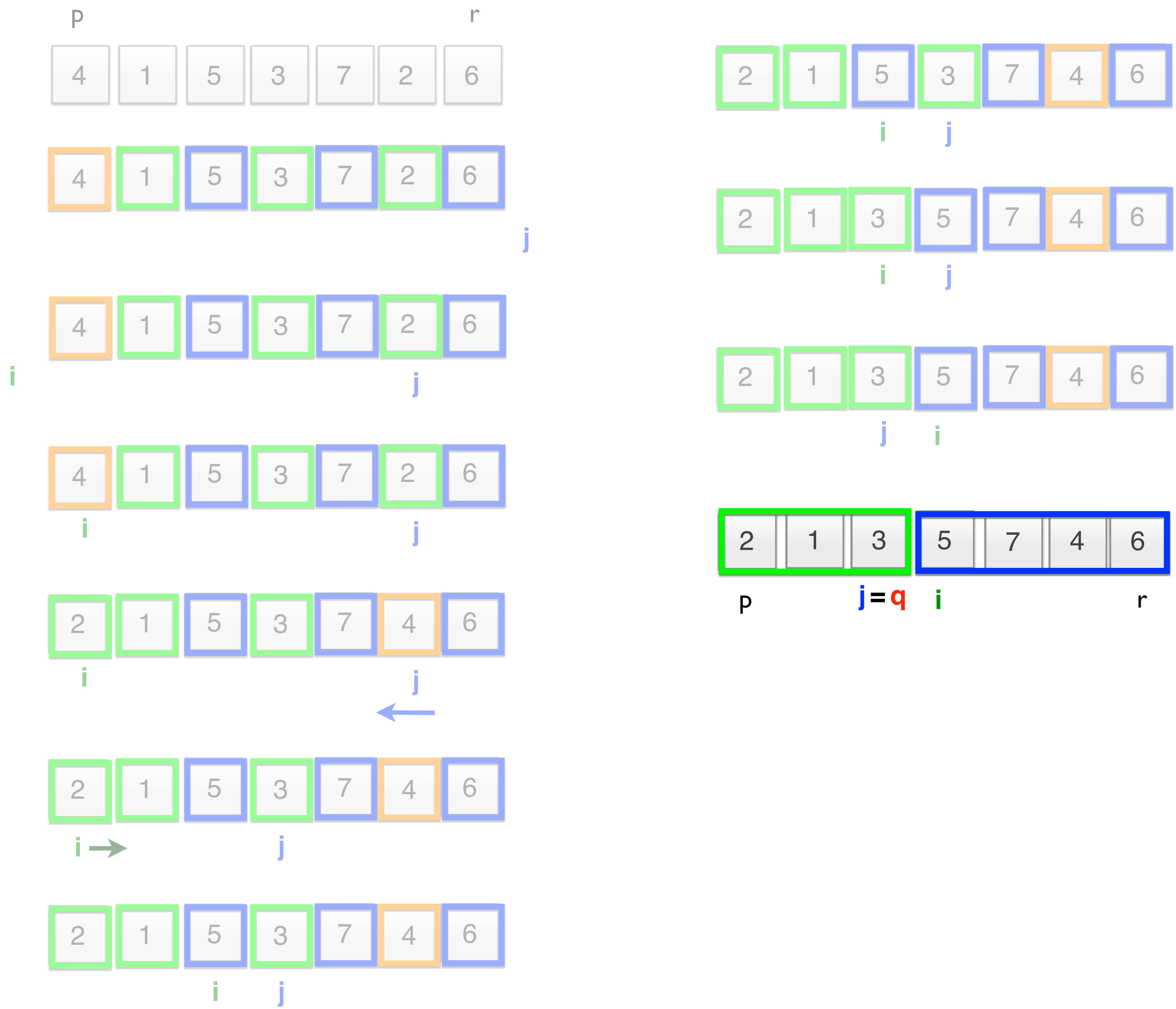
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 3 5 7 4 6
i
j

p
r
4 1 5 3 7 2 6
4 1 5 3 7 2 6
j
4 1 5 3 7 2 6
i
j
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 5 3 7 4 6
i
j
2 1 3 5 7 4 6
i
j
2 1 3 5 7 4 6
j
i


p
r
j=q
i

p
r
4 1 5 3 7 2 6
i
j
2 1 5 3 7 4 6
2 1 3 5 7 4 6
j=q
A:
≤ x
≥ x
q

```
PARTITION(A, p, r)
x = A[p]
i = p − 1
j = r + 1
while TRUE
        repeat
                j = j − 1
        until A[j] ≤ x
        repeat
                i = i + 1
        until A[j] ≥ x
        if i < j
                exchange A[i] with A[j]
        else return j
```

```
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$\boxed{x}$

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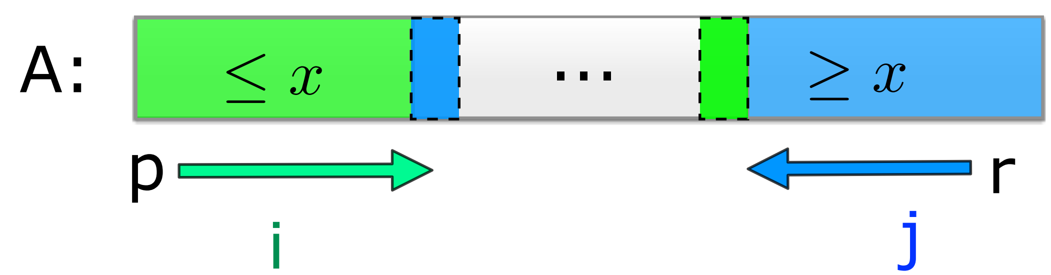

PARTITION(A, p, r)

$x = A[p]$

$i = p - 1$

$j = r + 1$

while TRUE

    repeat

        $j = j - 1$

    until $A[j] \leq x$

    repeat

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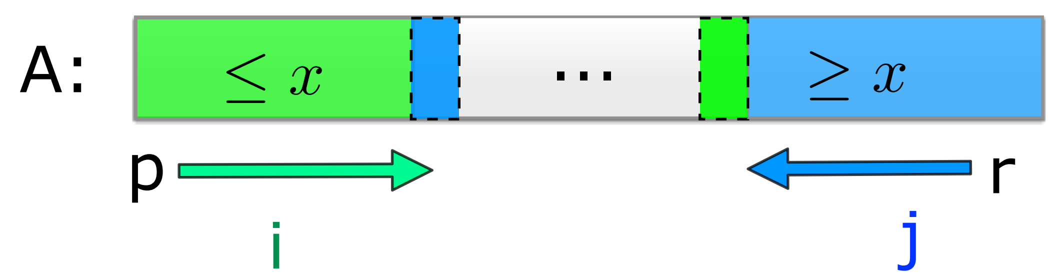

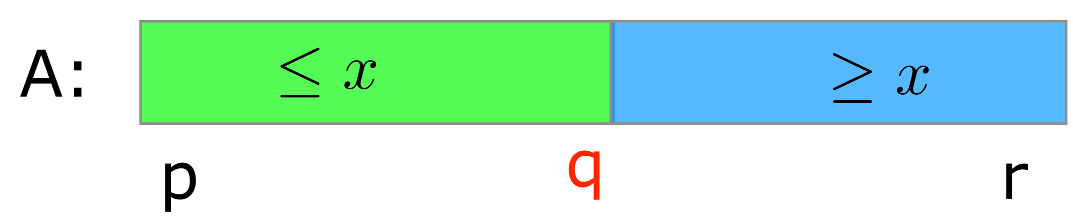

Key points

- Several ways to do the partition (check lecture handout, experiment)

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- Number of comparisons depends on the choice of the pivot, i.e., data.

  - Best case: we partition in halves all along

  - Worst case: ordered list (partition sizes: 1, n-1)

Key points

- Several ways to do the partition (check lecture handout, experiment)

- Number of comparisons depends on the choice of the pivot, i.e., data.

    - Best case: we partition in halves all along

    - Worst case: ordered list (partition sizes: 1, n-1)

- The best case is to be expected. On average quicksort is very efficient.

Summing up

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- Divide and conquer
  - Divide: Partition *in-place*
  - Conquer: Order each partition recursively

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- <u>Divide and conquer</u>
  - <u>Divide</u>: Partition *in-place*
  - <u>Conquer</u>: Order each partition recursively
- Partition algorithms <u>vary</u>, and affect the process dramatically
- Efficient <u>on average</u>

Thank you.

More info: lecture handout

Literature:

- Leiserson, Charles E., Ronald L. Rivest, and Clifford Stein. ***Introduction to algorithms*.** Edited by Thomas H. Cormen. The MIT press, 2001.
- Sedgewick, Robert. and Wayne, Kevin. ***Algorithms*.** Pearson Education, 2011.

garcia@evolbio.mpg.de

http://garciajulian.com