Quicksort

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Sorting

Input:

A sequence of *n* numbers

$\langle a_1, a_2, \ldots, a_n \rangle$

Output:

A permutation of the input sequence

$\langle a'_1, a'_2, \ldots, a'_n \rangle$

do it efficiently.

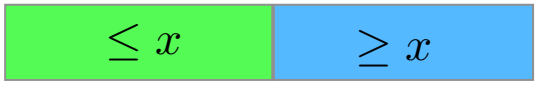
such that $a'_1 \leq a'_2 \leq \ldots \leq a'_n$

Quicksort

- Divide and conquer algorithm.
- Recursive
- Sorts in place (no extra arrays!).
- Popular: Top-10 algorithms 20th century (SIAM).

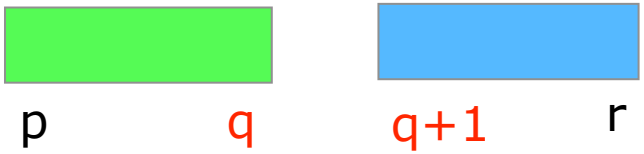
Quicksort strategy

Input: A: 

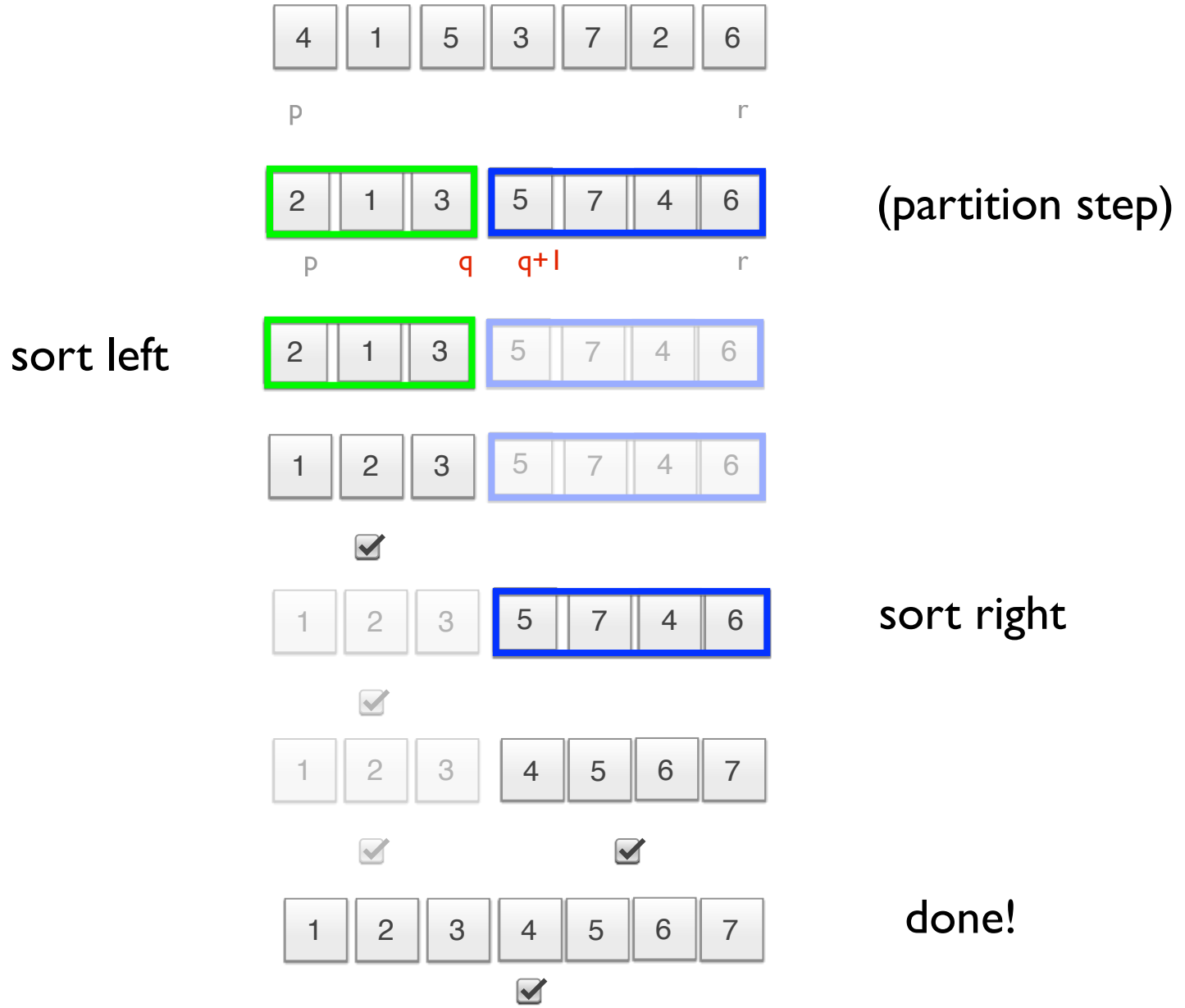
Divide: A: 

(partition step)

Conquer: Recursively solve two smaller problems.

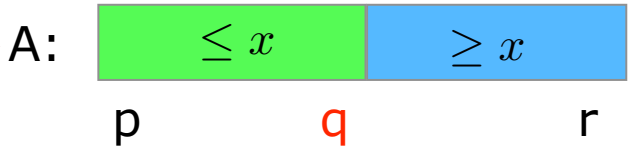


Combine: In-place! so we are done.


4 1 5 3 7 2 6
p r
2 1 3 5 7 4 6
(partition step)
p q q+1 r
sort left
2 1 3 5 7 4 6
1 2 3 5 7 4 6
1 2 3 5 7 4 6
sort right
1 2 3 4 5 6 7
1 2 3 4 5 6 7
done!

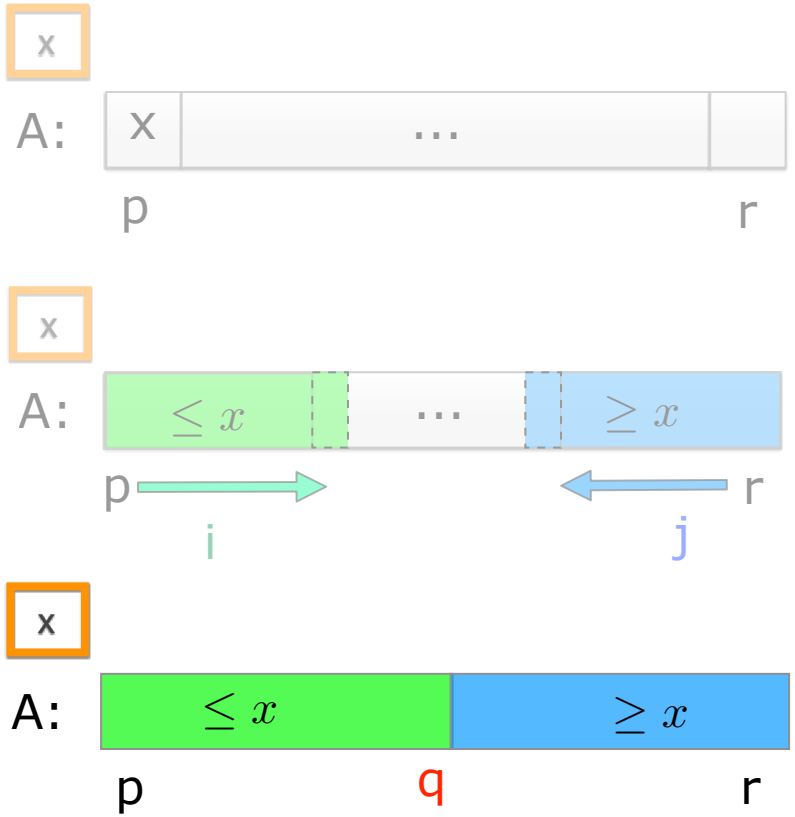
Is it non-trivial?
(exit door)

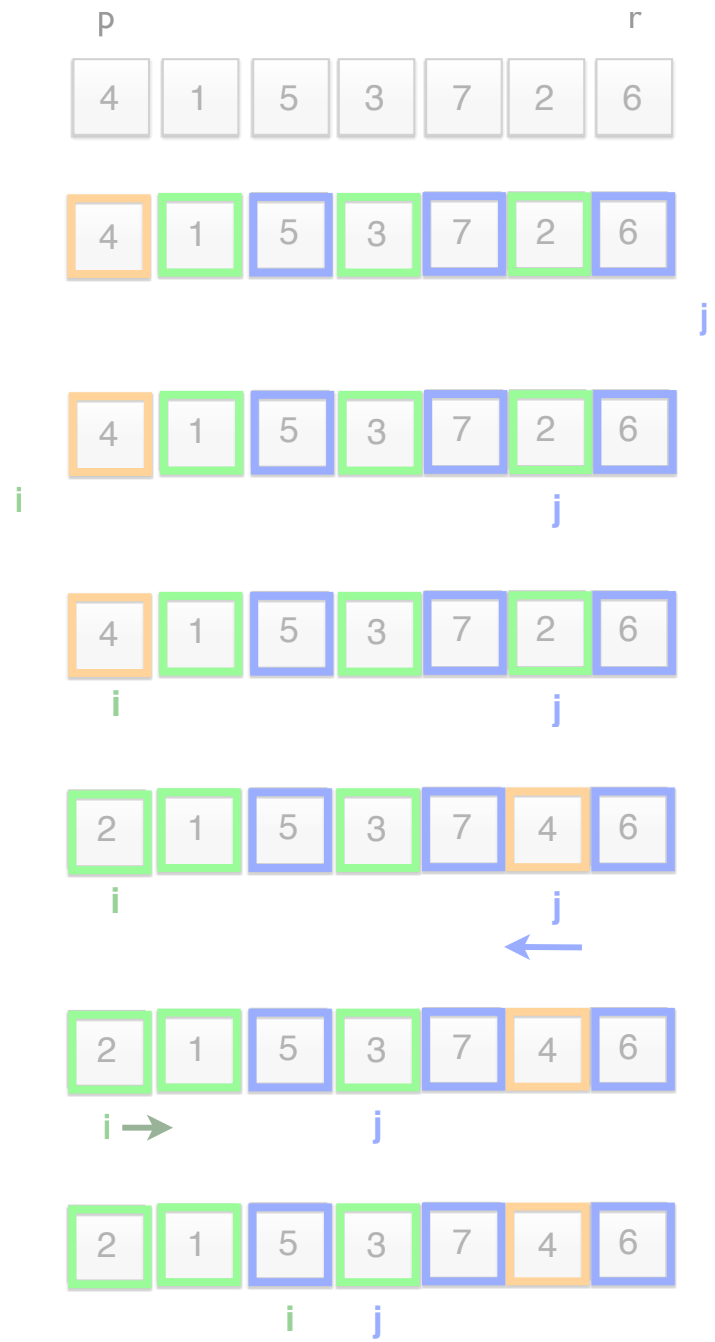
```
QUICKSORT(A, p, r)
1   if p < r
2       q = PARTITION(A, p, r)
3       QUICKSORT(A, p, q)
4       QUICKSORT(A, q + 1, r)
```

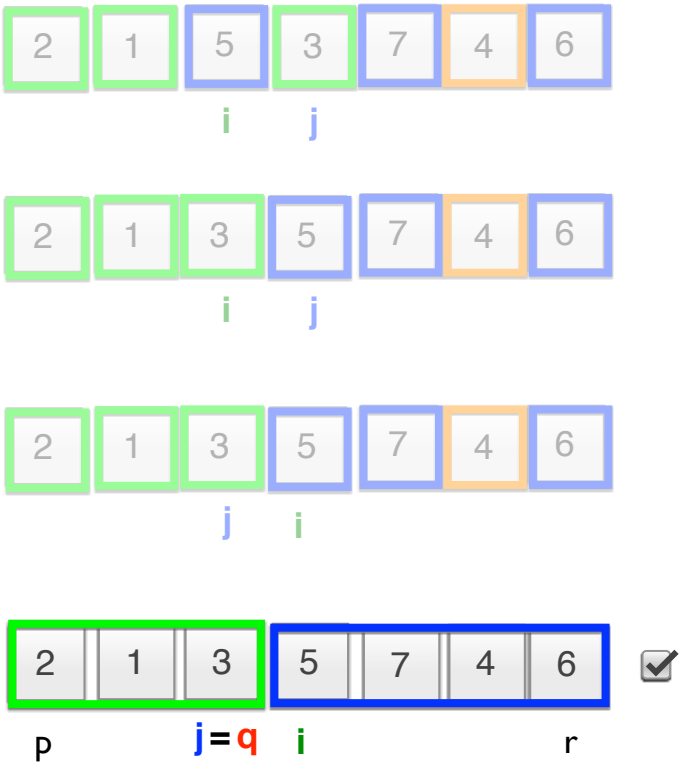


partition really does all the work!

PARTITION(A, p, r)



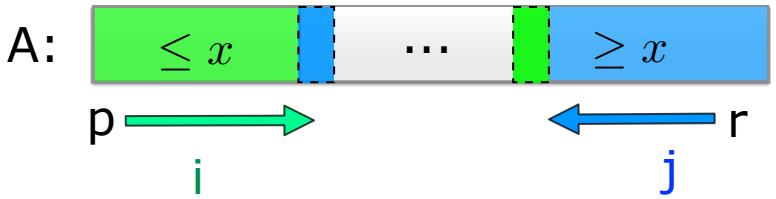

p
r
4 1 5 3 7 2 6
j
i
j
i
j
i
j
2 1 5 3 7 4 6
i
j
i
j
i
j

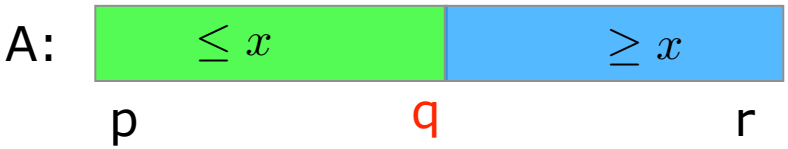

2 1 5 3 7 4 6
i j
2 1 3 5 7 4 6
i j
2 1 3 5 7 4 6
j i
2 1 3 5 7 4 6
p
j=q
i
r


A:
$\leq x$
$\geq x$
p
q
r

PARTITION(A, p, r)

```
1   x = A[p]
2   i = p − 1
3   j = r + 1
4   while TRUE
5       repeat
6           j = j − 1
7       until A[j] ≤ x
8       repeat
9           i = i + 1
10      until A[j] ≥ x
11      if i < j
12          exchange A[i] with A[j]
13      else return j
```





- Details of partition matter (tinker it, check other partitions, experiment)
- Number of comparisons depends on the choice of the pivot, i.e., data.
    - Best case: we partition in halves all along
    - Worst case: ordered list (partition sizes: 1, n-1)
- The best case is to be expected.
  On average quicksort is very efficient.

Summing up

- Divide and conquer
  - Divide: Partition *in-place*
  - Conquer: Order each partition recursively
- Partition algorithms vary, and affect the process dramatically
- Efficient on average

Thank you.

More info: lecture handout

Literature:

- Leiserson, Charles E., Ronald L. Rivest, and Clifford Stein. ***Introduction to algorithms*.** Edited by Thomas H. Cormen. The MIT press, 2001.
- Sedgewick, Robert. and Wayne, Kevin. ***Algorithms*.** Pearson Education, 2011.

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