Algebraic effects in Montague semantics

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Outline

1. Side effects in linguistic semantics
2. Algebraic effects and handlers
3. Making it Montagovian
4. Quantification and dynamism

We are here

1. Side effects in linguistic semantics
2. Algebraic effects and handlers
3. Making it Montagovian
4. Quantification and dynamism

Semantics

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- characterizing semantic knowledge…
  - …i.e., knowledge of *entailments*? *distributional properties*?
- describing how linguistic structure (i.e., syntax) gives rise to the things being characterized (whatever they are)
- describing how pragmatic stuff (e.g., presupposing something, referring to something, expressing something) should affect the things being characterized

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- described how linguistic structure gives rise to meanings, *compositionally*
  - simply typed λ-calculus
- no pragmatic stuff

Montague 1973:

Rules of functional application

S4. If $\alpha \in \mathrm{P}_{\mathrm{t/IV}}$ and $\delta \in \mathrm{P}_{\mathrm{IV}}$, then $F_4(\alpha, \delta) \in \mathrm{P}_{\mathrm{t}}$, where $F_4(\alpha, \delta) = \alpha\delta'$ and δ' is the result of replacing the first *verb* (i.e., member of B_{IV}, B_{TV}, $\mathrm{B}_{\mathrm{IV/t}}$, or $\mathrm{B}_{\mathrm{IV//IV}}$) in δ by its third person singular present.

Rules of functional application

T4. If $\delta \in \mathrm{P}_{\mathrm{t/IV}}, \beta \in \mathrm{P}_{\mathrm{IV}}$, and δ, β translate into δ', β' respectively, then $F_4(\delta, \beta)$ translates into $\delta'(\hat{}\beta')$.

T5. If $\delta \in \mathrm{P}_{\mathrm{IV/T}}$, $\beta \in \mathrm{P}_{\mathrm{T}}$, and δ, β translate into δ', β' respectively, then $F_5(\delta, \beta)$

$$
\begin{array}{c}
\\
\text{John} \quad \text{slept}
\end{array}
\quad \rightsquigarrow \quad
\begin{array}{c}
\lambda s.\mathbf{sleep}\mathbf{j}s \\
\lambda P, s.Ps(\lambda s.\mathbf{j}) \quad \lambda s, x.\mathbf{sleep}(xs)s
\end{array}
$$

Rules of quantification

S14. If $\alpha \in P_T$ and $\phi \in P_t$, then $F_{10,n}(\alpha, \phi) \in P_t$, where either (i) α does not have the form $\mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing the first occurrence of $\mathbf{he}_n$ or $\mathbf{him}_n$ by α and all other occurrences of $\mathbf{he}_n$ or $\mathbf{him}_n$ by $\left\{\begin{array}{l}\mathbf{he}\\ \mathbf{she}\\ \mathbf{it}\end{array}\right\}$ or $\left\{\begin{array}{l}\mathbf{him}\\ \mathbf{her}\\ \mathbf{it}\end{array}\right\}$ respectively, according as the gender of the first B_{CN} or B_T in α is $\left\{\begin{array}{l}\text{masc.}\\ \text{fem.}\\ \text{neuter}\end{array}\right\}$, or
(ii) $\alpha = \mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing all occurrences of $\mathbf{he}_n$ or $\mathbf{him}_n$ by $\mathbf{he}_k$ or $\mathbf{him}_k$ respectively.

Rules of quantification

T14. If $\alpha \in P_T, \phi \in P_t$, and α, ϕ translate into α', ϕ' respectively, then $F_{10,n}(\alpha, \phi)$ translates into $\alpha'(\hat{x}_n\phi')$.

Quantifying in

Every dog slept

every dog $\quad$ he$_n$ slept $\quad \rightsquigarrow$

$$\lambda s.\forall u : \mathbf{dog}\,u s \to \mathbf{sleep}\,\underline{u} s$$

$$\lambda P, s.\forall u : \mathbf{dog}\,u s \to P(\lambda s.u) s \qquad \lambda s.\mathbf{sleep}(\underline{x_n} s) s$$

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Not compositional

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  - Monads (Shan, 2002; Charlow, 2014)
  - Idioms (Kobele, 2018)

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Theories of side effects (e.g., monads) provide interfaces to impure behavior.
    ```

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Linguistic side effects

The effectful approach:

- identity linguistic phenomenon that appears to behave “impurely”, i.e., by subverting compositionality
  - e.g., quantification, anaphora, conventional implicature…
- find an effectful interface that appropriately describes its behavior
- add it to your compositional repertoire!

This talk

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- present two monadic interfaces to side effects: one for quantification and one anaphora
  - the Continuation monad and the State monad, respectively
  - analyses inspired by Charlow (2014)
- show how they may and *may not* be combined
- introduce *algebraic effects*

Monads

a functor $\mathcal{M}$, equipped with two operators, $(\cdot)^\eta$ ('return') and $\gg\!=$ ('bind')

Definition ($\mathcal{M}$)

$$\mathcal{M} : \mathcal{T} \to \mathcal{T}$$
$$(\cdot)^\eta : a \to \mathcal{M}(b)$$
$$(\gg\!=) : \mathcal{M}(a) \to (a \to \mathcal{M}(b)) \to \mathcal{M}(b)$$

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Intuitively: $\mathcal{M}(a)$ is the space where the side effects of some value of type a happen.

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The operators must satisfy the Monad Laws.

$$v^{\eta} \gg\!= k = kv \tag{Left Identity}$$

$$m \gg\!= \lambda v.v^{\eta} = m \tag{Right Identity}$$

$$(m \gg\!= n) \gg\!= o = m \gg\!= \lambda v.nv \gg\!= o \tag{Associativity}$$

In the Continuation monad, scope-taking is a kind of side effect.

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$$(\ggg=) : ((a \to o) \to o) \to (a \to (b \to o) \to o) \to (b \to o) \to o$$
$$m \ggg= k = \lambda c.m(\lambda v.kvc)$$

How it works

1. Ashley hugged every dog.

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$$\begin{aligned} \texttt{ashley} &= \mathbf{a}^{\eta} : \mathcal{C}(e) \\ \texttt{hugged} &= \mathbf{hug}^{\eta} : \mathcal{C}(e \to t) \\ \texttt{every} &= \lambda P, c.\forall x : Px \to cx : (e \to t) \to (e \to t) \to t \\ \texttt{dog} &= \mathbf{dog} : e \to t \end{aligned} \qquad \text{(Lexicon)}$$

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$$\begin{aligned} (\triangleright) &: \mathcal{C}(a \to b) \to \mathcal{C}(a) \to \mathcal{C}(b) && \text{(Grammar)} \\ m \triangleright n &= m \ggg \lambda f.n \ggg \lambda x.(f\,x)^{\eta} \\ &= \lambda c.m(\lambda f.n(\lambda x.c(f\,x))) \\ (\triangleleft) &: \mathcal{C}(a) \to \mathcal{C}(a \to b) \to \mathcal{C}(b) \\ m \triangleleft n &= m \ggg \lambda x.n \ggg \lambda f.(f\,x)^{\eta} \\ &= \lambda c.m(\lambda x.n(\lambda f.c(f\,x))) \end{aligned}$$

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$$\mathtt{ashley} \triangleleft (\mathtt{hugged} \triangleright \mathtt{everydog})$$

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$$\mathbf{a}^{\eta} \lhd (\mathbf{hug}^{\eta} \rhd \mathtt{every}\,\mathbf{dog})$$

expand $\mathtt{every}\,\mathbf{dog}$...

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$$\mathbf{a}^{\eta} \lhd (\mathbf{hug}^{\eta} \rhd \lambda c.\forall x : \mathbf{dog}\, x \rightarrow c x)$$

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to obtain a proposition, apply to the identity function…

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$$\forall x : \mathbf{dog}\,x \to \mathbf{hug}\,x\,\mathbf{a}$$

Summary

Using continuations to manage scope-taking:

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- scopal expressions take scope over their continuations, which are reified as they compose

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Using continuations to manage scope-taking:

- scopal expressions take scope over their continuations, which are reified as they compose
- values take scope trivially (applying Montague's "lift")

In the State monad, we may read from and write to an environment.

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$$(\cdot)^{\blacktriangleright} : \mathcal{S}(e) \to \mathcal{S}(e)$$

$$m^{\blacktriangleright} = m \ggg \lambda x, s.\langle x{::}s, x \rangle$$

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$$\text{expand} \rhd \ldots$$

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expand $\triangleleft \ldots$

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$$\lambda s.\langle \mathbf{a}{::}s, \mathbf{hug}(\mathtt{sel}(\mathbf{a}{::}s))\mathbf{a}\rangle$$

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Using State to manage anaphora:

- expressions that introduce discourse referents or engage in anaphora engage with the environment
- values are trivially stateful, by passing the environment on, untouched

How might one do this?

Combining quantification and anaphora

How might one do this?

Answer: one may use *monad transformers* (the strategy adopted by Shan (2002), and then, by Charlow (2014)).

$\mathcal{C}$ and $\mathcal{S}$ are associated with corresponding monad transformers, $\mathcal{C}_T$ and $\mathcal{S}_T$.

Definition ($\mathcal{M}_T$)

$$\mathcal{M}_T : (\mathcal{T} \to \mathcal{T}) \to \mathcal{T} \to \mathcal{T}$$
$$(\cdot)^{\eta} : a \to \mathcal{M}_T(\mathcal{M}_0)(b)$$
$$(\ggg\!=) : \mathcal{M}_T(\mathcal{M}_0)(a) \to (a \to \mathcal{M}_T(\mathcal{M}_0)(b)) \to \mathcal{M}_T(\mathcal{M}_0)(b)$$

given one of $\mathcal{C}$ or $\mathcal{S}$ as the *underlying monad*, we may apply one of $\mathcal{S}_T$ or $\mathcal{C}_T$ to it…

Definition ($\mathcal{C}_T$)

$$\mathcal{C}_T(\mathcal{M}_0)(a) : (a \to \mathcal{M}_0(o)) \to \mathcal{M}_0(o)$$

$$(\cdot)^\eta : a \to (a \to \mathcal{M}_0(o)) \to \mathcal{M}_0(o)$$
$$v^\eta = \lambda c.cv$$

$$\begin{aligned}(\ggg=) : ((a \to \mathcal{M}_0(o)) \to \mathcal{M}_0(o))\\ \to (a \to (b \to \mathcal{M}_0(o)) \to \mathcal{M}_0(o))\\ \to (b \to \mathcal{M}_0(o)) \to \mathcal{M}_0(o)\end{aligned}$$
$$m \ggg= k = \lambda c.m(\lambda v.kvc)$$

The State monad transformer

Definition ($\mathcal{S}_T$)

$$\mathcal{S}_T(\mathcal{M}_0)(a) : s \to \mathcal{M}_0((s, a))$$

$$(\cdot)^\eta : a \to s \to \mathcal{M}_0((s, a))$$
$$v^\eta = \lambda s.\langle s, v\rangle^\eta$$

$$(\gg\!=) : (s \to \mathcal{M}_0((s, a)))$$
$$\to (a \to (s \to \mathcal{M}_0((s, b))))$$
$$\to s \to \mathcal{M}_0((s, b))$$
$$m \gg\!= k = \lambda s.ms \gg\!= \lambda p.\mathtt{let}\ \langle s', v\rangle = p\ \mathtt{in}\ kvs'$$

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  - but a generic meaning for *every* cannot be written...we are required to know what $\mathcal{M}_0$ is!
  - even then, the meaning the quantifier will be somewhat stipulative, e.g., to account for the data above (though, it can be made to follow from a small set of primitives, as in Charlow (2014))

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Might we salvage our individual analyses in some other way? In doing so, might we account for data like (1)?

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1 Side effects in linguistic semantics

2 Algebraic effects and handlers

3 Making it Montagovian

4 Quantification and dynamism

Background

Algebraic effects and handlers provide a means of writing extensible code, recently especially popular in functional programming.[1]

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- characterizing anaphora in purely algebraic terms
- sticking with a traditional analysis of quantifiers, i.e., whereon they denote sets of sets

Algebraic signatures

An algebraic signature is a set E of operations, each one associated with a *parameter* p and an *arity* a (both types), along with a special operation η ('return').

$$E = \{\mathrm{op}_{1_{p_1 \rightsquigarrow a_1}}, \ldots, \mathrm{op}_{n_{p_n \rightsquigarrow a_n}}, \eta\}$$

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To say operator $\mathrm{op}_{p \rightsquigarrow a}$ is in signature E means that it has the following type signature:

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η always has the following type signature:

$$\eta : v \to \mathcal{F}_E(v)$$

In addition to the signature, an algebra determines a set of equations that must hold among its elements, of the form

$$\mathrm{op}_i(p_i; \ldots) = \mathrm{op}_j(p_j; \ldots)$$

The State algebra (signature)

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Reading the environment twice is no better than reading it once:

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Putting twice overwrites:

$$\mathtt{put}_{s\rightsquigarrow\star}(g; \lambda\star.\mathtt{put}_{s\rightsquigarrow\star}(g'; k)) = \mathtt{put}_{s\rightsquigarrow\star}(g'; k)$$

The Quantifier algebra (signature)

one operation, $\text{scope}_{(e \to t) \to t \rightsquigarrow e}$

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Some example elements of the Quantifier algebra…

- $\mathtt{scope}_{(e\to t)\to t\rightsquigarrow e}(\mathtt{every}\,\mathbf{dog}; \lambda y.\eta(\mathbf{sleep}\,y)) : \mathcal{F}_{\{\mathtt{scope}_{(e\to t)\to t\rightsquigarrow e}\}}(t)$
- $\mathtt{scope}_{(e\to t)\to t\rightsquigarrow e}(\mathtt{every}\,\mathbf{dog}; \lambda y.\mathtt{scope}_{(e\to t)\to t\rightsquigarrow e}(\mathtt{every}\,\mathbf{cat}; \lambda z.\eta(\mathbf{chase}\,z\,y))) : \mathcal{F}_{\{\mathtt{scope}_{(e\to t)\to t\rightsquigarrow e}\}}(t)$

The Quantifier algebra (laws)

Quantifying in:

$$\mathrm{scope}_{(e \to t) \to t \rightsquigarrow e}(q; \lambda x.\eta(kx)) = \eta(qk)$$

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Commuting $\mathtt{scope}_{(e\to t)\to e\rightsquigarrow e}$ past $\mathtt{get}_{\star\rightsquigarrow s}$ and $\mathtt{put}_{s\rightsquigarrow\star}$:

$$\mathtt{scope}_{(e\to t)\to e\rightsquigarrow e}(q; \lambda x.\mathtt{get}_{\star\rightsquigarrow s}(\star; \lambda s.\mathtt{put}_{s\rightsquigarrow\star}(s'; \lambda\star.kxss')))$$

$$= \mathtt{get}_{\star\rightsquigarrow s}(\star; \lambda s.\mathtt{put}_{s\rightsquigarrow\star}(s; \lambda\star.\mathtt{scope}_{(e\to t)\to e\rightsquigarrow e}(q; \lambda x.kxss')))$$

We are here

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How to do it

What we want is an encoding of the operations, as well as a way of *translating* λ-terms with lots of operations into ones with fewer operations in a way that respects the algebraic laws.

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In the combined State/Quantifier algebra, the normal form for any element is determined by the laws to be

$$\mathtt{get}_{\star \rightsquigarrow s}(\star; \lambda s.\mathtt{put}_{\star \rightsquigarrow s}(f\,s; \eta(g s)))$$

for some $f : s \to s$ and $g : s \to v$.

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Pairs of such functions f and g can be represented as $\lambda s.\langle f\,s, g\,s\rangle$ …they are State monadic!

Encoding elements

To encode elements of an algebra, we define a family of functors $\mathcal{F} : \mathcal{T}^*_{\rightsquigarrow} \to \mathcal{T} \to \mathcal{T}$, where

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$$\begin{aligned}\mathcal{F}_{\epsilon}(v) &= v \\ \mathcal{F}_{p \rightsquigarrow a, l}(v) &= (p \to (a \to \mathcal{F}_l(v)) \to o) \to o\end{aligned}$$

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$$\eta : v \to \mathcal{F}_{\epsilon}(v)$$
$$\eta v = v$$

Operations construct “pairs”; returning just returns…

An element of the State/Quantifier algebra

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$$\begin{aligned}
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&\qquad\qquad \lambda s.\texttt{put}_{s\rightsquigarrow\star}(x{::}s;\\
&\qquad\qquad\qquad \lambda\star.\texttt{get}_{\star\rightsquigarrow s}(\star;\lambda s'.\eta(\mathbf{hug}(\texttt{sel}\,s')x)))))
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&\qquad\qquad \lambda s.\mathtt{put}_{s\rightsquigarrow\star}(x{::}s;\\
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\end{aligned}
$$

$$= \lambda h.h(\mathtt{every}\,\mathbf{dog})(\lambda x, h'.h' \star (\lambda s.\ldots))$$

1. Every dog hugged itself.

$$\begin{aligned}&\texttt{scope}_{(e\to t)\to t\rightsquigarrow e}(\texttt{every}\,\mathbf{dog};\\ &\qquad \lambda x.\texttt{get}_{\star\rightsquigarrow s}(\star;\\ &\qquad\qquad \lambda s.\texttt{put}_{s\rightsquigarrow\star}(x{::}s;\\ &\qquad\qquad\qquad \lambda\star.\texttt{get}_{\star\rightsquigarrow s}(\star;\lambda s'.\eta(\mathbf{hug}(\texttt{sel}\,s')x)))))\end{aligned}$$

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This will be an expression of type

$$\begin{aligned}&\mathcal{F}_{(e\to t)\to t\rightsquigarrow e,\star\rightsquigarrow s,s\rightsquigarrow\star,\star\rightsquigarrow s}\\ =\ &(((e\to t)\to t)\to(e\to((\star\to(s\to\ldots)\to o')\to o'))\to o)\to o\end{aligned}$$

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We need a family of functions

$$\texttt{handleSentence}_l : \mathcal{F}_l(t) \to \mathcal{F}_{\star \to s, s, \to \star}(t)$$

where $l \in \{(e \to t) \to t \rightsquigarrow e, \;\; \star \rightsquigarrow s, \;\; s \rightsquigarrow \star\}^*$.

We are here

1. Side effects in linguistic semantics
2. Algebraic effects and handlers
3. Making it Montagovian
4. Quantification and dynamism

Predictions

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In sum

Our algebraic laws predict the contrasts! Crucial is the law that commutes $\mathtt{scope}_{(e \to t) \to t \rightsquigarrow e}$ past $\mathtt{get}_{\star \rightsquigarrow s}$ and $\mathtt{put}_{\star \rightsquigarrow s}$.

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This law destroys a quantifier's dynamic potential, rendering it externally static.

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This gives us a new and precise way of characterizing certain old semantic problems about quantification and dynamism:

- when combining algebras, where do any new laws come from? can they come for free?

References

Barker, Chris. 2002. Continuations and the Nature of Quantification. *Natural Language Semantics* 10:211–242. https://doi.org/10.1023/A:1022183511876.

Barker, Chris, and Chung-chieh Shan. 2014. *Continuations and natural language*, volume 53. Oxford studies in theoretical linguistics.

Charlow, Simon. 2014. On the semantics of exceptional scope. PhD Thesis, NYU, New York. https://semanticsarchive.net/Archive/2JmMWRjY.

Cooper, R. 1983. *Quantification and Syntactic Theory*. Studies in Linguistics and Philosophy. Springer Netherlands. https://www.springer.com/gp/book/9789027714848.

References

de Groote, Philippe. 2006. Towards a Montagovian Account of Dynamics. *Semantics and Linguistic Theory* 16:1–16. https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/2952, number: 0.

Heim, Irene, and Angelika Kratzer. 1998. *Semantics in Generative Grammar*. Malden: Blackwell.

Hendriks, H. L. W. 1993. *Studied flexibility : categories and types in syntax and semantics*. AmsterdamInstitute for Logic, Language and Computation. https://dare.uva.nl/search?identifier=2a784df2-19f2-4d8a-8096-6a4c05db0316.

References

Keller, William R. 1988. Nested Cooper Storage: The Proper Treatment of Quantification in Ordinary Noun Phrases. In *Natural Language Parsing and Linguistic Theories*, ed. U. Reyle and C. Rohrer, Studies in Linguistics and Philosophy, 432–447. Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-009-1337-0_15.

Kobele, Gregory M. 2018. The Cooper Storage Idiom. *Journal of Logic, Language and Information* 27:95–131. https://doi.org/10.1007/s10849-017-9263-1.

Maršík, Jirka, and Maxime Amblard. 2016. Introducing a Calculus of Effects and Handlers for Natural Language Semantics. In *Formal Grammar*, ed. Annie Foret, Glyn Morrill, Reinhard Muskens, Rainer Osswald, and Sylvain Pogodalla, Lecture Notes in Computer Science, 257–272. Berlin, Heidelberg: Springer.

Maršík, Jiří. 2016. Effects and handlers in natural language. phdthesis, Université de Lorraine. https://hal.inria.fr/tel-01417467.

Maršík, Jiří, and Maxime Amblard. 2014. Algebraic Effects and Handlers in Natural Language Interpretation. In *Natural Language and Computer Science*, ed. Valeria de Paiva, Walther Neuper, Pedro Quaresma, Christian Retoré, Lawrence S. Moss, and Jordi Saludes, volume TR 2014-002 of *Joint Proceedings of the Second Workshop on Natural Language and Computer Science (NLCS'14) & 1st International Workshop on Natural Language Services for Reasoners (NLSR 2014)*. Vienne, Austria: Center for Informatics and Systems of the University of Coimbra. https://hal.archives-ouvertes.fr/hal-01079206.

References

Montague, Richard. 1973. The Proper Treatment of Quantification in Ordinary English. In *Approaches to Natural Language: Proceedings of the 1970 Stanford Workshop on Grammar and Semantics*, ed. K. J. J. Hintikka, J. M. E. Moravcsik, and P. Suppes, Synthese Library, 221–242. Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-010-2506-5_10.

Plotkin, Gordon, and John Power. 2001. Semantics for Algebraic Operations. *Electronic Notes in Theoretical Computer Science* 45:332–345. http://www.sciencedirect.com/science/article/pii/S1571066104809708.

Plotkin, Gordon, and John Power. 2003. Algebraic Operations and Generic Effects. *Applied Categorical Structures* 11:69–94. https://doi.org/10.1023/A:1023064908962.

References

Shan, Chung-chieh. 2002. Monads for natural language semantics. *arXiv:cs/0205026* `http://arxiv.org/abs/cs/0205026`, arXiv: cs/0205026.

Steedman, Mark. 2000. *The syntactic process*, volume 24. MIT press Cambridge, MA.