Algebraic effects in Montague semantics

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Outline

- Side effects in linguistic semantics
- Algebraic effects and handlers
- Making it Montagovian
- Quantification and dynamism

We are here

- Side effects in linguistic semantics
- 2 Algebraic effects and handlers
- Making it Montagovian
- 4 Quantification and dynamism

Semantics is for...

• characterizing semantic knowledge...

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 - ▶ ...i.e., knowledge of *entailments? distributional properties?*

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- characterizing semantic knowledge...
 - ...i.e., knowledge of entailments? distributional properties?
- describing how linguistic structure (i.e., syntax) gives rise to the things being characterized (whatever they are)
- describing how pragmatic stuff (e.g., presupposing something, referring to something, expressing something) should affect the things being characterized





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- described how linguistic structure gives rise to meanings, compositionally
 - simply typed λ-calculus
- no pragmatic stuff

Functional application

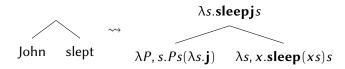
Montague 1973:

Rules of functional application

S4. If $\alpha \in P_{t/IV}$ and $\delta \in P_{IV}$, then $F_4(\alpha, \delta) \in P_t$, where $F_4(\alpha, \delta) = \alpha \delta'$ and δ' is the result of replacing the first *verb* (i.e., member of B_{IV} , B_{TV} , $B_{IV/t}$, or $B_{IV//IV}$) in δ by its third person singular present.

Rules of functional application

- T4. If $\delta \in P_{1/IV}$, $\beta \in P_{IV}$, and δ, β translate into δ', β' respectively, then $F_4(\delta, \beta)$ translates into $\delta'(\hat{\beta}')$.
- T5 If $\delta \in P_{m,m}$ $\beta \in P_m$ and δ β translate into δ' β' respectively then $F_{\epsilon}(\delta,\beta)$



Quantifying in

Rules of quantification

S14. If $\alpha \in P_T$ and $\phi \in P_t$, then $F_{10,n}(\alpha,\phi) \in P_t$, where either (i) α does not have the form \mathbf{he}_k , and $F_{10,n}(\alpha,\phi)$ comes from ϕ by replacing the first occurrence of \mathbf{he}_n or \mathbf{him}_n by α and all other occurrences of \mathbf{he}_n or

$$\begin{array}{l} \mbox{him}_n \mbox{ by } \left\{ \begin{array}{l} \mbox{he} \\ \mbox{she} \\ \mbox{it} \end{array} \right\} \mbox{ or } \left\{ \begin{array}{l} \mbox{him} \\ \mbox{her} \\ \mbox{it} \end{array} \right\} \mbox{ respectively, according as the gender of the} \\ \mbox{first } \mbox{B}_{CN} \mbox{ or } \mbox{B}_{T} \mbox{ in } \alpha \mbox{ is } \left\{ \begin{array}{l} \mbox{masc.} \\ \mbox{fem.} \\ \mbox{powter} \end{array} \right\}, \mbox{ or } \\ \mbox{powter} \end{array}$$

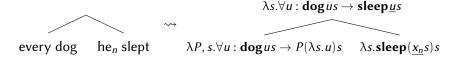
(ii) $\alpha = \mathbf{he}_k$, and $F_{10,n}(\alpha, \phi)$ comes from ϕ by replacing all occurrences of \mathbf{he}_n or \mathbf{him}_n by \mathbf{he}_k or \mathbf{him}_k respectively.

Rules of quantification

T14. If $\alpha \in P_T$, $\phi \in P_t$, and α , ϕ translate into α' , ϕ' respectively, then $F_{10,n}(\alpha,\phi)$ translates into $\alpha'(\hat{x}_n\phi')$.

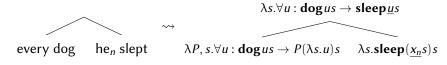
Quantifying in

Every dog slept



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Not compositional

Many techniques since Montague for establishing seemingly non-local quantifier-variable dependencies...

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- Grammars with interfaces to side effects
 - Continuations (Barker, 2002; Barker and Shan, 2014)
 - Monads (Shan, 2002; Charlow, 2014)
 - Idioms (Kobele, 2018)

Programming languages may exhibit "impure" behaviors.

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Theories of side effects (e.g., monads) provide interfaces to impure behavior.

The effectful approach:

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- identity linguistic phenomenon that appears to behave "impurely", i.e., by subverting compositionality
 - e.g., quantification, anaphora, conventional implicature...
- find an effectful interface that appropriately describes its behavior
- add it to your compositional repertoire!

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- show how they may and may not be combined
- introduce algebraic effects

a functor \mathcal{M} , equipped with two operators, $(\cdot)^{\eta}$ ('return') and \gg ('bind')

Definition (M)

$$\mathcal{M}: \mathcal{T} \to \mathcal{T}$$

$$(\cdot)^{\eta}: a \to \mathcal{M}(b)$$

$$(\gg): \mathcal{M}(a) \to (a \to \mathcal{M}(b)) \to \mathcal{M}(b)$$

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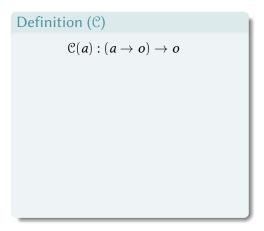
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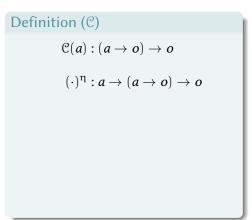
The operators must satisfy the Monad Laws.

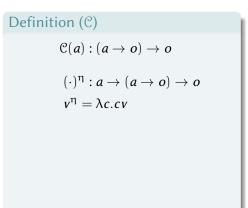


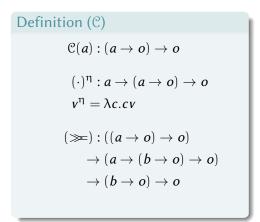
The Monad Laws

$$v^{\eta} \gg k = kv$$
 (Left Identity) $m \gg \lambda v. v^{\eta} = m$ (Right Identity) $(m \gg n) \gg o = m \gg \lambda v. nv \gg o$ (Associativity)









Definition (C)
$$C(a): (a \to o) \to o$$

$$(\cdot)^{\eta}: a \to (a \to o) \to o$$

$$v^{\eta} = \lambda c. cv$$

$$(>=): ((a \to o) \to o)$$

$$\to (a \to (b \to o) \to o)$$

$$\to (b \to o) \to o$$

$$m >= k = \lambda c. m(\lambda v. kvc)$$

$$\begin{aligned} & \text{ashley} = \mathbf{a}^{\eta} : \mathcal{C}(e) & \text{(Lexicon)} \\ & \text{hugged} = \mathbf{hug}^{\eta} : \mathcal{C}(e \to t) & \\ & \text{every} = \lambda P, c. \forall x : Px \to cx : (e \to t) \to (e \to t) \to t \\ & \text{dog} = \mathbf{dog} : e \to t & \end{aligned}$$

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$$(\triangleright) : \mathcal{C}(a \to b) \to \mathcal{C}(a) \to \mathcal{C}(b) \tag{Grammar}$$

$$m \triangleright n = m \gg \lambda f. n \gg \lambda x. (fx)^{\eta}$$

$$= \lambda c. m(\lambda f. n(\lambda x. c(fx)))$$

$$(\triangleleft) : \mathcal{C}(a) \to \mathcal{C}(a \to b) \to \mathcal{C}(b)$$

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Ashley hugged every dog.

 $ashley \triangleleft (hugged \triangleright everydog)$

$$\boldsymbol{a}^{\eta} \triangleleft (\boldsymbol{hug}^{\eta} \triangleright \operatorname{every} \boldsymbol{dog})$$

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expand every \mathbf{dog} ...

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expand ⊳...

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expand ⊲...

$$\lambda c. \forall x : \mathbf{dog} x \to c(\mathbf{hug} x \mathbf{a})$$

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to obtain a proposition, apply to the identity function...

$$\forall x : \mathbf{dog} x \to \mathbf{hug} x \mathbf{a}$$

Using continuations to manage scope-taking:

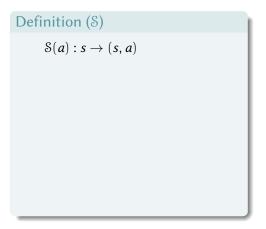
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- values take scope trivially (applying Montague's "lift")

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$$= \lambda s. \text{let } \langle f, s' \rangle = ms \text{ in let } \langle x, s'' \rangle = ns' \text{ in } \langle fx, s'' \rangle$$

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expand $\triangleright \dots$

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expand ⊲...

$$\lambda s. \langle a::s, hug(sel(a::s))a \rangle$$

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- expressions that introduce discourse referents or engage in anaphora engage with the environment
- values are trivially stateful, by passing the environment on, untouched

Combining quantification and anaphora

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Answer: one may use *monad transformers* (the strategy adopted by Shan (2002), and then, by Charlow (2014)).

Monad transformers

 ${\mathfrak C}$ and ${\mathfrak S}$ are associated with corresponding monad transformers, ${\mathfrak C}_{\mathcal T}$ and ${\mathfrak S}_{\mathcal T}.$

$$\begin{split} & \operatorname{Definition}\left(\mathcal{M}_{\mathcal{T}}\right) \\ & \mathcal{M}_{\mathcal{T}}: (\mathfrak{T} \to \mathfrak{T}) \to \mathfrak{T} \to \mathfrak{T} \\ & (\cdot)^{\mathfrak{n}}: a \to \mathcal{M}_{\mathcal{T}}(\mathcal{M}_{0})(b) \\ & (\gg): \mathcal{M}_{\mathcal{T}}(\mathcal{M}_{0})(a) \to (a \to \mathcal{M}_{\mathcal{T}}(\mathcal{M}_{0})(b)) \to \mathcal{M}_{\mathcal{T}}(\mathcal{M}_{0})(b) \end{split}$$

given one of \mathcal{C} or \mathcal{S} as the *underlying monad*, we may apply one of \mathcal{S}_T or \mathcal{C}_T to it...

The Continuation monad transformer

Definition (\mathcal{C}_T)

$$\mathcal{C}_{T}(\mathcal{M}_{0})(a):(a \to \mathcal{M}_{0}(o)) \to \mathcal{M}_{0}(o)$$

$$(\cdot)^{\eta}:a \to (a \to \mathcal{M}_{0}(o)) \to \mathcal{M}_{0}(o)$$

$$v^{\eta} = \lambda c.cv$$

$$(\gg):((a \to \mathcal{M}_{0}(o)) \to \mathcal{M}_{0}(o))$$

$$\to (a \to (b \to \mathcal{M}_{0}(o)) \to \mathcal{M}_{0}(o))$$

$$\to (b \to \mathcal{M}_{0}(o)) \to \mathcal{M}_{0}(o)$$

The State monad transformer

Definition (S_T) $S_T(\mathcal{M}_0)(a): s \to \mathcal{M}_0((s,a))$ $(\cdot)^{\eta}: a \to s \to \mathcal{M}_0((s,a))$ $v^{\eta} = \lambda s. \langle s, v \rangle^{\eta}$ $(\gg): (s \to \mathcal{M}_0((s,a)))$ $\rightarrow (a \rightarrow (s \rightarrow \mathcal{M}_0((s,b))))$ $\rightarrow s \rightarrow \mathcal{M}_0((s,b))$ $m \gg k = \lambda s. ms \gg \lambda p. 1 \text{et } \langle s', v \rangle = p \text{ in } kvs'$

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If we adopt the transformers approach from the start...

- we throw out our generalized quantifier meaning for every dog
- the type of *every dog* becomes $(e \to \mathcal{M}_0(t)) \to \mathcal{M}_0(t) \dots$

To summarize...

This general strategy can be made to work extremely well (Charlow, 2014).

But, how do we decide which monad transformer to apply to which monad?

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 - but a generic meaning for *every* cannot be written...we are required to know what M_0 is!
 - even then, the meaning the quantifier will be somewhat stipulative, e.g., to account for the data above (though, it can be made to follow from a small set of primitives, as in Charlow (2014))

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The transformers approach, when used generically, prevents us from writing meanings. When used non-generically, it loses extensibility.

Might we salvage our individual analyses in some other way? In doing so, might we account for data like (1)?

We are here

- Side effects in linguistic semantics
- Algebraic effects and handlers
- Making it Montagovian
- Quantification and dynamism

Algebraic effects and handlers provide a means of writing extensible code, recently especially popular in functional programming.¹

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- staying in STLC (with unit)
 - characterizing anaphora in purely algebraic terms
 - sticking with a traditional analysis of quantifiers, i.e., whereon they denote sets of sets

An algebraic signature is a set E of operations, each one associated with a parameter p and an arity a (both types), along with a special operation η ('return').

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η always has the following type signature:

$$\eta: v \to \mathcal{F}_E(v)$$



Algebraic laws

In addition to the signature, an algebra determines a set of equations that must hold among its elements, of the form

$$op_i(p_i; \ldots) = op_j(p_j; \ldots)$$

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Reading the environment twice is no better than reading it once:

$$\mathtt{get}_{\star \leadsto s}(\star; \lambda g. \mathtt{get}_{\star \leadsto s}(\star; \lambda g'. kgg')) = \mathtt{get}_{\star \leadsto s}(\star; \lambda g. kgg)$$

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Putting twice overwrites:

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The Quantifier algebra (signature)

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Some example elements of the Quantifier algebra...

- $\bullet \ \operatorname{scope}_{(e \to t) \to t \leadsto e}(\operatorname{every} \operatorname{dog}; \lambda y. \eta(\operatorname{sleep} y)) : \mathcal{F}_{\{\operatorname{scope}_{(e \to t) \to t \leadsto e}\}(t)}$
- $\begin{array}{l} \bullet \ \operatorname{scope}_{(e \to t) \to t \leadsto e}(\operatorname{everydog}; \lambda y. \operatorname{scope}_{(e \to t) \to t \leadsto e}(\operatorname{everycat}; \lambda z. \eta(\operatorname{chase}{zy}))): \\ \mathcal{F}_{\{\operatorname{scope}_{(e \to t) \to t \leadsto e}\}(t)} \end{array}$

The Quantifier algebra (laws)

Quantifying in:

$$scope_{(e \to t) \to t \leadsto e}(q; \lambda x. \eta(kx)) = \eta(qk)$$

is just a matter of

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Commuting $scope_{(e \to t) \to e \to e}$ past $get_{\star \to s}$ and $put_{s \to \star}$:

$$\mathtt{scope}_{(e \rightarrow t) \rightarrow e \leadsto e}(q; \lambda x. \mathtt{get}_{\star \leadsto s}(\star; \lambda s. \mathtt{put}_{s \leadsto \star}(s'; \lambda \star .kxss')))$$

$$= \mathtt{get}_{\star \leadsto s}(\star; \lambda s.\mathtt{put}_{s \leadsto \star}(s; \lambda \star .\mathtt{scope}_{(e \to t) \to e \leadsto e}(q; \lambda x.kxss')))$$

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What we want is an encoding of the operations, as well as a way of *translating* λ -terms with lots of operations into ones with fewer operations in a way that respects the algebraic laws.

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In the combined State/Quantifier algebra, the normal form for any element is determined by the laws to be

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for some $f : s \rightarrow s$ and $g : s \rightarrow v$.

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Pairs of such functions f and g can be represented as $\lambda s. \langle f s, g s \rangle \dots$ they are State monadic!

To encode elements of an algebra, we define a family of functors $\mathcal{F}: \mathcal{T}_{\sim}^* \to \mathcal{T} \to \mathcal{T}$, where

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• Υ^*_{\leadsto} is the free monoid (i.e., of lists) over $\Upsilon_{\leadsto} = \{p \leadsto a \mid p, a \in \Upsilon\}$

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Operations construct "pairs"; returning just returns...



Every dog hugged itself.

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```
scope_{(e \to t) \to t \leadsto e}(every \mathbf{dog}; \\ \lambda x. get_{\star \leadsto s}(\star; \\ \lambda s. put_{s \leadsto \star}(x :: s; \\ \lambda \star . get_{\star \leadsto s}(\star; \lambda s'. \eta(\mathbf{hug}(sels')x)))))
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This will be an expression of type

$$\mathfrak{F}_{(e \to t) \to t \leadsto e, \star \leadsto s, s \leadsto \star, \star \leadsto s} \\
= (((e \to t) \to t) \to (e \to ((\star \to (s \to \ldots) \to o') \to o')) \to o) \to o$$

Handling operations

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We need a family of functions

$$handleSentence_l: \mathcal{F}_l(t) \to \mathcal{F}_{\star \to s,s,\to \star}(t)$$

where
$$l \in \{(e \rightarrow t) \rightarrow t \rightsquigarrow e, \star \rightsquigarrow s, s \rightsquigarrow \star\}^*$$
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In sum

Our algebraic laws predict the contrasts! Crucial is the law that commutes $scope_{(e \to t) \to t \leadsto e}$ past $get_{\star \leadsto s}$ and $put_{\star \leadsto s}$.

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This law destroys a quantifier's dynamic potential, rendering it externally static.

Conclusion

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- allow us to study interactions between linguistic side effects, in terms of algebraic laws

This gives us a new and precise way of characterizing certain old semantic problems about quantification and dynamism:

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- are compositional, using traditional tools (like, e.g., monads do)
- are extensible (unlike monad transformers, where providing meanings came at the cost of expanding the grammar)
- are relatively conservative (e.g., quantifiers are still of type $(e \rightarrow t) \rightarrow t$)
- allow us to study interactions between linguistic side effects, in terms of algebraic laws

This gives us a new and precise way of characterizing certain old semantic problems about quantification and dynamism:

when combining algebras, where do any new laws come from? can they come for free?

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