Rational Speech Act models are utterance-independent updates of world priors

Jean-Philippe Bernardy, Julian Grove, and Christine Howes

Semdial 2022 - DubDial, August 23, 2022

CLASP, University of Gothenburg

Introduction to RSA

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

u = 'JP ate five cookies.'

$[\![u]\!] = n_{cookies} \geq 5$

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

u = 'JP ate five cookies.'

$[\![u]\!] = n_{cookies} \geq 5$

Three components:

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

u = 'JP ate five cookies.'

$[\![u]\!] = n_{cookies} \geq 5$

Three components:

- A literal listener L_0:
  $u \mapsto P_{L_0}(w|u)$

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

$u =$ 'JP ate five cookies.'

$[\![u]\!] = n_{cookies} \geq 5$

Three components:

- A literal listener L_0:
  $u \mapsto P_{L_0}(w|u)$
- A pragmatic speaker S_1:
  $w \mapsto P_{S_1}(u|w)$

RSA (Frank and Goodman, 2012; Goodman and Frank, 2016)

u = 'JP ate five cookies.'

$[\![u]\!] = n_{cookies} \geq 5$

Three components:

- A literal listener L_0:
  $u \mapsto P_{L_0}(w|u)$
- A pragmatic speaker S_1:
  $w \mapsto P_{S_1}(u|w)$
- A pragmatic listener L_1:
  $u \mapsto P_{L_1}(w|u)$

The literal listener L_0

$$P_{L_0}(w|u) \propto \mathbb{1}(w \geq n) \times P(w)$$

w	5	6	7
'JP ate 5 cookies'	1/3	1/3	1/3
'JP ate 6 cookies'	0	1/2	1/2
'JP ate 7 cookies'	0	0	1

The pragmatic speaker S_1

$$P_{S_1}(u \mid w) \propto \frac{P_{L_0}(w \mid u)^\alpha}{e^{\alpha \times C(u)}}$$

(Assume $\alpha = 1$, and that $C(u)$ is constant.)

w	5	6	7
'JP ate 5 cookies'	1	0.16	0.01
'JP ate 6 cookies'	0	0.84	0.06
'JP ate 7 cookies'	0	0	0.93

The pragmatic listener L_1

$$P_{L_1}(w|u) \propto P_{S_1}(u|w) \times P(w)$$

w	5	6	7
‘JP ate 5 cookies’	0.85	0.14	0.01
‘JP ate 6 cookies’	0	0.93	0.07
‘JP ate 7 cookies’	0	0	1

Plan

- Reformulate the components of RSA as *filters* on reconceived prior distributions (using information gain).

Plan

- Reformulate the components of RSA as *filters* on reconceived prior distributions (using information gain).
- Discuss some consequences:

Plan

- Reformulate the components of RSA as *filters* on reconceived prior distributions (using information gain).
- Discuss some consequences:
    - for computing RSA models;

Plan

- Reformulate the components of RSA as *filters* on reconceived prior distributions (using information gain).
- Discuss some consequences:
    - for computing RSA models;
    - for the algorithmic plausibility of RSA (Marr, 1982).

RSA via information gain

Information gain as K-L divergence

Kullback-Leibler (K-L) divergence (between distributions P and Q over $\mathcal{X}$):

$$D_{\mathrm{KL}}(Q \parallel P) = -\sum_{x \in \mathcal{X}} Q(x) \log\left(\frac{P(x)}{Q(x)}\right)$$

Information gain as K-L divergence

Kullback-Leibler (K-L) divergence (between distributions P and Q over $\mathcal{X}$):

$$D_{\mathsf{KL}}(Q \parallel P) = -\sum_{x \in \mathcal{X}} Q(x) \log\left(\frac{P(x)}{Q(x)}\right)$$

Higher values: more information is gained by going from P to Q.

A theorem

Theorem
If $Q(x) \propto f(x) \times P(x)$, and the range of f is $\{0, 1\}$, then:

$$D_{\mathrm{KL}}(Q \parallel P) = -\log\left(\sum_x f(x) \times P(x)\right)$$

A theorem

Theorem
If $Q(x) \propto f(x) \times P(x)$, and the range of f is $\{0, 1\}$, then:

$$D_{\mathrm{KL}}(Q \parallel P) = -\log\left(\sum_x f(x) \times P(x)\right)$$

In words: if f updates P merely by *filtering* it (resulting in Q), then the information gain of Q is the negative log of the expected value of f.

A theorem

Theorem
If $Q(x) \propto f(x) \times P(x)$, and the range of f is $\{0, 1\}$, then:

$$D_{\mathrm{KL}}(Q \parallel P) = -\log\left(\sum_{x} f(x) \times P(x)\right)$$

In words: if f updates P merely by *filtering* it (resulting in Q), then the information gain of Q is the negative log of the expected value of f.

- Important: the expected value of f is *also* the normalizing constant for Q:

$$Q(x) = \frac{f(x) \times P(x)}{\sum_{x'} f(x') \times P(x')}$$

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u, w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u, w') \times P(w')}$$

where $l(u, w) = 1$ or 0, depending on whether u is true at w

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u,w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u,w') \times P(w')}$$

where $l(u,w) = 1$ or 0, depending on whether u is true at w

Let us define the *literal information gain* of u:

$$G_{L_0}(u) = D_{\mathsf{KL}}(P_{L_0}(w|u) \parallel P(w))$$

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u, w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u, w') \times P(w')}$$

where $l(u, w) = 1$ or 0, depending on whether u is true at w

Let us define the *literal information gain* of u:

$$G_{L_0}(u) = D_{\mathrm{KL}}(P_{L_0}(w|u) \parallel P(w))$$

Using our theorem:

$$G_{L_0}(u) = -\log \sum_{w' \in \mathcal{W}} l(u, w') \times P(w')$$

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u,w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u,w') \times P(w')}$$

where $l(u,w) = 1$ or 0, depending on whether u is true at w

Let us define the *literal information gain* of u:

$$G_{L_0}(u) = D_{\mathrm{KL}}(P_{L_0}(w|u) \parallel P(w))$$

Using our theorem:

$$G_{L_0}(u) = -\log \sum_{w' \in \mathcal{W}} l(u,w') \times P(w')$$

$$P_{L_0}(w|u) = l(u,w) \times P(w) \times e^{G_{L_0}(u)}$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$P_{S_1}(u \mid w) \propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}}$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$
\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}}
\end{aligned}
$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$\begin{aligned} P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^\alpha}{e^{\alpha \times C(u)}} \\ &\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^\alpha}{e^{\alpha \times C(u)}} \\ &\propto l(u, w)^\alpha \times P(w)^\alpha \times e^{\alpha \times (G_{L_0}(u) - C(u))} \end{aligned}$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto l(u, w)^{\alpha} \times P(w)^{\alpha} \times e^{\alpha \times (G_{L_0}(u) - C(u))} \\
&\propto l(u, w) \times e^{\alpha \times (G_{L_0}(u) - C(u))}
\end{aligned}$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$
\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto l(u, w)^{\alpha} \times P(w)^{\alpha} \times e^{\alpha \times (G_{L_0}(u) - C(u))} \\
&\propto l(u, w) \times e^{\alpha \times (G_{L_0}(u) - C(u))}
\end{aligned}
$$

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto l(u, w)^{\alpha} \times P(w)^{\alpha} \times e^{\alpha \times (G_{L_0}(u) - C(u))} \\
&\propto l(u, w) \times e^{\alpha \times (G_{L_0}(u) - C(u))}
\end{aligned}$$

Upshot:

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$
\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u, w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto l(u, w)^{\alpha} \times P(w)^{\alpha} \times e^{\alpha \times (G_{L_0}(u) - C(u))} \\
&\propto l(u, w) \times e^{\alpha \times (G_{L_0}(u) - C(u))}
\end{aligned}
$$

Upshot:

- $l(u, w)$ is our filter

Reformulating the pragmatic speaker

The pragmatic speaker S_1:

$$
\begin{aligned}
P_{S_1}(u \mid w) &\propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto \frac{\left(l(u,w) \times P(w) \times e^{G_{L_0}(u)}\right)^{\alpha}}{e^{\alpha \times C(u)}} \\
&\propto l(u,w)^{\alpha} \times P(w)^{\alpha} \times e^{\alpha \times (G_{L_0}(u) - C(u))} \\
&\propto l(u,w) \times e^{\alpha \times (G_{L_0}(u) - C(u))}
\end{aligned}
$$

Upshot:

- $l(u, w)$ is our filter
- $P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$ is our *new* “prior”!

Reformulating the pragmatic speaker: example

$$[\![\text{'JP ran } u \text{ km'}]\!] = w \geq u$$

Reformulating the pragmatic speaker: example

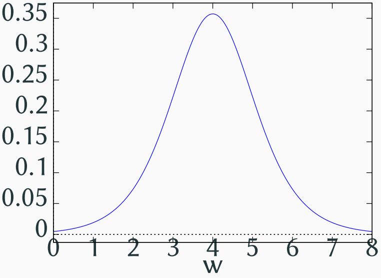
$$[\![\text{`JP ran } u \text{ km'}]\!] = w \geq u$$

$$P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$$

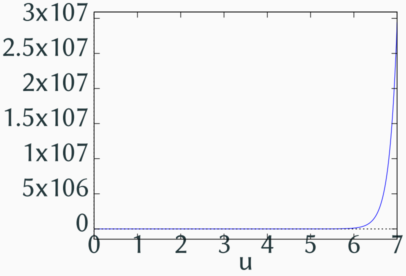
Reformulating the pragmatic speaker: example

$$[\![\text{‘JP ran } u \text{ km’}]\!] = w \geq u$$

$$P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$$



Prior over JP’s running distance



$P_{S_1}(u|w = 7)$ with $\alpha = 4$

Making the normalizing constant explicit

As a proportion:

$$P_{S_1}(u|w) \propto l(u,w) \times P_{S_1}(u)$$

Making the normalizing constant explicit

As a proportion:

$$P_{S_1}(u|w) \propto l(u,w) \times P_{S_1}(u)$$

Normalizing:

$$P_{S_1}(u|w) = \frac{l(u,w) \times P_{S_1}(u)}{\sum_{u' \in \mathcal{U}} l(u',w) \times P_{S_1}(u')}$$

Making the normalizing constant explicit

As a proportion:

$$P_{S_1}(u|w) \propto l(u, w) \times P_{S_1}(u)$$

Normalizing:

$$\begin{aligned} P_{S_1}(u|w) &= \frac{l(u, w) \times P_{S_1}(u)}{\sum_{u' \in \mathcal{U}} l(u', w) \times P_{S_1}(u')} \\ &= l(u, w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \end{aligned}$$

Making the normalizing constant explicit

As a proportion:

$$P_{S_1}(u|w) \propto l(u, w) \times P_{S_1}(u)$$

Normalizing:

$$\begin{aligned} P_{S_1}(u|w) &= \frac{l(u, w) \times P_{S_1}(u)}{\sum_{u' \in \mathcal{U}} l(u', w) \times P_{S_1}(u')} \\ &= l(u, w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \end{aligned}$$

Making the normalizing constant explicit

As a proportion:

$$P_{S_1}(u|w) \propto l(u, w) \times P_{S_1}(u)$$

Normalizing:

$$\begin{aligned} P_{S_1}(u|w) &= \frac{l(u, w) \times P_{S_1}(u)}{\sum_{u' \in \mathcal{U}} l(u', w) \times P_{S_1}(u')} \\ &= l(u, w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \end{aligned}$$

$$G_{S_1}(w) = -\log \sum_{u \in \mathcal{U}} l(u, w) \times P_{S_1}(u)$$

the *specificity* of w

Reformulating the pragmatic listener

The pragmatic listener L_1:

$$P_{L_1}(w|u) \propto \; P_{S_1}(u|w) \times P(w)$$

Reformulating the pragmatic listener

The pragmatic listener L_1:

$$\begin{aligned} P_{L_1}(w|u) &\propto P_{S_1}(u|w) \times P(w) \\ &\propto l(u, w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \times P(w) \end{aligned}$$

Reformulating the pragmatic listener

The pragmatic listener L_1:

$$\begin{aligned} P_{L_1}(w|u) &\propto P_{S_1}(u|w) \times P(w) \\ &\propto l(u,w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \times P(w) \\ &\propto l(u,w) \times e^{G_{S_1}(w)} \times P(w) \end{aligned}$$

Reformulating the pragmatic listener

The pragmatic listener L_1:

$$\begin{aligned}
P_{L_1}(w|u) &\propto P_{S_1}(u|w) \times P(w) \\
&\propto l(u,w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \times P(w) \\
&\propto l(u,w) \times e^{G_{S_1}(w)} \times P(w)
\end{aligned}$$

Reformulating the pragmatic listener

The pragmatic listener L_1:

$$
\begin{aligned}
P_{L_1}(w|u) &\propto P_{S_1}(u|w) \times P(w) \\
&\propto l(u,w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \times P(w) \\
&\propto l(u,w) \times e^{G_{S_1}(w)} \times P(w)
\end{aligned}
$$

Define $P_{L_1}(w) = e^{G_{S_1}(w)} \times P(w)$ as our *new* prior!

Reformulating the pragmatic listener

The pragmatic listener L_1:

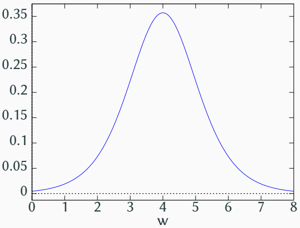
$$
\begin{aligned}
P_{L_1}(w|u) &\propto P_{S_1}(u|w) \times P(w) \\
&\propto l(u,w) \times P_{S_1}(u) \times e^{G_{S_1}(w)} \times P(w) \\
&\propto l(u,w) \times e^{G_{S_1}(w)} \times P(w)
\end{aligned}
$$

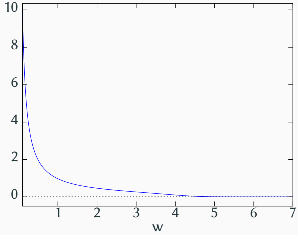
Define $P_{L_1}(w) = e^{G_{S_1}(w)} \times P(w)$ as our *new* prior!

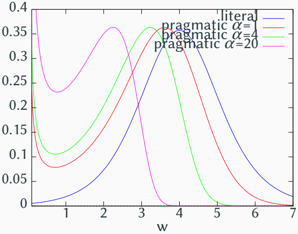
Upshot:

$$P_{L_1}(w|u) \propto l(u,w) \times P_{L_1}(w)$$

Reformulating the pragmatic listener: example







$P(w)$ $\times$ $e^{G_{S_1}(w)}$ $=$ $P_{L_1}(w)$

(JP's distance prior) (specificity) (L_1 "pragmatic" prior)

Some consequences

Computing RSA models

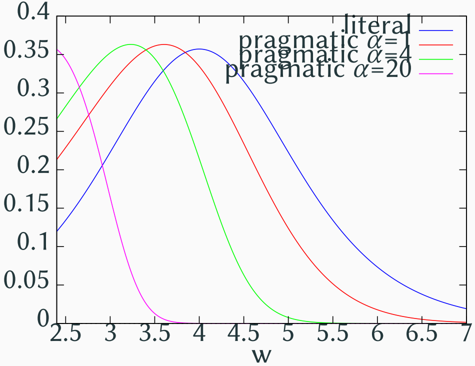
Main result:

$$P_{L_1}(w|u) \propto l(u, w) \times P_{L_1}(w)$$

Main result:

$$P_{L_1}(w|u) \propto l(u, w) \times P_{L_1}(w)$$

The "pragmatic" interpretation of 'JP ran 2.4 km' is gotten by simply cropping $P_{L_1}(w)$ and normalizing:

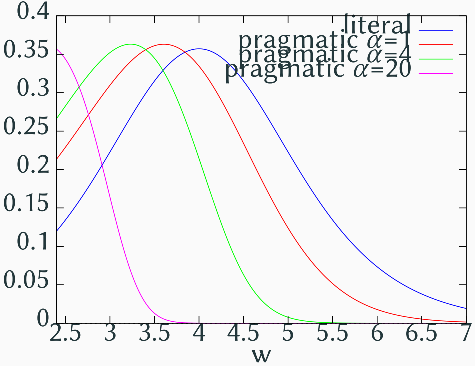


Computing RSA models

Main result:

$$P_{L_1}(w|u) \propto l(u,w) \times P_{L_1}(w)$$

The "pragmatic" interpretation of 'JP ran 2.4 km' is gotten by simply cropping $P_{L_1}(w)$ and normalizing:



(Side note: the expected implicature is not always generated!)

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

- For literal interpretations, this prior is $P(w)$.

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

- For literal interpretations, this prior is $P(w)$.
- For pragmatic interpretations, this prior is $P_{L_1}(w)$.

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

- For literal interpretations, this prior is $P(w)$.
- For pragmatic interpretations, this prior is $P_{L_1}(w)$.

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

- For literal interpretations, this prior is $P(w)$.
- For pragmatic interpretations, this prior is $P_{L_1}(w)$.

(RSA-style) pragmatic interpretations can be *learned*, just as probability distributions over events can; pragmatic distributions are additionally sensitive to an event's specificity.

References

Frank, Michael C., and Noah D. Goodman. 2012. Predicting pragmatic reasoning in language games. *Science* 336:998 – 998.

Goodman, Noah D., and Michael C. Frank. 2016. Pragmatic Language Interpretation as Probabilistic Inference. *Trends in Cognitive Sciences* 20:818–829. `https://www.sciencedirect.com/science/article/pii/S136466131630122X`.

Marr, David. 1982. *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information.* Cambridge: MIT Press.