

# **Rational Speech Act models are utterance-independent updates of world priors**

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# Introduction to RSA

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- A **pragmatic speaker**  $S_1$ :  
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- A **pragmatic listener**  $L_1$ :  
 $u \mapsto P_{L_1}(w|u)$

## The literal listener $L_0$

$$P_{L_0}(w|u) \propto \mathbb{1}(w \geq n) \times P(w)$$

$w$	5	6	7
‘JP ate 5 cookies’	1/3	1/3	1/3
‘JP ate 6 cookies’	0	1/2	1/2
‘JP ate 7 cookies’	0	0	1



## The pragmatic speaker $S_1$

$$P_{S_1}(u \mid w) \propto \frac{P_{L_0}(w \mid u)^\alpha}{e^{\alpha \times C(u)}}$$

(Assume  $\alpha = 4$ , and that  $C(u)$  is constant.)

$w$	5	6	7
‘JP ate 5 cookies’	1	0.16	0.01
‘JP ate 6 cookies’	0	0.84	0.06
‘JP ate 7 cookies’	0	0	0.93

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$w$	5	6	7
‘JP ate 5 cookies’	0.85	0.14	0.01
‘JP ate 6 cookies’	0	0.93	0.07
‘JP ate 7 cookies’	0	0	1

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  - for computing RSA models;
  - for the algorithmic plausibility of RSA (Marr, 1982).

## **RSA via information gain**

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## Information gain as K-L divergence

Kullback-Leibler (K-L) divergence (between distributions  $P$  and  $Q$  over  $\mathcal{X}$ ):

$$D_{\text{KL}}(Q \parallel P) = - \sum_{x \in \mathcal{X}} Q(x) \log \left( \frac{P(x)}{Q(x)} \right)$$



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Higher values: more information is gained by going from  $P$  to  $Q$ .

## A theorem

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If  $Q(x) \propto f(x) \times P(x)$ , and the range of  $f$  is  $\{0, 1\}$ , then:

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- **Important:** the expected value of  $f$  is *also* the normalizing constant for  $Q$ :

$$Q(x) = \frac{f(x) \times P(x)}{\sum_{x'} f(x') \times P(x')}$$

## Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u, w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u, w') \times P(w')}$$

where  $l(u, w) = 1$  or  $0$ , depending on whether  $u$  is true at  $w$

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Let us define the *literal information gain* of  $u$ :

$$G_{L_0}(u) = D_{\text{KL}}(P_{L_0}(w|u) \parallel P(w))$$

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Upshot:

- $l(u, w)$  is our filter
- $P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$  is our *new* “prior”!



## Reformulating the pragmatic speaker: example

$$\llbracket \text{'JP ran } u \text{ km'} \rrbracket = w \geq u$$

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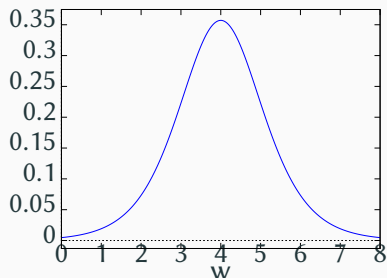
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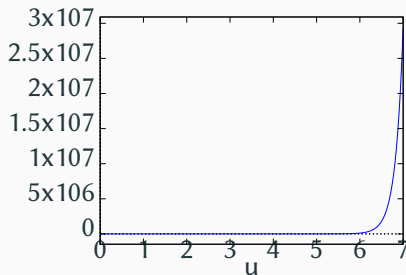
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Prior over JP's running distance



$P_{S_1}(u|w=7)$  with  $\alpha=4$

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As a proportion:

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$$G_{S_1}(w) = -\log \sum_{u \in \mathcal{U}} l(u, w) \times P_{S_1}(u)$$

the *specificity* of  $w$



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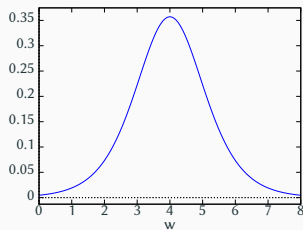
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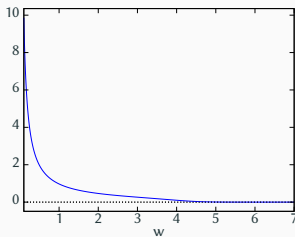
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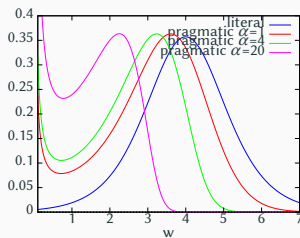
$P(w)$

(JP's distance prior)



$\times e^{G_{S_1}(w)}$

(specificity)



$= P_{L_1}(w)$

( $L_1$  "pragmatic" prior)

## **Some consequences**

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# Computing RSA models

Main result:

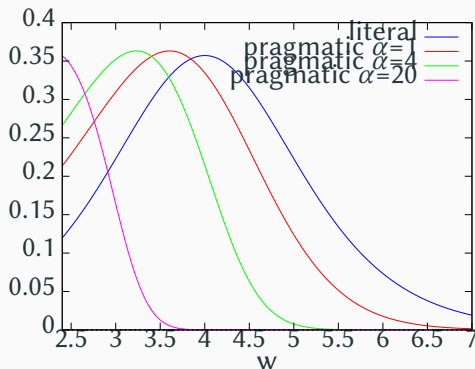
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The “pragmatic” interpretation of ‘JP ran 2.4 km’ is gotten by simply cropping  $P_{L_1}(w)$  and normalizing:

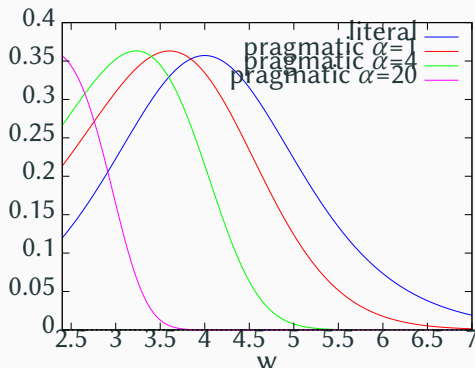


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(Side note: the expected implicature is not always generated!)

# Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

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(RSA-style) pragmatic interpretations can be *learned*, just as probability distributions over events can; pragmatic distributions are additionally sensitive to an event's specificity.



### References

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- Frank, Michael C., and Noah D. Goodman. 2012. Predicting pragmatic reasoning in language games. *Science* 336:998 – 998.
- Goodman, Noah D., and Michael C. Frank. 2016. Pragmatic Language Interpretation as Probabilistic Inference. *Trends in Cognitive Sciences* 20:818–829. <https://www.sciencedirect.com/science/article/pii/S136466131630122X>.
- Marr, David. 1982. *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information*. Cambridge: MIT Press.