Presupposition projection as a scope phenomenon

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   - Karlos has a car. (presupposition)

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  - The "projection problem" (Langendoen and Savin, 1971)

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- Presupposition triggers in the scopes of propositional attitude verbs

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1. The satisfaction theory

2. A scopal account

Rough sketch of how it works

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    - Might amount to set intersection (of sets of worlds, assignments, ...)

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  - *Karlos brought his car* $\rightsquigarrow$ *Karlos has a car*

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- But sometimes it shouldn’t happen.

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  - E.g., the type $e_{\#}$ is that of something which is either an individual (e.g., Karlos) or undefined (#).

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  - E.g., the type $e_{\#}$ is that of something which is either an individual (e.g., Karlos) or undefined (#).
  - This move allows us to treat partial functions as total; e.g., a partial function of type $e \to t$ is now a total function of type $e \to t_{\#}$ that maps the part of its domain on which it is not defined to #.

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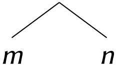

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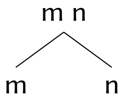

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We need something more powerful: a monad.

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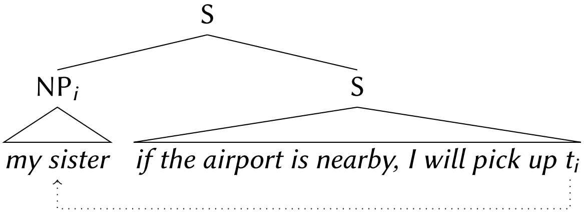

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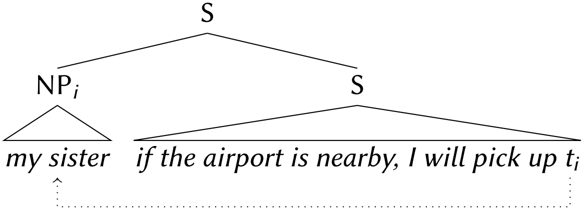

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- As well as make Functional Application a bit more general:
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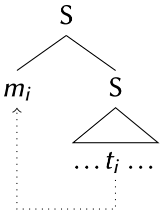
- In general, we can use “Quantifier Raising” metaphorically:



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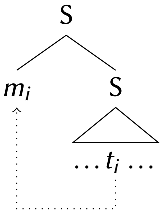

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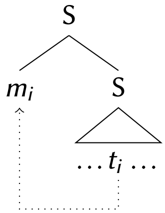

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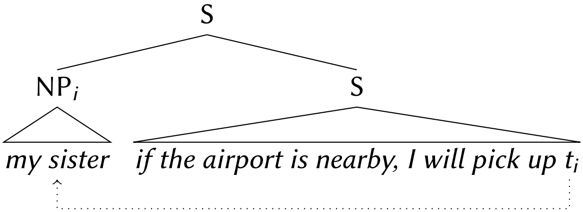

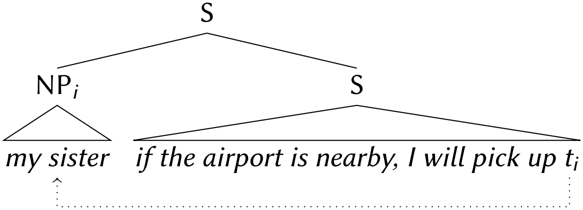

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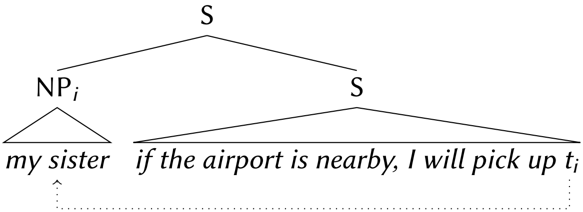

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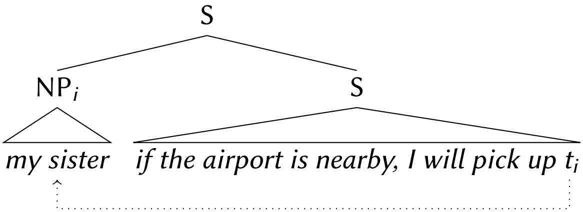

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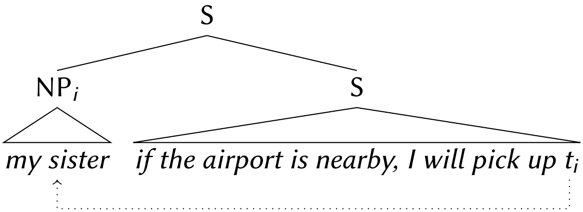

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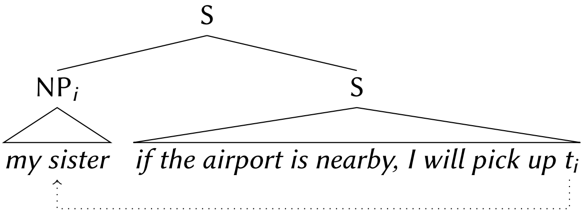

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  - $\rightsquigarrow$ *I have a sister* ☺

Filtering…

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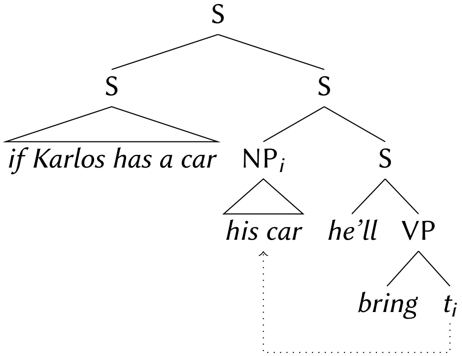

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    - ★ by composing via continuations (Barker, 2002; Barker and Shan, 2014)

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  - Accomplished in terms of $(\cdot)^{\uparrow}$, μ, and Functional Application.

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