Rational Speech Act models are utterance-independent updates of world priors

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Introduction to RSA

RSA (Frank and Goodman, 2012)

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- A pragmatic listener L_1:
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The literal listener L_0

$$P_{L_0}(w|u) \propto \mathbb{1}(w \geq n) \times P(w)$$

w	5	6	7
'JP ate 5 cookies'	1/3	1/3	1/3
'JP ate 6 cookies'	0	1/2	1/2
'JP ate 7 cookies'	0	0	1

The pragmatic speaker S_1

$$P_{S_1}(u \mid w) \propto \frac{P_{L_0}(w \mid u)^{\alpha}}{e^{\alpha \times C(u)}}$$

(Assume $\alpha = 1$, and that $C(u)$ is constant.)

w	5	6	7
'JP ate 5 cookies'	1	0.16	0.01
'JP ate 6 cookies'	0	0.84	0.06
'JP ate 7 cookies'	0	0	0.93

The pragmatic listener L_1

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w	5	6	7
‘JP ate 5 cookies’	0.85	0.14	0.01
‘JP ate 6 cookies’	0	0.93	0.07
‘JP ate 7 cookies’	0	0	1

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- Discuss some consequences:
  - for computing RSA models;
  - for the algorithmic plausibility of RSA (Marr, 1982).

RSA via information gain

Information gain as K-L divergence

Kullback-Leibler (K-L) divergence (between distributions P and Q over $\mathcal{X}$):

$$D_{\mathsf{KL}}(Q \parallel P) = -\sum_{x \in \mathcal{X}} Q(x) \log\left(\frac{P(x)}{Q(x)}\right)$$

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Higher values: more information is gained by going from P to Q.

A theorem

Theorem
If $Q(x) \propto f(x) \times P(x)$, and the range of f is $\{0, 1\}$, then:

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In words: if f updates P merely by *filtering* it (resulting in Q), then the information gain of Q is the negative log of the expected value of f.

- Important: the expected value of f is *also* the normalizing constant for Q:

$$Q(x) = \frac{f(x) \times P(x)}{\sum_{x'} f(x') \times P(x')}$$

Reformulating the literal listener

$$P_{L_0}(w|u) = \frac{l(u,w) \times P(w)}{\sum_{w' \in \mathcal{W}} l(u,w') \times P(w')}$$

where $l(u,w) = 1$ or 0, depending on whether u is true at w

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Let us define the *literal information gain* of u:

$$G_{L_0}(u) = D_{\mathrm{KL}}(P_{L_0}(w|u) \parallel P(w))$$

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Reformulating the pragmatic speaker

The pragmatic speaker S_1:

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Upshot:

- $l(u, w)$ is our filter
- $P_{S_1}(u) \propto e^{\alpha \times (G_{L_0}(u) - C(u))}$ is our *new* “prior”!

Reformulating the pragmatic speaker: example

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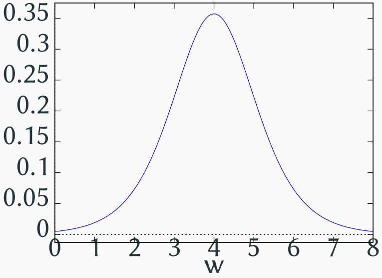
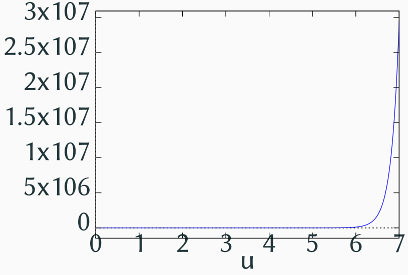


Prior over JP's running distance



$P_{S_1}(u|w = 7)$ with $\alpha = 4$

Making the normalizing constant explicit

As a proportion:

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$$G_{S_1}(w) = -\log \sum_{u \in \mathcal{U}} l(u,w) \times P_{S_1}(u)$$

the *specificity* of w

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Define $P_{L_1}(w) = e^{G_{S_1}(w)} \times P(w)$ as our *new* prior!

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Upshot:

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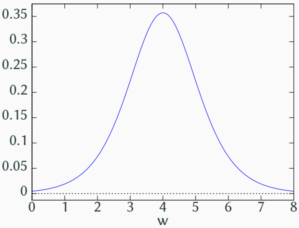

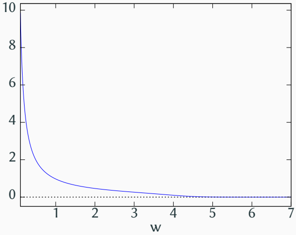

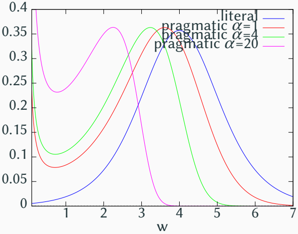

$P(w) \quad \times \quad e^{G_{S_1}(w)} \quad = \quad P_{L_1}(w)$

(JP's distance prior) (specificity) (L_1 posterior)

Some consequences

Computing RSA models

Main result:

$$P_{L_1}(w|u) \propto l(u,w) \times P_{L_1}(w)$$

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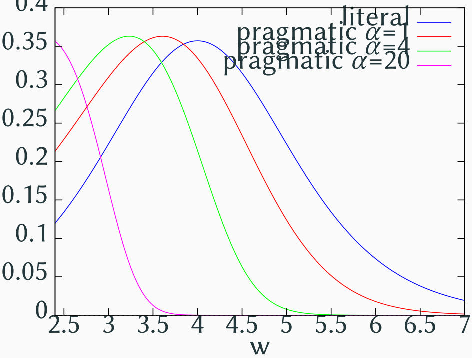
The “pragmatic” interpretation of ‘JP ran 2.4 km’ is gotten by simply cropping $P_{L_1}(w)$ and normalizing:



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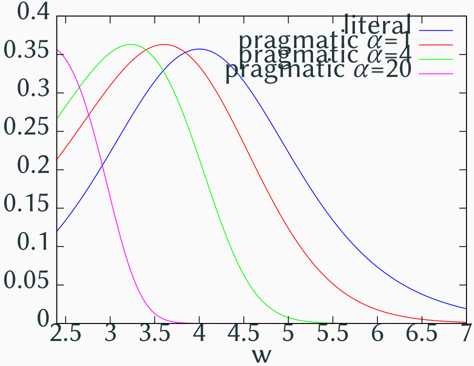

(Side note: the expected implicature is not always generated!)

Algorithmic plausibility

From an algorithmic perspective (Marr, 1982), pragmatic interpretations (as according to RSA models) can be obtained the same way as literal interpretations: by filtering a certain prior by the literal meaning of the utterance.

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(RSA-style) pragmatic interpretations can be *learned*, just as probability distributions over events can; pragmatic distributions are additionally sensitive to an event's specificity.

References

Frank, Michael C., and Noah D. Goodman. 2012. Predicting pragmatic reasoning in language games. *Science* 336:998 – 998.

Marr, David. 1982. *Vision: A Computational Investigation into the Human Representation and Processing of Visual Information.* Cambridge: MIT Press.