Probabilistic compositional semantics, purely

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CLASP, University of Gothenburg

Motivation

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Such programming languages are often *impure*: they allow for probabilistic effects, like sampling and marginalization, to occur at any point in a program.

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- Achieve a seamless integration with approaches to meaning based on higher-order logic.

End up with a characterization of meanings as *probabilistic programs*, which are, nevertheless, pure (i.e., no real probabilistic effects).

Such programs *describe* probability distributions over logical meanings.

Schematically…

$$[\![\textit{someone is tall}]\!] \;=\; \exists x : \mathrm{human}(x) \wedge \mathrm{height}(x) \geq \theta_{tall}$$

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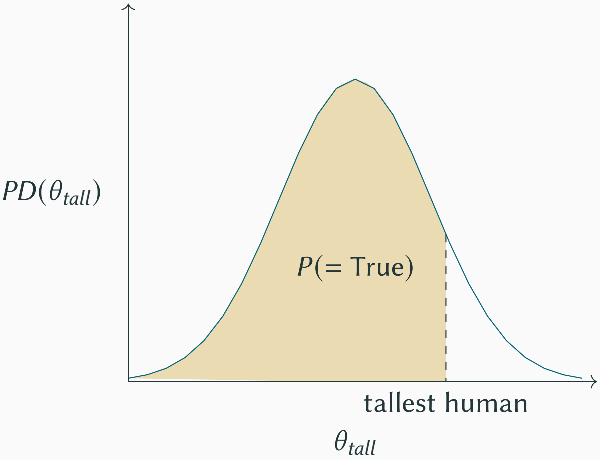
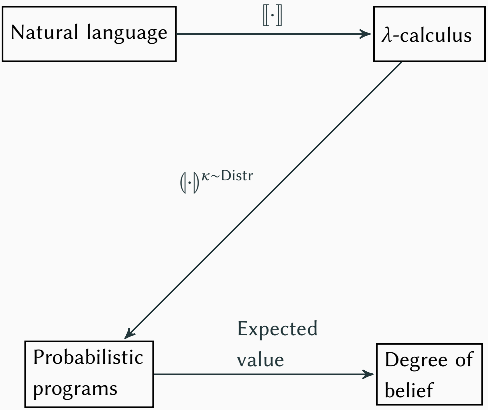
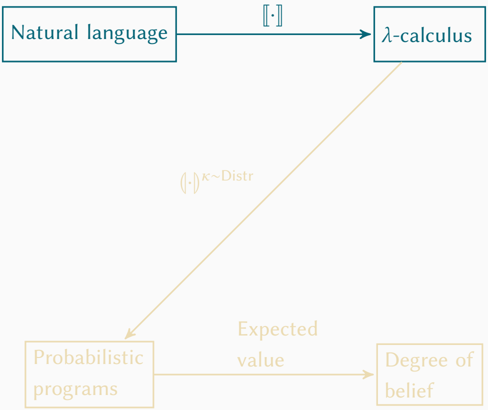
$$[\![\textit{someone is tall}]\!] = \exists x : \text{human}(x) \wedge \text{height}(x) \geq \theta_{tall}$$



Our system


Natural language
$[\![\cdot]\!]$
λ-calculus
$(\!|\cdot|\!)^{\kappa\sim\mathrm{Distr}}$
Probabilistic programs
Expected value
Degree of belief

Up next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Formal semantics

Two strategies to formally interpret natural language, inherited from Montague:

Logics and sets

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Two strategies to formally interpret natural language, inherited from Montague:

- direct: right into set theory
  - denotations (entities, functions, etc.) are elements of sets
- indirect: into a formal logic, e.g., the simply-typed λ-calculus/higher-order logic

Indirect interpretation

(1) Someone is tall.

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[\![\textit{someone}]\!] &= \lambda k.\exists x : \text{human}(x) \land k(x) \\
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Functional application and β-reduction:

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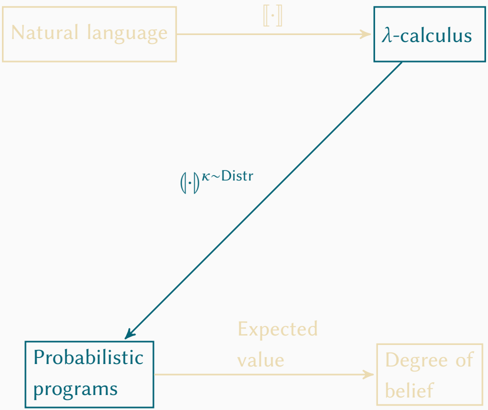
Functional application and β-reduction:

- $[\![\textit{someone}]\!]([\![\textit{is}]\!]([\![\textit{tall}]\!])) \quad \rightarrow_\beta \quad \exists x : \text{human}(x) \wedge \text{height}(x) \geq \theta_{tall}$

Up next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The probabilistic interpretation

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Let us assume that the non-logical constants of the logical language are finite in number and are ordered.

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- A *context* (κ) is a tuple of type $\alpha_1 \times ... \times \alpha_n$, where α_i is the type of the i^{th} constant.
- A context for this language would be of type $(e \longrightarrow d_{tall}) \times (e \longrightarrow t) \times (r \longrightarrow r \longrightarrow t) \times d_{tall}$.

A λ-homomorphism in a context

Given some context κ:

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$$\begin{aligned}
(\!|x|\!)^\kappa &= x && \text{(variables)}\\
(\!|\lambda x.M|\!)^\kappa &= \lambda x.(\!|M|\!)^\kappa && \text{(abstractions)}\\
(\!|MN|\!)^\kappa &= (\!|M|\!)^\kappa(\!|N|\!)^\kappa && \text{(applications)}\\
(\!|\langle M, N\rangle|\!)^\kappa &= \langle(\!|M|\!)^\kappa, (\!|N|\!)^\kappa\rangle && \text{(pairing)}\\
(\!|M_i|\!)^\kappa &= (\!|M|\!)^\kappa_i && \text{(projection)}
\end{aligned}$$

Etc. ($\diamond$, logical constants)

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Goal: allow the context to be a random variable.

Probabilistic programs

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  - Represents a normal distribution with mean μ and standard deviation σ.
  - $\mathcal{N}(\mu, \sigma)(f) = \int_{-\infty}^{\infty} \mathrm{PDF}_{\mathcal{N}(\mu,\sigma)}(x) * f(x) dx$

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- The probabilistic program that returns Jean-Philippe with a probability of 1.

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You can pass the value returned by a probabilistic program *m* to a function *k* from values to probabilistic programs, in order to make a bigger, sequenced probabilistic program.

$$\begin{aligned} (\star) &: ((\alpha \to r) \to r) && \text{(`bind')} \\ &\quad \to (\alpha \to (\beta \to r) \to r) \\ &\quad \to (\beta \to r) \to r \\ m \star k &= \lambda f.m(\lambda x.k(x)(f)) \end{aligned}$$

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"Run *m*, computing *x*. Then feed *x* to *k*."

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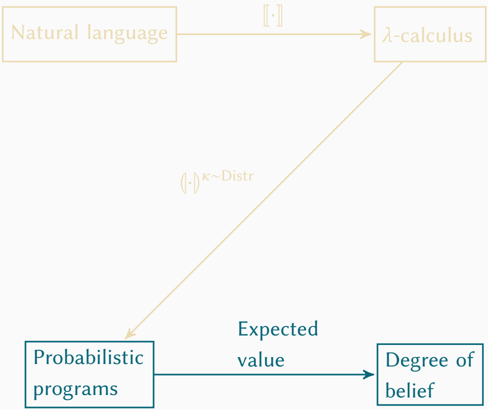
- If a context is of type $\alpha_1 \times \ldots \times \alpha_n$, then we seek a probabilistic program K of type $(\alpha_1 \times \ldots \times \alpha_n \to r) \to r$.
- Then, for a sentence ϕ in the logical language, we may do:

$$K \star \lambda\kappa.\eta(\llparenthesis\phi\rrparenthesis^{\kappa}) : (t \to r) \to r$$

Up next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- In the above, it picks out the mass assigned to $\top$.
- $\lambda b.1$ picks out the total mass (assigned to $\top$ and $\bot$).
- So, $P(p)$ is the probability that p returns $\top$.

An example

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Say our constants are:

1. height : $e \to d_{tall}$
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Define K as:

$$K = \mathcal{N}(72, 3) \star \lambda d.\eta(height, human, (\geq), d)$$

An example (cont'd)

$$K \star \lambda\kappa.\eta(\llparenthesis \exists x : \text{human}(x) \wedge \text{height}(x) \geq \theta_{tall} \rrparenthesis^{\kappa})$$

An example (cont'd)

$$
\begin{aligned}
& K \star \lambda\kappa.\eta(\llparenthesis \exists x : \mathrm{human}(x) \wedge \mathrm{height}(x) \geq \theta_{tall} \rrparenthesis^{\kappa}) \\
& \vdots \\
= \; & \lambda f.\, \mathcal{N}(72, 3)(\lambda d. f(\exists x : human(x) \wedge height(x) \geq d)) \\
= \; & \lambda f.\, \int_{-\infty}^{\infty} \mathrm{PDF}_{\mathcal{N}(72,3)}(y) * f(\exists x : human(x) \wedge height(x) \geq y) dy
\end{aligned}
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$$= \int_{-\infty}^{\infty} \mathrm{PDF}_{\mathcal{N}(72,3)}(y) * \mathbb{1}(\exists x : \mathit{human}(x) \wedge \mathit{height}(x) \geq y) dy$$

…the mass of $\mathcal{N}(72, 3)$ less than or equal to the height of the tallest human

Bayesian inference (e.g., RSA)

RSA

RSA models: a popular application of probabilistic semantics.

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- …who infers a distribution over meanings m from an utterance u, based on the probability that a pragmatic speaker, S_1, would make the utterance u to convey m.
- Given a meaning m, the probability that S_1 would make the utterance u to convey m is related to the probability that a literal listener, L_0, would infer m, given a literal interpretation of u.

Factoring by a weight / observing a premise

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Conclusion

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…using the same logical language one uses to characterize linguistic meanings.

References

Goodman, Noah D., and Michael C. Frank. 2016. Pragmatic Language Interpretation as Probabilistic Inference. *Trends in Cognitive Sciences* 20:818–829.

Goodman, Noah D., Vikash K. Mansinghka, Daniel Roy, Keith Bonawitz, and Joshua B. Tenenbaum. 2008. Church: a language for generative models. In *Proceedings of the Twenty-Fourth Conference on Uncertainty in Artificial Intelligence*, UAI'08, 220–229. Arlington, Virginia, USA: AUAI Press.

Goodman, Noah D., and Andreas Stuhlmüller. 2013. Knowledge and Implicature: Modeling Language Understanding as Social Cognition. *Topics in Cognitive Science* 5:173–184.

Lassiter, Daniel, and Noah D. Goodman. 2013. Context, scale structure, and statistics in the interpretation of positive-form adjectives. *Semantics and Linguistic Theory* 23:587–610. Number: 0.

Lassiter, Daniel, and Noah D. Goodman. 2017. Adjectival vagueness in a Bayesian model of interpretation. *Synthese* 194:3801–3836.