(i) $((p \vee q) \,\&\, (q \vee r)) \,\&\, (p \vee s)$
(ii) $(\sim p \,\&\, (\sim p \rightarrow q)) \rightarrow q$
(iii) $(p \vee q) \rightarrow ((r \leftrightarrow s) \,\&\, p)$

(b) Translate into standard notation:
(i) ApCKNpNqKpEqr
(ii) KANKAKEE$pqrspqrs$
(iii) NCAKE$pqrst$

(c) Express DeMorgan's Laws in Polish notation.

Chapter 7

Predicate Logic

7.1 Syntax

We now turn to the second of the logical languages we will examine: predicate logic. In it we will be able to analyze arguments such as (5-1) and (5-2) as well as all the arguments of the statement calculus.

In predicate logic an elementary statement can be composed of a *predicate* and a number of *terms*. For example, $H(s)$ contains a (one-place) predicate H and the term s; the statement $L(j, m)$ is composed of a (two-place) predicate L and two terms, j and m. The former might serve, for example, as the translation of *Socrates is human*, where *Socrates* is represented by s and *is human* by the predicate H. Similarly, $L(j, m)$ might be the predicate logic counterpart of *John loves Mary*.

Predicates are specified as one-place, two-place, etc. according to the number of terms they require to form a statement. In the examples above, H was one-place and L was two-place. Combining a predicate with the wrong number of terms results in an expression which is not well formed, e.g., $H(j, m)$; $L(s)$. Predicates will typically be represented by upper case letters but will not ordinarily carry any explicit indication of the number of terms they require. There is no limit on the number of places for a predicate so long as the number is finite.

We should note here, incidentally, that a predicate in the logical language need not correspond to a predicate in the grammatical sense in a natural language. Although the (logical) predicate H above was used to translate the (grammatical) predicate *is human* in the sentence *Socrates is human*, the logical predicate L corresponds to the transitive verb *loves* in *John loves Mary*, and indeed nothing prevents us from translating *John loves Mary* into predicate logic as $G(m)$, where G is a one-place predicate corresponding to *John loves*, which presumably is not a grammatical constituent at all.

Terms come in two varieties. The first is *individual constants* exemplified by s, m, and j above. As the name suggests, in the semantics of the predicate logic these will denote specific individuals, and in translating natural language statements they will typically show up as the correspondents of proper names such as *John, Mary*, and *Socrates*. The second kind of term is the *individual variable* (or simply *variable*), for which we use lower case letters from the end of the alphabet—v, w, x, y, z—with primes and/or subscripts attached if we need to mention more of them. When a predicate is combined with one or more variables, e.g., $H(x)$, $L(m, y)$, the result is not a statement but an expression called an *open statement* or *propositional function*.

An open statement can be made into a statement by prefixing an appropriate number of quantifiers, thus: $(\forall x)H(x)$, $(\exists y)L(m, y)$. The *universal quantifier* is denoted by $\forall$ and is the correspondent of English expressions such as *all, each*, and *every*. The existential quantifier, represented by $\exists$, corresponds to *some* (in the sense of "at least one, possibly more"). The x written alongside the universal quantifier in $(\forall x)H(x)$ indicates that the quantification is with respect to that variable in the expression which follows. This labelling of quantifiers is necessary since an expression may in general contain more than one quantifier and more than one variable. For example, in $(\forall x)(\exists y)L(x, y)$ the first position in $L(x, y)$ is universally quantified and the second existentially, but in $(\exists x)(\forall y)L(x, y)$ it is the other way around.

Letting H correspond once again to *is human*, we might gloss $(\forall x)H(x)$ as *Every individual is human* or *Everything is human*. $(\exists x)H(x)$ would correspond to *Some (at least one) individual is human*, or briefly, *Something is human*. Letting m correspond to *Mary* and L to *loves*, $(\exists y)L(m, y)$ could be the translation of *There is at least one individual whom Mary loves*, or more briefly, *Mary loves something* (or *someone*, if all individuals we happen to be talking about are human). Similarly, $(\forall y)L(m, y)$ would correspond to *Mary loves every individual*, or, again, if all the individuals happen to be human, *Mary loves everyone*.

Note incidentally that in many instances the particular choice of variable letters is not important. Instead of $(\forall x)H(x)$ we could equally well have written $(\forall y)H(y)$ or $(\forall z)H(z)$, etc. Similarly, $(\exists x)L(m, x)$ would do

just as well as $(\exists y)L(m, y)$. Of course when more than one variable is involved, we must use different letters for variables which may be distinct, e.g., in $(\forall x)(\exists y)L(x, y)$. To write $L(x, x)$ and then quantifiy, say existentially, to produce $(\exists x)L(x, x)$ would give a statement which we might gloss as *There is at least one individual which loves itself* (or *himself*). Here the same term, x, occupies both positions required by the two-place predicate L. In $(\exists x)(\forall y)L(x, y)$, *There is at least one individual who loves every individual*, the x and the y may take on values which are distinct individuals but they may also be the same; indeed, this statement will be true only if there is at least one individual who loves every individual, including that individual itself (or himself). Still, we should keep in mind that the choice of variable letter is immaterial so long as the same quantifiers are associated with the same predicate positions; i.e., $(\exists y)(\forall x)L(y, x)$ is an alphabetic variant of $(\exists x)(\forall y)L(x, y)$; but $(\exists y)(\forall x)L(y, x)$ and $(\exists x)(\forall y)L(y, x)$ would *not* be alphabetic variants because the quantifiers are associated with different positions in the predicate L.

We will return to further details of the use of quantifiers and variables below, but let us now observe that statements and open statements may be joined by the connectives $\sim, \&, \vee, \rightarrow$, and $\leftrightarrow$; for example:

(7–1) (i) $\sim H(x)$
(ii) $\sim H(s)$
(iii) $((\forall x)H(x) \,\&\, L(j, m))$
(iv) $(\sim H(s) \rightarrow \sim(\forall x)H(x))$

Expressions (ii), (iii), and (iv) might translate, respectively, *Socrates is not human, Everything is human and John loves Mary*, and *If Socrates is not human, then not everything is human*. Formula (i) is not a statement but an open statement since it contains an unquantified variable, and, not being a statement, it would presumably not serve as the translation of any declarative English sentence (perhaps *He is not human*, where the referent of *he* is not specified would come closest).

An open statement, even if internally complex, may always have quantifiers prefixed, so we may, if we choose, convert $\sim H(x)$ into $(\forall x)\sim H(x)$ or $(\exists x)\sim H(x)$ (corresponding to *Every individual is not human* — which is ambiguous for many speakers, but the intended sense here is that each individual fails to be human — and *At least one individual is not human*, respectively.)

Now that we have informally introduced the syntax of the language of predicate logic by illustration and example, we give a precise formulation of the syntactic rules.

The vocabulary consists of

(7-2)
(i) individual constants: $j, m, \ldots$
(ii) individual variables: $x, y, z, \ldots$ (sometimes subscripted)
The individual constants and variables together are called the set of *terms*.
(iii) predicates: $P, Q, R, \ldots$, each with a fixed finite number of argument places, called its *arity*.
(iv) the five connectives of the logic of statements: $\sim, \vee, \&, \to, \leftrightarrow$
(v) two quantifiers: $\forall$ and $\exists$
(vi) auxiliary symbols: (,) and [,]

The syntactic rules generate the set of *formulas* of the language of predicate logic. We will define the set of *statements* as a proper subset of this set of formulas.

(7-3)
(i) If P is an n-ary (i.e., n-place) predicate and $t_1, \ldots, t_n$ are terms, then $P(t_1, \ldots, t_n)$ is a formula.
(ii) If φ and ψ are formulas, then $\sim\varphi$, $(\varphi \,\&\, \psi)$, $(\varphi \vee \psi)$, $(\varphi \to \psi)$, and $(\varphi \leftrightarrow \psi)$ are formulas.
(iii) If φ is a formula and x is an individual variable, then $(\forall x)\varphi$ and $(\exists x)\varphi$ are formulas.
(iv) The formulas of the language of predicate logic can only be generated by finite numbers of applications of rules (i)–(iii).

The first rule generates *atomic* formulas (containing no connectives or quantifiers) like $R(x,y)$, $P(c)$, $K(m,x)$, and $S(x,z,m)$. Note that connectives may combine formulas with or without quantifiers, and that any quantifier plus variable may be prefixed to a formula, even when that variable does not occur within the formula (e.g., $(\forall x)P(y)$ is well-formed.) The syntax thus allows *"vacuous" quantification*. (Some systems do not allow vacuous quantification, but they pay a price: the set of generative rules is more complex if quantifier prefixes are allowed only when the quantified variable occurs inside the formula. We prefer to have a simpler syntax and render vacuous quantification meaningless or harmless in the semantics.)

If x is any variable and φ is a formula to which a quantifier is attached by rule (iii) above to produce $(\forall x)\varphi$ or $(\exists x)\varphi$, then we say that φ is the *scope* of the attached quantifier and that φ or any part of φ *lies in the scope* of that quantifier. We also refer to the φ as the *matrix* of the expression $(\forall x)\varphi$ or $(\exists x)\varphi$. Some examples are given in (7-4) below, where the scope of each quantifier is underlined.

(7-4)
(i) $(\exists x)\underline{P(x)}$
(ii) $(\exists y)\underline{R(x,y)} \,\&\, P(y)$
(iii) $(\exists y)\underline{(R(x,y) \,\&\, P(y))}$
(iv) $(\exists x)\underline{(P(m) \,\&\, R(j,y))}$
(v) $(\exists x)\underline{(\forall y)\underline{(R(x,y) \to K(x,x))}}$ (scope of $(\exists x)$) (scope of $(\forall y)$)
(vi) $(\exists x)\underline{[Q(x) \,\&\, (\forall y)\underline{(P(y) \to (\exists z)\underline{S(x,y,z)})}]}$ (scope of $(\exists x)$) (scope of $(\forall y)$) (scope of $(\exists z)$)

In (ii) note that $(\exists y)$ was attached to $R(x,y)$, which is therefore the scope of the existential quantifier, and the result conjoined with $P(y)$, which is outside the scope of $(\exists y)$. In (iii), on the other hand, the existential quantifier was attached to $(R(x,y) \,\&\, P(y))$, which thereby becomes its scope. Note that in (iv) a quantifier has the following formula as its scope even if the quantification is vacuous. In (v) and (vi) we see cases involving quantification of formulas which already contain quantifiers. Thus the scope of one quantifier may be contained within the scope of another.

This notion of quantifier scope is crucial in the following definition.

(7-5) DEFINITION 7.1 *An occurrence of a variable x is* bound *if it occurs in the scope of $(\exists x)$ or $(\forall x)$. A variable is* free *if it is not bound.* ■

Binding is hence a relation between a prefixed quantifier and an occurrence of a variable. For example, in $P(x)$ the x is free, but it is bound in $(\exists x)P(x)$. In (7-4)(ii), the y in $R(x,y)$ is bound (by the $(\exists y)$) but the y in $P(y)$ is free. In (iii) both occurrences of y are bound. In both (ii) and (iii) the x in $R(x,y)$ is free, not being in the scope of a quantifier associated with x. Similarly, the y in (iv) is free because the only quantification is by $(\exists x)$. We now see that what we have called "vacuous" quantification is vacuous in the sense that it does not give rise to any binding of variables. Note incidentally that constants, e.g., m and j in (iv), are not said to be bound or free; binding applies only to variables. In (v) and (vi) all variable occurrences are bound. Note that a variable which may be free in a subformula may become bound in a larger formula, e.g., the x and y in $S(x,y,z)$ in (vi).

Any occurrence of a variable in a formula is either bound or free; there is no middle ground. A variable may only be bound once, however. We might wonder, for example, whether the x in the $M(x)$ of the formula $(\forall x)(P(x) \rightarrow (\exists x)M(x))$ is bound both by the $(\exists x)$ and again by the $(\forall x)$. It is not; it is bound by the $(\exists x)$ only, and the x in $P(x)$ is then bound by the $(\forall x)$. The formula would surely be less confusingly written if we had chosen different variables, thus: $(\forall x)(P(x) \rightarrow (\exists y)M(y))$, which is an alphabetic variant of the original. In general, it is good practice in writing formulas to avoid using the same variable letter for distinct variables, even in cases such as this one where the intervening quantification assures their distinctness. We will return to the subject of alphabetic variants in Sec 7.3.

A *statement* of the predicate logic is defined as a formula that does not contain any free variables. Every occurrence of a variable in a statement is hence bound by some quantifier in the formula. The set of statements is sometimes called the set of *sentences, propositions*, or *closed formulas* of the predicate logic. A formula with at least one free variable is, as we have said, called an *open formula*, or *propositional function*.

7.2 Semantics

We give here an informal account of the semantics of predicate logic. A more formal treatment will be presented in Part D (Chapter 13).

As with propositional logic, a statement in the predicate calculus bears one of the truth values 1 (true) or 0 (false). If the statement is composed of predicates and terms, and possibly quantifiers also, then its truth value is determined by the *semantic values* (which are not necessarily truth values) of its components. For example, the statement $H(s)$, composed of the one-place predicate H and the constant s, receives its truth value in the following way: s has as its semantic value some individual chosen from a set D of individuals presumed to be fixed in advance. (D is like the domain of discourse of set theory and is often referred to in that way). Suppose, for example, that D is the set of all human beings, living or dead, and the individual assigned to s is Socrates. The predicate H has as its semantic value some *set* of individuals from D—let us say, for example, {Socrates, Aristotle, Plato, Mozart, Beethoven}. The statement $H(s)$ now gets the truth value *true* by virtue of the fact that the individual corresponding to s is a member of the set corresponding to H. On the other hand, had H had as its value the set {Mahler, Proust, Michelangelo}, $H(s)$ would have been false, Socrates not being a member of this set.

We use the double brackets $[\![\alpha]\!]$ to indicate the semantic value of the expression α. Thus, in the preceding example $[\![s]\!]$ = Socrates , $[\![H]\!]$ = {Socrates, Aristotle, Plato, Mozart, Beethoven}, and $[\![H(s)]\!] = 1$.

A two-place predicate L has as its semantic value a *set of ordered pairs* of individuals from D, i.e., a subset of $D \times D$. A statement such as $L(j,m)$ is true just in case the ordered pair of individuals $\langle x,y \rangle$ is in this set, where x is the semantic value of j and y is the semantic value of m. For example, if $[\![j]\!]$ is John Donne and $[\![m]\!]$ is Mary Queen of Scots, then $L(j,m)$ is true if ⟨John Donne, Mary Queen of Scots⟩ is in the set which is the semantic value of L; otherwise $L(j,m)$ is false. In general symbolic terms, for any two-place predicate K and terms a and b, $[\![K(a,b)]\!] = 1$ iff $\langle [\![a]\!], [\![b]\!] \rangle \in [\![K]\!]$.

Clearly the truth value of any statement in predicate logic will depend on the domain of discourse and the choice of semantic values for the constants and predicates. When these are fully specified, we say that we have a *model* for predicate logic. More specifically, a model consists of a set D and a function F which assigns:

(i)	to each individual constant	a member of D
(ii)	to each one-place predicate	a subset of D
(iii)	to each two-place predicate	a subset of $D \times D$
	and in general	
(iv)	to each n-place predicate	a subset of $\underbrace{D \times D \times \ldots \times D}_{n}$

(i.e., a set of ordered n-tuples of elements from D)

Thus, a statement in predicate logic such as $H(s)$ or $L(j,m)$ is not simply true or false, but true or false *relative to a particular model M*. If we want to emphasize this fact in our notation, we can add the name of the model as a superscript, thus: $[\![H(s)]\!]^M = 1, [\![s]\!]^M$ = Socrates, etc. Certain statements will turn out to be true or false irrespective of the model chosen, and such statements constitute the tautologies and contradictions, respectively, of predicate logic. Inasmuch as the connectives &, $\vee$, etc. have the same truth tables as in statement logic, it follows at once that an expression such as $H(s) \vee \sim H(s)$ is a tautology in this system, and $H(s) \,\&\, \sim H(s)$ is a contradiction. That is, whatever the model, $H(s)$ and $\sim H(s)$ will have opposite truth values, thus $H(s) \vee \sim H(s)$ will always be true, etc. A statement such as $H(s)$ or $L(j,m)$, whose truth value varies from model to model, is contingent. We will shortly encounter examples in the predicate calculus which are not such straightforward analogs of tautologies and contradictions in the logic of statements.

The semantics of quantified expressions is somewhat more complex than that of statements composed simply of predicates and terms. We sketch the basic ideas here informally and defer the detailed formalism to Chapter 13.

A formula in which all occurrences of variables are bound, such as $(\forall x)$ $H(x)$ or $(\exists y)L(m,y)$, is a statement and should accordingly be true or false with respect to the chosen model. Such statements, however, are composed syntactically of a quantifier (plus variable) and an open statement, e.g., $H(x)$ or $L(m,y)$, which is not a statement and does not, strictly speaking, have a truth value. In evaluating quantified expressions we nonetheless let these propositional functions take on truth values temporarily by letting the quantified variable range over all the individuals in the domain D one by one and determining the truth value the propositional function *would have* for each of those individuals. For example, to determine the truth value of $(\forall x)H(x)$ we let x range over all individuals in D and for each such assignment of a value to x determine the truth value $H(x)$ would have: true if $[\![x]\!]$ is in $[\![H]\!]$ in the model and false otherwise. Then $(\forall x)H(x)$ is true iff $[\![x]\!]$ is in $[\![H]\!]$ for *all* individuals in D. If for some individual, $[\![x]\!] \notin [\![H]\!]$, then $(\forall x)H(x)$ is false. To put it another way, $(\forall x)H(x)$ is true (in a particular model) iff $H(x)$ is true as x takes on as successive values every individual in D. Correspondingly, $(\exists x)H(x)$ is true iff $H(x)$ is true *for at least one* individual in D when x assumes that value.

Let us consider a small partial model. Predicates and terms not mentioned are also assumed to have values in the model, but for clarity we ignore them.

(7–6) Let $D = \{\text{Socrates, Aristotle, Plato, Mozart, Beethoven, Tolstoy}\}$

$$\begin{array}{llll}
F(s) & = \text{Socrates} & F(m) & = \text{Mozart} \\
F(a) & = \text{Aristotle} & F(b) & = \text{Beethoven} \\
F(p) & = \text{Plato} & F(t) & = \text{Tolstoy}
\end{array}$$

$$\begin{array}{ll}
F(H) = & \{\text{Socrates, Aristotle, Plato}\} \\
F(M) = & \{\text{Socrates, Aristotle, Plato, Mozart,} \\
& \text{Beethoven, Tolstoy}\} = D \\
F(L) = & \{\langle\text{Socrates, Socrates}\rangle, \langle\text{Socrates, Aristotle}\rangle, \\
& \langle\text{Mozart, Beethoven}\rangle, \langle\text{Beethoven, Mozart}\rangle, \\
& \langle\text{Tolstoy, Plato}\rangle, \langle\text{Plato, Mozart}\rangle, \langle\text{Aristotle, Tolstoy}\rangle\}
\end{array}$$

The reader may now verify that the following statements, among others, are true in this model:

$$H(s), H(a), H(p), M(s), M(b), L(s,s), L(t,p)$$

while the following are false:

$$H(m), H(b), H(t), L(a,s), L(m,m)$$

The statement $(\forall x)M(x)$ is true in this model since $M(x)$ is true whenever x takes as its value each of the members of D, i.e., $M(s)$, $M(a)$, $M(p)$, $M(m)$, $M(b)$, and $M(t)$ are all true. The statement $(\exists x)H(x)$ is true, since $H(x)$ is true for at least one value of x—in fact, it is true when $[\![x]\!]$ is Socrates or Aristotle or Plato. (Note carefully that the semantic values x takes are individuals from the set D and not constant letters s, a, p, etc., of the language.) Similarly, it is easy to see that since $(\forall x)M(x)$ is true (and the universe of discourse is not empty), $(\exists x)M(x)$ is true also.

The statement $(\exists y)L(m,y)$ is true since there is at least one value of y (just Beethoven, in fact) such that the ordered pair $\langle$ Mozart, $[\![y]\!]\rangle$ is in the set assigned to L, i.e., $[\![L]\!]$. However, $(\forall y)L(m,y)$ is false since $\langle$ Mozart, $[\![y]\!]\rangle$ is not in $[\![L]\!]$ for every value of y ($\langle$ Mozart, Socrates $\rangle$ is lacking, for example).

Given the truth values of the statements already mentioned, we can determine the truth values of complex statements such as $(H(s) \,\&\, H(m))$, $((\forall x)M(x) \vee H(a))$, and $(H(p) \rightarrow (\exists y)L(m,y))$ in the usual way according to the truth tables for the sentential connectives. The reader should verify that these three examples are, respectively, false, true, and true in the assumed model.

Evaluating an expression such as $(\exists x)(H(x) \,\&\, M(x))$ in which the connective lies inside the scope of the quantifier is not quite so straightforward. By the rule for evaluating existentially quantified expressions, we must determine whether there is any value of x in D which makes the complex propositional function $H(x) \,\&\, M(x)$ true. Here we must try each individual in D as a value for x and determine whether both $H(x)$ and $M(x)$ are true for that value. If at least one such value is found, $(\exists x)(H(x) \,\&\, M(x))$ is true; otherwise, it is false. In the model given, this formula is true since there are in fact three individuals—Socrates, Aristotle, and Plato—which are both in $[\![H]\!]$ and $[\![M]\!]$. On the other hand, $(\forall x)(H(x) \,\&\, M(x))$ is false; not every individual is in both $[\![H]\!]$ and $[\![M]\!]$. We see, however, that $(\forall x)(H(x) \rightarrow M(x))$

is true in this model. There is no individual which, when assigned to x, makes the conditional $H(x) \rightarrow M(x)$ false; i.e., no individual which is in $[\![H]\!]$ but not in $[\![M]\!]$.

Problem: What is the truth value in this model of $(\forall x)(L(m,x) \rightarrow H(x))$? of $(\forall x)(L(m,x) \rightarrow M(x))$?

Expressions containing quantifiers within the scope of other quantifiers add an extra degree of complexity in the evaluation. The same rules apply, but the expression is evaluated, so to speak, from the outside in. $(\forall x)(\exists y) L(x,y)$, for example, will be true just in case for every possible value of x in D the expression $(\exists y)L(x,y)$ is true. When is the latter true? If there is at least one value of y for which $L(x,y)$ is true, where x has the value fixed in the previous step. That is, we let x range over all the individuals in D, and at each value we determine the truth value of $(\exists y)L(x,y)$ by again letting y range over all the individuals in D. $(\exists y)L(x,y)$ might be true for some values of x and false for others, but the entire expression $(\forall x)(\exists y)L(x,y)$ is true only if $(\exists y)L(x,y)$ is true for every value of x.

In the chosen model, $(\forall x)(\exists y)L(x,y)$ happens to be true. To see this, let x be Socrates; then $(\exists y)L(x,y)$ is true when y is Socrates or Aristotle. If x is Aristotle, then $(\exists y)L(x,y)$ is true when y is Tolstoy, and so on. We find ultimately that for each value of x we can always find some value of y which makes $L(x,y)$ true. Or to put it another way, each member of D appears at least once as first member in the set of ordered pairs assigned to L. Thus, $(\forall x)(\exists y)L(x,y)$ is true in this model.

Note, on the other hand, that in this model $(\exists y)(\forall x)L(x,y)$ is false. For this formula to be true, we would have to find at least one value of y for which $(\forall x)L(x,y)$ is true, i.e., some individual which appears as second member with every individual in D as first member in the set of ordered pairs assigned to L. It is easy to see from inspection of $[\![L]\!]$ that no such individual exists, so $(\exists y)(\forall x)L(x,y)$ is false in this model.

These last two examples demonstrate that the order in which quantifiers appear in an expression when one is universal and the other existential can have semantic significance. That is to say that there may in general be models, as here, in which one statement is true and the other, with the order of quantifiers reversed, is false. This is immediately evident if we choose a slightly less artificial model. Let D be the set of all living persons and let $L = \{\langle x,y\rangle \mid x \text{ loves } y\}$. Then $(\forall x)(\exists y)L(x,y)$ is true if for each person there is at least one individual (possibly himself or herself) whom that person loves, but $(\exists y)(\forall x)L(x,y)$ would be true only if there were at least one person who is universally loved, i.e., loved by everyone. It is easy to imagine that the former could be true while the latter is false.

We can now see how to translate certain types of English statements into predicate logic. For *All cats are mammals*, for example, we will need one-place predicates, call them C and M, to correspond to *is a cat* and *is a mammal*. We can then represent the English statement by $(\forall x)(C(x) \rightarrow M(x))$, which we can gloss roughly as follows: for every individual in the universe of discourse, if that individual is a cat, then it is also a mammal; or more briefly, everything which is a cat is a mammal. (Note, by the way, that in our predicate calculus rendition the statement is true in case there are no cats in the universe of discourse, for then $C(x)$ will be false for all values of x and hence the conditional will always be true.) In the same vein, *No cats are mammals* might come out as $(\forall x)(C(x) \rightarrow\sim M(x))$: everything which is a cat fails to be a mammal. Again, this is true in case the universe of discourse contains no cats, unlike the English statement, which we might well regard as inappropriate or nonsensical in such an instance. We are once again in familiar territory where English statements and their nearest logical correspondents do not match up perfectly.

Some cats are mammals could be translated as $(\exists x)(C(x)\,\&\,M(x))$, which is true iff there is at least one individual in the domain of discourse which is both a cat and a mammal. In this case the absence of cats from the universe of discourse makes the predicate calculus statement false, whereas we might say that the English statement suffers from presupposition failure (with whatever consequences attach to this defect). Note that $(\exists x)(C(x) \rightarrow M(x))$, with a conditional as in the translation of the universal statement, would not do. This statement is true when there are no cats (the antecedent is always false) or when there is at least one mammal (the consequent is true). Although we are prepared to accept a certain amount of disparity between English statements and their logical translations, we would not want to say that *Some cats are mammals* gets the same translation as *Either there are no cats or there is at least one mammal*.

The negative existential statement *Some cats are not mammals* could be rendered as $(\exists x)(C(x)\,\&\,{\sim}M(x))$, (which is also false when the universe of discourse contains no cats).