Factivity, presupposition projection, and the role of discrete knowledge in gradient inference judgments

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Joint work

Factivity, presupposition projection, and the role of discrete knowledge in gradient inference judgments*

Julian Grove and Aaron Steven White University of Rochester

Abstract We investigate whether the factive presuppositions associated with some clause-embedding predicates are fundamentally discrete in nature—as classically assumed—or fundamentally gradient—as recently proposed (Tonhauser, Beaver, and Degen 2018). To carry out this investigation, we develop statistical models of presupposition projection that implement these two hypotheses, fit these models to existing inference judgment data aimed at measuring factive presuppositions (Degen and Tonhauser 2021), and compare the models' fit to the data using standard statistical model comparison metrics. We



Aaron Steven White UofR

Motivation

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We need linking assumptions...

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- We will develop a compositional probabilistic semantics that allows us to formulate Bayesian models of inference data (following Grove and Bernardy (2023)).
- We will use this semantics to *combine* theories of factivity with linking assumptions seamlessly.

Factivity and gradience

Factivity

(1) Jo loves that Mo Left.

 \sim Mo left.

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This inference patterns like a presupposition, using family-of-sentence tests (Chierchia and McConnell-Ginet 1990):

- (2) a. Jo doesn't love that Mo Left.
 - b. Does Jo love that Mo left?
 - c. If Jo loves that Mo Left, she'll also love that Bo left.
 - \sim Mo left.

Gradience

What sorts of inference patterns arise from uses of factive predicates in an experimental setting?

• E.g., if you ask someone to rate the likelihood that Mo left, given that *Jo loves that Mo left* is true.

White and Rawlins (2018)

'Someone {discovered, didn't discover} that a particular thing happened.'

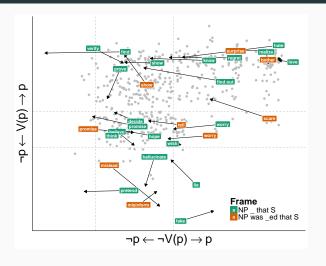
White and Rawlins (2018)

'Someone {discovered, didn't discover} that a particular thing happened.'

'Did that thing happen?'

(yes, maybe or maybe not, no)

White and Rawlins (2018)

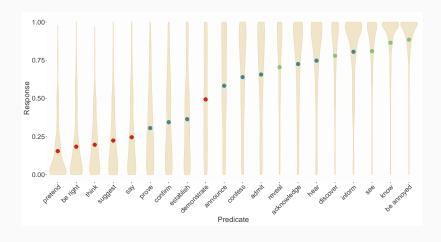


x-axis: negative polarity; *y*-axis: positive polarity

Degen and Tonhauser (2022)

Helen asks: "Did Amanda discover that Danny ate the last cupcake?"		
Is	Helen certain that Danny ate the last cupcake	e?
no		yes
	Next	

Degen and Tonhauser (2022)



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Another possibility:

- Predicates are generally ambiguous between being factive or non-factive. But different predicates are factive with different frequencies, and it is these frequencies which differ among one another in a gradient fashion.
- It is just that *discover*'s average factivity is less than *know*'s.

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The Fundamental Discreteness Hypothesis
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- The Fundamental Discreteness Hypothesis
 Factivity is a discrete semantic property of at least some token occurrences clause-embedding predicates. (A given occurrence of a particular predicate either triggers a projective inference, or it does not trigger a projective inference.)
- The Fundamental Gradience Hypothesis
 The gradient distinctions among predicates reflect the different gradient contributions specific predicates make to inferences about the truth of their complement clauses.

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Hypotheses

The fundamental discreteness hypothesis represents the classical view (Kiparsky and Kiparsky 1970; Karttunen 1971, i.a.).

The fundamental gradience hypothesis represents a more recent view, i.e., that presupposition triggers can trigger inferences gradiently (Tonhauser, Beaver, and Degen 2018).

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- Provide a modular probabilistic semantics that allows the two hypotheses to be stated precisely.
 - I.e., give a characterization of Bayesian models that encode the two hypotheses, and formulate explicit linking assumptions using the very same semantic repertoire.
- Fit these models to inference data and compare the fits.

A modular probabilistic semantics

Following Grove and Bernardy (2023), we provide an interface for stating a probabilistic semantics using *monads*.

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$$(\sim): P\alpha \to (\alpha \to P\beta) \to P\beta$$

allowing us to bind a probabilistic program of type $P\alpha$ to a value of type α used inside another probabilistic program.

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· An operator 'return'

$$(\cdot)$$
: $\alpha \to P\alpha$

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But where does the threshold *d* come from?

Solution: the meaning of *tall* is represented by a probability distribution.

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- Returns the property true of an individual if their height exceeds the threshold d.
 - But now d takes on the probability distribution represented by thresholdPrior.
 - So the meaning of tall represents a probability distribution over properties, each one fixed by some threshold d.

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- Model linguistic meanings as programs, not of type $P\alpha$, but of type $P(P\alpha)$.
 - The "inner" P P(Pα) represents uncertainty that arises on individual occasions of use and interpretation, even after the meanings of expressions have been fixed.
 - The "outer" $P P(P\alpha)$ represents *metalinguistic* uncertainty; that is, uncertainty about what interpretation to apply in the first place.

Say you're in a noisy bar. Someone says Jo is -all...

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- Having fixed one adjective or the other, there is uncertainty that arises from the relevant degree threshold.

Models of factivity

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 For any given complement clause, a background fact is provided which is intended to make the clause either likely or unlikely to be true...

Fact (which Elizabeth knows): Zoe is a math major. Elizabeth asks: "Did Tim pretend that Zoe calculated the tip?"		
no	Is Elizabeth certain that Zoe calculated the tip	? yes
	Next	

They additionally conduct a separate norming study intended to assess the prior certainties associated with such complement clauses, paired with their background facts.

Fact: Zoe is 5 years old.			
How likely is it that Zoe calculated the tip?			
impossible	definitely		
Continue			

Factivity and world knowledge

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This gives us four models...

The discrete-factivity model

```
discrete-factivity : P(P\kappa)

discrete-factivity = \langle \mathbf{v}, \mathbf{w} \rangle \sim \text{priors}

\tau_{\mathbf{v}} \sim \text{Bernoulli}(\mathbf{v})

\tau_{\mathbf{w}} \sim \text{Bernoulli}(\mathbf{w})

\tau_{\mathbf{v}} \vee \tau_{\mathbf{w}}
```

Important: The Bernoulli variable $\tau_{\mathbf{v}}$ associated determining whether or not a predicate is factive is sampled at the "outer" P.

The wholly-gradient model

```
wholly-gradient : P(P\kappa) wholly-gradient = \langle \mathbf{v}, \mathbf{w} \rangle \sim \text{priors}  \boxed{ \begin{aligned} \tau_{\mathbf{v}} &\sim \text{Bernoulli}(\mathbf{v}) \\ \tau_{\mathbf{w}} &\sim \text{Bernoulli}(\mathbf{w}) \\ \left\langle \tau_{\mathbf{v}}, \tau_{\mathbf{w}} \right\rangle \end{aligned} }
```

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The discrete-world model

The wholly-discrete model

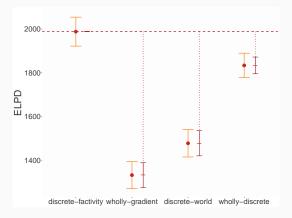
```
wholly-discrete : P(P\kappa) wholly-discrete = \langle \mathbf{v}, \mathbf{w} \rangle \sim \text{priors} \tau_{\mathbf{v}} \sim \text{Bernoulli}(\mathbf{v}) \tau_{\mathbf{w}} \sim \text{Bernoulli}(\mathbf{w}) \boxed{\langle \tau_{\mathbf{v}}, \tau_{\mathbf{w}} \rangle}
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Comparisons

We compare the four models in terms of their expected log pointwise predictive densities (ELPDs) computed under the widely applicable information criterion (Gelman, Hwang, and Vehtari 2014; Watanabe 2013).

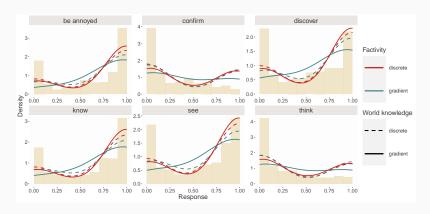
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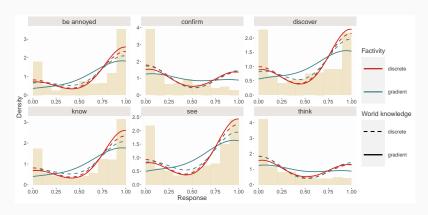
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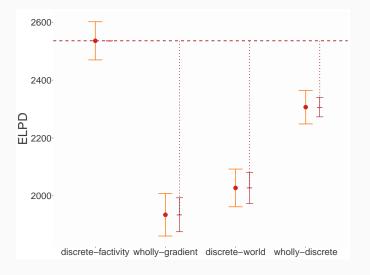
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- Modify the methodology slightly, so that complement clauses have minimal lexical content.
 - · Helps get rid of the effect of world knowledge.

Our replication

Given data from 288 new participants:

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First modification: the bleached task

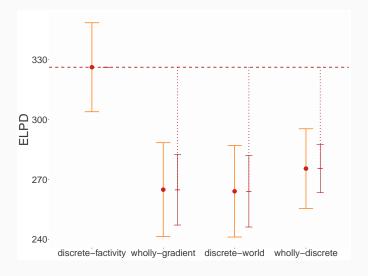
You are at a party. You walk into the kitchen and overhear Linda ask somebody else a question. Linda doesn't know you and wants to be secretive, so speaks in somewhat coded language.

Linda asks: "Did Tim pretend that a particular thing happened?"

	Is Linda certain that that thing happened?	
no		yes
	Next	

Bleached comparisons

Given data from 47 participants:



Second modification: the templatic task

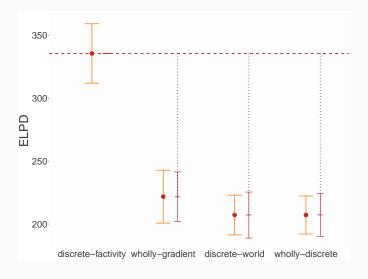
You are at a party. You walk into the kitchen and overhear William ask somebody else a question. The party is very noisy, and you only hear part of what is said. The part you don't hear is represented by the 'X'.

William asks: "Did Ray pretend that X happened?"

	Is William certain that X happened?	
no		yes
	Next	

Templatic comparisons

Given data 49 participants:



Summing up

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- But we have to integrate semantic analyses into theories of inference carefully... here, we choose to integrate them in a modular fashion, using monads.

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- But we have to integrate semantic analyses into theories of inference carefully... here, we choose to integrate them in a modular fashion, using monads.
- The strategy of using an already-available formal apparatus allows linking assumptions to be made explicit and testable.

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