Economics of networks

DSIER [/d□'za□ər/] — Summer 2025

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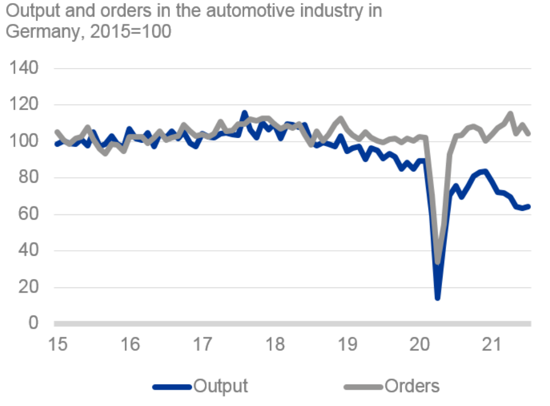
Why should I care?

- **Big picture**: networks turn isolated data points into a map of economic interdependence.
- **Key insight**: centrality and paths reveal who really drives trade, prices and growth.
- **Today’s skill**: load a real network in R and compute its core stats.

Roadmap for today

1. Concepts & data
2. Metrics: walks, paths, diameter
3. Centrality: degree, closeness, betweenness
4. Null–model baseline
5. Application: Economic Complexity

Disruptions in the Automotive Industry



Source: Deutsche Bank

Chip shortage global supply-chains

Forbes

LEADERSHIP STRATEGY

Supply Chain Economics: Car Chip Shortage

Bill Conerly Senior Contributor
I connect the dots between the economy ... and business!

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Jul 13, 2021, 07:20am EDT

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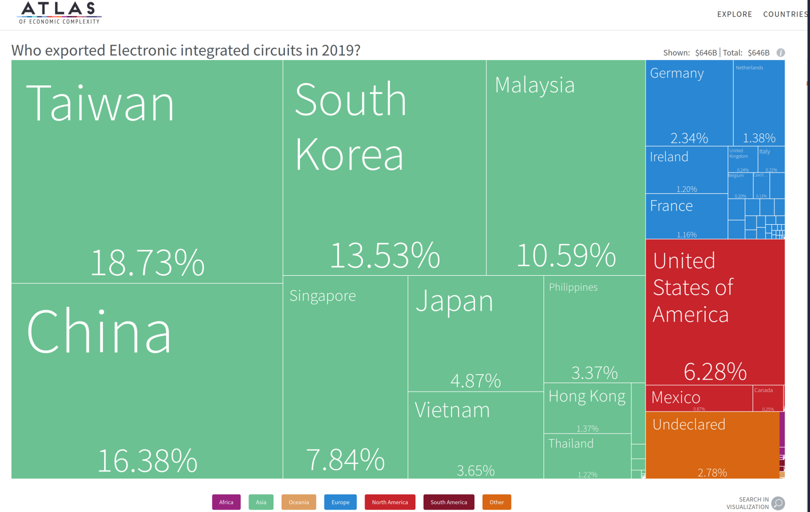
Bosch's new semiconductor factory (Photo by Robert Michael/picture alliance via Getty Images)
DPA/PICTURE ALLIANCE VIA GETTY IMAGES



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Smartphone GVC before vs after 2017

2009–12

2017–20

How has the network thinned?

Characteristics of the Supply Chain network

	[2009:2012]	**[2013:2016]**	**[2017:2020]**
# of different countries	21	16	15
# of different Buyers	18	12	11
# of different Sellers	13	10	9
Number of supply links	224	154	130

What is a network?

1. A set of nodes (vertices)
2. A set of edges (links) connecting pairs of nodes

Trade illustration

- Nodes = countries
- Edge $i \to j$ if country i exports to j
- Collect edges in an adjacency matrix A with $A_{ij} = 1$ when the flow exists

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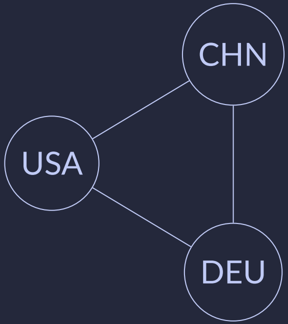
From simple to richer graph structures

Level 0 – Simple

- Undirected, unweighted
- No self-loops ($A_{ii} = 0$)
- Captures trade relationship (bilateral)

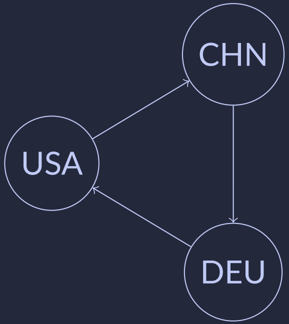


Undirected, unweighted

From simple to richer graph structures

Level 1 – Directed

- Order matters: $A_{ij} \neq A_{ji}$
- Captures export *flows*

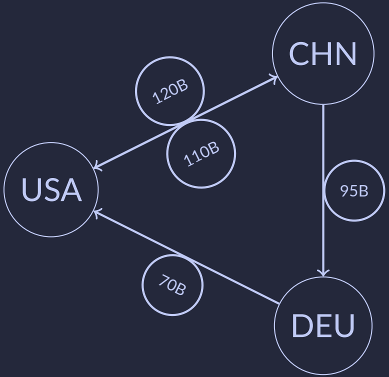


Directed edges = export flows

From simple to richer graph structures

Level 2 – Weighted

- Edge values = intensity (volume, tariff)
- Can even be negative (cost/friction)



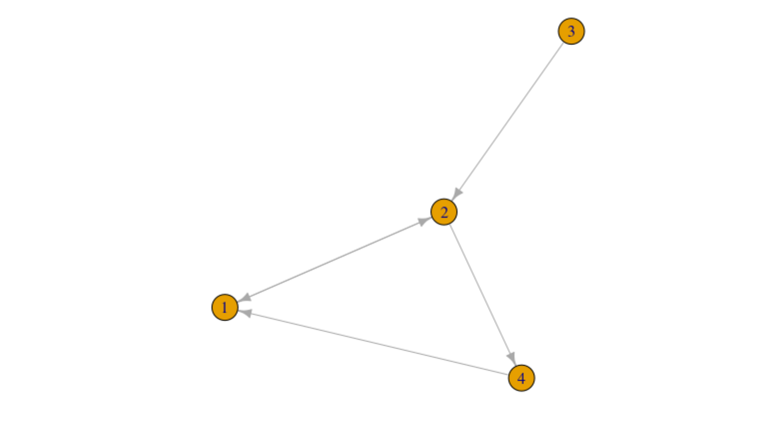
Edge weights = trade volume (USD billions)

Example of Directed Graph

```
> library(igraph)
> node_list <- tibble(id = 1:4)
> edge_list <- tibble(from = c(1, 2, 2, 3, 4), to = c(2, 3, 4, 2, 1))
> directed_g<- graph_from_data_frame(d = edge_list,
                         vertices = node_list, directed = TRUE)
> get.adjacency(directed_g)
4 x 4 sparse Matrix of class "dgCMatrix"
  1 2 3 4
1 . 1 . .
2 . . 1 1
3 . 1 . .
4 1 . . .
```

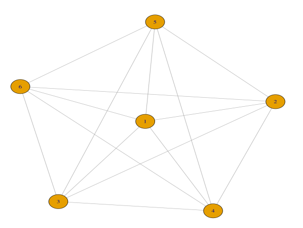
Example of Directed Graph

```
> plot(directed_g, edge.arrow.size = 0.2)
```

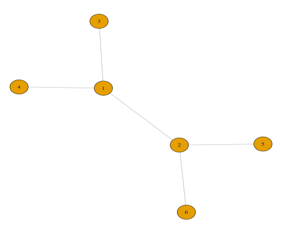


Other type of graphs

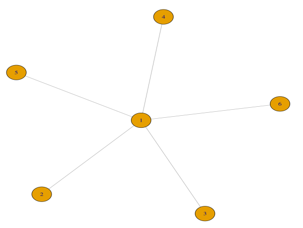
Complete Graph



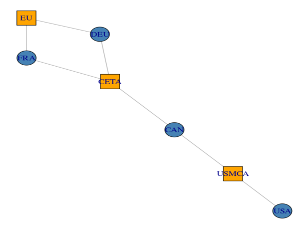
Tree



Star



Bipartite Network



Practical Corner: Bipartite Network

```r
# generate a dataframe to represents all the edges of your bipartite ntw
d <- data.frame(country=c("DEU", "DEU", "FRA", "FRA", "CAN","CAN", "USA"),
            trade_agr=c("CETA","EU", "EU", "CETA","CETA","USMCA","USMCA"))
# trasform it in a graph
g <- graph_from_data_frame(d, directed = FALSE)
# define color and shape mappings to distinguish nodes type
V(g)$label <- V(g)$name
V(g)$type <- 1
V(g)[name %in% d$trade_agr]$type <- 2
col <- c("steelblue", "orange")
shape <- c("circle", "square")
plot(g,
     vertex.color = col[V(g)$type],
     vertex.shape = shape[V(g)$type]
)
```

Walks, paths geodesics

1. **Walk** Any ordered sequence of edges: CHL $\rightarrow$ BRA $\rightarrow$ DEU $\rightarrow$ BRA
2. **Path** A walk with *no* repeated node: CHL $\rightarrow$ BRA $\rightarrow$ DEU
3. **Length** Number of edges in the path from i to j (above: 2)
4. **Geodesic** Shortest path between two nodes; its length is the graph distance, which we denote with $\ell(i, j)$. For CHL–DEU the geodesic is CHL $\rightarrow$ BRA $\rightarrow$ DEU.

Economic reading — The geodesic length tells you how many trade "hops" a Chilean export shock needs to reach Germany.

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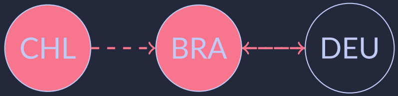
Seeing the metrics (toy trade network)



walk

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path

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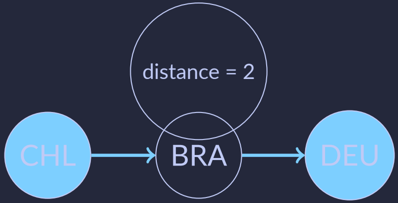
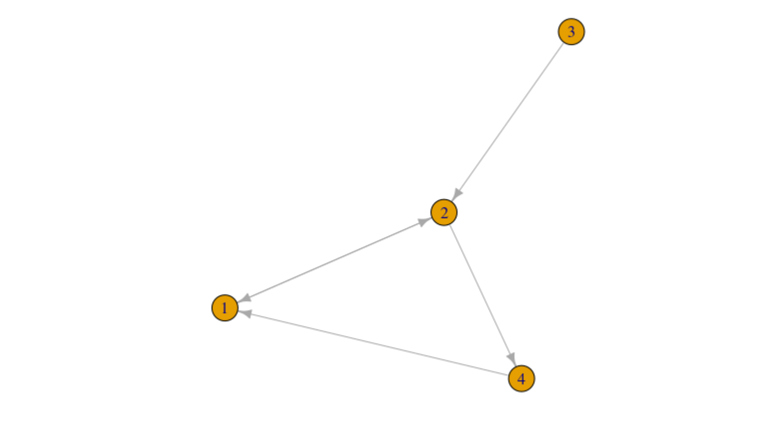


geodesic

Test corner

How many paths from 3 to 1? Which is shortest?



Stats for graphs

```
> igraph::all_simple_paths(directed_g, 3, 1)
[[1]]
+ 3/4 vertices, named, from 2c34291:
[1] 3 2 1

[[2]]
+ 4/4 vertices, named, from 2c34291:
[1] 3 2 4 1

> igraph::shortest_paths(directed_g, 3, 1)
$vpath
$vpath[[1]]
+ 3/4 vertices, named, from 2c34291:
[1] 3 2 1
```

How connected is a trade network?

1. **Density (no self loop)**

$$\delta = \frac{2m}{n(n-1)}$$

   where m = edges, n = nodes. [Guess for the trade network btw countries?]

2. **Giant component size** Fraction of nodes in the largest connected piece. *Ex.* 94 % of countries belong to one export web.
3. **Diameter** $\max_{i,j} \ell(i,j)$. "Farthest two countries need 6 hops."
4. **Average path length** $\bar{\ell} = \frac{2}{n(n-1)} \sum_{i>j} \ell(i,j)$. Real trade: $\bar{\ell} = 3.1$

Reading — Low diameter + high giant-component share imply shocks can spread globally; low density curbs redundancy.

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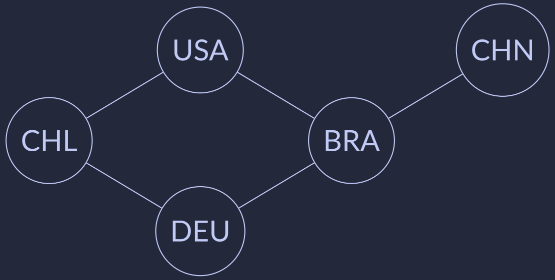
Connectedness on our toy graph



Metric	**Value**
Nodes n	5
Edges m	5
Density δ	0.50
Giant component share	100%
Diameter	3
Avg. path length $\bar{\ell}$	1.9

All five countries sit in one component; any shock crosses the network in ≤3 hops.

Key Network Structures in Economics

Network Type	Structure / Intuition	Density	Avg. Path	Economic Context
Star (hub-and-spoke)	One central hub connected to all others	Low	Very short	Logistics, supply chains, platform economies
Core–periphery	Dense central group + sparse outer nodes	Medium	Short to moderate	Global trade hierarchy: developed vs. emerging
Modular (community)	Dense internal clusters with few inter-cluster links	Medium	Moderate	Regional trade blocs, innovation clusters
Scale-free	Hubs dominate; many nodes with few links	Low	Very short	Financial contagion, tech networks
Bipartite (countries–products)	Two node types (e.g., exporters and goods)	Structured	Varies	Economic complexity, RCA-based trade analysis

Different network shapes reflect different economic dynamics—efficiency, fragility, inequality, or specialization.

Application 1: Buyer–Supplier Network

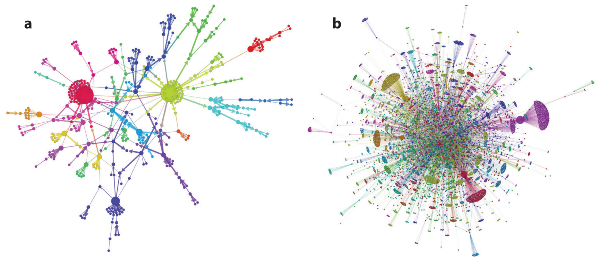


Figure 1

Firm-to-firm trade. Every node represents a firm, and the arrows show the direction of trade. The size of the node represents how many relationships a given firm has, while the size of the arrow shows the value of the transaction. (*a*) Relationships between US importers and Norwegian exporters of HS product 847990 in 2006. (*b*) Relationships between US importers and Norwegian exporters of all products in 2006.

Bernard et al. (2018) – Firm-to-firm trade between US and Norwegian firms

(a) HS 847990 – One Product

Sparse, modular network with clear firm clusters. Suggests specialized, non-overlapping supply chains. Shocks likely stay localized unless a hub is affected.

(b) All Products

Dense, tangled core with many interconnections. Reflects scale-free or core–periphery structure. Efficient but more exposed to contagion via central firms.

Application 2: Environmental cooperation agreements network

Carattini et al. (2022): Countries as nodes and edges represent whether there is an environmental agreement between that country pair.

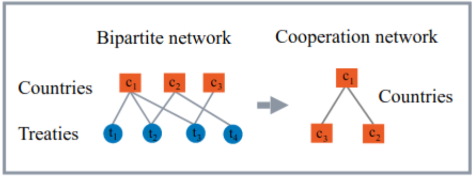
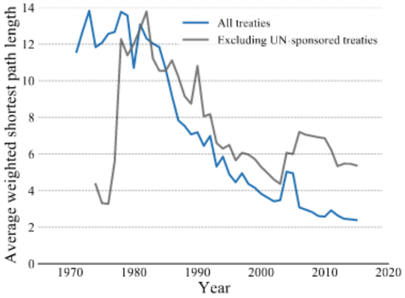


Figure 1: **Network construction**

Application 2: Environmental cooperation agreements network



(b) Average shortest path length

Reference at this link

From "how far?" to "who matters?"

1. We've learned to **measure distance**
    - walks, paths, geodesics tell us *how trade shocks travel*
2. Next question: **which nodes shape those shocks most?**
    - Does a hub with many partners matter more than a broker on the only East–West route?
3. Centrality measures
    - *Degree:* direct reach
    - *Closeness:* speed of access
    - *Betweenness:* brokerage power
    - *Eigenvector:* inherited prestige

Take-away — centrality turns raw topology into economic influence.

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Local vs. global reach

1. **Degree centrality** (direct reach)

$$k_i = \sum_{j=1}^{n} A_{ij}$$

- *Notation* $A_{ij} = 1$ if country j exports to i (otherwise 0).
- *Concept* "How many direct partners do I trade with?"
- *Ex.* China has $k_{\text{CHN}} \approx 140$: wide export option set dampens a single-partner shock.

2. **Closeness centrality** (inv. average distance)

$$C_i = \frac{n-1}{\sum_{j \neq i} d(i,j)}$$

- *Notation* $d(i,j)$ is the length of the shortest trade path (number of hops) from i to j.
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Bridges and prestige hubs

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$$e_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij} \, e_j$$

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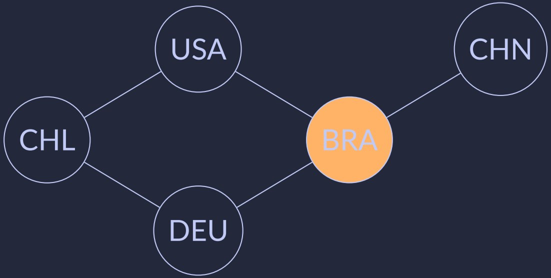
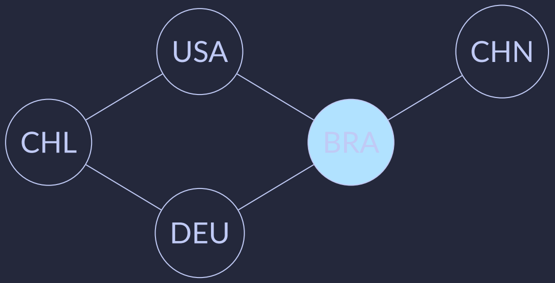
Degree centrality (direct partners)



Country	k_i
CHL	2
USA	2
BRA	3
DEU	2
CHN	1

BRA links to three partners ⁇ widest direct reach.

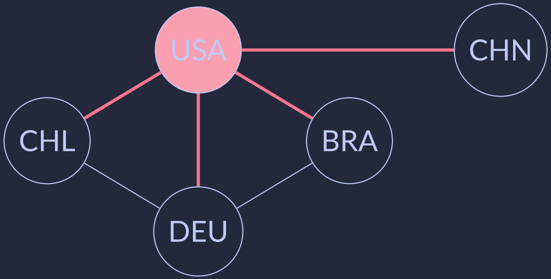
Closeness (avg. trade hops to everyone)



Country	C_i
CHL	0.57
BRA	0.80
USA	0.67
DEU	0.57
CHN	0.50

BRA never farther than 2 hops ⍰ quickest access to all.

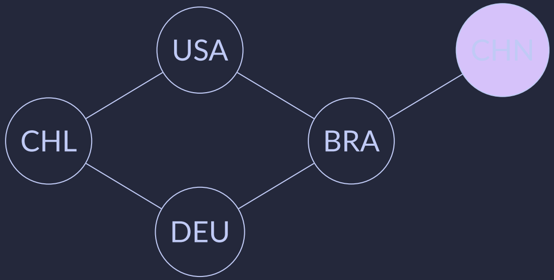
Betweeness (Bridge vs. prestige)



Country	B_i
CHL	0.5
BRA	1.0
USA	4.0
DEU	1.0
CHN	0

USA lies on 4 of 5 shortest cross-region routes ⍰ choke-point.

Betweeness (Bridge vs. prestige)



Country	e_i
CHL	0.22
BRA	0.28
USA	0.30
DEU	0.25
CHN	0.35

CHN gains status from trading with high-score USA BRA.

How are B_i and e_i computed? (toy graph)

1. Betweenness B_i

1. List every unordered pair (s,t) CHL–USA, CHL–BRA, ..., DEU–CHN $(10$ pairs$)$
2. For each pair, find *all* shortest paths.

   CHL–CHN: CHL–USA–CHN (3 hops)
   CHL–DEU–BRA–CHN (3 hops)

3. Credit a node $\frac{1}{\sigma_{st}}$ if it lies *inside* the path (endpoints not counted).
4. Sum over the 10 pairs ? the table values.

Why USA = 4? USA is on

CHL–USA–CHN, CHL–USA–BRA, BRA–USA–DEU, BRA–USA–CHN

and no other node shares those 4 obligations.

2. Eigenvector e_i

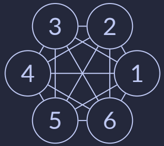
Power-iteration on the (row-normalised) adjacency matrix:

$$e^{(t+1)} = Ae^{(t)} \quad \longrightarrow \quad e$$

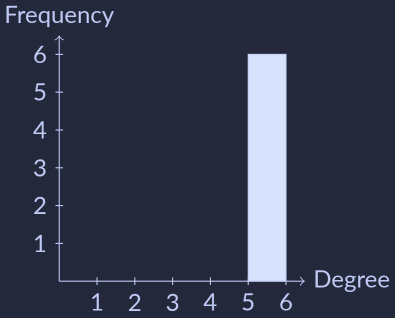
1. Start with $e^{(0)} = (1,1,1,1,1)^\top$
2. Multiply, renormalise, repeat until the vector stabilises
3. The stabilised entry e_i is larger when a node is linked to *other* large-e nodes.

Why CHN = 0.35? CHN's only partner is BRA, whose score is already high, so the recursion keeps pushing CHN upward even with degree 1.

Degree Distributions



Complete graph
All nodes have degree 5



Degree distribution
Uniform — no heterogeneity

Real trade networks are rarely this uniform: most have few links, a few have many.

Why Use Null Models?

- Real networks mix basic constraints (size, activity) and meaningful structure (hubs, communities).
- Null models = random benchmarks matching basic constraints (e.g., node degrees).
- Compare real network metrics (like path length) to the null: Is the observed structure surprising?
- **Example:** Is avg. path length 3.1 in trade shorter/longer than expected by chance, given country degrees?

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Key Null Models

Two common benchmarks:

1. **Erdős-Rényi** $G(n, p)$ (Simplest)
   - Randomly connects nodes with fixed probability p.
   - *Preserves:* Avg. density. Ignores node specifics.
   - *Use:* Baseline for ”purely random” connections.

2. **Configuration Model (CM)**
   - Randomly connects nodes while keeping *each node's exact degree* the same as observed.
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Interpreting Deviations from Configuration Model

Compare Real Network Metric vs. CM Metric (e.g., Avg. Path Length $\bar{\ell}$)

Purpose: Does structure beyond individual node degrees matter?

Common Interpretations for $\bar{\ell}$:

- Real $\bar{\ell}$ < CM $\bar{\ell}$: Network is **more efficiently connected** than expected by chance (given degrees). $\rightarrow$ Suggests specific organizing principles (e.g., hubs).
- Real $\bar{\ell}$ > CM $\bar{\ell}$: Network is **more fragmented / distant** than expected by chance (given degrees). $\rightarrow$ Suggests barriers or clustering.

$\implies$ Null models help isolate the impact of non-random network topology.

Which null model answers which question?

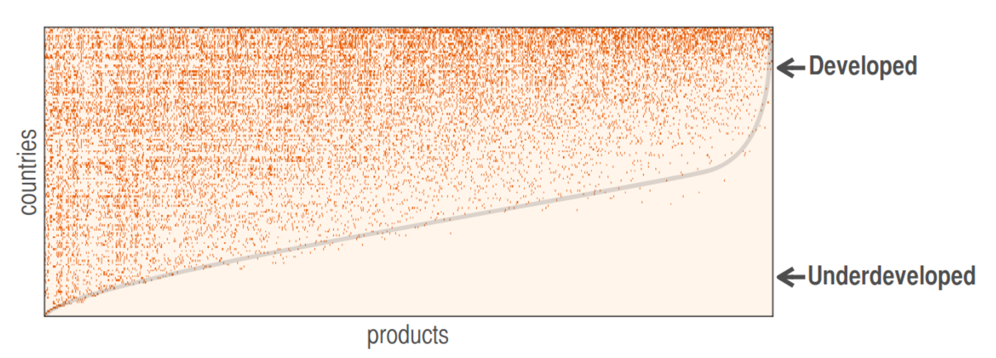
Model	Best suited question	Keeps fixed	igraph call
Erdős–Rényi $G(n,p)$	"Is the WTO network denser than random?"	n, p (avg. density)	`sample_gnp(n,p)`
Configuration model	"Is avg. path length surprisingly short/long given node degrees?"	exact degree seq.	`sample_degseq()`
Edge-rewiring	"Are specific motifs (e.g., feed-back cycles) more common than chance, given degrees?"	in-/out-deg. seq.	`rewire(…, keeping_degseq())`
Gravity-constrained†	"Does digital-services trade structure exceed a gravity baseline?"	Node attributes (GDP, dist.), expected flows	simulate from gravity model, then build graph

†Common in econ: Fit gravity model (e.g., Poisson), predict flows, threshold to get links, analyse the resulting graph as the nu

APPLICATIONS

“The productivity of a country resides in the diversity of its available non-tradable capabilities, and therefore, cross-country differences in income can be explained by differences in economic complexity, as measured by the diversity of capabilities present in a country and their interactions.”
Hidalgo and Hausmann 2009

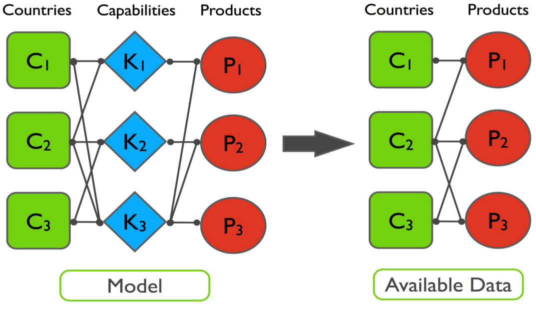
Matrix of diversification of countries



Source: Cristelli, Tacchella, Pietronero (2014)

The theory of hidden capabilities

A country is able to produce a product when it has the capabilities to do it (Hausmann & Hidalgo 2009)



Source: Hidalgo et al. (2009)

Network structure

Let us index countries with $c = 1, \ldots, n$ and products with p
The bipartite network is represented by means of a biadjacency matrix B of size $n \times p$

$$B_{cp} = \begin{cases} 1, & \text{if country } c \text{ is a significant exporter of the product } p, \\ 0, & \text{otherwise} \end{cases}$$

Significant exporter, when

$$RCA_{cp} = \frac{\frac{q_{cp}}{\sum_p q_{cp}}}{\frac{\sum_c q_{cp}}{\sum_c \sum_p q_{cp}}} > 1 \tag{1}$$

which is whenever the share of product p in the country export basket is larger than its share in the world trade

Method of Reflections

MoR consists of iteratively calculating the average value of the previous-level properties of a node's neighbors and is defined as the set of observables:

$$k_{c,N} = \frac{1}{k_{c,0}} \sum_p B_{cp}\, k_{p,N-1}$$

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for $N \geq 1$. With initial conditions given by the degree, or number of links, of countries and products, $k_{c,0} = \sum_p B_{cp}$ (diversification) and $k_{p,0} = \sum_c B_{cp}$ (ubiquity)

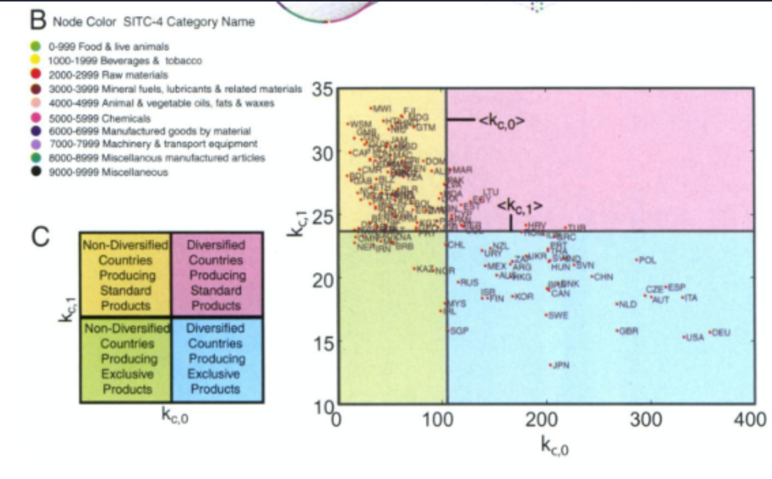
Methods of Reflections

Definition	Working Name	**Description:** Short summary Question Form
$k_{a,0}$	Diversification	Number of products exported by country a. How many products are exported by country a?
$\kappa_{\alpha,0}$	Ubiquity	Number of countries exporting product α. How many countries export product α?
$k_{a,1}$	$k_{c,1}$	Average ubiquity of the products exported by country a. How common are the products exported by country a?
$\kappa_{\alpha,1}$	$k_{p,1}$	Average diversification of the countries exporting product α. How diversified are the countries that export product α?
$k_{a,2}$	$k_{c,2}$	Average diversification of countries with an export basket similar to country a How diversified are countries exporting goods similar to those of country a?
$\kappa_{\alpha,2}$	$k_{p,2}$	Average ubiquity of the products exported by countries that export product α. How ubiquitous are the products exported by product's α exporters?

Table S 1 Interpretation of the bipartite network description obtained from the method of reflections.

For countries, even variables (kc,0 ,kc,2 ,kc,4,…) are generalized measures of diversification, whereas odd variables (kc,1 ,kc,3 ,kc,5,…) are generalized measures of the ubiquity of their exports.

Results



Source: Hidalgo et al. (2007)

Null Model

They construct two random matrices

- availability of capabilities (a)

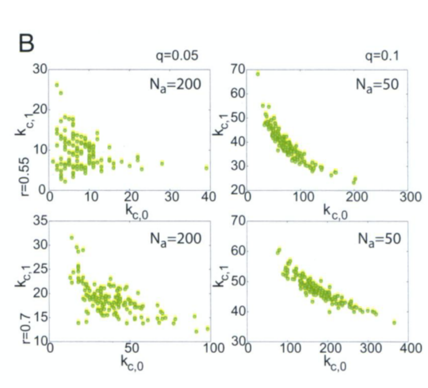
$$C_{ca} = \begin{cases} 1, & \text{with prob. r} \\ 0, & \text{with prob 1-r} \end{cases}$$

- necessary capabilities to produce products

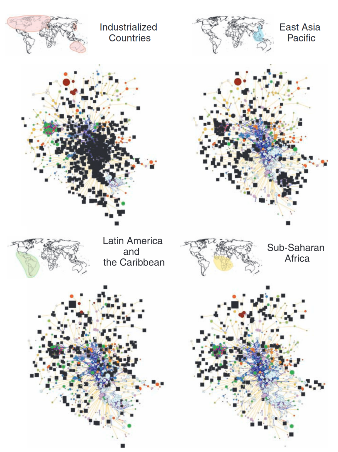
$$\Pi_{pa} = \begin{cases} 1, & \text{with prob. q} \\ 0, & \text{with prob 1-q} \end{cases}$$

$\hat{B}_{cp} = 1$ if $\sum_a \Pi_{pa} = \sum_a \Pi_{pa} C_{ca}$, 0 otherwise.

Results of the null model



Countries in the Product Space



Source: Hidalgo et al. (2007)

Projecting the Bipartite Network

Let B be the $n \times p$ biadjacency matrix where $B_{cp} = 1$ if country c significantly exports product p.

Step: Project onto products Define a product–product relatedness matrix M as:

$$M_{pp'} = \sum_c B_{cp} \cdot B_{cp'}$$

This gives the number of countries that export both products p and p'. The matrix M is symmetric and captures the *co-export* intensity between products.

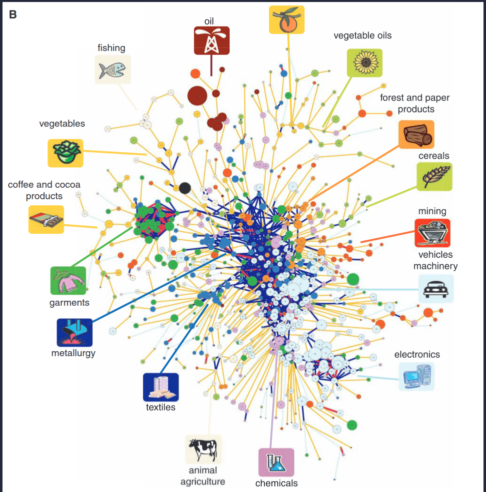
Weighted version: Normalize by product ubiquity:

$$\phi_{pp'} = \frac{\sum_c B_{cp} \cdot B_{cp'}}{\max(k_{p,0}, k_{p',0})}$$

where $k_{p,0} = \sum_c B_{cp}$ is the number of exporters of product p.

Interpretation: Two products are close in the product space if many countries export them both.

The Product Space of Trade



Source: Hidalgo et al. (2007)

Sources

- Jackson, Matthew O. Social and economic networks. Vol. 3. Princeton: Princeton university press, 2008.
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Suggested references:

- Networks: An Introduction, by MEJ Newman