Modelado y Simulación

Julián Jiménez Cárdenas

Facultad de Matemáticas
FUKL

July 30, 2024

Introduction to Modeling I

Mathematical modeling is a powerful tool that:

- Translates real-world problems into mathematical language
- Provides insights into complex systems
- Enables predictions and decision-making
- Facilitates optimization and efficiency

Key applications include:

- Physics and engineering
- Economics and finance
- Biology and medicine
- Climate science and ecology

Elaborating in some applications, we have

Introduction to Modeling II

- **Celestial Mechanics**
  - Understanding motion of stars, planets, comets
  - Applications in agriculture, navigation, calendars
  - Modern astrophysics: universe formation and development
- **Energy Supply**
  - Modeling energy consumption trends
  - Planning for future energy needs
- **Everyday Scenarios**
  - E.g., safe driving distances at various speeds
- **Climate Change**
  - Modeling global warming impacts
  - Predicting changes in weather patterns
  - Assessing human impact on climate

Introduction to Modeling III

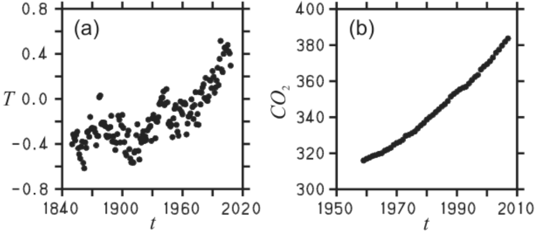

Figure: An illustration of global warming. (a) The HadCRUT3 global temperature anomaly data (consisting of annual differences from 1961-90 normals) are given in °C (Rayner et al, 2003, Brohan et al, 2006); (b) the Mauna Loa data (Tans 2008) of atmospheric CO_2 concentrations are given in ppmv.

Linear Models I

First we introduce the simplest modeling approach: **the linear functions**.

- Suppose we aim to set a relation between two variables, **x** and **y**, and we have paired data from these variables of the form

$$\mathcal{D} = \{(x_i, y_i) \ : \ i = 1, 2, \ldots, n\}\,.$$

- Thus, we can choose any pair of these points $(x_i, y_i), (x_j, y_j) \in \mathcal{D}$ with $i \neq j$ to find the line that passes through these, which algebraically can be written as

$$y(x) = y_i \frac{x - x_j}{x_i - x_j} + y_j \frac{x - x_i}{x_j - x_i}.$$

- Note that $y(x_i) = y_i$ and $y(x_j) = y_j$, as expected.

Linear Models II

Example (U.S. energy comsumption)

The following table relates the U.S. energy consumption C (in 10^15 Btu) in time t (U.S. Dept, of Energy 2008).

t	C	t	C
1950	34.616	1980	78.122
1955	40.208	1985	76.491
1960	45.087	1990	84.652
1965	54.017	1995	91.173
1970	67.844	2000	98.975
1975	71.999	2005	100.506

- Fit a line passing through the data.
- Graph the error $e = (y - y^{(mod)})/y^{(mod)}$ of the line with respect to the real data. $y^{(mod)}$ is the value predicted by the model.

Exponential Model

Example (Orbital period and distance)

Consider the following data.

Table: Orbital periods and mean distances of planets from the Sun (World Almanac 2010). Here r is the mean distance from the sun in 10^9 km. T_p is the period in earth years $a = 365.25$ days.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
r	0.0579	0.1082	0.1496	0.2280	0.7785	1.4335	2.8718	4.4948
T_p	0.2408	0.6152	1.0000	1.8808	11.8618	29.4566	84.0107	164.7858

- Fit a straight line for this data. Is it adequate?
- Assume $T_p = Ar^B$ and use logarithms to transform this relation to a linear one. Fit a straight line. Is it adequate?

Polynomial methods I

The linear approach is useful because of its simplicity, but it has a rather limited range of applicability, as most models cannot be developed in terms of linear functions. Thus, we will consider polynomial models.

- **Linear Polynomials**. There is a unique line passing through a pair $(x_1, y_1), (x_2, y_2)$ of points. Algebraically, let $y = a_0 + a_1 x$ be this line. The requirement of this line passing throught these points imply the following relations

$$y_1 = a_0 + a_1 x_1, \quad y_2 = a_0 + a_1 x_2.$$

  This system can be written as

$$\begin{pmatrix} 1 & x_1 \\ 1 & x_2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$

Polynomial methods II

and this can be inverted (almost always) in order to find a_0, a_1, *a.k.a.*, the line, which is

$$P_1(x) = y_1 \frac{x - x_2}{x_1 - x_2} + y_2 \frac{x - x_1}{x_2 - x_1}.$$

- **Non-Linear Polynomials**. Now, if we have a set $\mathcal{P} = \{(x_, y_0), (x_1, y_1), \dots, (x_n, y_n)\}$ of $n + 1$ points, there is (almost always) a polynomial of degree n passing through these points, and it has the form

$$P_n(x) = y_0 L_0(x) + y_1 L_1(x) + \cdots + y_n L_n(x),$$

with

$$L_k(x) = \prod_{i=0, i \neq k}^{n} \frac{x - x_i}{x_k - x_i}.$$

Polynomial methods III

The previous polynomial formula is called the lagrangian form of polynomials. Let us deepen on these ideas in the board.