Second Midterm

Scientific Computing II
Fundación Universitaria Konrad Lorenz
April 27, 2024

- We are evaluating your parallel programming skills (in C/C++, Python), so only parallel implementations will be assessed. **Purely sequential implementations will not receive grade.**
- Given your proficiency in profiling, optimizing, and modularizing code, **any obvious inefficiencies will result in penalties.**
- The exam will be available from April 27th, 2024, 8.15am. From that time on you will have 2 hours and 15 minutes to complete and upload it to the respective assignment in Aula Virtual. Midterms sent through email will not be graded.
- You can ask any question of this assignment during class or through email: `julian.jimenezc@konradlorenz.edu.co`.

Mandelbrot Set

The Mandelbrot set is a famous mathematical set named after Benoit Mandelbrot, who first studied it in the late 1970s. It is defined in the complex plane by iterating the function:

$$f_c(z) = z^2 + c$$

where z is a complex number and c is a constant complex number. The Mandelbrot set, denoted by M, consists of all complex numbers c for which the sequence defined by $z_{n+1} = z_n^2 + c$ does not diverge to infinity when iterated from $z_0 = 0$.

Your task will be to graph the Mandelbrot Set. Below you can find some hints that will guide you through this task.

1. A complex number can be defined as an ordered pair $z = a + ib$, where $i = \sqrt{-1}$. Then, given two complex numbers $z = a + ib$ and $w = c + id$, the sum of these two is $z+w = (a+c)+i(b+d)$. On the other hand, squaring z gives $z^2 = (a+ib)^2 = (a^2-b^2)+i(2ab)$. Furthermore, the norm of z is the distance from 0 to z, and is $|z| = \sqrt{a^2 + b^2}$.

2. To determine the convergence or divergence of the Mandelbrot sequence $z_{n+1} = z_n^2 + c$, with $z_0 = 0$ and a given complex number c, iterate through the sequence up to a specified maximum number of iterations (e.g., 1000). If, at any point, the absolute value of z_n exceeds 2, consider the sequence diverged, and return the number of iterations (which is 1000 in case of non-divergence).

3. Consider a meshgrid of the complex plane for all $z = a+ib$ with values of $(a, b) \in [-1, 1]^2$. This meshgrid should contain at least 5×10^7 complex numbers. Calculate the previous step for all these numbers.

4. Use the previous data to plot your results in Python using *plt.imshow()* or *plt.pcolor*. Customize the colormap for better visualization.