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OCES 2003 :

Descriptive Physical Oceanography

Lecture 15: intro to waves



Outline

- ▶ Recap: circulation and dependence on small-scale dynamics
- ▶ **waves**: fundamental concepts
 - periodicity, crest/trough/node
 - **wavelength** + **period**
 - **frequency** + **wavenumber**
 - **restoring force** + **dispersion relation**
 - propagation, **phase/group velocity**

Key terms: waves, wavenumber, frequency, period, dispersion relation, phase/group velocity

Reacp: MOC

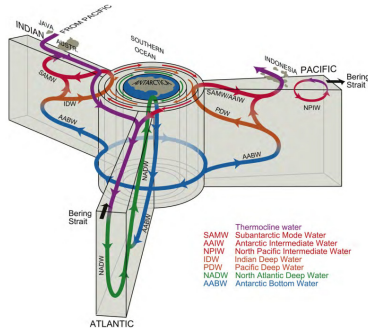


Figure: Schematic of the 3d MOC with watermass distributions. From Talley *et al.* (2011), *Descriptive Physical Oceanography*; see more in their Fig. 14.11. Format after Arnold Gordon (1991).

- MOC important for climate, carbon storage, ecology, etc.
 - e.g. warming of Western Europe by AMOC
 - e.g. carbon storage by deep water formation
- mostly along-isopycnal flow

- isolated places for watermass transformation + deep/abyssal water formation (deep convection)

Recap: what goes down must come up

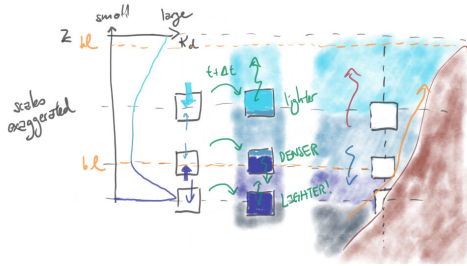


Figure: Schematic of the diffusive upwelling.

- ▶ diapycnal mixing contribute upwelling, strongest in boundary layers
→ broad diffusive boundary intensified upwelling

what causes the boundary intensification of κ_d ?

Recap: what goes down must come up

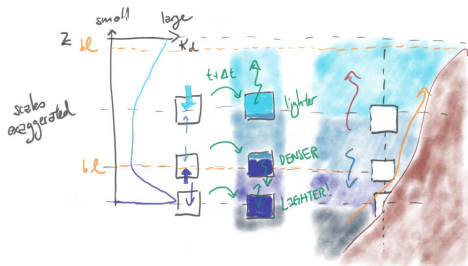


Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers
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what causes the boundary intensification of κ_d ? **dynamics!**

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) → internal gravity waves (Lec. 16) → shear instabilities (Lec. 17)

Recap: form stress and SO overturning

Role of **baroclinic instability** (Lec. 13, see also Lec. 17), important for

- ▶ vertical **momentum** transfer by **interfacial form stress**
- ▶ scale transfer of **energy**
→ **mesoscale eddies**, conduit between large-scales and **submesoscales**
- ▶ **along-isopycnal mixing** and also MOC

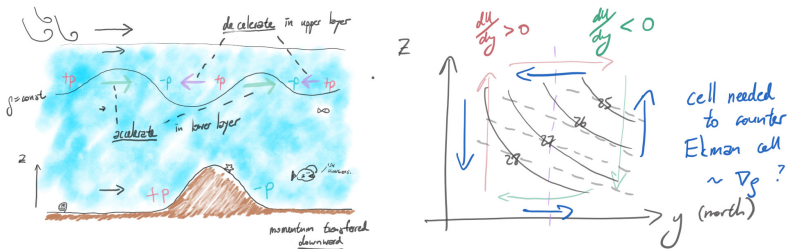


Figure: Schematic of form stress and eddy induced overturning cell in Southern Ocean (see Lec. 14)

Outlook of the next few lectures

Dynamics important, next four lectures on

- ▶ **waves** (this Lec. + 16, 18) and **instabilities** (Lec. 17)

→ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- ▶ **how to describe waves** (Lec. 15)
- ▶ types of waves (Lec. 16)
 - consequence + leading to instabilities
- ▶ instabilities (Lec. 17)
 - **parcel**-type (mechanistic) arguments for instability
- ▶ tides (particularly as **internal gravity waves**) (Lec. 18)

Examples of waves

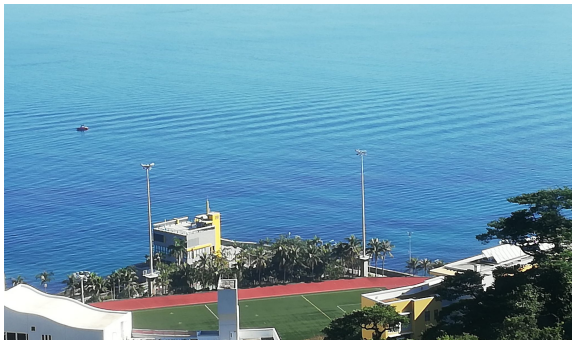


Figure: Gravity waves with signal at the sea surface (as darker and lighter bands). Taken at HKUST.

Surface gravity waves¹

► restoring force is buoyancy

→ treat air-sea as one fluid with a giant jump in density

¹ According to Richard Feynman, while water waves are “...easily seen by everyone... are the worse possible example [of waves], because... they have all the complications that waves can have (From Feynman, Leighton & Sands, 1971, *Lectures of Physics*).

Examples of waves

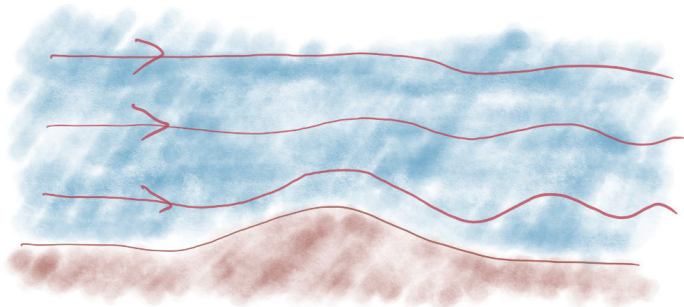


Figure: Flow over topography (e.g. tidal motion) leading to wave generation.

Tides and/or internal gravity waves forced by tidal motion

- ▶ restoring force is still buoyancy
- wave breaking contributing to mixing

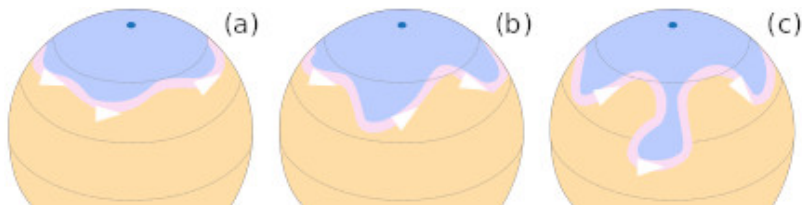


Figure: Example of jet stream meanders as Rossby waves. Figure from Wikipedia.

Rossby waves (cf. same Rossby as Rossby number)

- **restoring force** is Coriolis (or background gradient in vorticity)
 - planetary-scale waves, generic in rotating systems (e.g. rotating tanks, Jupiter, galactic disks)

Examples of waves

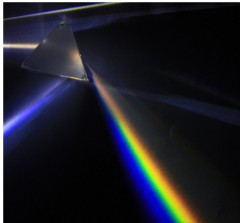
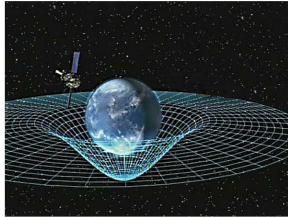
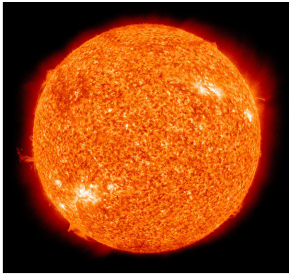


Figure: Example of some other waves in systems that support waves: Alfvén waves (magnetic field + Lorentz force), gravitational waves (spacetime + gravity), electromagnetic waves (but also **wave-particle duality**), sound waves (mechanical forcing + any medium). All figures from Wikipedia except the cello one.

Features of waves

Some observations:

- ▶ waves have some **oscillation/periodicity**
→ want a measure of **period**
- ▶ waves to **propagate**
→ **speed/velocity** associated with waves
- ▶ waves need a **medium** to travel through
→ subtlety with Electro-Magnetic waves (not touched on here)
- ▶ characterised by a **restoring force** (follows from medium)
- ▶ waves can increase in **amplitude** and **steepen**
→ wave breaking and mixing
- ▶ can **disperse, refract, interfere** etc. (used in Lec. 17, 20)

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physics \Rightarrow **dispersion relation**, identifies the type of waves

Features of waves

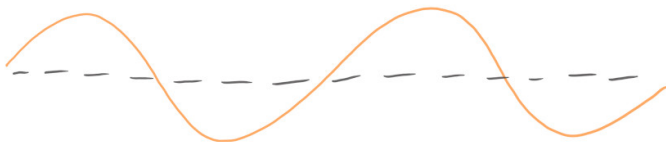


Figure: Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement η described by (could also be sine)

$$\eta \sim \cos(x)$$

Features of waves

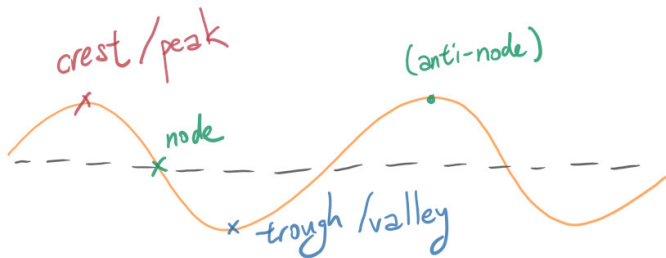


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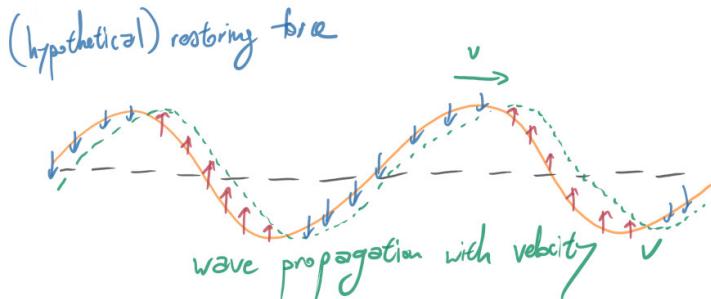


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- **displacement** η described by (could also be sine)

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Features of waves

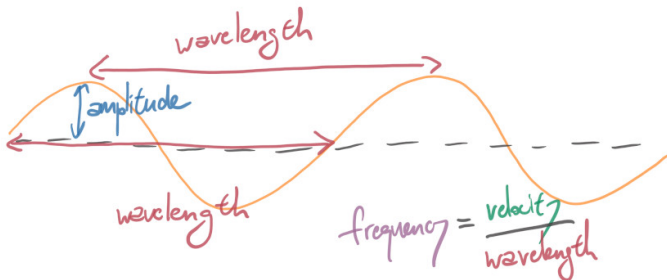


Figure: Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement η described by (could also be sine)

$$\eta \sim A \cos(x - vt), \quad \gamma = v/\lambda$$

Features of waves

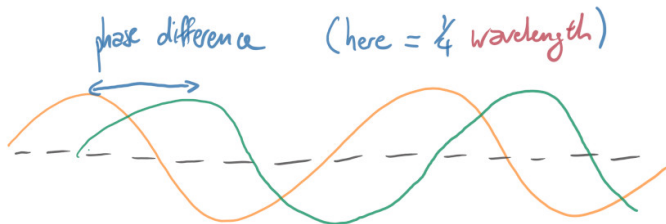


Figure: Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement η described by (could also be sine)

$$\eta \sim A \cos(x), \quad \eta \sim A \cos(x - \lambda/4)$$

Features of waves

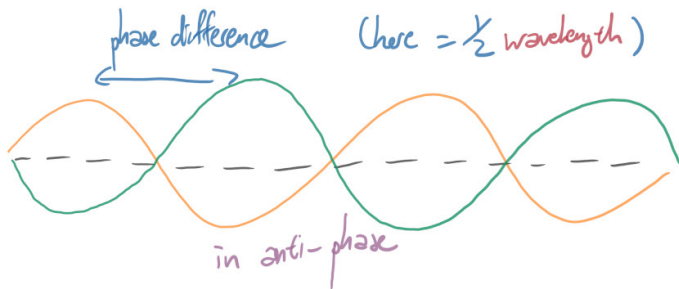


Figure: Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement η described by (could also be sine)

$$\eta \sim A \cos(x), \quad \eta \sim A \cos(x - \lambda/2) \sim -A \sin(x)$$

Features of waves

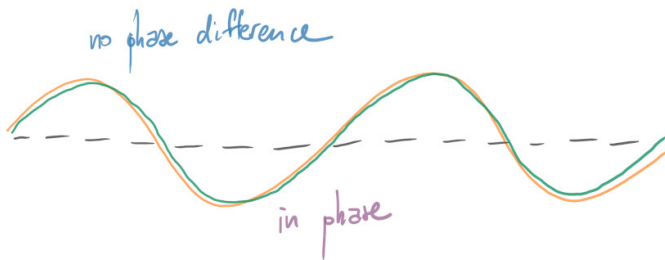


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Features of waves

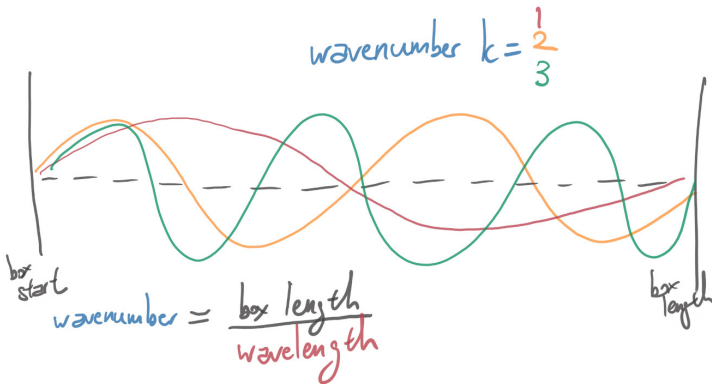


Figure: Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement η described by (could also be sine)

$$\eta \sim A \cos(2x), \quad \eta \sim A \cos(1x), \quad \eta \sim A \cos(3x)$$

Features of waves

$$\gamma = \frac{v}{\lambda}, \quad k = \frac{2\pi}{\lambda}$$

- ▶ γ the **frequency** (units: $\text{s}^{-1} = \text{Hz}$)
→ how quickly the wave oscillates
- ▶ $v = c_p$ the **phase velocity**
→ how fast the wave itself moves around
- ▶ λ the **wavelength**
→ how long the wave is
- ▶ k the **wavenumber**
→ intuitively how many waves can you fit in a box (so $k \sim \lambda^{-1}$)
→ does not necessarily have to be an integer

Features of waves: dispersion relation

Usually describe waves in terms of **wavenumber** k and the **angular frequency** $\omega = 2\pi\gamma$, i.e.

$$\eta = A \cos(kx - \omega t)$$

- generally, for $\mathbf{x} = (x, y, z)$, we would have

$$\eta = A \cos(\mathbf{k} \cdot \mathbf{x} - \omega t)$$

→ k is the **wavevector**

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Aside: If you know your **complex numbers**, the above is neatly encapsulated as

$$\eta = \text{Real} \left[A e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t - \theta_0)} \right],$$

where e is Euler's number, $i = \sqrt{-1}$, and θ_0 denotes a phase shift if any (could be sucked into the amplitude). For calculating the **dispersion relation** this form is substantially nicer to deal with (don't have to keep track of sines and cosines when taking derivatives).

Features of waves: dispersion relation

Note that

$$\omega = 2\pi\gamma = 2\pi\frac{v}{\lambda} = vk$$

- ▶ the physics tells you how $v = v(k)$
- ▶ the **dispersion relation** is given by

$$\omega = \mathcal{F}(k; \dots)$$

for some function \mathcal{F}

→ the dispersion identifies the types of wave (see Lec. 16), e.g.

$$\omega = \sqrt{gk}, \quad \omega = -\frac{\beta}{k}, \quad \omega = B_0 k, \quad \omega = \frac{\hbar k^2}{2m},$$

Superposition

(Linear) waves can be superimposed, leading to **interference**

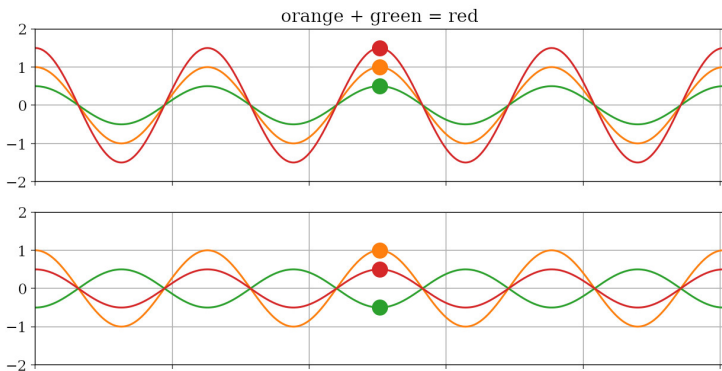


Figure: Interference of waves with **Red** = **Orange** + **Green**. For waves in phase (**constructive** interference) and waves in anti-phase (**destructive** interference).

Q. but what about waves not quite in phase or anti-phase?

Superposition

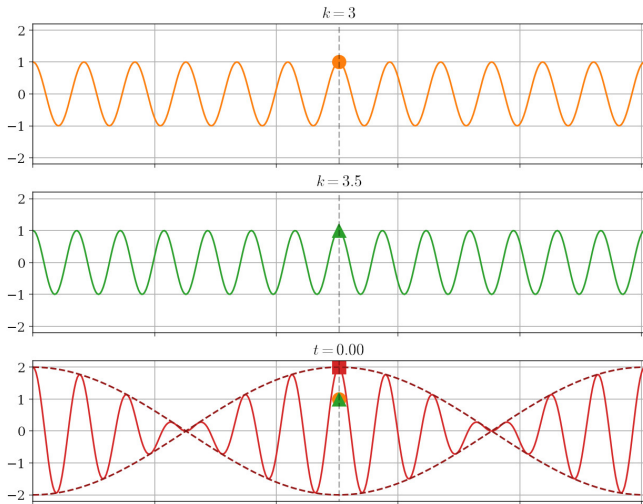


Figure: Superposition of two waves slightly out of phase, again with Red = Orange + Green. The crests have a marker marked on to track its progress later.

Wave propagation

Recall that $v = v(k)$ is the wave velocity. More precisely,

- ▶ the **phase speed** in a direction ($= v$) is defined as

$$c_{p,x} = \frac{\omega}{k}$$

→ how the wave by itself travels

Wave propagation

Recall that $v = v(k)$ is the wave velocity. More precisely,

- ▶ the **phase speed** in a direction ($= v$) is defined as

$$c_{p,x} = \frac{\omega}{k}$$

→ how the wave by itself travels

- ▶ the **group velocity** c_g is defined as

$$c_{g,x} = \frac{\partial \omega}{\partial k}$$

- ▶ in higher space dimensions,

$$c_{p,x} = \frac{\omega}{k_x}, \quad c_{p,y} = \frac{\omega}{k_y}, \quad c_{p,z} = \frac{\omega}{k_z}, \quad c_g = \nabla_k \omega$$

→ NOTE! Phase propagates in the direction of k

(subtlety: $c_{p,x}$ is not a component of the velocity that phases propagate at; see Ch.5 App of Vallis (2006))

Example: 1d Rossby waves (animation)

Wave propagation

- ▶ group velocity describes
 - how a collection of waves travel as a **group** or **wavepacket**
 - velocity that “stuff” propagates at
- ▶ a type of wave is **non-dispersive** if

$$c_g = c_p$$

- e.g. $\omega = B_0 k$ and $\omega = k\sqrt{gH}$ are non-dispersive
- if non-dispersive, wavepacket and phase travel together

- ▶ example just now is **dispersive**
 - 1d Rossby waves, $\omega = -\beta/k$ (exercise: show $c_g = -c_p$ for this case)

Wave propagation

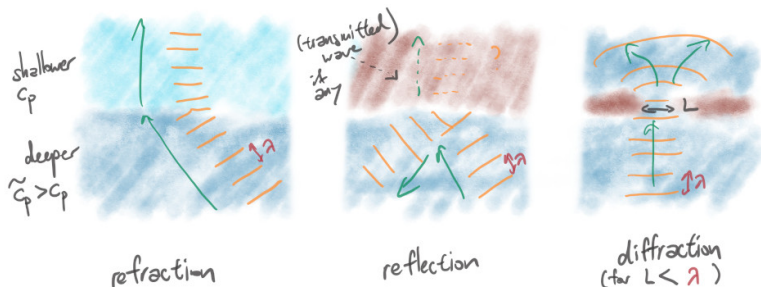


Figure: Schematic of **refraction**, **reflection** (and **transmission**), and **diffraction**, nominally using **monochromatic** (i.e. one choice of k) surface gravity wave as an example. The orange lines are phase lines (e.g. think wave crests).

refraction, **reflection**, and **diffraction** (used in Lec. 17, 20)

- ▶ resulting interference of waves can lead to **wave steepening** and wave breaking

Wave propagation

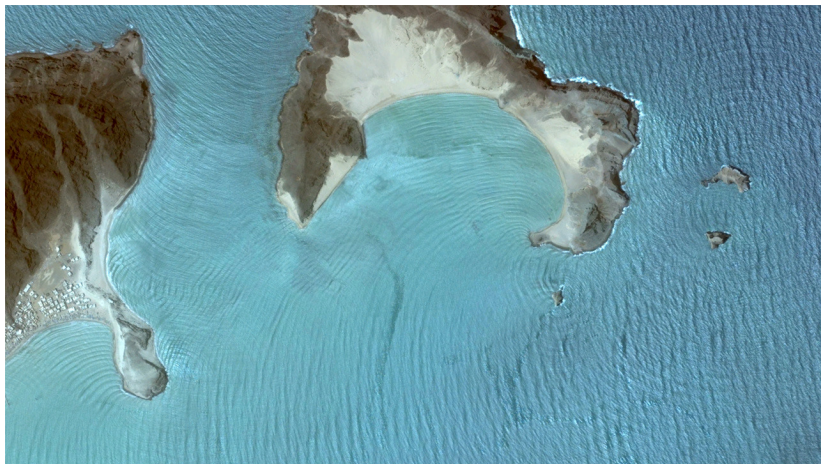


Figure: Picture of (presumably non-monochromatic) waves over the Arabian sea. Image taken from <https://www.earthglance.com/post/133835790223/wave-diffraction-on-the-arabian-sea>.

Summary

smaller-scale dynamics affects large-scale circulation

- ▶ **waves** are ubiquitous physical features
 - depends on physics
- ▶ wave described by the **dispersion relation** $\omega = \mathcal{F}(k)$
 - physics of the system dictates what $\mathcal{F}(k)$ is
 - usually use **angular frequency** ω and **wavenumber** k
(absorbs factors of 2π floating around)
 - $k \sim \lambda^{-1}$ sometimes used to characterise **scale of motion**
(more on this in Lec. 17)
- ▶ difference in c_p and c_g
 - individual (former) and collective (latter) velocity

wave breaking contributes to diapycnal mixing (see Lec. 17)