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The repository principally contains the compiled products rather than the source for size reasons.

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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

## Lecture 17: (equilibrium) tides



# Outlook of the next few lectures

Dynamics important

- ▶ **waves** (this Lec. + 16, 18) and **instabilities** (Lec. 17)

→ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- ▶ how to describe waves (Lec. 15)

- ▶ types of waves (Lec. 16)

→ consequence + leading to instabilities

- ▶ **tides (particularly as internal gravity waves)** (Lec. 17)

# Outline

## ► tides

- as deformation of sea surface arising from gravitational attraction **only** (rotation leads to evolution but does not **partake** in force balance)
- observations and features
- frequency + period
- forcing
- propagation

## ► internal tides (basically **internal gravity waves**)

- dynamics and transfer
- statistics (**Garrett–Munk spectrum**)

**Key terms:** high/low/spring/neap tides, tidal forcing, internal tides, wavenumber and length-scales Garrett–Munk spectrum

# Recap: abyssal upwelling (Lec. 14)

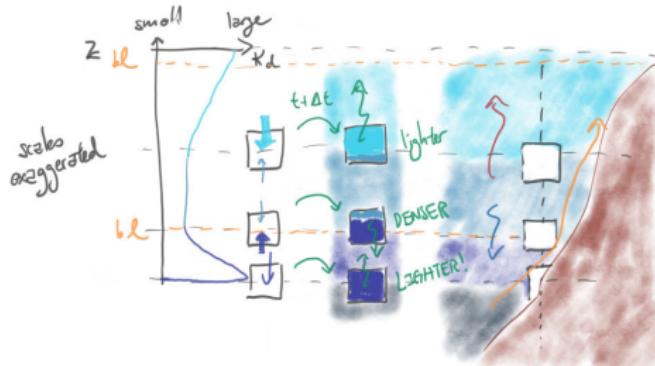


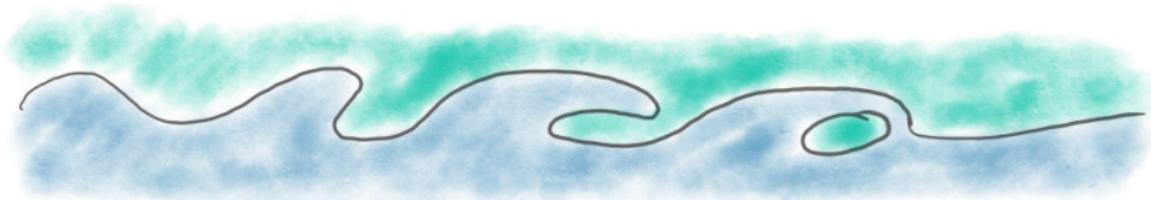
Figure: Schematic of the diffusive upwelling.

- ▶ diapycnal mixing contribute upwelling, strongest in boundary layers  
→ broad diffusive boundary intensified upwelling

what causes the abyssal intensification of  $\kappa_d$ ? dynamics!

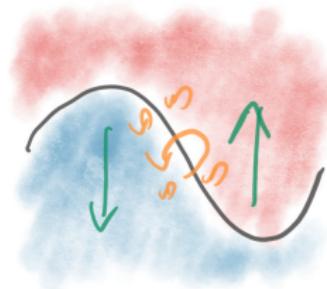
- ▶ internal gravity waves leading to shear instabilities which leads to diapycnal mixing

# Recap: shear instabilities and diapycnal mixing (Lec. 17)



**Figure:** Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- ▶ growing disturbance, **instability**
  - mixing of material **across** isopycnals after reconnection, i.e. **diapycnal mixing**
- ▶ shear instabilities can be interpreted as a **constructive interference** of waves
  - key role of **vorticity** and induced velocity



**Figure:** Velocity shear from waves can lead to mixing.

# Recap: internal gravity waves (Lec. 16)

Recall that since  $N \gg f$  on Earth, so **internal inertia-gravity waves** satisfy

$$\omega \approx \pm \sqrt{f^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- ▶ **internal gravity waves influenced by rotation**
  - main restoring force is **gravity**, with influences from Coriolis effect
  - remember  $|f| \leq |\omega| \leq |N|$

## Recap: internal gravity waves (Lec. 16)

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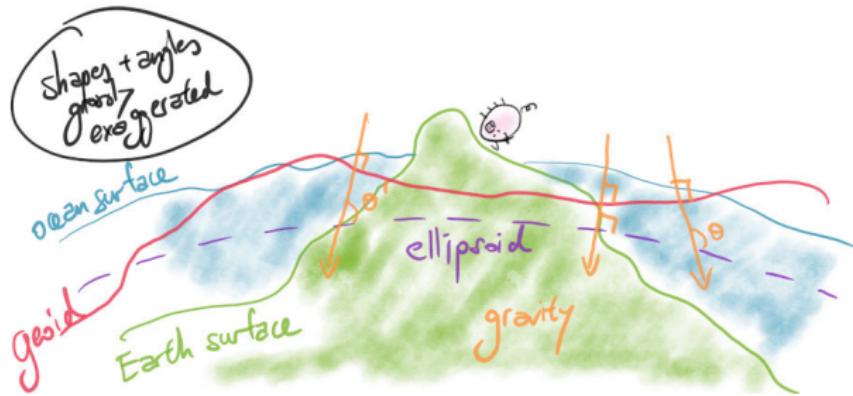
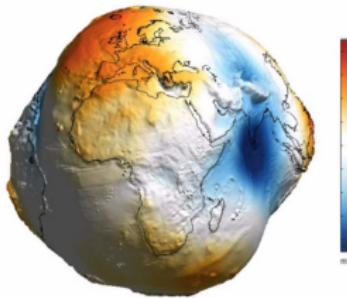
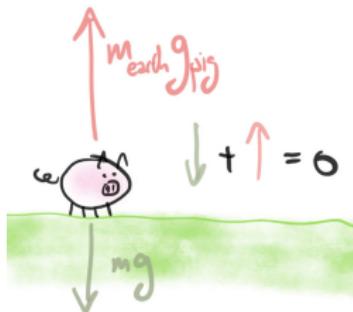
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- ▶ **internal gravity waves influenced by rotation**
  - main restoring force is **gravity**, with influences from Coriolis effect
  - remember  $|f| \leq |\omega| \leq |N|$

**internal tides = internal gravity waves with tidal forcing**

# Recap: gravity and geoid

(Lec. 7)



# Tides: observations



**Figure:** Tides at (top) St. Michael Mount in Cornwall (near Penzance), and Holy Island causeway (Lindisfarne) in Northumberland, UK. Images by Michael Marten (<http://www.michaelmarten.com/>), image taken from ([twistedsifter.com](http://twistedsifter.com)).

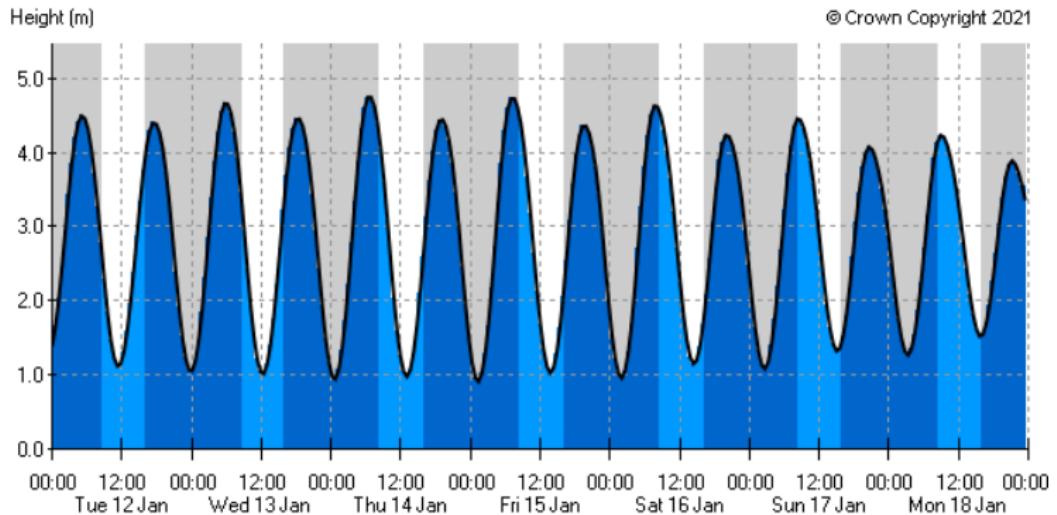
# Tides: observations



**Figure:** High (or flood) and low (or ebb) tide at Tobermory, Isle of Mull, Scotland, using the pastel pink and red house as references. Modified images from [www.thechaoticscot.com](http://www.thechaoticscot.com) (left) and from myself (right).

- ▶ tidal signal most obvious in coastal sea level
  - but generally in SSH  $\eta$  (recall Lec. 7)
  - $\eta/H$  largest near coasts (because of  $H \ll 1$ )

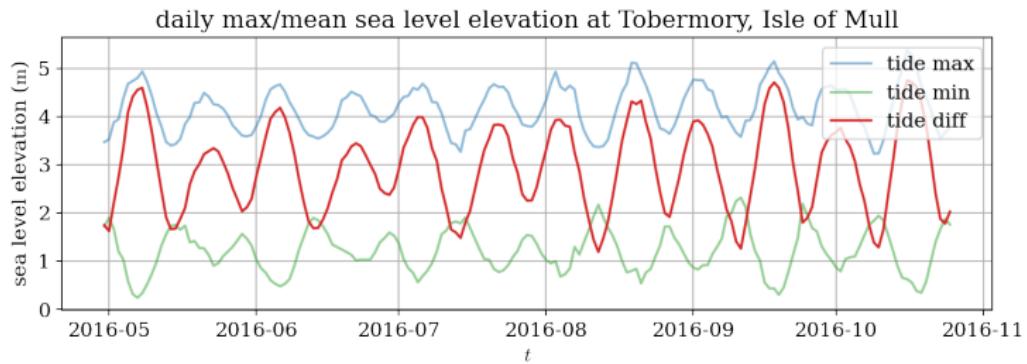
# Tides: observations



**Figure:** Predicted (!) tidal charts at Tobermory, Isle of Mull, Scotland between over a period of seven days. Taken from <http://www.ukho.gov.uk>.

- ▶ high and low tide at around 12 hr periods  
→ longer term variability (cf. wave packets, Lec. 15)

# Tides: observations

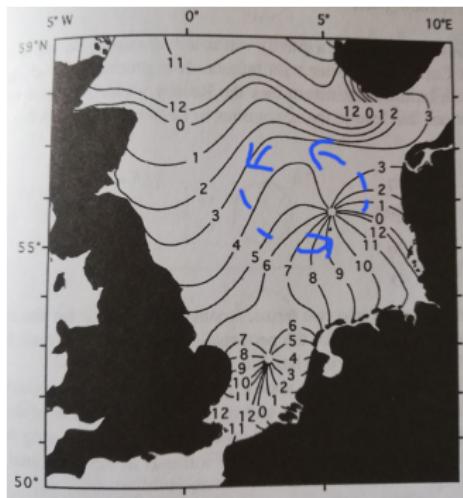


**Figure:** Daily maximum and minimum sea surface elevation (blue and green) and their difference (red) over a six month period. Data from BODC, see `tobermory_tides.ipynb`.

- ▶ longer term variability
- ▶ oscillation of around **two weeks** between max and min difference
  - **spring** and **neap tides**

(spring not to do with the season, more like the boing-boing mechanical spring)

# Tides: observations



**Figure:** Phase lines (denoting peak of SSH, in hours in GMT) of the semi-diurnal tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- ▶ tides over the whole globe  
→ wave-like structure
- ▶ propagation characterised by **phase lines** cf.  $c_p$  in Lec. 15)
  - **anti-clockwise** propagation in this NH example (cf. **cyclonic**)
  - note signal propagates with the boundary to the **right** of it
  - **Coriolis effect again?**

<https://www.youtube.com/watch?v=u6ZNnGnc-cQ>

# Tides: forcing



$$F_1 = G \frac{m_1 m_2}{r^2} = G \frac{m_2 m_1}{r^2} = F_2$$

Figure: Schematic of gravitational attraction for two masses, from Lec. 7.

- ▶ driving and restoring force being gravity, what objects drive the tides then?

# Esoteric pop culture interlude



**Figure:** What is the thing that unites all of these? Bonus: can you name all of these? (Copyright probably with Aardman studios, Square Enix, Naoko Takeuchi + Toei animation, Clamp, Nickelodeon, and two not sures).

# Tides: lunar forcing



**Figure:** Phases of the moon. From [www.dreamstime.com](http://www.dreamstime.com) (by user Miroslav Nemecek).

- ▶ the moon drives most of the Earth's tides
- ▶ Earth rotates with a period of a day  
→ daily = **diurnal**
- ▶ principal tide around every 12 hours (**semi-diurnal**)
- ▶ moon's orbit of Earth around 28 days

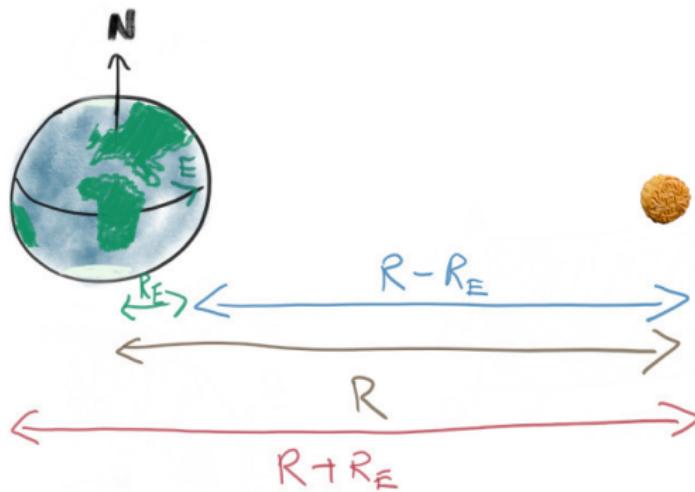
# Tides: forcing



**Figure:** Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here. Consider also free-floating ball bearing picture from Needham.

- ▶ suppose we have an Earth and moon(cake)

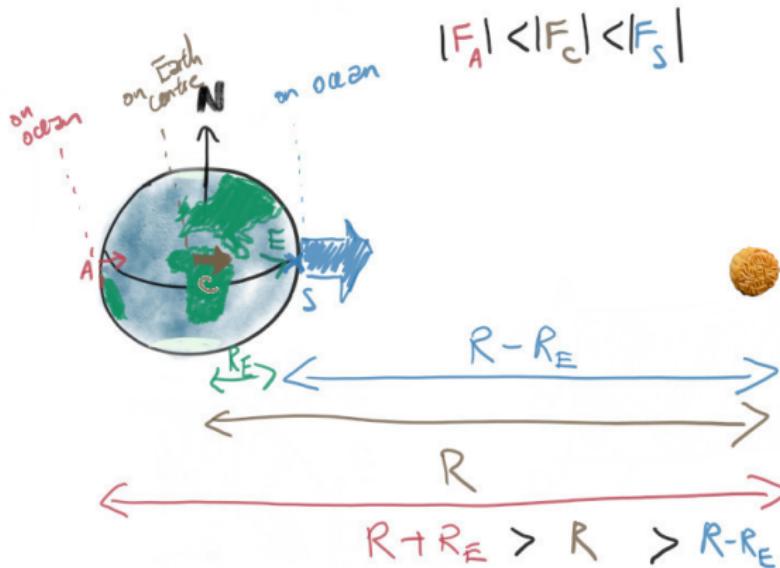
# Tides: forcing



**Figure:** Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here. Consider also free-floating ball bearing picture from Needham.

► define some distances

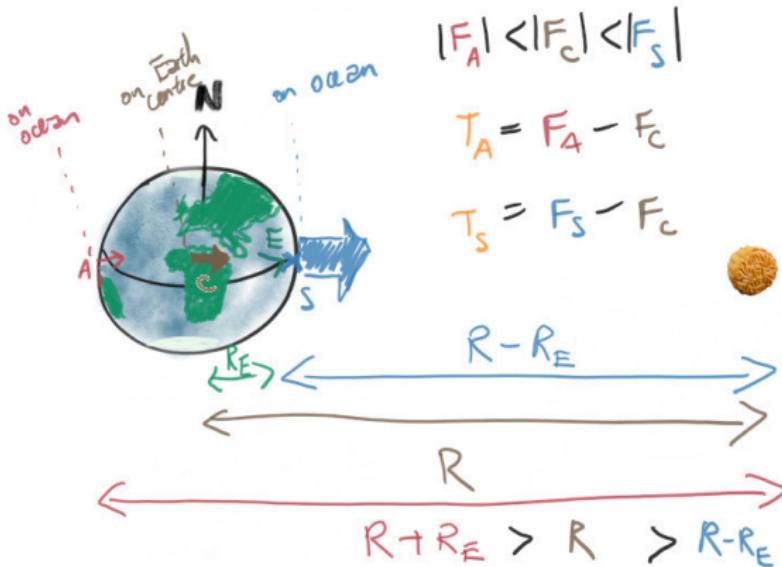
# Tides: forcing



**Figure:** Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here. Consider also free-floating ball bearing picture from Needham.

- differences in gravitational force because distances

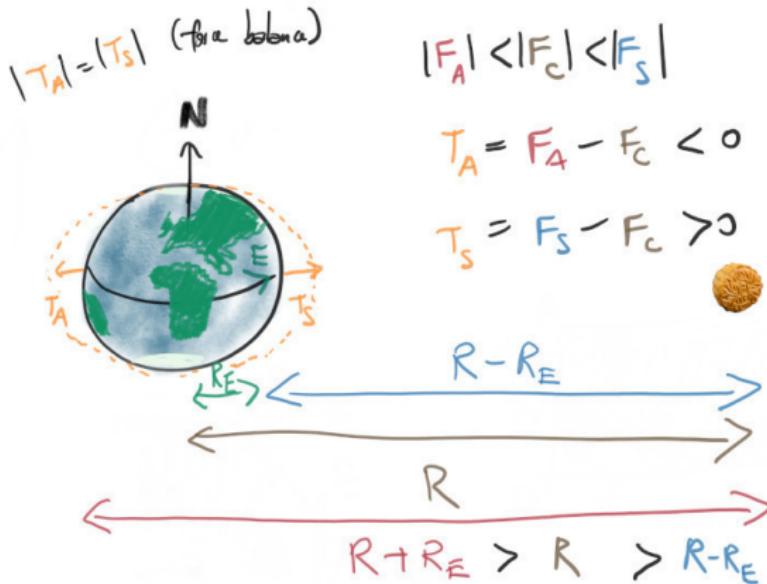
# Tides: forcing



**Figure:** Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here. Consider also free-floating ball bearing picture from Needham.

► tide generating force = force on ocean - force on center

# Tides: forcing



**Figure:** Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here. Consider also free-floating ball bearing picture from Needham.

- ▶ force balance, equal and opposite, pile water  $\Rightarrow$  two bulges

# Tides: forcing

- ▶ Earth rotates with period 24 hours, one bulge when moon overhead and one when on far side  
→ i.e. **semi-diurnal** signal
- ▶ above apply equally to the Sun too  
→ note while  $M_{\text{Sun}} \gg M_{\text{moon}}$ ,  $R_{\text{Sun-Earth}} \gg R_{\text{moon-Earth}}$ , so since tidal acceleration is roughly

$$a_{\text{tide}} \sim \frac{M}{r^3},$$

**magnitude** of Solar influence turns out to only be about half as strong (homework: show this explicitly)

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**magnitude** of Solar influence turns out to only be about half as strong (homework: show this explicitly)

- !!! above is for **equilibrium** theory, **dynamic** theory absolutely cares about rotation because then we need to care about wave propagation

# Tides: forcing

symbol	period (in solar hrs)	rel. amp (to $M_2$ )	name
$M_2$	12.42	1	principal lunar (semi-diurnal)
$K_1$	23.93	0.58	luni-solar (diurnal)
$S_2$	12.00	0.47	principal solar (semi-diurnal)
$O_1$	25.82	0.42	principal lunar (diurnal)
$N_2$	12.66	0.19	larger lunar elliptic (semi-diurnal)
:	:	:	:
$Mf$	327.85 ( $\approx$ 14 days)	0.09	lunar fortnightly
$Mm$	661.30 ( $\approx$ 28 days)	0.05	lunar monthly
$SSa$	4382.86	0.04	solar semi-annual

**Table:** Some sample tidal forcings sorted by relative amplitude to the  $M_2$  tide (which is the largest forcing for Earth). Subset of Table 6.2 given in Wunsch (2015). The last few entries are weak and long term but they are there.

- ▶  $M_2$  and  $K_1$  the dominant ones
  - usually do include these two in numerical models
- ▶ notice the periods are close to multiples of 12 hours

# Tides: springs and neaps



**Figure:** Schematic of spring and neap tides.

- ▶ moon(cake) and Sun in phase (or anti-phase)

# Tides: springs and neaps



**Figure:** Schematic of spring and neap tides.

- **spring tide:** max difference in highs and lows

# Tides: springs and neaps



**Figure:** Schematic of spring and neap tides.

- moon(cake) and Sun in **quadrature**

# Tides: springs and neaps



**Figure:** Schematic of spring and neap tides.

- **neap tide:** min difference in highs and lows

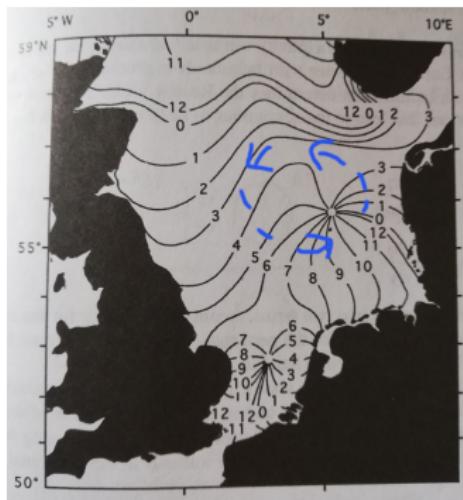
# Tides: springs and neaps



**Figure:** Schematic of spring and neap tides.

- ▶ occurs every two weeks or so (in line with lunar rotation)

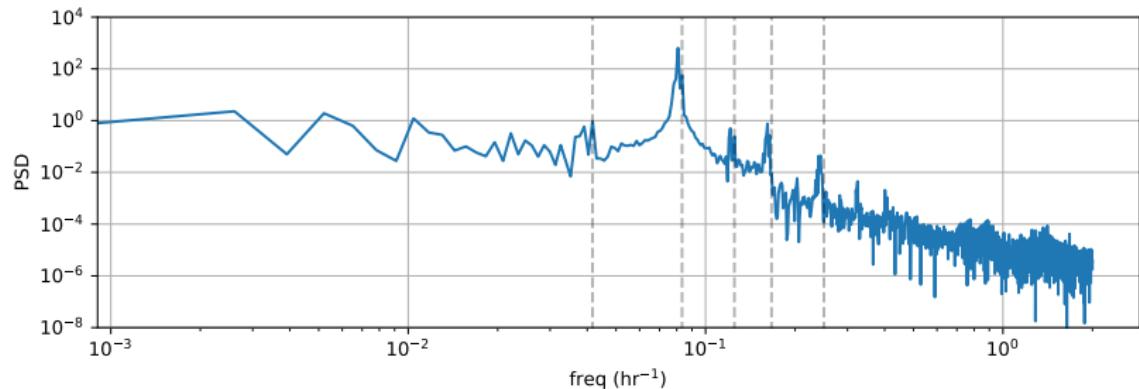
# Tides: propagation + Kelvin waves



**Figure:** Phase lines (denoting peak of SSH, in hours in GMT) of the semi-diurnal tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- ▶ anti-clockwise signal in NH from SSH from **Kelvin waves** (see Lec 16)
  - Kelvin waves propagate **cyclonically**
  - disturbance from tide generating force
- ▶ fast propagation (because gravity wave speed)
- ▶ non-dispersive
- ▶ decays over **Rossby deformation radius**
- ▶ cares about rotation
  - **dynamic theory of tides** (lec 16)

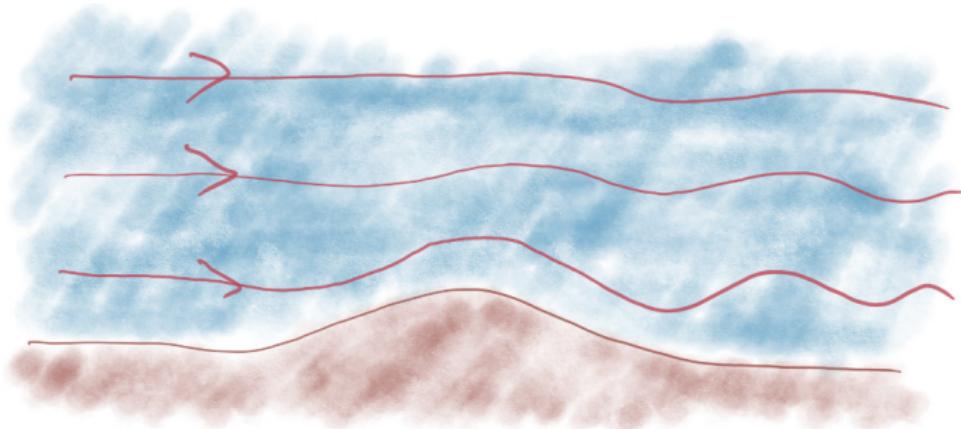
# Tides: power spectrum (see OCES 3301)



**Figure:** Power spectrum density (PSD) of the Tobermory tide signal, with peaks marked on. See OCES 3301 lec 8.

- ▶ way to quantify the strength of oscillations in time
  - big peak around the 12 hour frequency (e.g. M2 semi-diurnal tide)
  - peak at the 24 hour mark (e.g. K1 diurnal tide)
  - some **sub-harmonics**

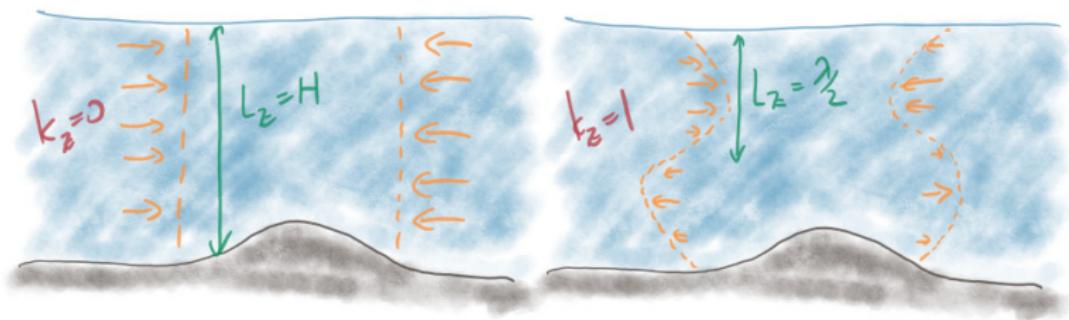
# Internal tides: generation



**Figure:** Flow over topography (e.g. tidal motion) leading to wave generation.

- ▶ flow over topography can lead to wave generation  
→ **internal gravity waves** (Lec. 16)
- ▶ repeated “sloshing” by tide generating force  
→ period of “sloshing” (e.g.  $M_2$  is around 12 hrs)

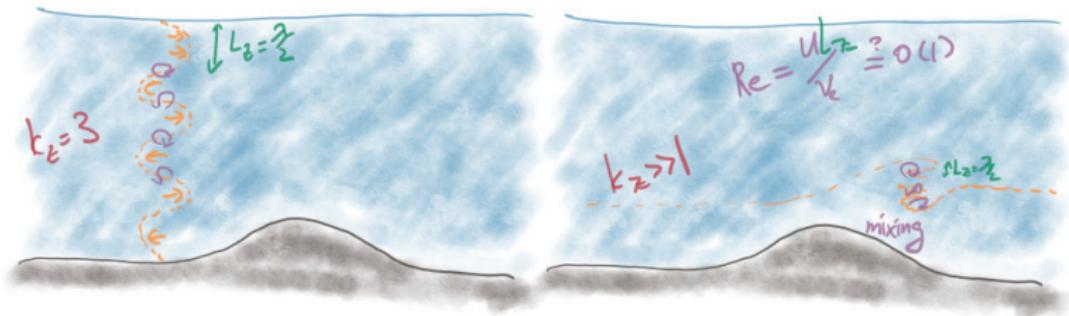
# Internal tides: decreasing in scale



**Figure:** Motion associated to theoretical  $k_z = 0$  and  $k_z = 1$  (with boundary as ocean surface). Note associated with  $k_z$  is a length-scale.

- ▶ tidal forcing is to first approximation uniform over depth of ocean  
→ think  $k_z = 0$   
(commonly **barotropic** but I don't like "barotropic" used like this...)
- ▶ generating waves of increasing vertical variation (**barotropic conversion**, but, again...)  
→ e.g.  $k_z = 1$ , increase in frequency, **decrease** in vertical wavelength or **vertical scale**

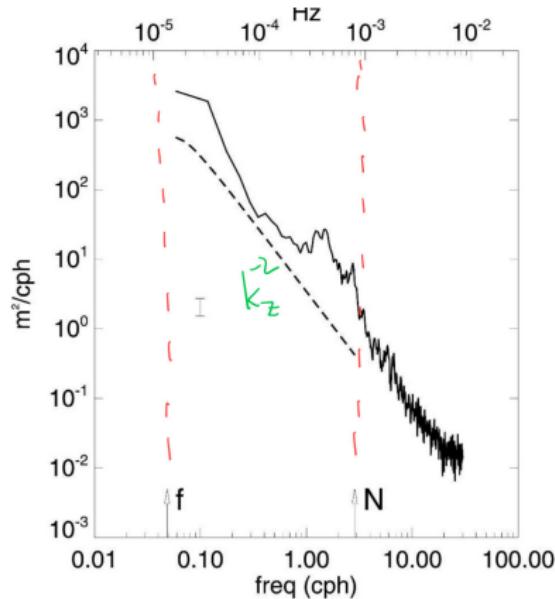
# Internal tides: breaking



**Figure:** Motion associated to theoretical  $k_z = 3$  and a more realistic case with nominal  $k_z \gg 1$ , via the observation that  $k_z$  can be linked to a length-scale.

- ▶ instabilities lead to more small-scales
  - e.g.  $k_z$  keeps increasing, so vertical scale keeps decreasing...
- ▶ eventually a dynamical  $Re$  (Lec. 10) starts becoming important
  - reconnecting isopycnals, diapycnal mixing (Lec. 10, 14, 16)

# Internal tides: statistics



**Figure:** Observation of ocean displacement as a power spectrum, with the Garrett–Munk wavenumber (so frequency) spectrum dependence put in. Note also the frequency boundaries marked on since interal gravity waves should satisfy  $f < \omega < N$ . From Stevens *et al.* (2005), *J. Phys. Oceanogr.*, modified from their Fig. 3.

- ▶ general characteristics of internal waves?
    - Garrett–Munk spectrum
- (Garrett & Munk, 1972, *Geophys. Fluid. Dyn.*)

$$E \sim \omega^{-2} \sim k_z^{-2} \quad (\omega \gg f)$$

- ▶ universal behaviour?
- ▶ initially fit to observations, but can derive from first principles?

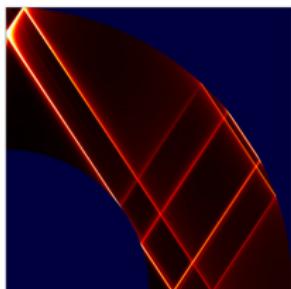
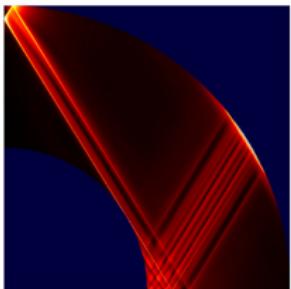
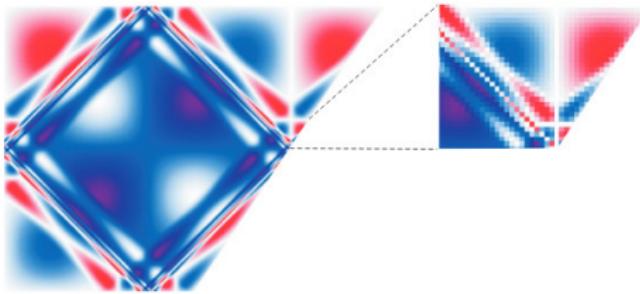
→ e.g. wave turbulence formalism

(way beyond here; see works by Victor L'vov or book by Sergey Nazarenko)

# Summary

- ▶ tides as deformation arising solely from gravitational attraction (Lec. 4 + 7)
  - tide generating force as a differential force
- ▶ signal particularly in SSH (Lec. 7), but also can generate internal waves (Lec. 16)
  - low mode forcing to high mode motion (decreasing length scale) (Lec. 15)
  - Garrett–Munk spectrum,  $E \sim k_z^{-2}$
  - wave breaking lead to diapycnal mixing (Lec. 10), feedback onto MOC (Lec. 14)

# Summary



**Figure:** Internal gravity wave attractors in a uniformly stratified channel. Image from Maas (2005), *Int. J. Bifurcat. Chaos*, reworking of Maas et al. (1997), *Nature*.

- ▶ what do waves do when they break?
- ▶ where do wave break?
- ▶ how do waves break?
- ▶ how to model the above?
- ▶ implications for overall system?

**Figure:** Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc.*).