OCES 2003 Assignment 3, Spring 2024

Julian Mak (jclmak@ust.hk)

Set on: Mon 15th Apr; due: Mon 22nd Apr

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

  - zero on the question concerned
  - a fixed penalty starting from around 1/3 of the total marks
  - zero for the whole assignment/midterm/final

  The following counts as plagiarism (and is a non-exhaustive list):

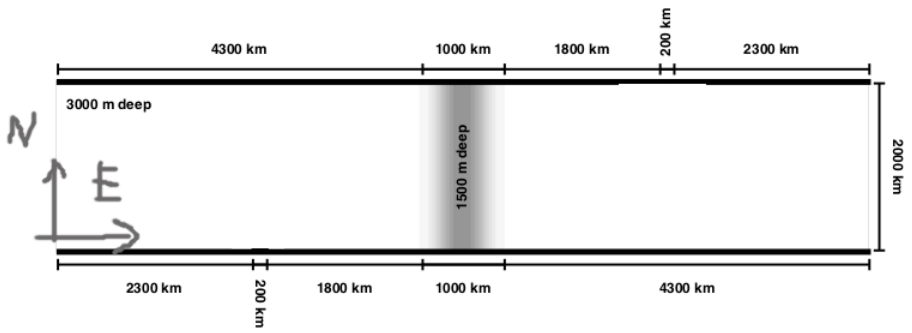
  - copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
    * use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
    * any more than around three usages in text is probably excessive
  - copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
  - changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. Question concerns geostrophic flow and constraints by potential vorticity.

   (a) Suppose we have a bathymetry like the one below for a Southern Hemisphere β plane (courtesy of Dave Munday; from Munday *et al.*, 2015):



   Assuming geostrophic balance, argue (by words, maths, pictures, or otherwise) why when the flow goes from left to right (so West to East) and hits the bathymetry we should expect the flow to deflect towards the North (in this case going up the page).

   Argue that, regardless of the hemisphere we are in, we should have a *equator*-ward deflection as the flow moves from West to East and hits the bathymetry in a configuration like the above. (Hint: see next part of the question.)

   [3 marks]

   (b) Recall from Lec 12 that, to leading order, potential vorticity is given by

$$q = \frac{\zeta + f}{H} = \frac{\boldsymbol{e}_z \cdot \nabla \times \boldsymbol{u} + (f_0 + \beta y)}{H} \sim \frac{U/L + f_0}{H},$$

   where $\zeta = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{u}$ is the vertical component of the vorticity, U and L are typical velocity and length scales for the flow, and the other symbols have their usual meanings. Provide some numerical estimates of U/L and values of f_0 for a geostrophic eddy, and argue why we can generally just consider $q \approx f/H$.

   [2 marks]

2. Miscellaneous questions relating to waves.

   (a) Sound waves in fluids (which we will use in Lec 20 relating to *sonar* for example) are slightly different to the waves we have talked about in the lectures, as they are *compression*-type or *longitudinal* waves (the ones in class are *transverse* waves); you can think of sounds waves in fluids as pressure waves. The sound speed is given by the Newton–Laplace formula

$$c_{\text{sound}} = \sqrt{\frac{K_s}{\rho}},$$

   where K_s is a measurement of stiffness of medium, and ρ is the density (not going to make a distinction which density here). Noting that c_{sound} is about four times larger in water than in air, is it that the speed of sound is larger in seawater because seawater is denser, or because seawater is more 'stiff'? Explain your answer.

   [1 mark]

(b) Because of the temperature distribution in the ocean, sound waves tend to change direction as they travel because of the change in seawater density (e.g., submarines tend to measure the surrounding temperature because it affects the performance of sonar). What is this wave phenomenon called?

[1 mark]

(c) If a sound wave is non-dispersive, would the above speed be interpreted as a phase speed, group speed, both, or neither? Explain your answer.

[1 mark]

(d) Sound waves have been proposed for use as a way to observe long term changes in the ocean *temperature* (and thus ocean heat content in relation to global warming). Using your answers to the previous parts and what you know so far from the course, speculate how this might actually work, and possible challenges of the methodology. Don't write more than a page on this (excluding pictures in that count), and cite any sources you do use.

(Hint: try searching for *acoustic tomography*, which I normally don't have time to go through in Lec 20. The idea of acoustic tomography was killed in the 8/90s by animal protection groups for its possible effect on large marine mammals such as whales, but this might stem from an error of sorts in the units of calculation...)

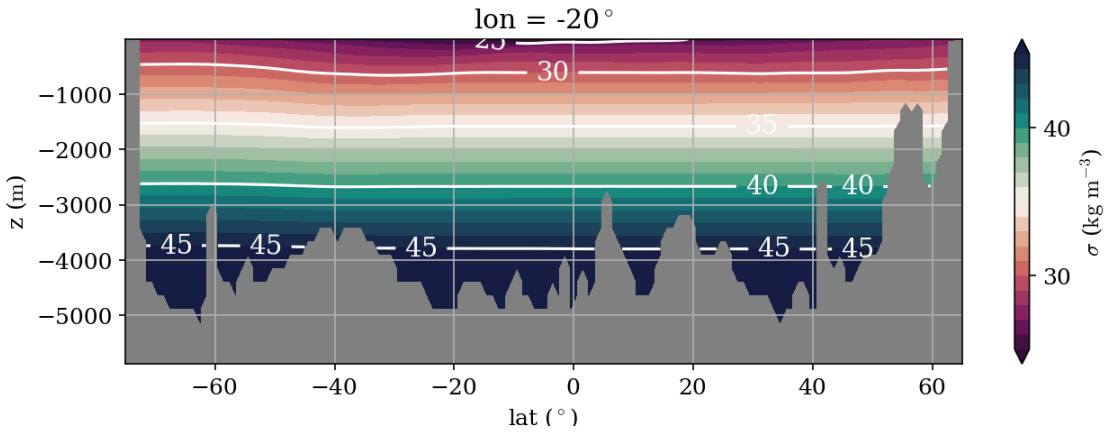
[4 marks]

3. Questions related to thermal wind shear relation.

(a) Describe what thermal wind shear relation entails and under what conditions thermal wind shear holds in less than thirty words (do not use equations).

[2 mark]

(b) Revisiting material from Lec. 5 and 6, the following is a meridional section when using in-situ density:

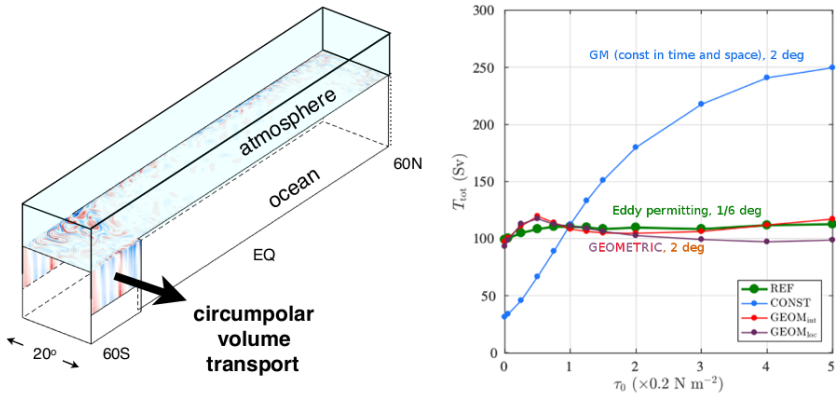


Explain what the thermal wind shear relation using in-situ density would imply for the Antarctic Circumpolar Current in the region between 70 to about 40° S, as compared to using potential density say (see the lectures for the relevant plots). Argue for yourself again why in-situ density is not the dynamically relevant variable of interest over most of the ocean. Use equations if you like, but don't write for more than half a page; excessive and/or irrelevant content will be penalised.

[2 marks]

(c) There is an ongoing research question in physical and numerical oceanography relating to how the Southern Ocean (thermal wind) circumpolar transport depends on the changes in forcing. In

particular, there is the phenomenon of *eddy saturation*, which finds that when eddies are modelled accordingly we can get a circumpolar transport that is insensitive to large changes in wind forcing. Graphically, we get a flat line of the diagnosed transport against the wind (e.g., Munday *et al.* 2013; Mak *et al.*, 2018) that looks something like the one below:



While flat lines are generally not that interesting, argue by thermal wind balance and connectivity of Southern Ocean and global stratification what the green line would imply relative to the blue line (should hopefully tell you why flat lines can in cases be interesting). Don't write more than half a page on this, and cite sources as appropriate if you do use it (hint: some of the papers I've been lead author of might help.)

(Note: You could consider the blue line as a typical ocean climate model, and the red and purple lines are our proposed fixes for the 'inconsistent' sensitivity with the model 'truth' given by the green line. See Mak *et al.*, 2022 (the one where all authors names begin with M) for the fix in action in a 'realistic' model.)

[4 marks]

(d) (Relatively technical question.) In satellite altimetry that we will visit in Lec 20, the idea is that you measure the SSH, from which you work out the pressure via hydrostatic balance, and then compute some *derivatives* for the surface geostrophic flow. Suppose you have perfect measures of interior density. How might one infer for the interior geostrophic flow combined with satellite altimetry using thermal wind relation? There is a fundamental technical issue that means there can be a non-vanishing and possibly arbitrarily large flow in the interior, to do with the procedure using the thermal wind relation rather than the data availability or quality. Try and state what that is.

(Hint: this is a case where writing the equations out would actually help. The only word that is emphasised in this question might give you a hint.)

[2 marks]

!? (Bonus question, no marks + for interest only, and probably don't try this yourself at home.) Relating to Q.2, inhaling helium tends to make your voice sound all squeaky (while gases like sulfur hexaflouride makes your voice sound really deep). Why? Look this up yourself. Some stuff in Q.2 would be relevant, but the answer is actually slightly more complicated than you think, relating to natural resonances and *timbre*.

Some people talk about the ocean as a drum/whistle that responds to external forcing, with a natural frequency that it likes to resonate at (e.g., waves). Under warming scenarios, how might the natural frequency change in relating to answer in the previous part?