Boring but important disclaimers:

- If you are not getting this from the GitHub repository or the associated Canvas page (e.g. CourseHero, Chegg etc.), you are probably getting the substandard version of these slides Don't pay money for those, because you can get the most updated version for free at

  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
- I do not claim the compiled products and/or code are completely mistake free (e.g. I know I don't write Pythonic code). Use the material however you like, but use it at your own risk.
- As said on the repository, I have tried to honestly use content that is self made, open source or explicitly open for fair use, and citations should be there. If however you are the copyright holder and you want the material taken down, please flag up the issue accordingly and I will happily try and swap out the relevant material.

OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 7: Mechanical forcing 1 (pressure and gravity)

Outline

- recall forcing on ocean
  $\rightarrow$ thermodynamic (T and $S \Rightarrow \rho$ and buoyancy)
  $\rightarrow$ mechanical (wind, gravity, pressure, rotation etc.)

- gravity + pressure (alluded to last Lec.)
  $\rightarrow$ geoid (see also Lec. 18)
  $\rightarrow$ sea surface height (SSH)
  $\rightarrow$ weight
  $\rightarrow$ hydrostatic pressure
  $\rightarrow$ some consequences for flow

Key terms: geoid, SSH, hydrostatic pressure

Recap: forces

- Newton's second law: objects are in steady state (at rest or steady speed) unless there is a net force

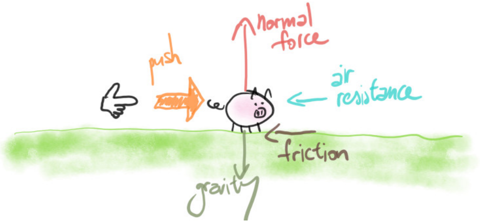

Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vacuum because we have air resistance + abuse of animal rights).

- thermodynamic forcing: affects T and S
- mechanical forcing: affects momentum

thermodynamic variables affects momentum via pressure

Recap: buoyancy forces

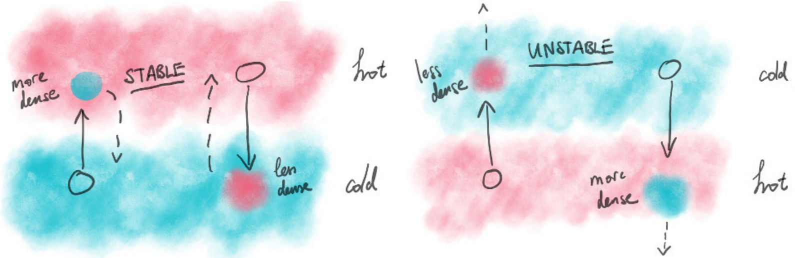

Figure: (Un)stable temperature configurations.

- buoyancy of fluid ultimately depends on density
  $\rightarrow$ lighter density water, more 'floaty'
  $\rightarrow$ heavier things (less buoyant water) = more weight, imbalance and sinks

Recap: in-situ vs. potential/neutral density

- $\rho = \rho(T, S, p)$ via the EOS, but want to neglect p contribution to ρ because non-dynamic (from a work done point of view)
- in-situ density ρ says you basically have no up-down motion in the deep

  $\rightarrow$ but we know we have a bit!

  $\rightarrow$ contributions from p included here
- potential densities referenced to different levels says you can

  $\rightarrow$ **some** p contribution removed

- **you want the p resulting from ρ but without p in the ρ** (otherwise, a circular argument?)

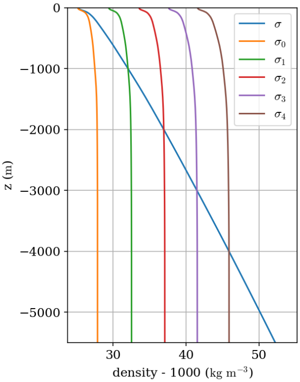

Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{D}_{\boldsymbol{u}} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Gravity

- attraction between bodies of different masses
  → note it is a purely attractive force (cf. magnetism)

$$F = G\frac{m_1 m_2}{r^2}, \qquad G = 6.674 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$$

→ G the gravitational constant
(exercise: check the LHS and RHS units agree)

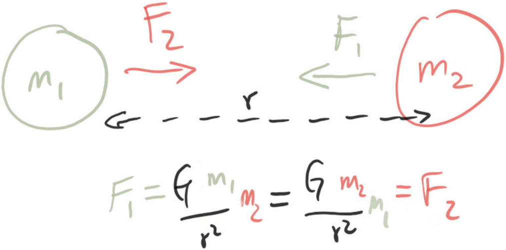

Figure: Schematic of gravitational attraction for two masses. If $m_1 \gg m_2$ (e.g. Earth and a pig) then forces on each body are equal, but its effect on the pig is much larger than it is for the Earth (recall $F = ma$).

Gravity and weight

- Let's take Earth as an example:

$$F = G\frac{m_{\text{earth}}}{r^2_{\text{earth}}}m,$$

taking (units!)

$G = 6 \times 10^{-11}$

$m_{\text{earth}} = 6 \times 10^{24}$

$r_{\text{earth}} = 6400 \text{ km} \approx 6 \times 10^{6} \text{ m}$

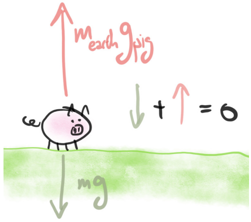

Figure: Gravity as applied on Earth. Note that g_{pig} is tiny (exercise: make an estimate of g_{pig}).

$$F = 6 \times 10^{-11}\frac{6 \times 10^{24}}{(6 \times 10^{6})^2}m = \frac{6^2}{6^2} \times 10^{-11+24-12}m = 10m \equiv mg$$

- the gravitational acceleration **on Earth** (recall $F = ma$) is $g \approx 10 \text{ m s}^{-2}$ (exercise: don't drop the decimal places like I did above and repeat the calculation)

Mass vs. weight

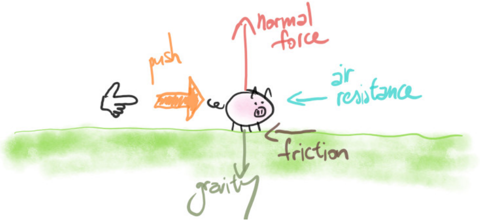

Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vacuum because we have air resistance + abuse of animal rights).

- the pig above with mass m has weight mg

  → mass is how much 'stuff' a body has

  → weight is a **force** and dependent on value of g

  → e.g. pig has same mass on moon but **weighs less** there because $g_{\text{moon}} \approx (1/6)g$ (exercise: why is g_{moon} smaller?)

Geoid

- picture for spherical bodies with uniform mass (then r is distance between centre of gravity), but Earth is not quite spherical...

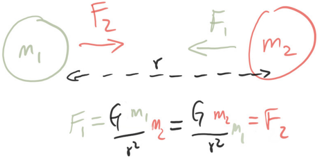

Figure: Schematic of gravitational attraction for two masses.

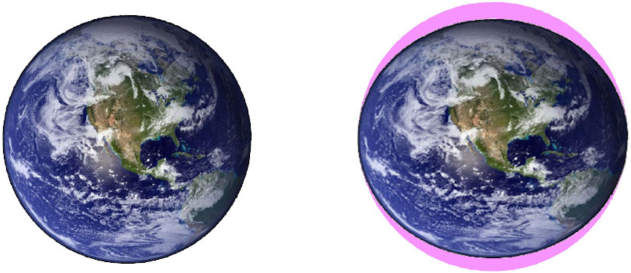

Figure: Cartoon of spherical vs. ellipsoid earth (it's inflated slightly at the Equator from Earth's spinning). Modified picture from NASA.

Geoid

- ...nor is the mass uniformly distributed!

  → where there is more mass there is more gravitational attraction
- geoid is the surface that the ocean surface would trace out if we only had gravity and rotation

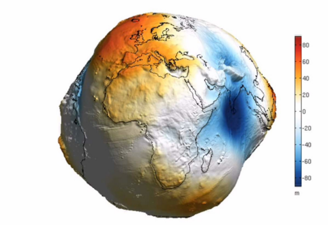

Figure: The “lumpy potato” Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

→ or, geoid is the surface where gravity is everywhere perpendicular to it (I like this one more...)

→ wind and tidal action move sea surface around

→ important concept in dynamics + sea level science

Geoid

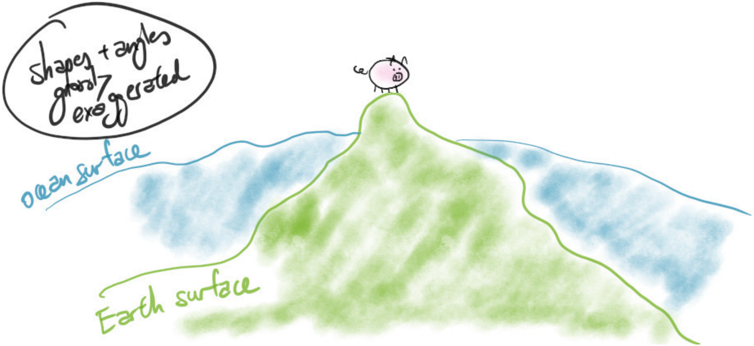

Figure: Schematic of the ellipsoid and geoid.

Geoid

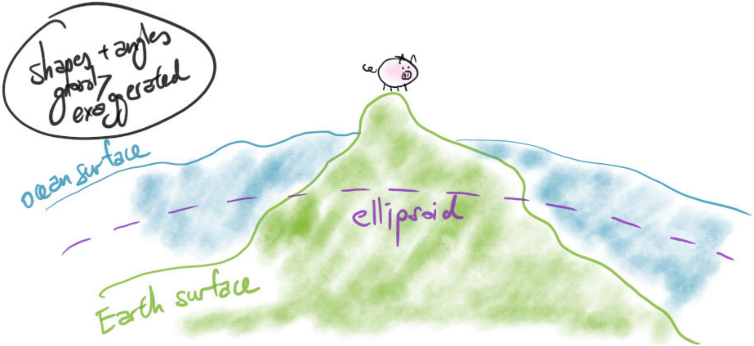

Figure: Schematic of the ellipsoid and geoid.

Geoid

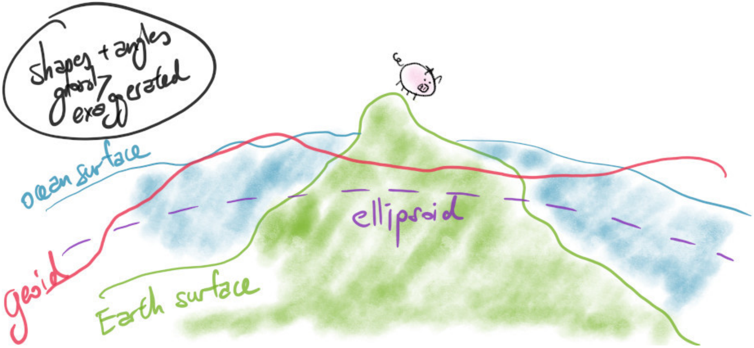

Figure: Schematic of the ellipsoid and geoid.

Geoid

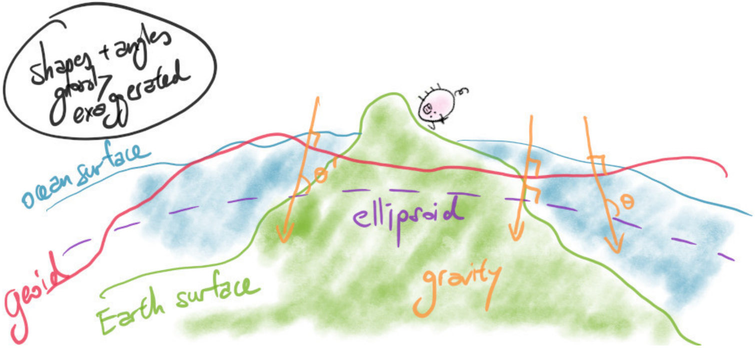

Figure: Schematic of the ellipsoid and geoid.

Gravity + weight

- differences greatly exaggerated above, in reality gravity variations are very small

  $\rightarrow$ that's why it was very difficult to get the geoid!

  $\rightarrow$ really needed satellites (see Lec. 20)

Gravity + weight

- differences greatly exaggerated above, in reality gravity variations are very small

  $\rightarrow$ that's why it was very difficult to get the geoid!

  $\rightarrow$ really needed satellites (see Lec. 20)
- for most intents and purposes we can take g to be a constant

  $\rightarrow$ remember the ocean is quite thin ($H/L \ll 1$)

Gravity + weight

- differences greatly exaggerated above, in reality gravity variations are very small

  $\rightarrow$ that's why it was very difficult to get the geoid!

  $\rightarrow$ really needed satellites (see Lec. 20)
- for most intents and purposes we can take g to be a constant

  $\rightarrow$ remember the ocean is quite thin ($H/L \ll 1$)

- it does matter when we are talking about things like sea level (see OCES 4001)

  $\rightarrow$ sea level change but **relative to what**?

  $\rightarrow$ e.g. ellipsoid? ground? geoid?

SSH to be <u>instantaneous</u> height relative to <u>ellipsoid</u>

Pressure (recall last Lec.)

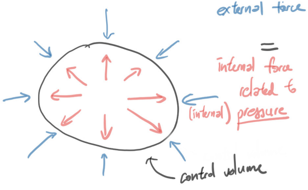

Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume $\Rightarrow$ steady
- steady $\Rightarrow$ in force balance, no net force

- pressure = force per area,

$$p = F/A, \qquad \text{units: N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^5 Pa (Pascals) (see e.g. Wikipedia for others)

$\rightarrow$ cf. millibars (mbar) in atmosphere

$\rightarrow$ lines of constant pressure = isobar

Pressure: atmospheric example

日期/Date: 14.10.2020 香港時間/HK Time: 14:00 香港天文台 Hong Kong Observatory

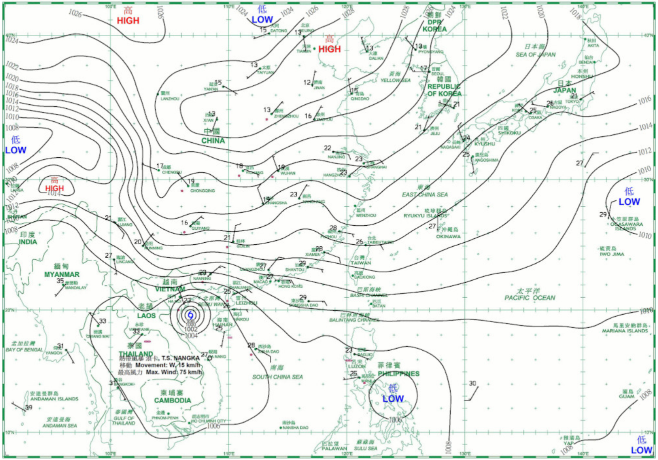

Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

Hydrostatic balance (recall last Lec.)

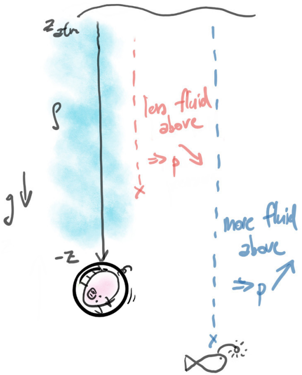

Figure: Schematic of hydrostatic pressure

- hydrostatic approximation: pressure **approximately equal** to weight above when static

  → weight is $F = mg$ so for force balance,

$$F = mg = \boxed{g \int_{-z}^{z_{\mathrm{atm}}} \rho(z')\, \mathrm{d}z' = p},$$

with $g \approx 9.81\ \mathrm{m\ s^{-2}}$

→ if $\rho = \mathrm{const}$ then $p = \rho g z + p_{\mathrm{atm}}$

Horizontal effect?

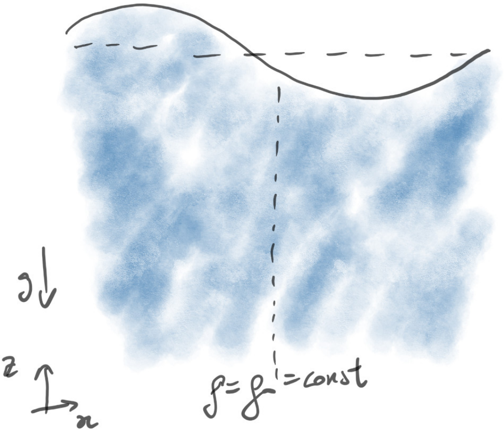

Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?

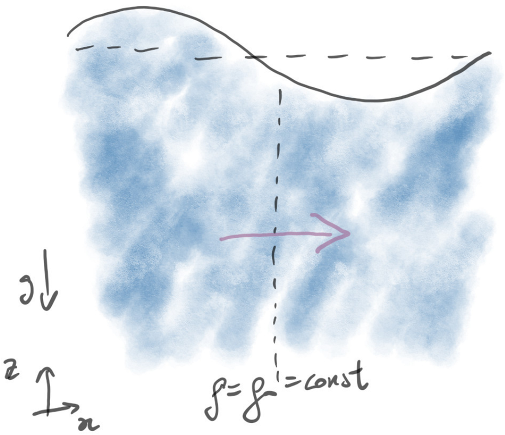

Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?

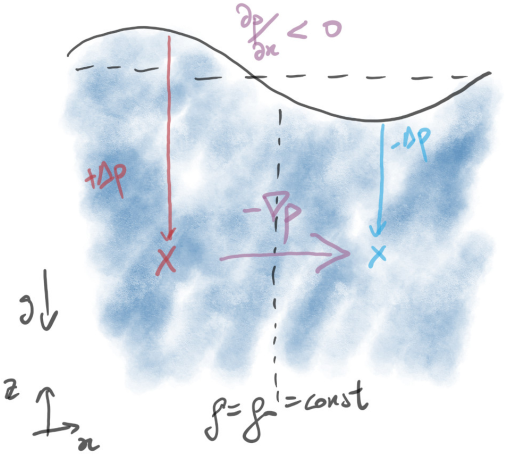

Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?

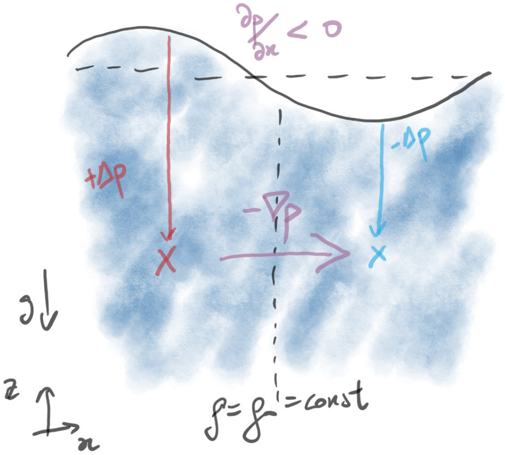

Figure: Horizontal effect because of hydrostatic pressure.

- assuming hydrostatic balance, water moves from $+\Delta p$ to $-\Delta p$ because there is a **net force** (negative pressure gradient $-\nabla p$)

  $\rightarrow$ important later for geostrophic flows (see next Lec.)

Atmospheric example revisited

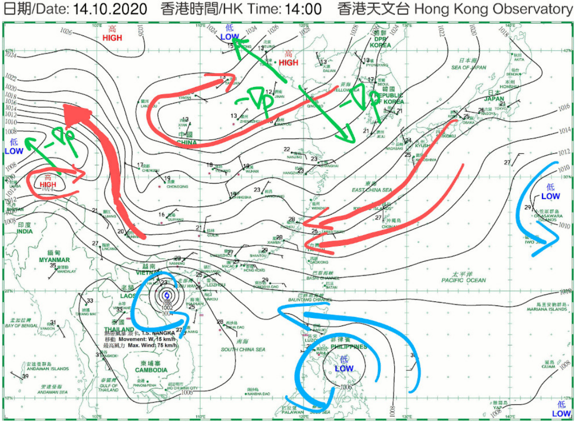

Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

- note that flow doesn't go in the direction of $-\nabla p$!

  $\rightarrow$ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Oceanic example

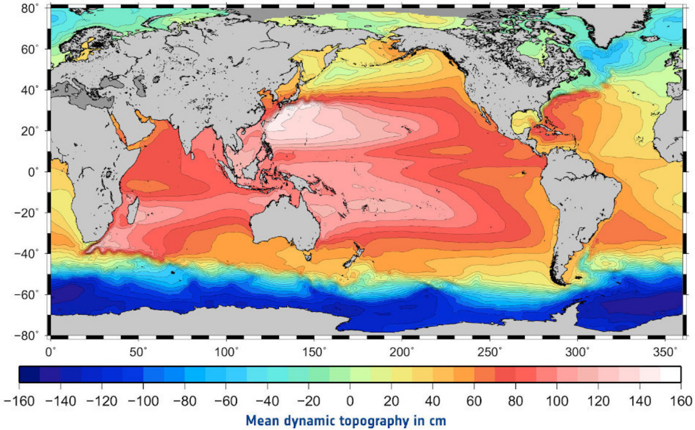

Figure: Time-mean global SSH (also called mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

Oceanic example

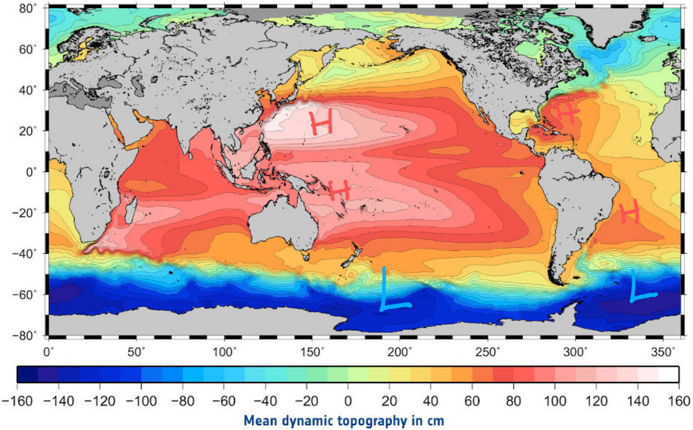

Figure: Time-mean global SSH (also called mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via hydrostatic balance

Oceanic example

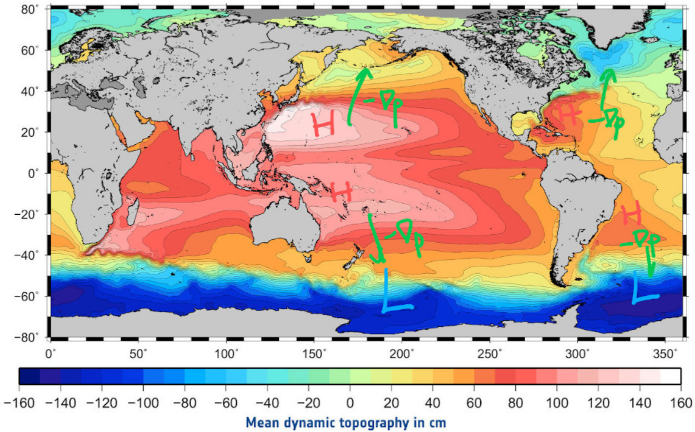

Figure: Time-mean global SSH (also called mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via hydrostatic balance

Oceanic example

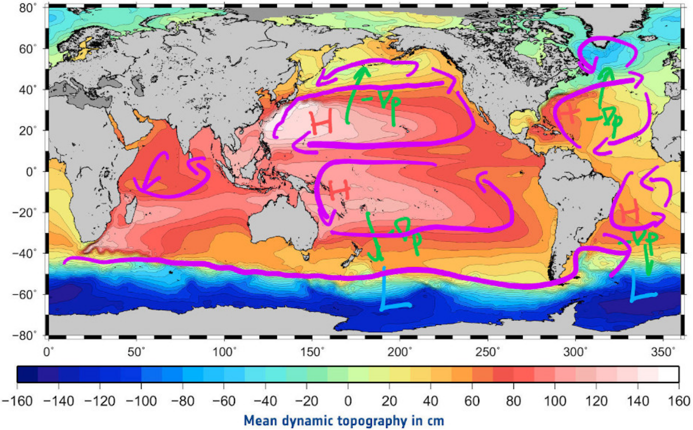

Figure: Time-mean global SSH (also called mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via hydrostatic balance

  $\rightarrow$ flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Summary

- gravity + geoid

  → astronomical forcing on ocean

  (see Lec. 18)

  → geoid important for e.g. sea level change (see Lec. 18 + OCES 4001)

- hydrostatic pressure

  → pressure proportional to weight of fluid above

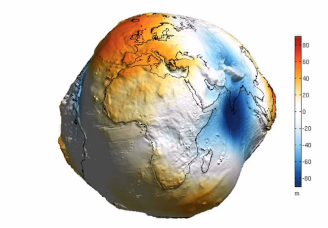

Figure: The "lumpy potato" Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

- buoyancy (thermodynamic stuff) affects pressure...
- ...leading to pressure **gradients** (mechanical force) driving a **flow**...

  → ...but rotation can influence resulting flow! (see next Lec.)