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The repository principally contains the compiled products rather than the source for size reasons.

- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 11: Gyres 1 (overview + Sverdrup balance)



Outline

- ▶ (wind driven?) gyres and features
 - subtropical/subpolar gyres
 - Western Boundary Currents (WBCs) (e.g. Gulf stream, Kuroshio)
- ▶ β -plane + Sverdrup balance
 - simple model of wind-driven gyre (on β -plane)
 - wind balancing gradient of Coriolis
- ▶ depth-independent model with no topography: Sverdrup balance
 - interior dynamics

Key terms: subtropical/subpolar gyres, $f = f_0 + \beta y$, Sverdrup balance

Aim of these two lectures

combine material so far for a theory of **wind-driven gyres**

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plan of attack:

1. recall material on **gyres** (Lec 2)
2. construct model (wind, rotation, friction; Lec. 4, 7-10)
 - **β -plane**
 - assumption for density (but see next Lec.) (Lec. 5 + 6)
3. analyse and deduce via balance arguments
 - **Sverdrup balance** (wind + rotation; Lec. 8 + 9)
 - **mass conservation** (Lec. 4?)
 - **vorticity balance** (wind + friction; Lec. 9 + 10)

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“All models are wrong, but some are useful”

– attributed to George Box

Recap: gyres

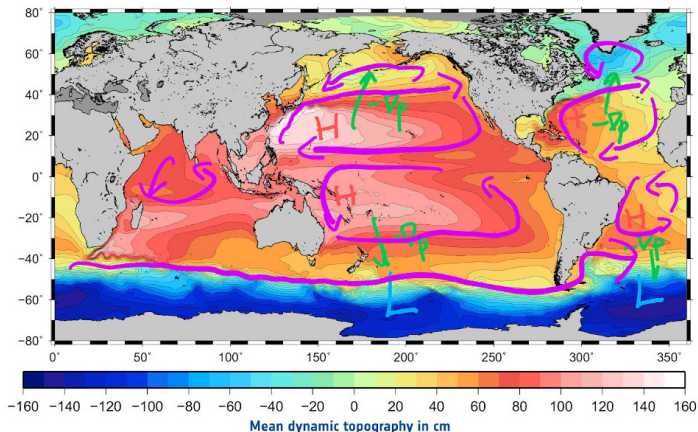


Figure: Time-mean global SSH (also called **mean dynamic topography**, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res. Oceans.

- contours of SSH + geostrophic balance \Rightarrow flow, important part of **MOC**

Recap: gyres + WBCs

- ▶ **subtropical and subpolar gyres** (former shown here)
 - **anti-cyclonic and cyclonic** respectively (in both hemispheres) (see Lec 8 + 9)
- ▶ **Western Boundary Current** as a part of system
 - Gulf stream here, Kuroshio in Pacific
 - transports **tropical + warm** water towards high latitudes

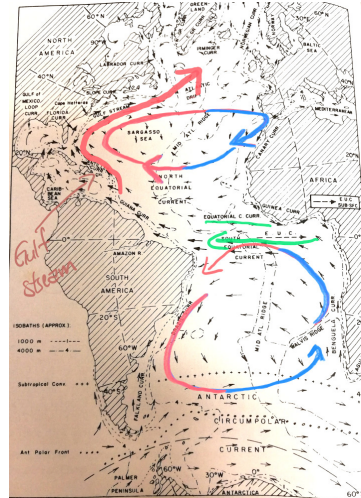
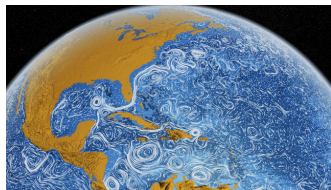
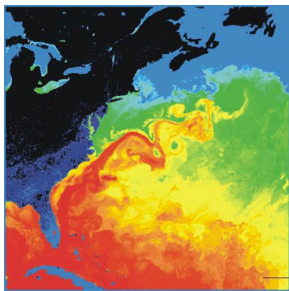


Figure: Features in the Atlantic Ocean. Modified Figure 7.9 from Pickard & Emery (1990), 5th edn.

Recap: gyres + WBCs

- Q. why Western and not Eastern? (see next Lec.)
- Q. processes leading to eddies? (see Lec. 17)
- Q. fluctuations + role in climate? (see Lec. 17 + OCES 4001)



Gulf stream in temperature (left) and surface current speed, from NASA

Recall: Coriolis effect and parameter (Lec. 8)

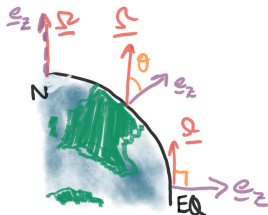


Figure: Mis-alignment of Ω and e_z used locally for depth.

- ▶ for a spherical Earth we take rotation axis to be z -axis, i.e. $\Omega = \Omega e_z$ (this a vector), but locally, z is depth...
- ▶ introduce the latitudinally varying **Coriolis parameter**

$$f = 2\Omega \sin(\text{latitude})$$

- ▶ want to further simplify this, spherical (i.e. (lon, lat, depth)) to Cartesian geometry (i.e. (x, y, z)) (cf. Lec 8, when rationalising Coriolis effect)

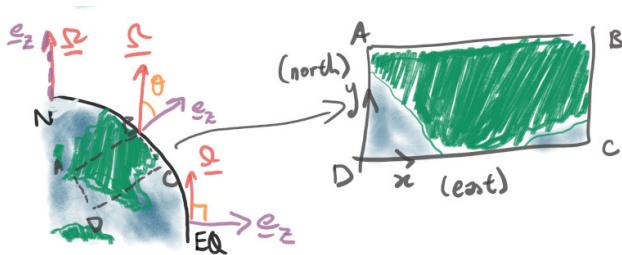
β -plane

Figure: β -plane schematic. $(\text{lon}, \text{lat}) \rightarrow (x, y)$ with $f = f_0 + \beta y$ on the plane.

- ▶ $x \leftrightarrow \text{lon}, y \leftrightarrow \text{lat}, z \leftrightarrow \text{depth}$
- ▶ $f = 2\Omega \sin(\text{lat}) \leftrightarrow f = f_0 + \beta y$
 - f_0 (units: s^{-1} , same as f) the uniform rotation frequency
 - $\beta = \partial f / \partial y$ (units: exercise)
 - **not** haline contraction (context should be clear)

will use β -plane extensively

Model preliminaries

For simplification, going to assume:

- ▶ NH β -plane ($f = f_0 + \beta y > 0$), domain is square
- ▶ subtropical wind profile (for subtropical gyre)
 - only for simplicity, will extend
 - note: negative **wind stress curl**
- ▶ assume **depth-independence** (either $\rho = \rho_0$ or vertically integrate)
 - sometimes **barotropic** (I don't like this term for technical reasons)
- ▶ lateral frictional boundary layers
 - main **sink** of stuff is going to be over boundary layers

Single (subtropical) gyre: schematic

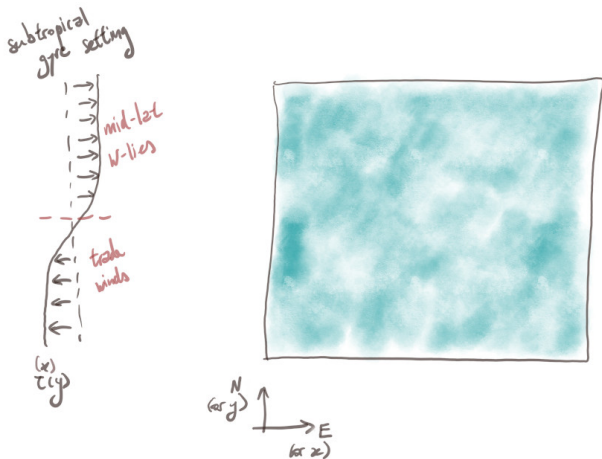


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β -plane, square, homogeneous in density)

Equations

Original equations something like (in vertical vorticity ω):

$$\frac{\partial \omega}{\partial t} + \underbrace{\mathbf{u} \cdot \nabla \omega}_{\text{inertia}} + \underbrace{\beta v}_{\text{Coriolis}} = \underbrace{-r\omega}_{\text{drag}} + \underbrace{F_\tau(x, y)}_{\text{wind}}$$

- ▶ 2d equations in (x, y) (assumed no vertical variation)
- ▶ no pressure gradients (took a $\nabla \times$ of momentum equation)
- ▶ Coriolis effect appears through βv (as $f = f_0 + \beta y$)
→ it is **meridional** velocity v that shows up here
- ▶ $F_\tau = \mathbf{e}_z \cdot \nabla \times \boldsymbol{\tau}$ is **wind stress curl**
→ $F_\tau < 0$ for wind profile in previous slide
- ▶ friction **parameterised** as **linear drag**
→ important near boundaries

Sverdrup balance

Throw away time derivative and inertia term (cf. $Ro \ll 1$) gives

Stommel's model (Stommel, 1948, *Trans. Amer. Geophys. Union*)

$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_{\tau}(x, y)}_{\text{wind}}$$

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In **interior** friction is unimportant, so we have **Sverdrup balance**

$$\beta v \sim F_{\tau}(x, y)$$

- Coriolis balancing wind stress curl
→ geostrophy is Coriolis balancing **pressure gradient** (but no pressure in vorticity equation)

$\beta > 0$ so v related to wind stress curl in interior

Note: The standard derivation involves looking at Ekman up/downwellings associated with the wind, then imply the Sverdrup interior

Single (subtropical) gyre: Sverdrup interior

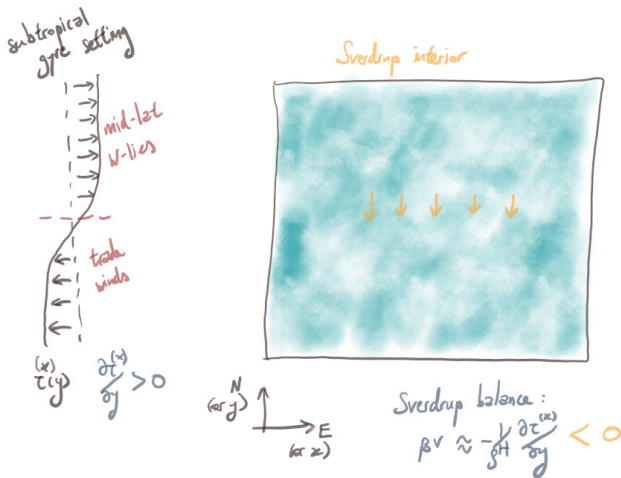


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β -plane, square, homogeneous in density)

Single (subtropical) gyre: mass conservation

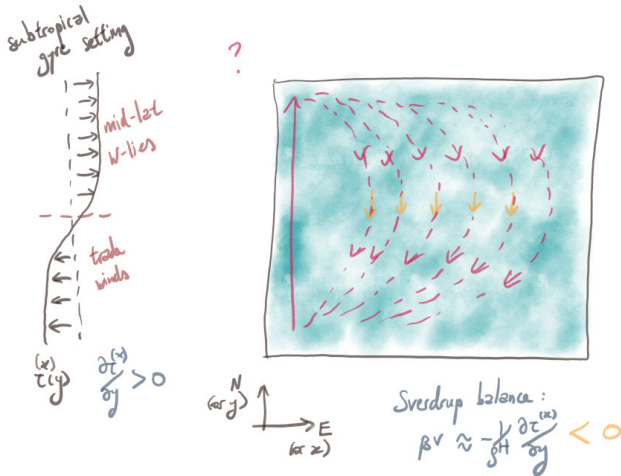


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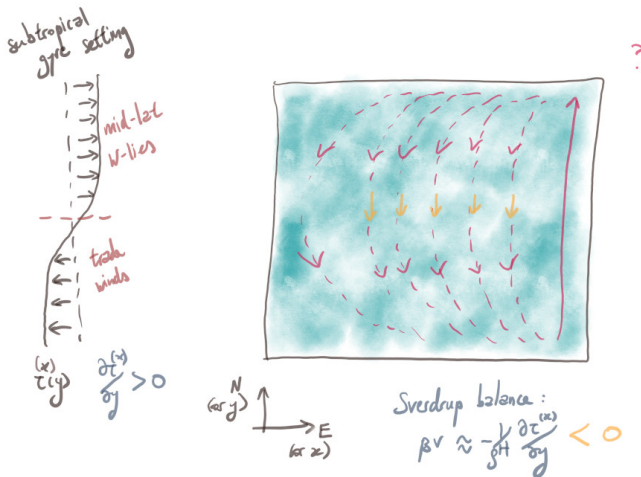


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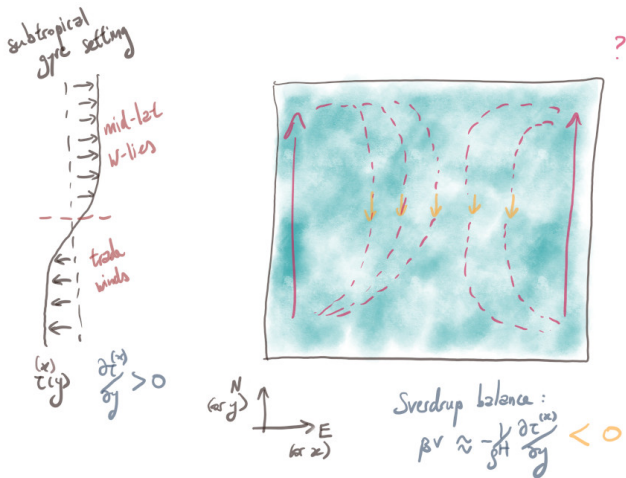


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Double (subtropical + subpolar) gyre

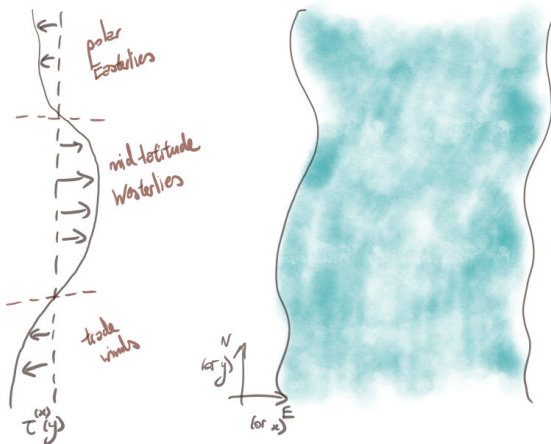


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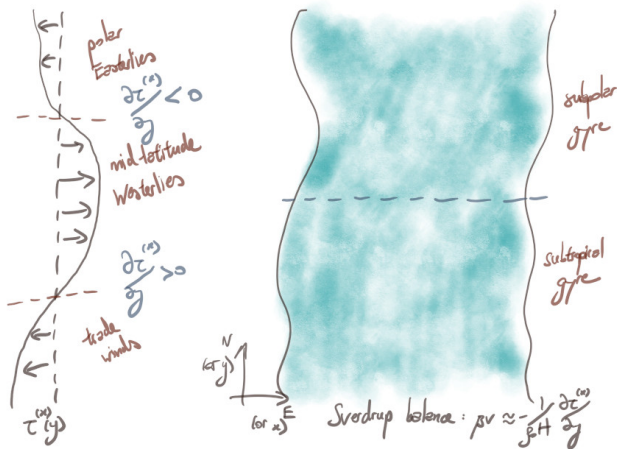


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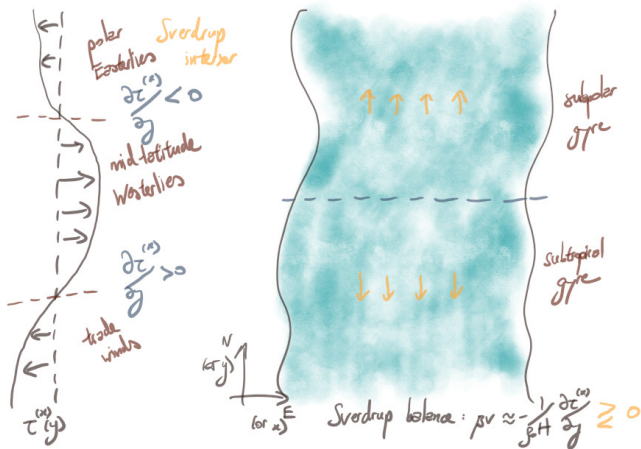


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Summary

- Sverdrup balance:

$$\beta v \sim F_{\tau}(x, y)$$

→ Sverdrup interior,
meridional flow related to
wind stress curl

- mass balance + continuity
implies essentially two
possibilities

- intuition: flow should be western intensified

→ energetic argument, flow should go in direction of wind

next lecture: support intuition by vorticity balance argument

