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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 17: Dynamics 3 (instabilities and mechanisms)

# Outlook of the next few lectures

Dynamics important, next two lectures on

- ▶ waves (this Lec. + 16, 18) and instabilities (Lec. 17)

→ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- ▶ how to describe waves (Lec. 15)

- ▶ types of waves (Lec. 16)

→ consequence + leading to instabilities

- ▶ instabilities (Lec. 17)

→ parcel-type (mechanistic) arguments for instability

- ▶ tides (particularly as internal gravity waves) (Lec. 18)

# Outline

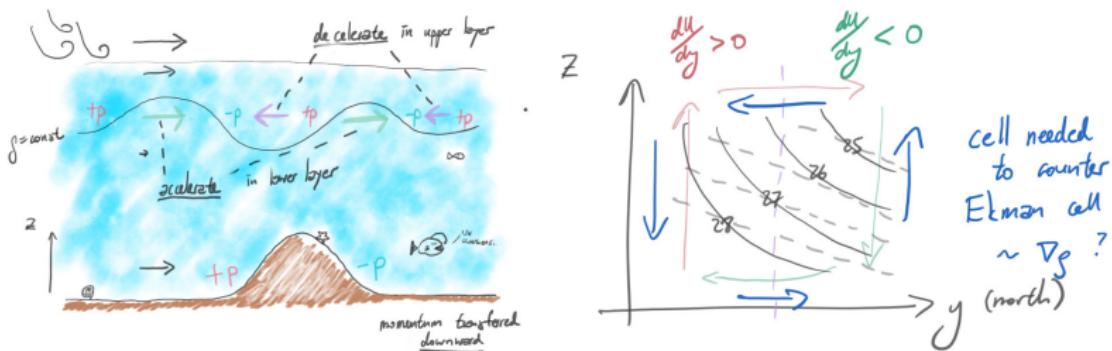
- ▶ static/convective instabilities
  - largely buoyancy related
  - Rayleigh-Taylor and double diffusive instabilities
  - parcel argument revisited (recall Lec. 5)
- ▶ shear instabilities
  - largely flow shear (thus vorticity gradient) related
  - baroclinic instabilities (MOC, Lec. 13)
  - Kelvin-Helmholtz (diapynal mixing, Lec. 14)
  - a unifying picture (?): constructive interference of two vorticity waves

**Key terms:** instabilities, static, shear, wave interference, action-at-a-distance

# Recap: role of waves and instabilities

Role also of **baroclinic instability** (Lec. 13, see also Lec. 17), important for

- ▶ vertical momentum transfer by **interfacial form stress**
- ▶ scale transfer of **energy**  
→ mesoscale eddies, conduit between large-scales and submesoscales
- ▶ along-isopycnal mixing and also MOC



**Figure:** Schematic of form stress and eddy induced overturning cell in Southern Ocean (see Lec. 14)

# Recap: role of waves and instabilities

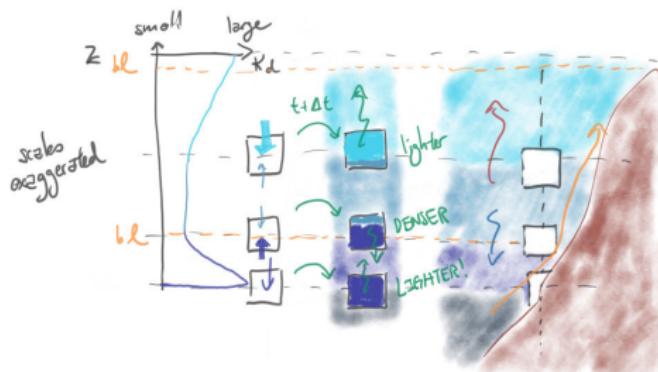


Figure: Schematic of the diffusive upwelling.

- ▶ diapycnal mixing contribute upwelling, strongest in boundary layers  
→ broad diffusive boundary intensified upwelling

what causes the boundary intensification of  $\kappa_d$ ? dynamics!

- ▶ at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- ▶ at the bottom, probably tidal conversion (Lec. 18) → internal gravity waves (Lec. 16) → shear instabilities (Lec. 17)

# Concept of stability



Figure: Pig being prodded (probably don't try this in real life).

- ▶ imagine a <sup>(spherical)</sup> pig is in a valley and you prod it a bit...

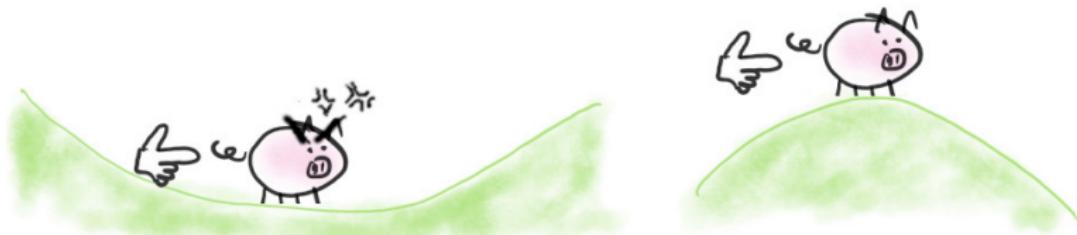
# Concept of stability



**Figure:** Pig being prodded (probably don't try this in real life).

- ▶ the pig wants to stay “close” (not defined rigourously but can be) **to its initial position, stable state** (relative to position anyway, you might want to run)

# Concept of stability



**Figure:** Pig being prodded (probably don't try this in real life).

- ▶ imagine a (point mass spherical?) pig now on a hill and you prod it a bit...

# Concept of stability

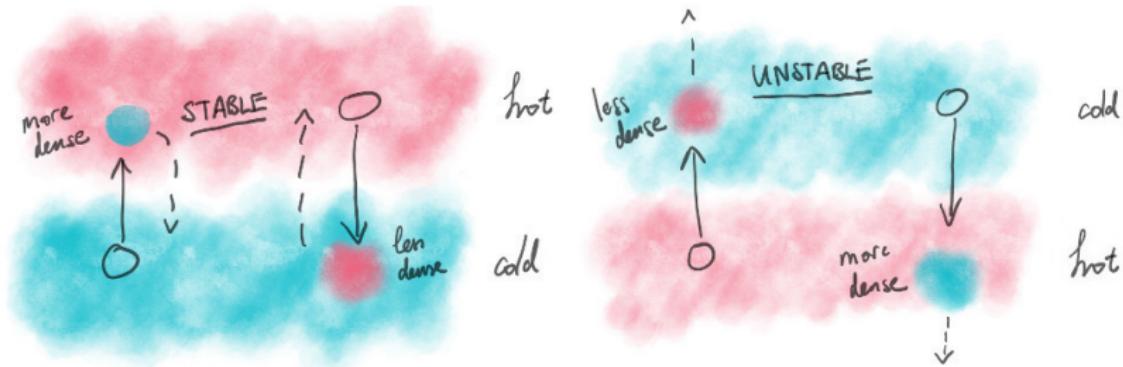


**Figure:** Pig being prodded (probably don't try this in real life).

- ▶ the <sup>(spherical)</sup> pig rolls down the hill, i.e. **not** staying close to its original position, unstable state

# Static instabilities: parcel argument

(recall Lec 5)



Essence of **parcel argument**:

- ▶ take a parcel and assume, if moved, conserves (carries) water properties with it
  - assume fast time-scales ( $\tau_{\text{diff}} \ll \tau_{\text{dyn}}$  i.e. weak mixing)
- ▶ consider forces and motion arising from **anomalies**
  - here temperature (or density) anomalies, leading to buoyant forces

# Static instabilities: $N^2 > 0$

- ▶ recall Brunt–Väisälä or buoyancy frequency

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}$$

- ▶  $N^2 > 0$ , stratification is **stable**

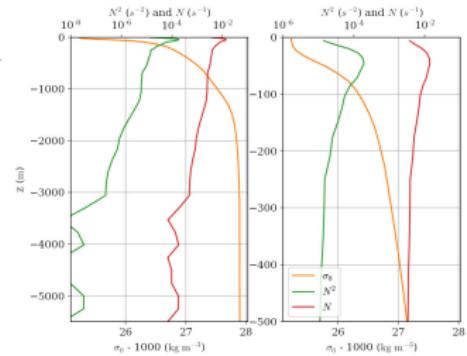
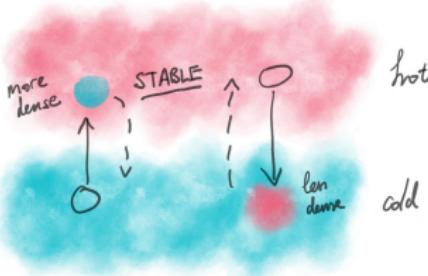
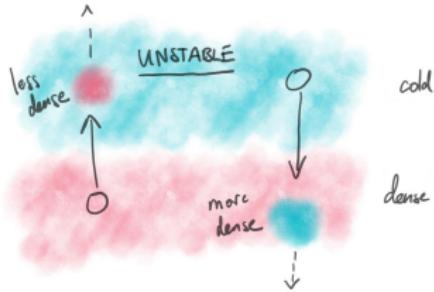


Figure:  $\sigma_0$  (see Lec. 6) and the associated  $N^2$  and  $N$ . See `plot_eos.ipynb`.



- ▶ if no damping, particle overshoots and oscillates, i.e. **wave-like motion**
- Q. what is the **frequency** of motion?

# Static instabilities: $N^2 < 0$



- ▶ if  $N^2 < 0$  then stratification is **unstable**
  - Rayleigh–Taylor instability (or just **static instability**)
  - ample videos/pictures online<sup>1</sup>

- ▶ instability is **very quick** in the ocean! ( $<$  months)
  - upper bound **growth rate** by  $|N^2|$  (e.g. Haine & Marshall, 1998, *J. Phys. Oceanogr.*)
  - overturns to re-establish stable stratification
  - **large transport** (i.e.  $\kappa_d \gg 1$  if you want to think of it that way)

<sup>1</sup> see e.g. Megan Davies Wykes's website <http://www2.eng.cam.ac.uk/~msd38/gallery.html> ▶

# Static instabilities: $N^2 < 0$

- ▶ deep convection not that easy to get in ocean (see Lec. 14)
  - intense cooling
  - some as part of overflows

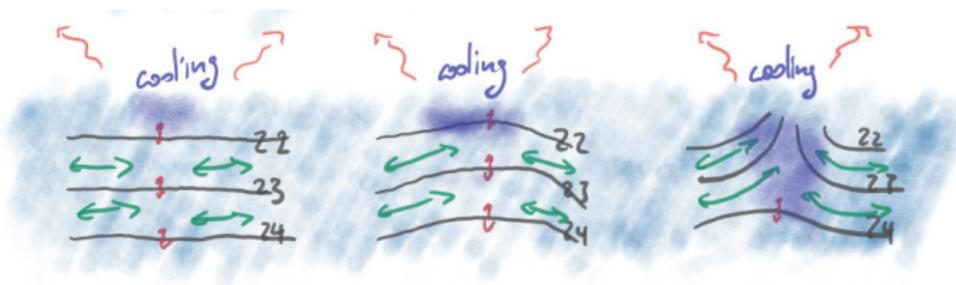
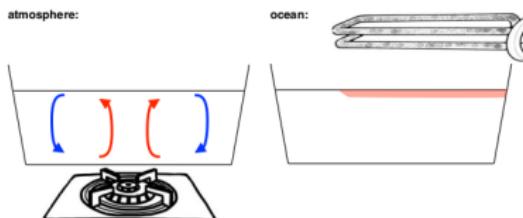


Figure: Watermass transformation by decreasing temperature.

# Double diffusive instability

Recall that  $\rho = \rho(T, S, p)$  and

$$\left( \frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = \kappa_T \nabla^2 T,$$

$$\left( \frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = \kappa_S \nabla^2 S$$

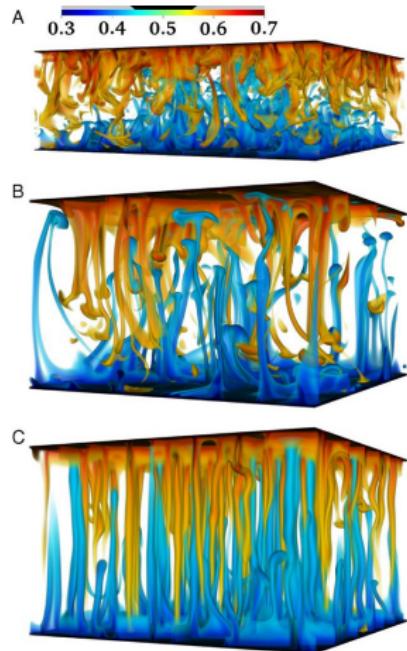
- ▶ generally  $\kappa_T \neq \kappa_S$  (hence double diffusion)

→ ocean has  $\kappa_T \gg \kappa_S$

→ sometimes known as semi-convection

(astrophysics, e.g. helium and heat in the Sun)

- ▶ can lead to non-negligible transport



**Figure:** Simulation in temperature stable but salt unstable regimes. From Yang, Verzicco & Lohse (2016), *Proc. Nat. Acad. USA*, modified from their Fig. 1.

# Double diffusive instability: salt fingering

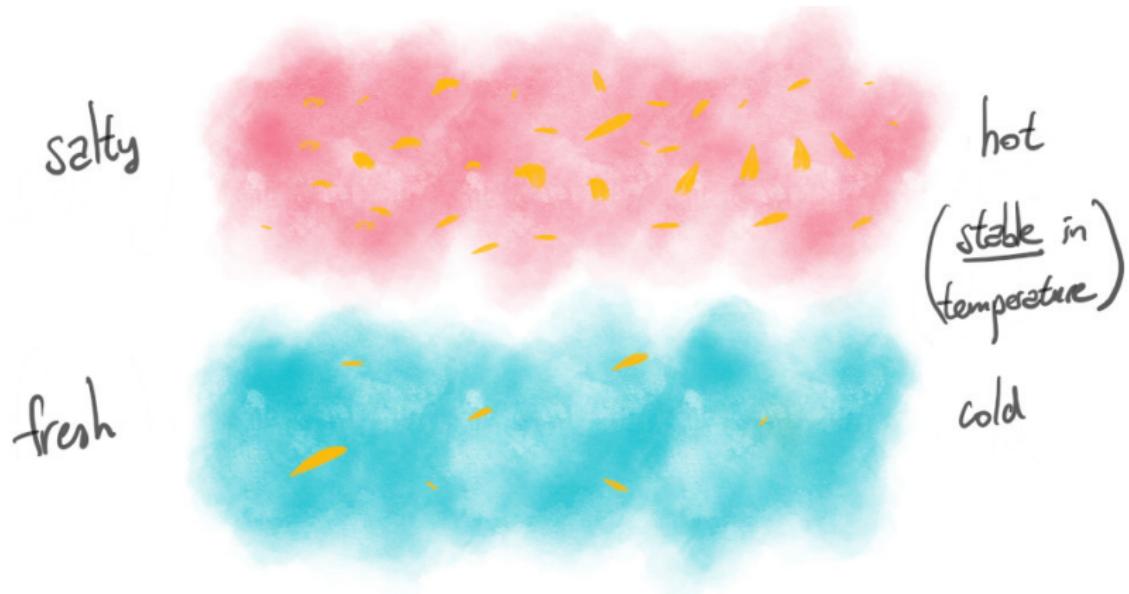


Figure: Salt fingering schematic: stable in temperature gradient but unstable in salinity gradient.

# Double diffusive instability: salt fingering

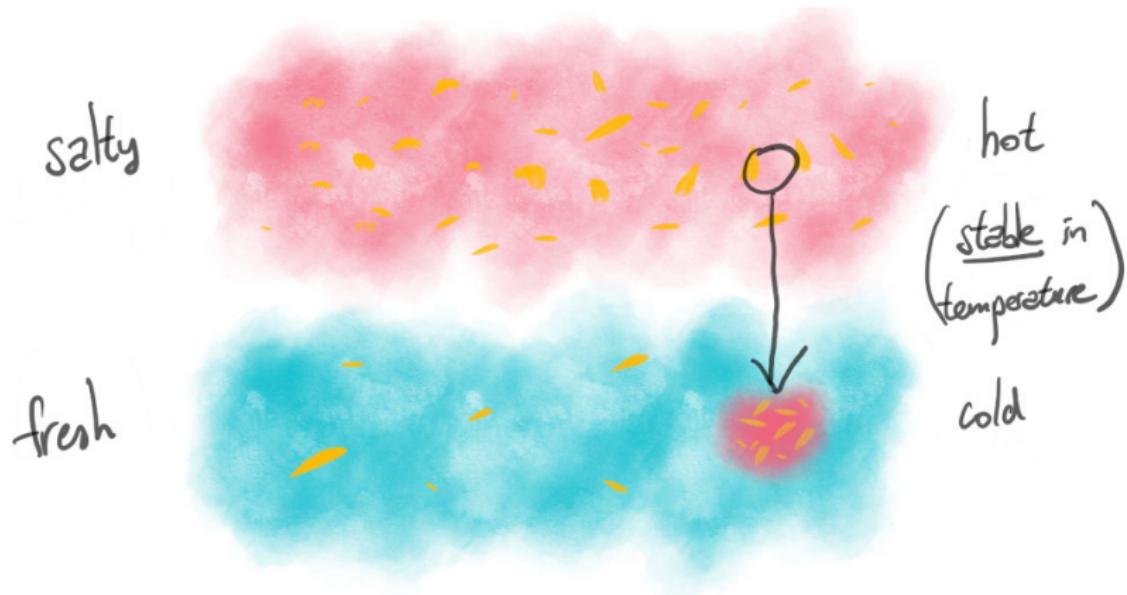


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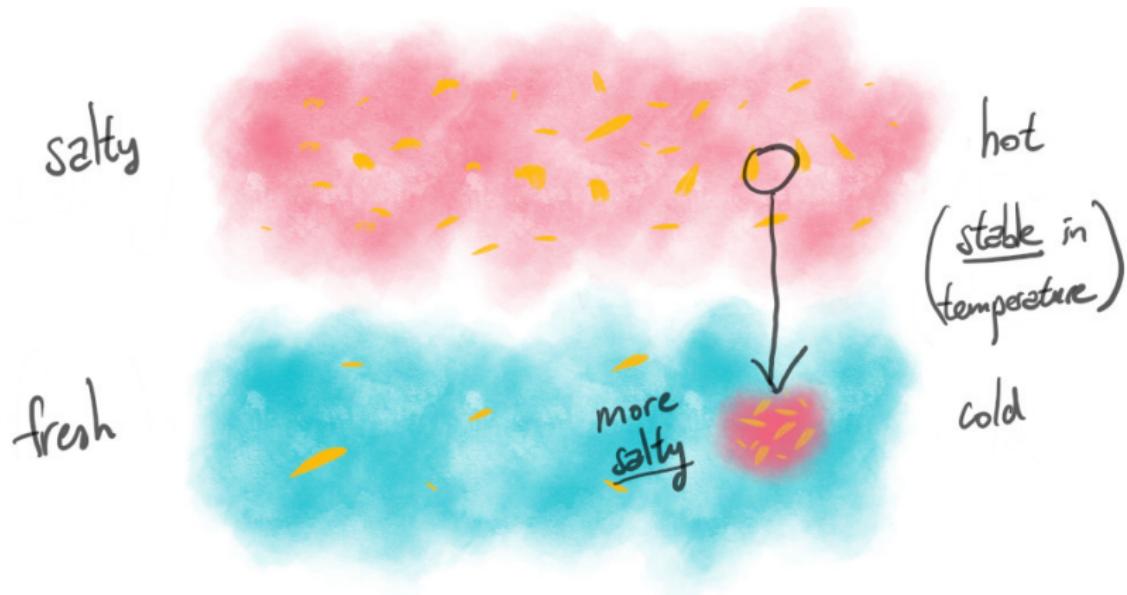


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# Double diffusive instability: salt fingering

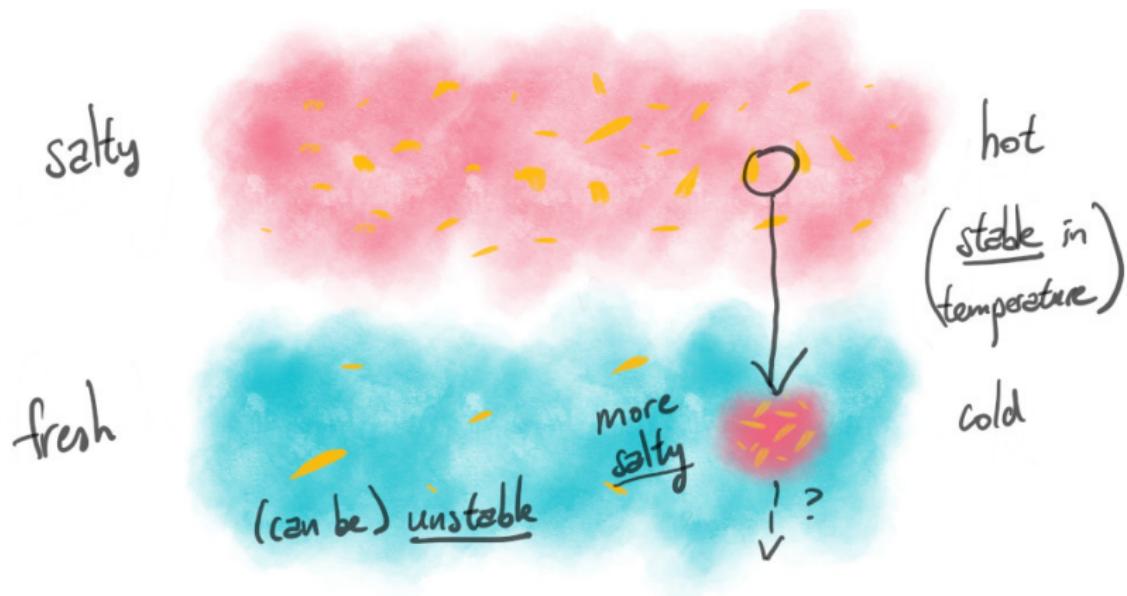
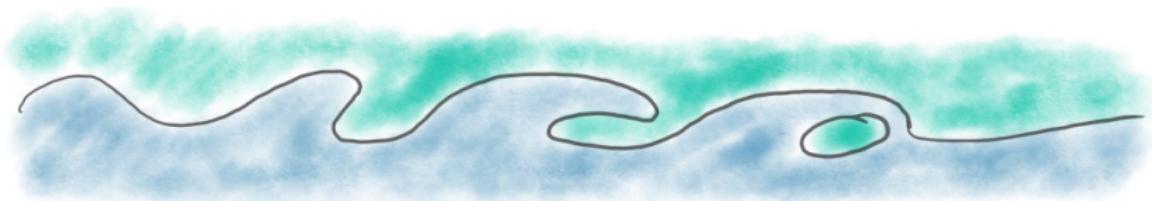


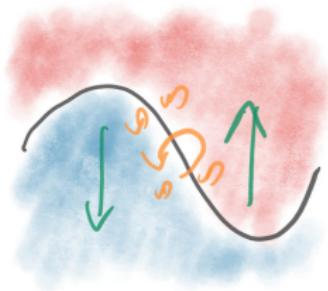
Figure: Salt fingering schematic: stable in temperature gradient but unstable in salinity gradient.

# Shear instabilities



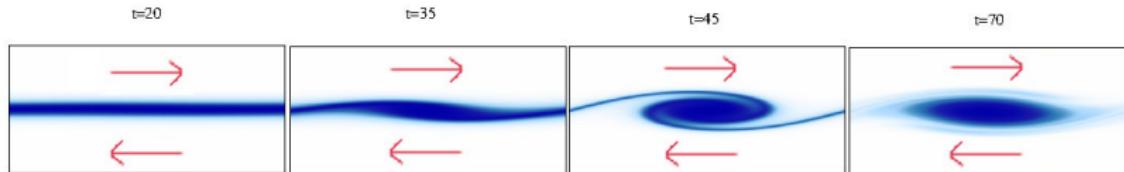
**Figure:** Schematic of mixing by (irreversible) wave breaking leading to e.g. diapycnal mixing.

- ▶ growing disturbance, **instability**
  - convective and/or shear (see Lec. 17)
  - mixing of material **across** isopycnals after reconnection, leading to **diapycnal mixing**
- ▶ feedback onto MOC (see Lec. 14)



**Figure:** Velocity shear from waves can lead to mixing.

# Shear instabilities



**Figure:** Roll-up of flow arising from a shear instability (horizontal, no stratification here). Adapted from Mak *et al.* (2017), *Phys. Rev. Fluids*, Fig.1.

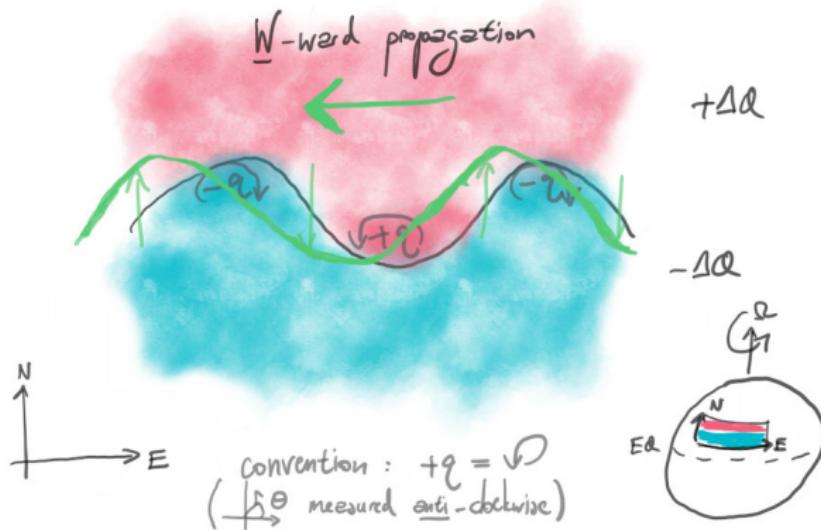
- ▶ Kelvin–Helmholtz instability
  - roll up into billows, stratification usually involved
- ▶ Holmboe instability
  - in estuaries?

# Shear instabilities

## Baroclinic instability (cf. Lec 13)

- ▶ sloping isopycnal surfaces  $\Leftrightarrow$  vertical shear flow (from thermal wind)
  - wants to flatten isopycnals, important for MOC (cf. Lec 13 + 14)
- ▶ mixing by **isopycnal** rather than **dipycnal**
- ▶ Eady + Charney paradigm (see more in Vallis, 2006)

# Shear instabilities as vorticity wave resonance



**Figure:** Parcel argument for Rossby wave propagation. (1) Assumes conservation of vorticity by wave. (2) Vorticity anomalies induces flow. (3) Flow leads to self-advection.

**How does / can this transfer to shear flows?**

# Vorticity wave propagation in background flow

$$U = U_{\infty} \text{ for now}$$



# Vorticity wave propagation in background flow

$$\Delta Q = 0$$

$$\Delta Q < 0$$

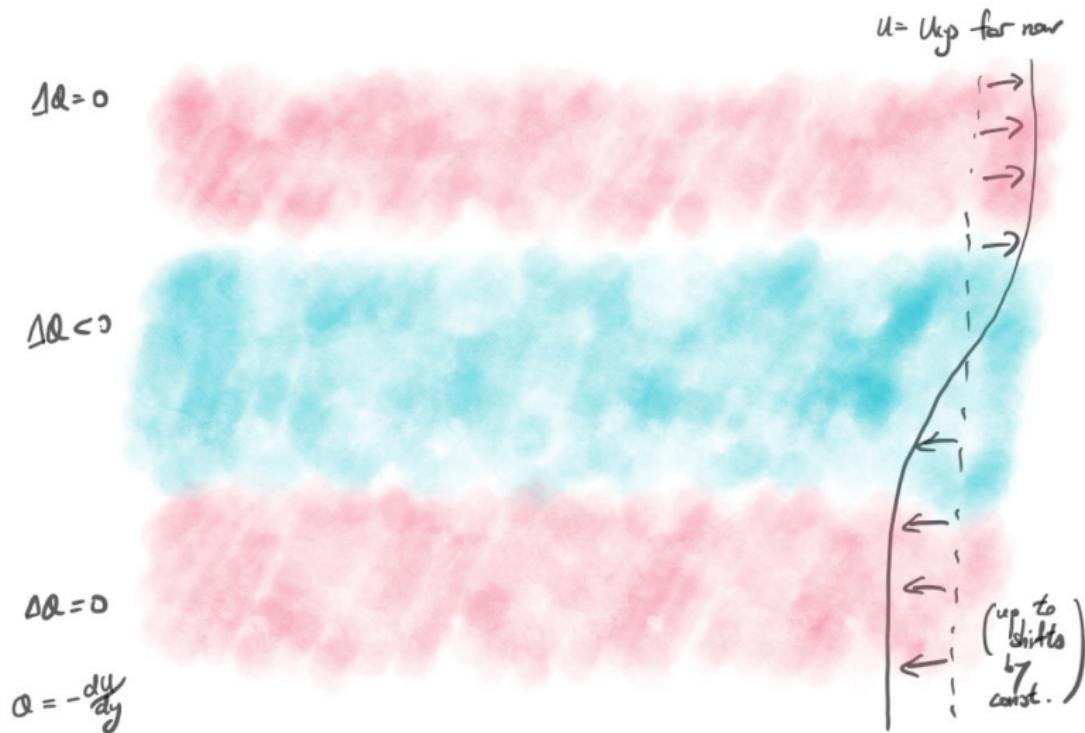
$$\Delta Q = 0$$

$$\Omega = -\frac{du}{dy}$$

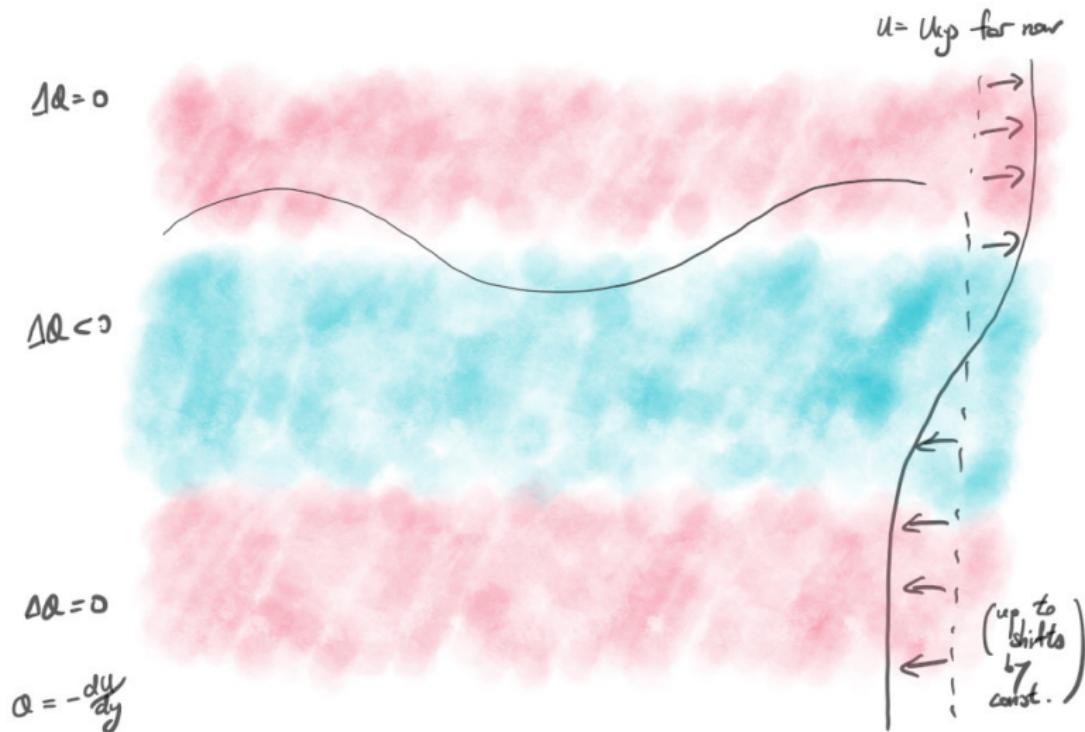
$$u = U_{bg} \text{ for now}$$



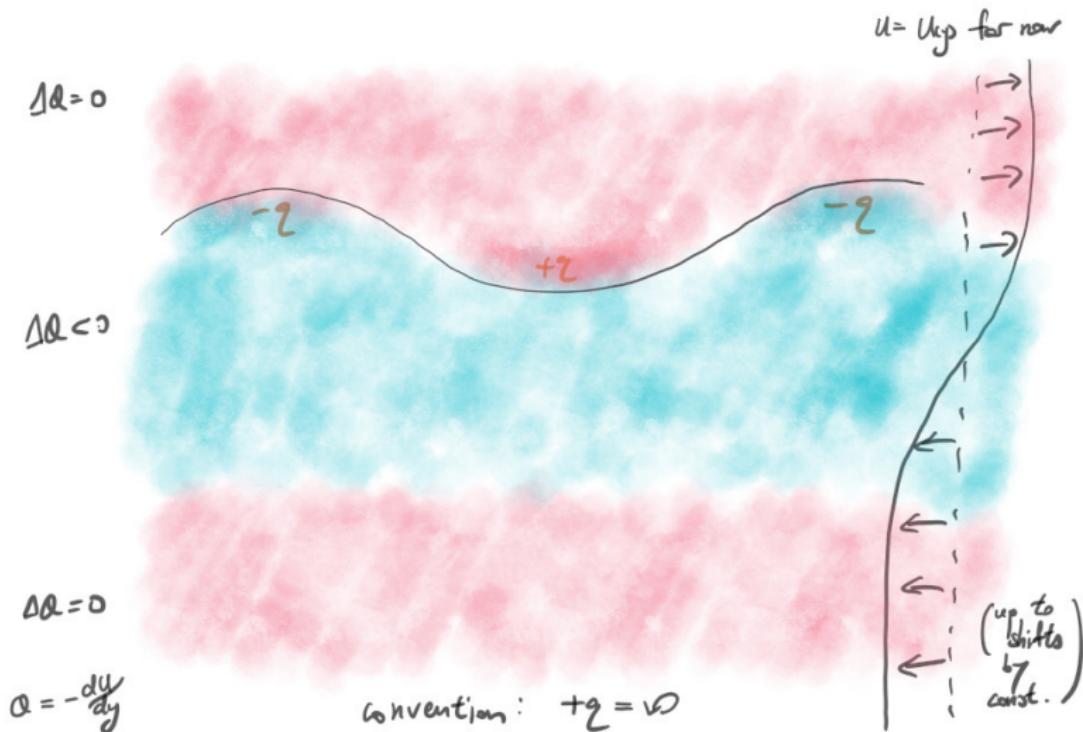
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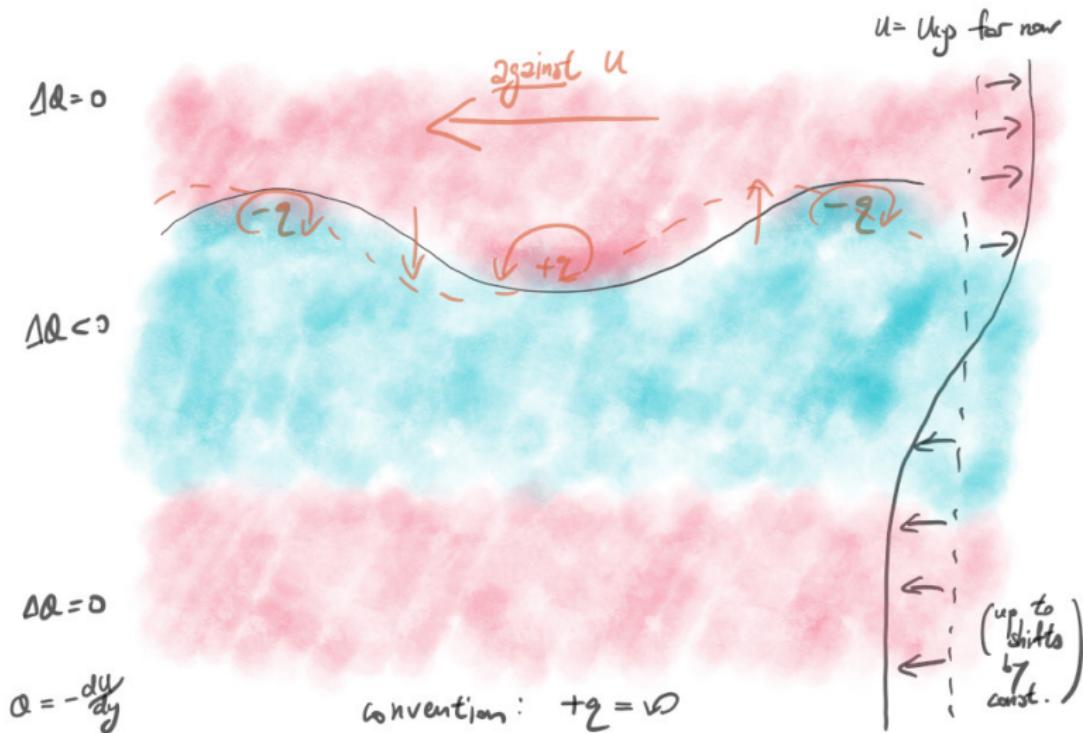
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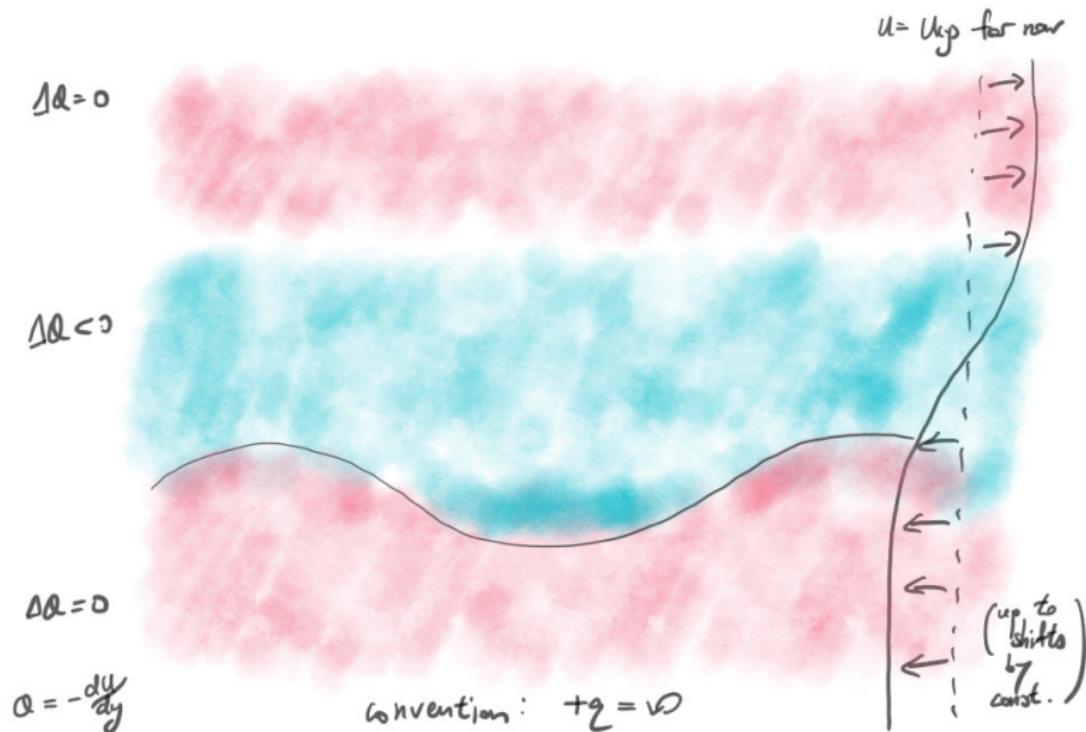
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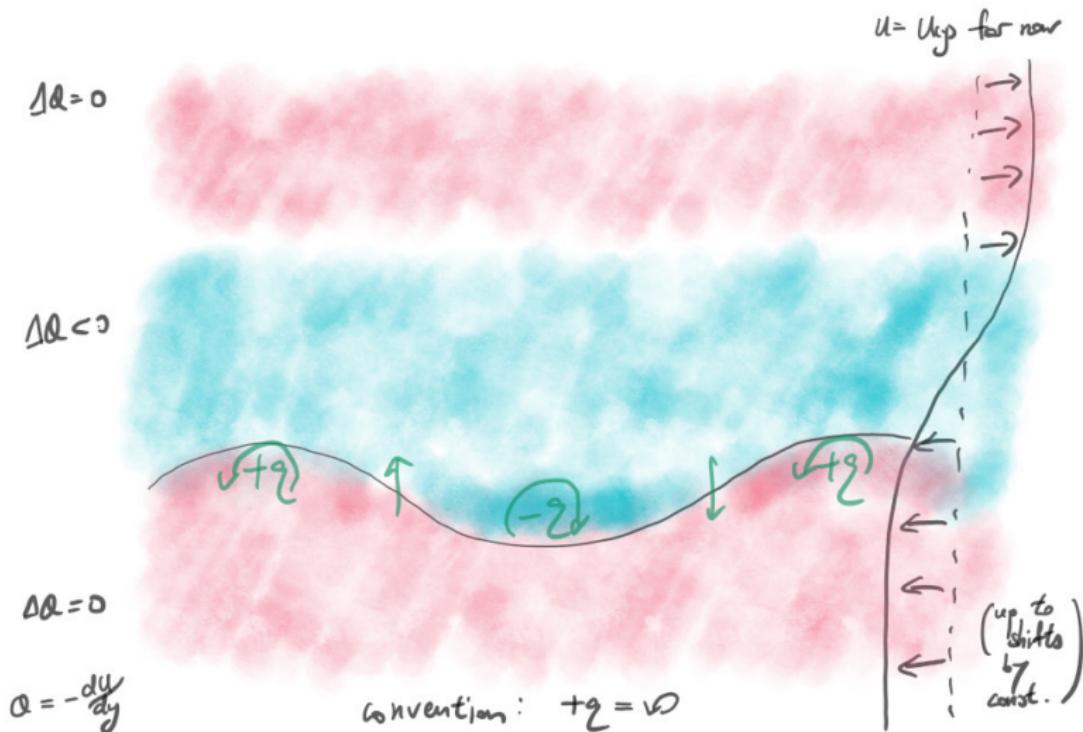
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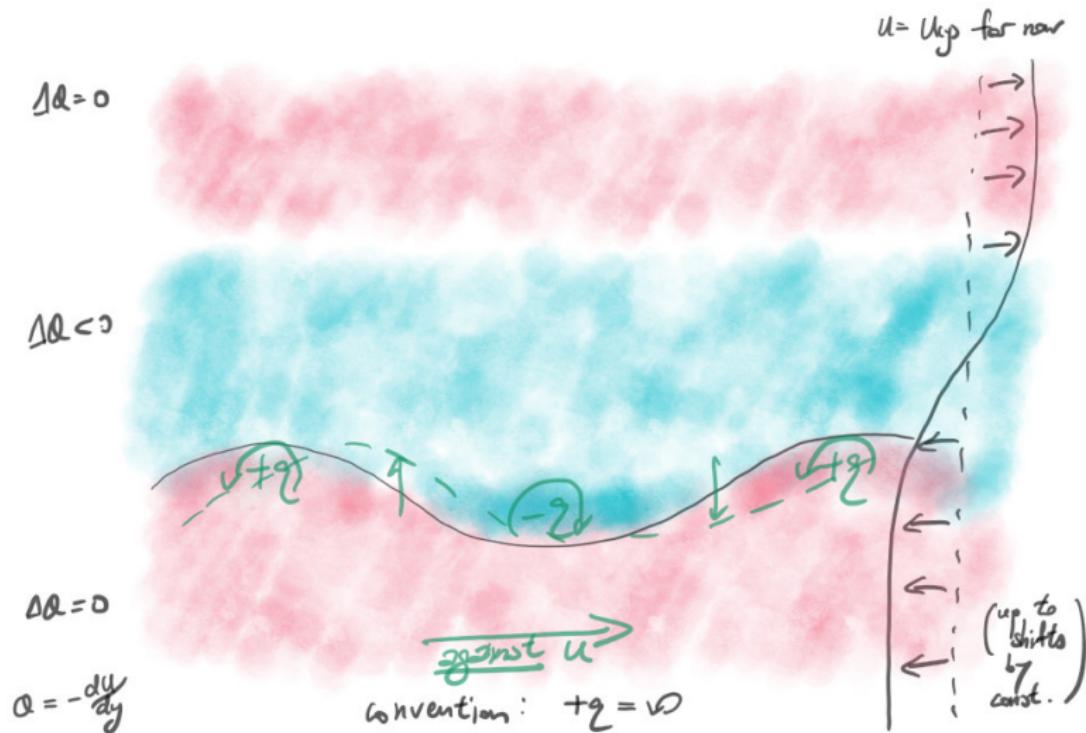
# Vorticity wave propagation in background flow



# Vorticity wave propagation in background flow



# Vorticity wave propagation in background flow



# Vorticity wave propagation in background flow

- ▶ fundamentally same mechanism
  - waves conserve vorticity
  - resulting anomalies induce flow
  - flow induces propagation
- ▶ can think of them as **generalised Rossby waves**
  - normal Rossby waves: background vorticity gradient from planetary rotation
  - here: background vorticity gradient from **background shear flow**

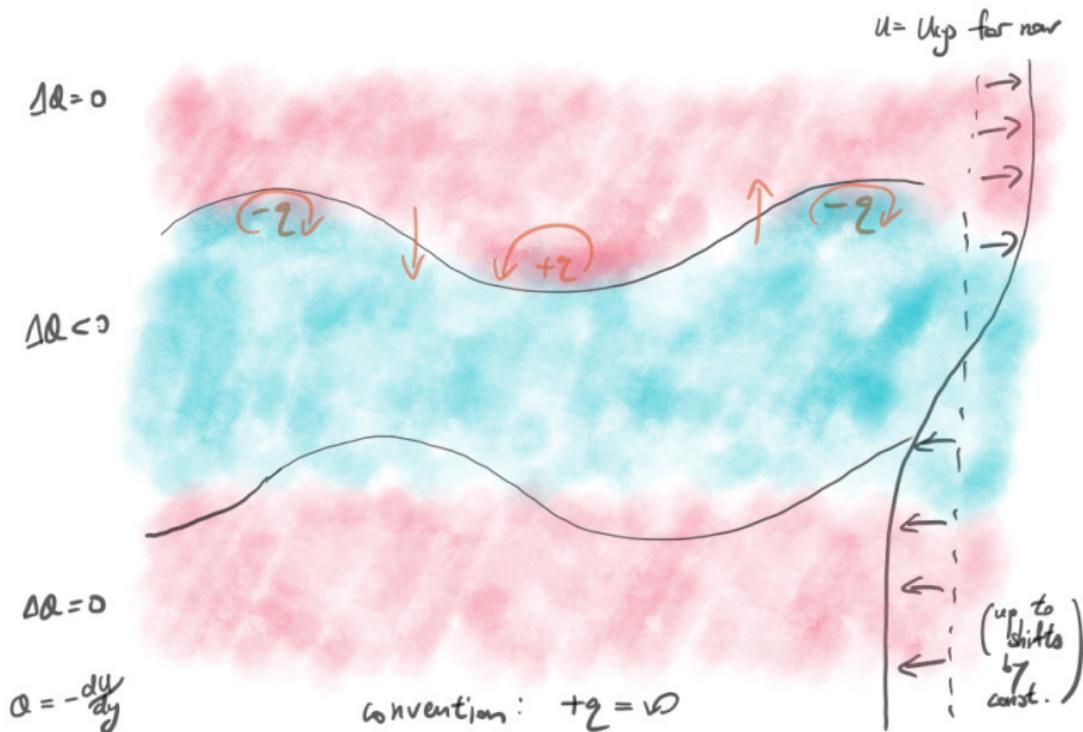
# Vorticity wave propagation in background flow

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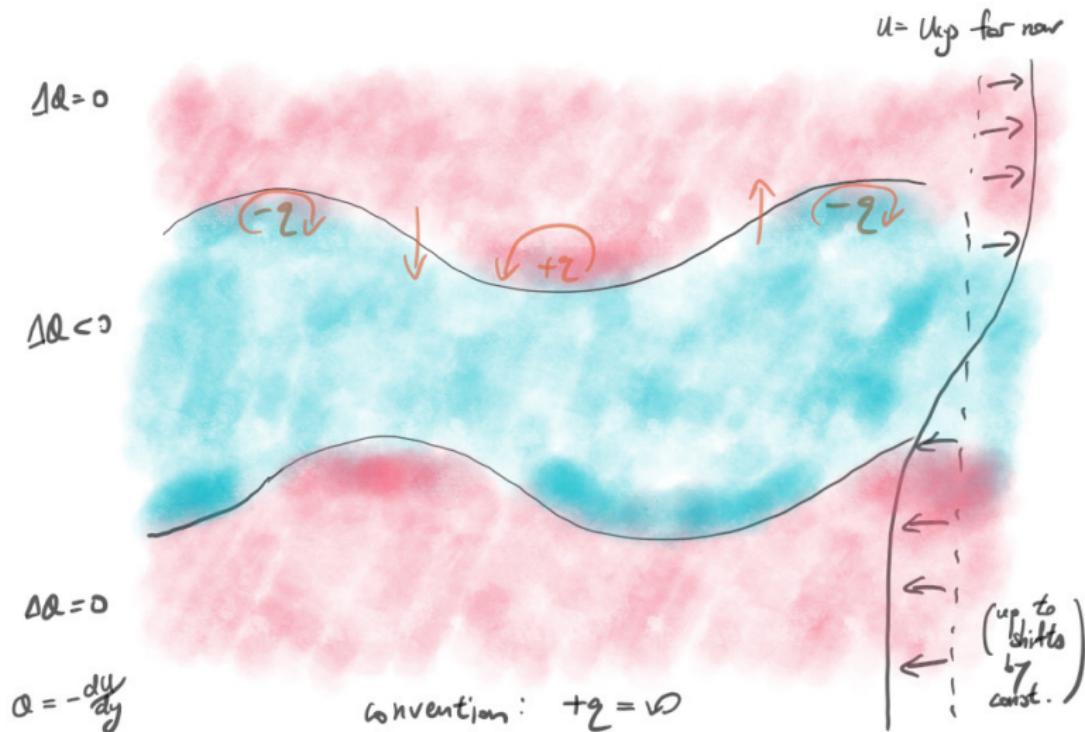
**How do these waves play with each other then?**

- ▶ depends on the **phase shift**

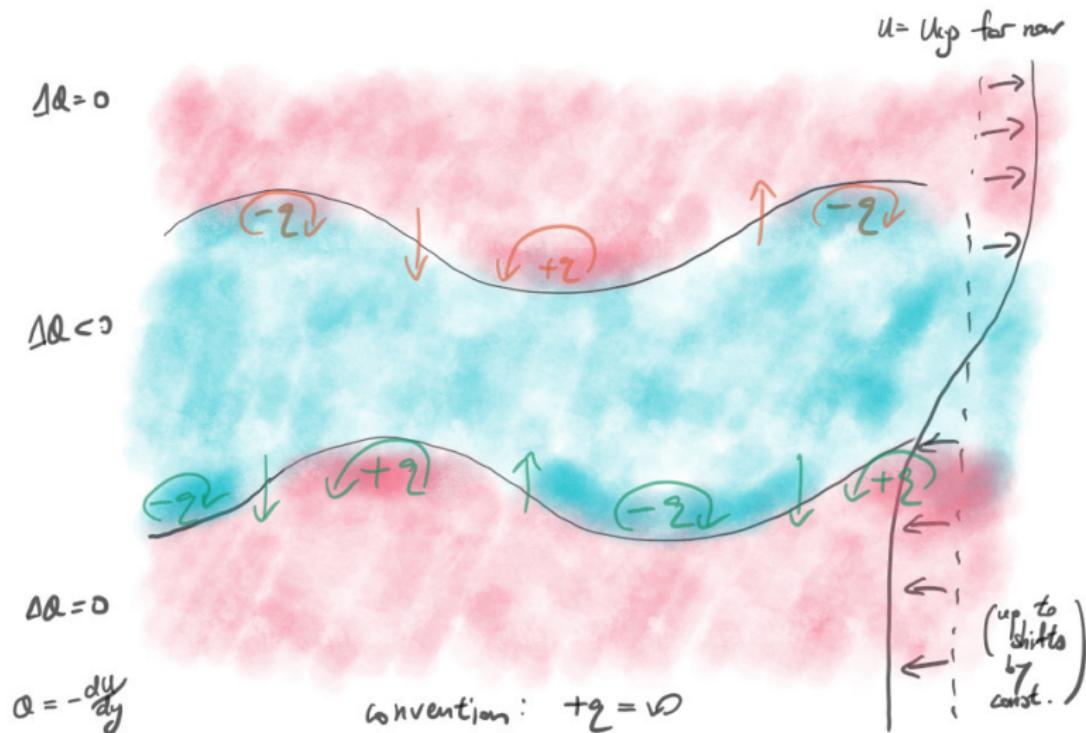
# Stable case (top wave lags bottom by $\pi/2$ )



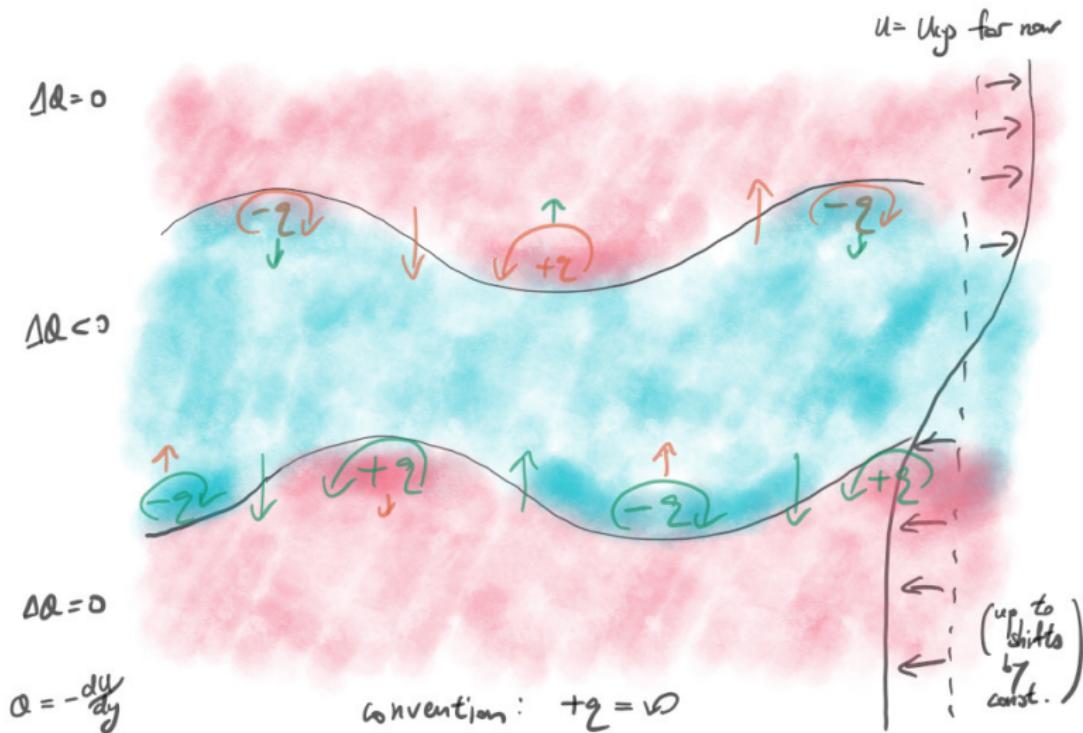
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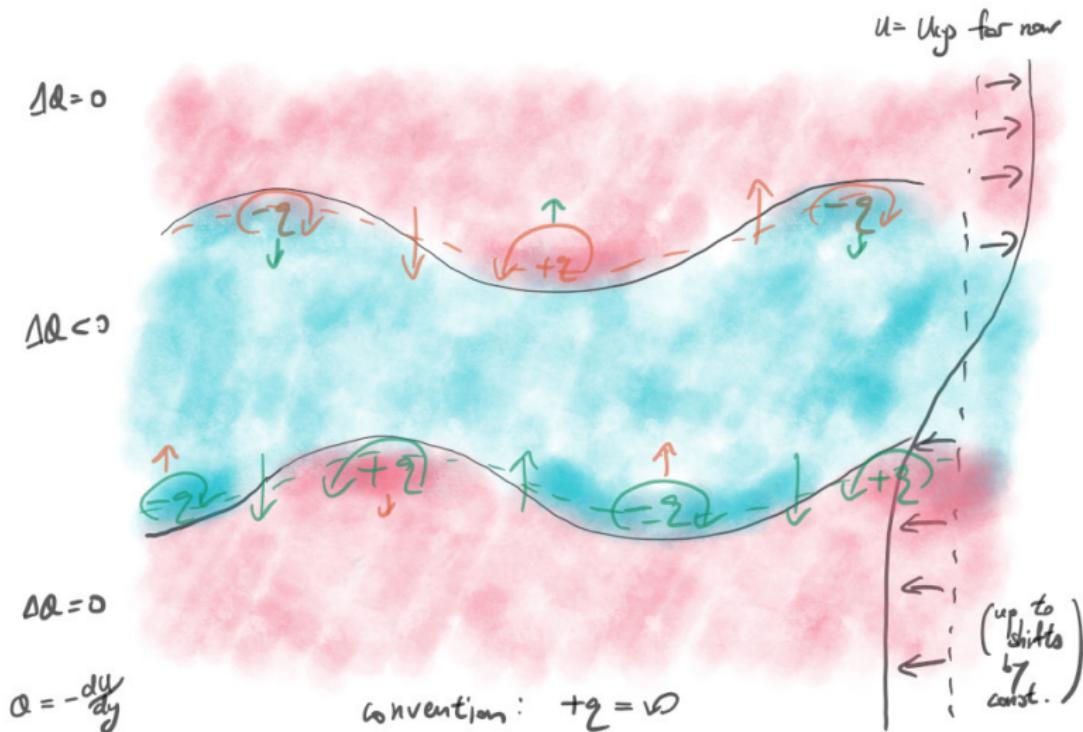
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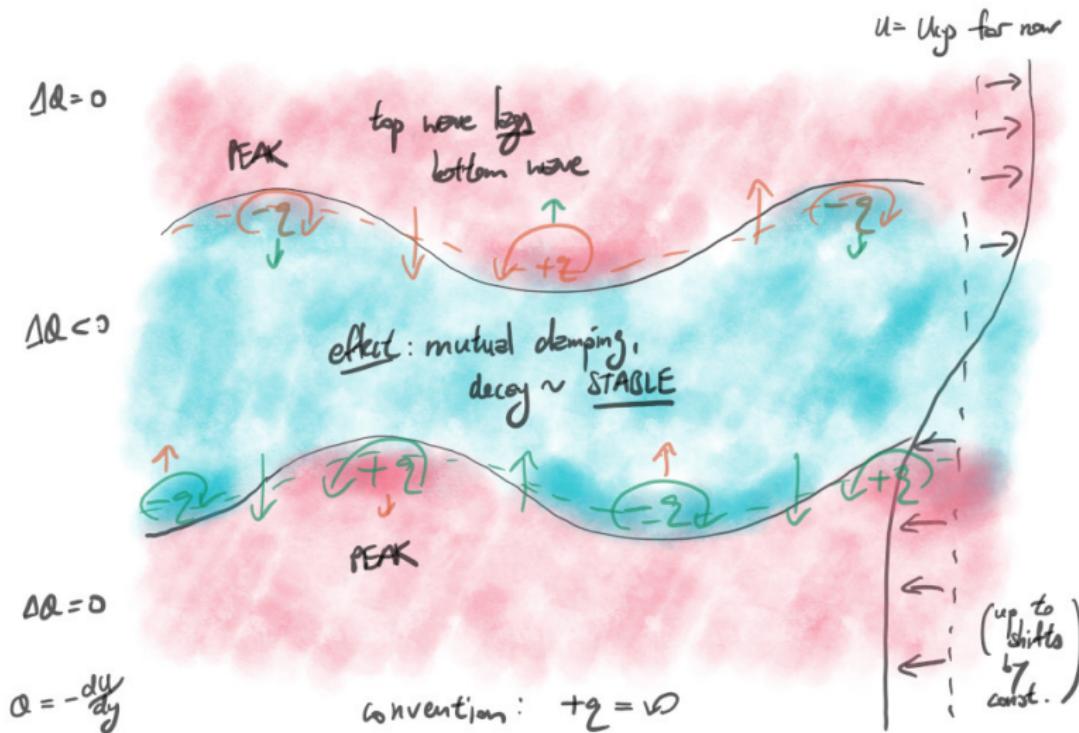
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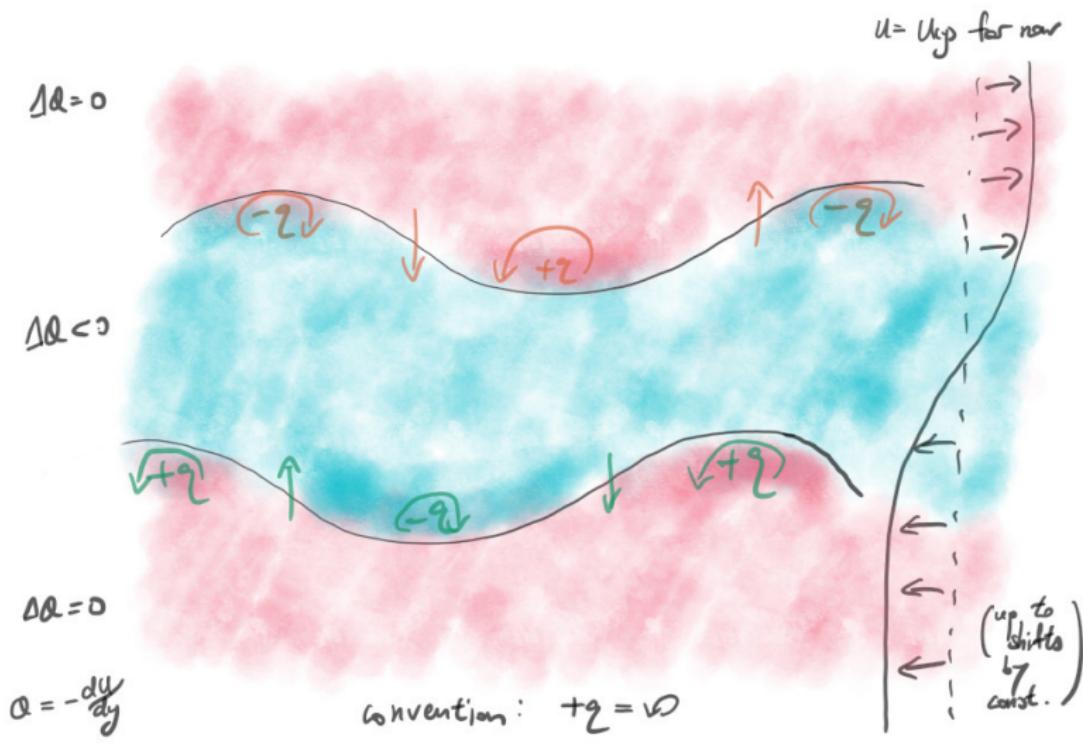
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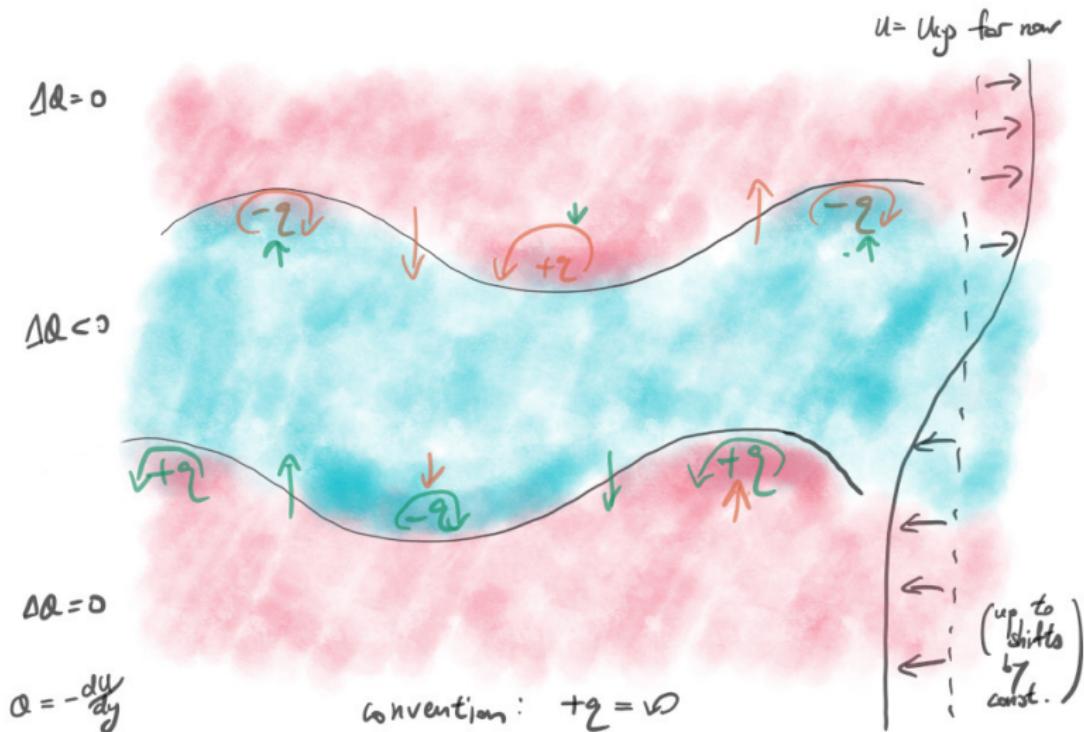
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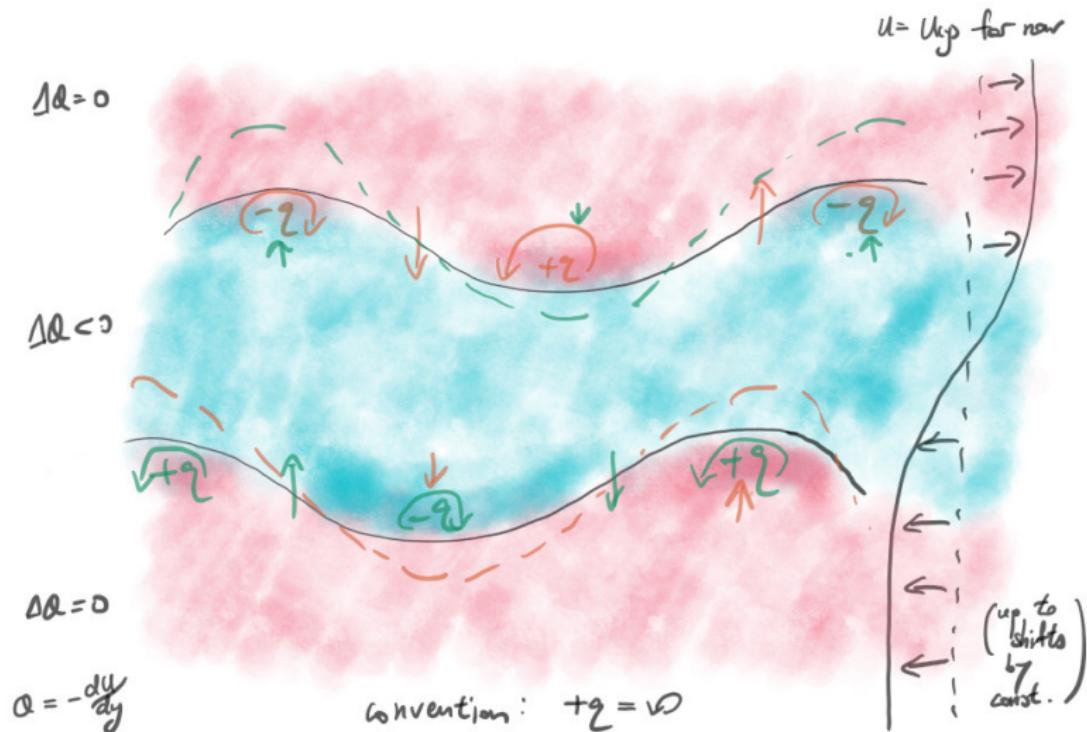
# Unstable case (top wave leads bottom by $\pi/2$ )



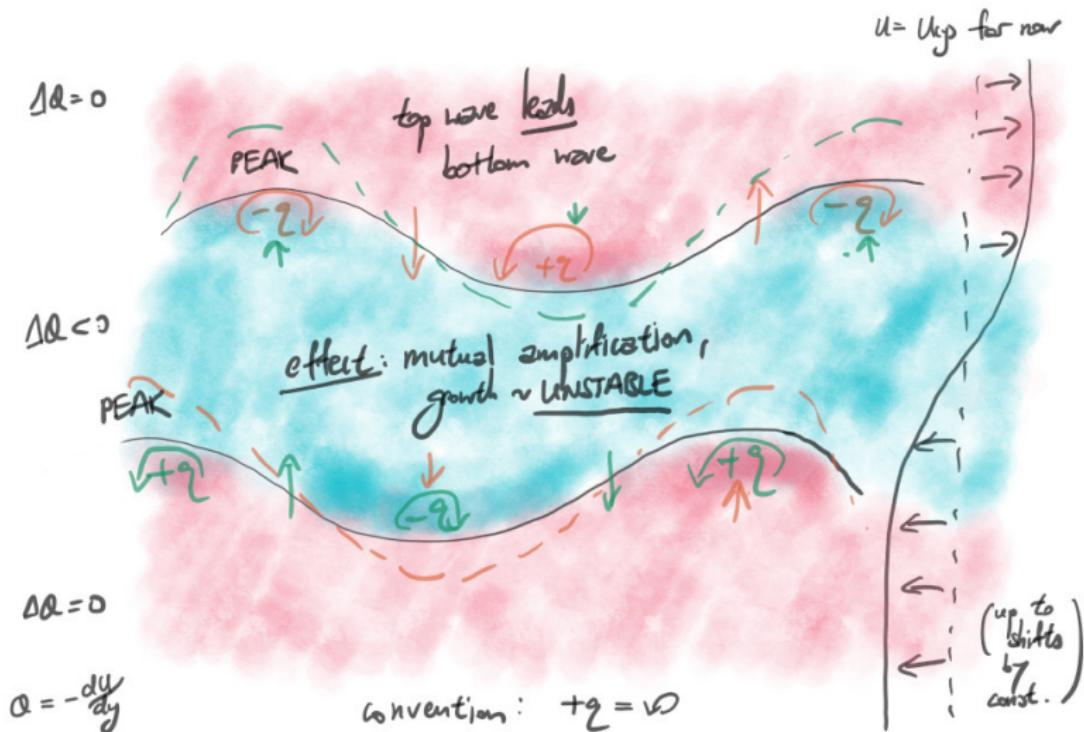
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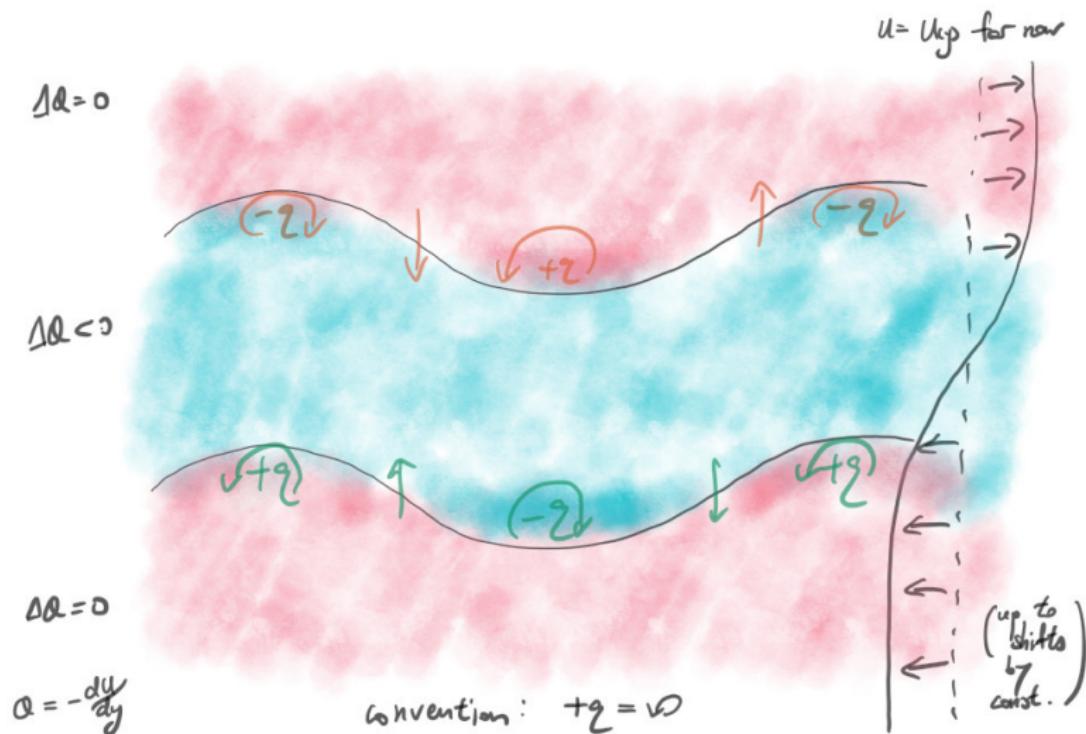
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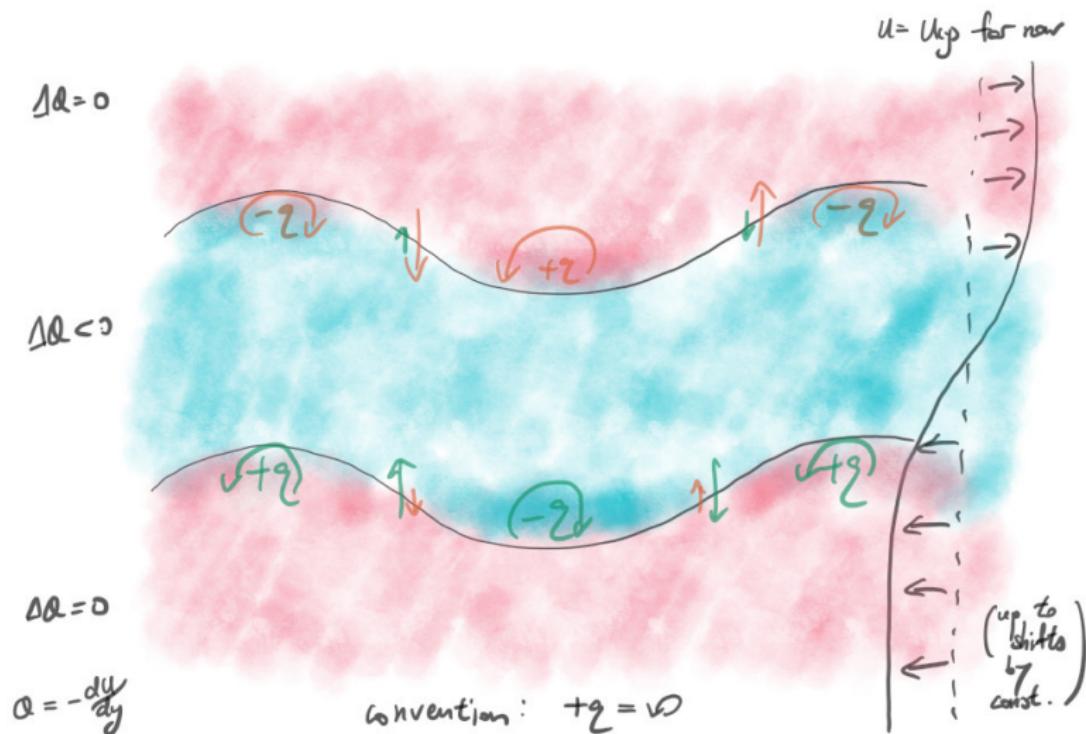
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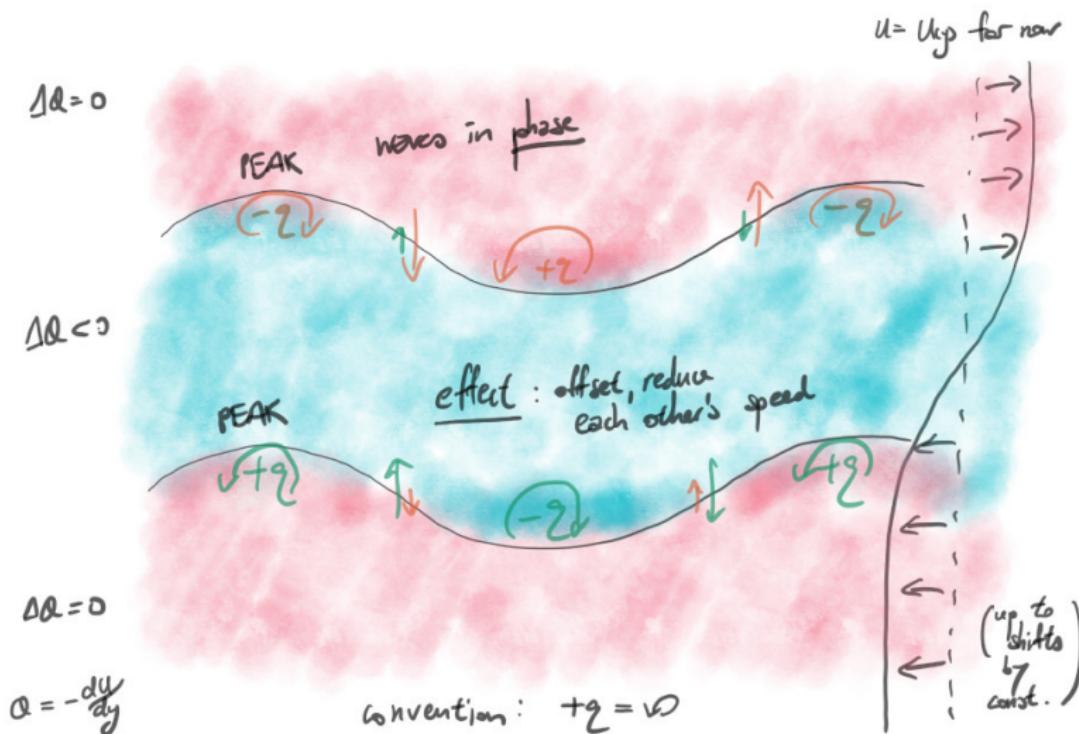
## Other cases (waves in phase)



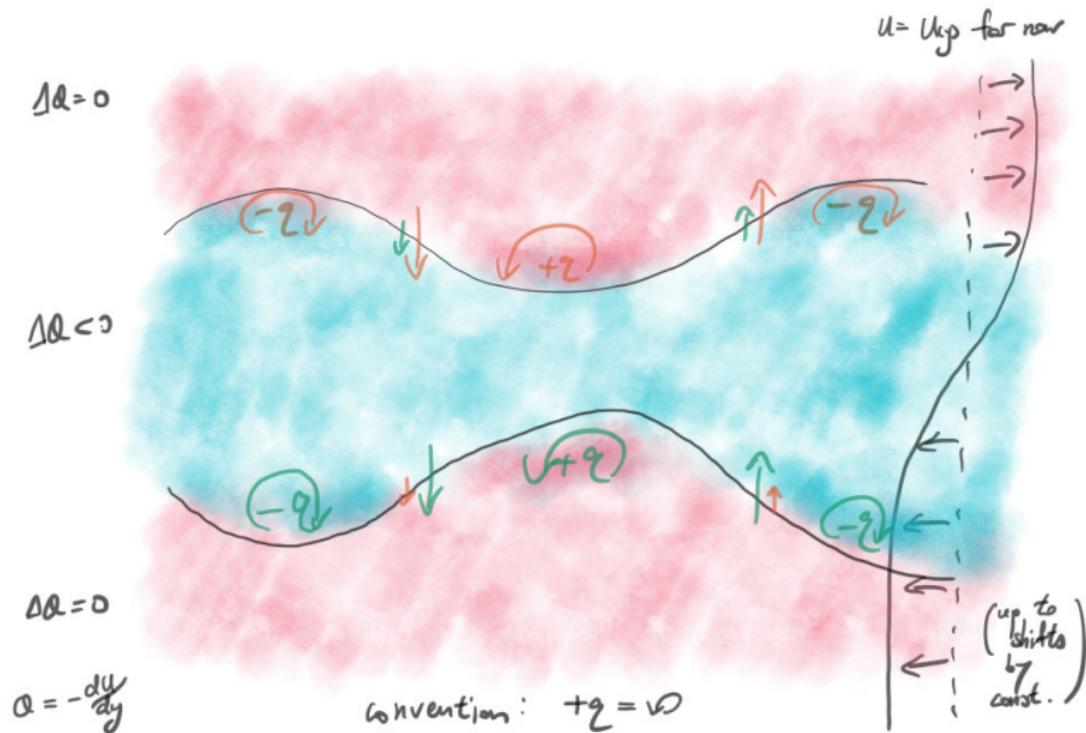
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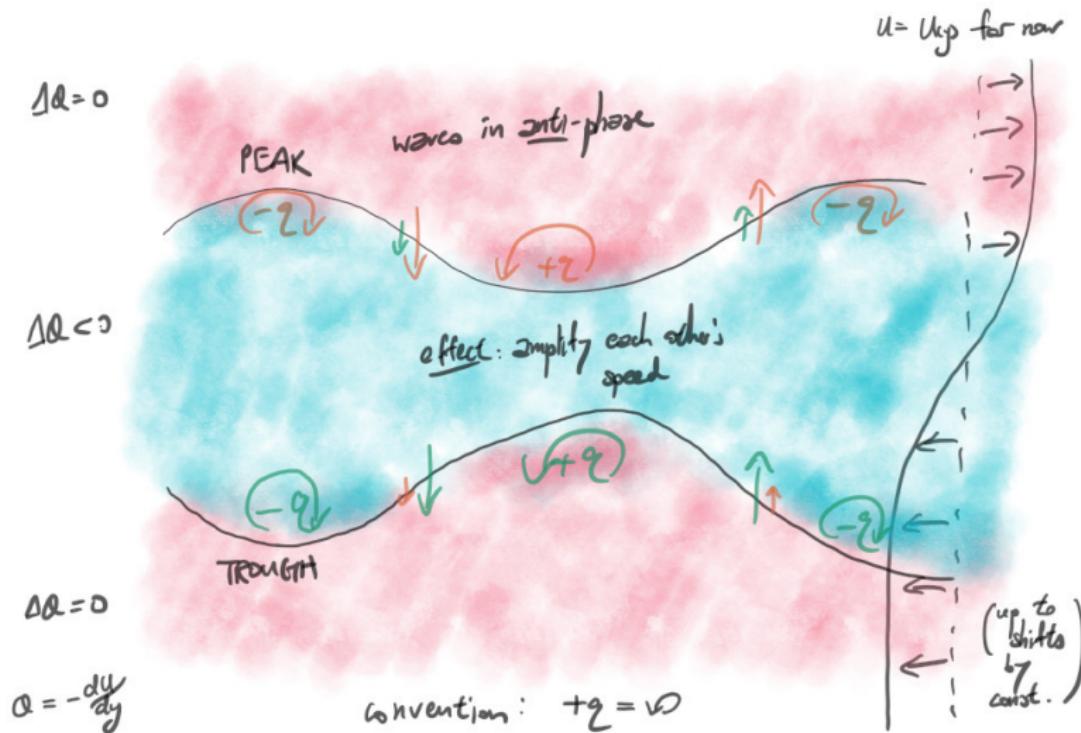
## Other cases (waves in phase)



## Other cases (waves in anti-phase)



## Other cases (waves in anti-phase)



# Summary

- ▶ concept of **stability** and **instability**
  - reservoir of “stuff” (e.g. free energy), can tap into it?
  - release of “stuff” leads to vigorous motion
- ▶ enhanced mixing (**isopycnal** and **diapycnal**)
  - important for MOC, energy/momentum cycles etc.
- ▶ some **kinematic** arguments for instability
  - parcel argument for static instability
  - constructive interference of waves for shear instability  
(generic?)
- ▶ all **instabilities** here generic, applies to all sorts of systems
  - ocean, planetary/stellar atmospheres/interiors, ...