OCES 2003 Assignment 1, Spring 2024

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Set on: Mon 19$^{\text{th}}$ Feb; due: Mon 26$^{\text{th}}$ Feb

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal places and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

- zero on the question concerned
- a fixed penalty starting from around 1/3 of the total marks
- zero for the whole assignment/midterm/final

The following counts as plagiarism (and is a non-exhaustive list):

- copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
  - use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
  - any more than around three usages in text is probably excessive
- copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
- changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. Recall that 1 Sverdrup is 1 million cubic meters per second, i.e. 1 Sv = 10^6 m^3 s^{-1}. The Antarctic Circumpolar Current has about 130 Sv of transport, while Western Boundary Currents have about 30 Sv of transport. The question asks for some minor calculations to put into perspective what "130 Sv" really means.

   (a) Suppose you have a bathroom tap that is flowing with a speed of 1.5 m s^{-1}, where the opening has a cross section of 1 cm^2 . Work out the tap's transport in units of Sv, giving your answer accurate to the 2 decimal places but in the form exponential form, i.e. $a.bc \times 10^d$. [1 mark]

   (b) The Pearl river flows pass the HK-Macau bridge. Suppose we take the length to about 30 km across, with an average depth of 10 m, and the river is flowing with an average speed of 5 miles hr^{-1} (reasonably fast flowing). Work out the transport in units of Sv, giving your answer accurate to 2 decimal places but in the form exponential form, i.e. $a.bc \times 10^d$. Use 1 mile = 1600 m. [1 mark]

   (c) Suppose the Mediterranean sea was completely empty (as it might have been once upon a time), leaving a volume of about 3.75×10^6 km^3 to be filled. The average flow rate into the Med sea by the surrounding rivers is about 0.01 Sv. Work out roughly how long it would take to fill up the Med sea, giving your answer to the nearest 10,000 years. [1 mark]

2. This question tests on understanding of derivatives. Suppose I have the map below showing height above ground $h(x, y)$. Lighter colours denoting higher ground, the lines are lines of constant height, and the green numbers are the labels for the contour lines. You can think of x as East-West and y as North-South, with $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ pointing East and North. We are looking for rough answers here that are valid on the coarse scale of this map.

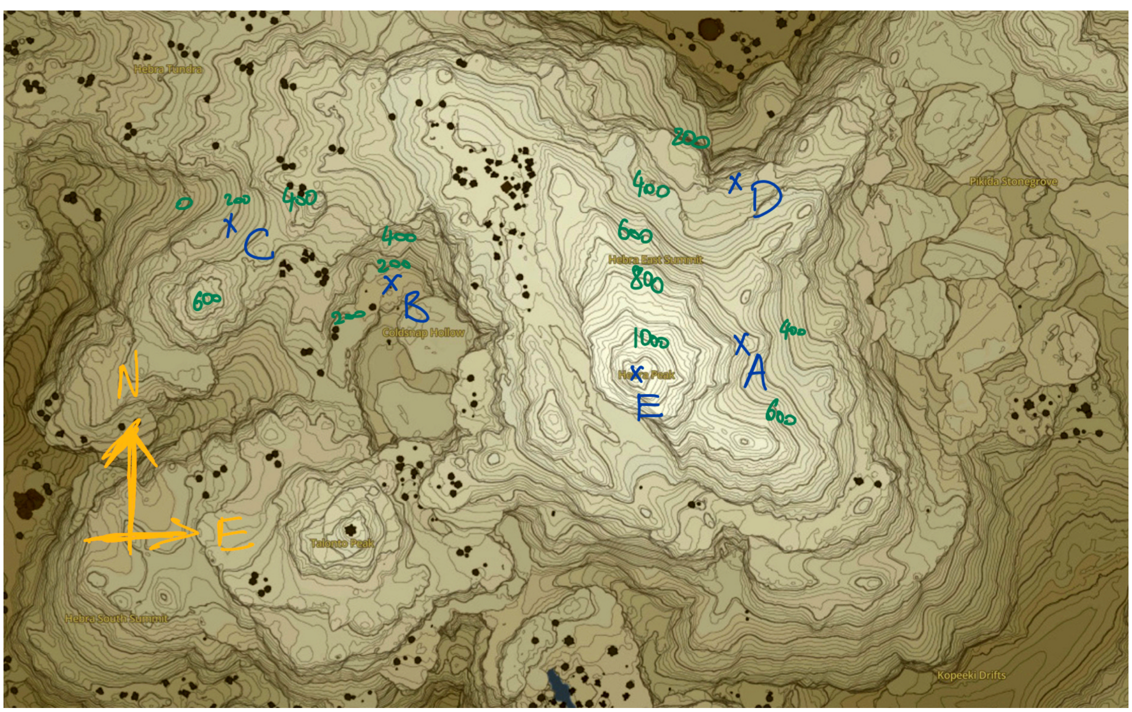

Figure 1: A map. Copyright with Nintendo.

(a) Suppose you are at point A, what is the sign of $\partial h/\partial x$?

(b) Suppose you are at point B, what is the sign of $\partial h/\partial y$?

(c) Suppose you are at point C, roughly which direction is such that $|\nabla h| = 0$?

(d) Suppose you are at point D, which direction is $|\nabla h|$ is *largest*?

(e) Suppose you are at point E, (i.e. on the top of a hill), which direction do you go so that both components of ∇h are strictly positive?

Given your answers in some combinations of Positive, Negative, Zero, East, South, West, North, No direction, and/or All direction. (Participation points for those who can point out where the map comes from.)

[5 marks]

3. Draw and/or state a 2d flow $\boldsymbol{u}$ where $\nabla \cdot \boldsymbol{u} = 0$ and $\nabla \times \boldsymbol{u} = 0$ everywhere. Justify your answer by a parcel argument like in the lectures, by directly computing or otherwise. [2 marks]

4. Arctic sea ice coverage is seen to be decreasing over time (e.g. link here).

   (a) If sea ice coverage is denoted $I(t)$ and t proceeds forwards in time, sketch out an expected graph for $I(t)$ as a function of t over seasonal cycles (to include two Northern Hemisphere summers and two winters). State the signs of $\partial I/\partial t$ between the peak summer and peak winter period, and between peak winter and peak summer period. Justify your answer briefly. [2 marks]

   (b) Above asks for $\partial I/\partial t$ over seasonal time-scales. Over yearly time-scales, if sea ice is decreasing, what is the sign of $\partial I/\partial t$? [1 mark]

   (c) Sea ice melting is not normally considered a significant factor to sea level changes, unlike *land* ice melting (e.g. the Greenland Ice Sheet). Give a one sentence answer for why that is the case. [1 mark]

   (d) Nevertheless, decreasing sea ice cover leads to other effects. What would happen to the salinity of the sea water when sea ice above it melts, how might that change the density, and would it make the sea water more or less *convectively stable* (cf. picture of the fluid parcel in Lecture 5)? [2 marks]

   (e) If sea ice melts, would the underlying water gain or lose heat? Explain your answer. [2 marks]

   (f) Search for the *ice-ocean governor* mechanism (e.g. on Google; might need to read a paper), and state how sea ice could lead to different changes in the *mechanical* forcing over the Arctic ocean, enabled by the absence of sea ice (from atmosphere-ocean interaction), and by the presence of sea ice (from ice-ocean interaction). Cite all sources you use, and you should not write more than half a page on this. [4 marks].

!? (No marks bonus question, but participation marks possible) For Q2*e*, what would signs of the second derivatives $\partial^2 h/\partial x^2$, $\partial^2 h/\partial x \partial y$ and $\partial^2 h/\partial y^2$ be? What if we are in a trough (in the valley) instead? What about on a *saddle point* (like a horse saddle; look this up if you can't visualise it)?

!? (No marks bonus question, but participation marks possible, and hint for Q3) Assuming you went for the "easy" option in Q3., can you come up with more complex 2d flow that satisfies the conditions in Q3? Are there general conditions for such flows? (These have some relevance to the classic theory of aeronautics, applicable to flow over airfoils for example, and are solutions of the *Cauchy–Riemann equations* in complex analysis for example.)