OCES 2003 Assignment 2, Spring 2024

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Model solutions and mark scheme

Problems

1. One would be that in-situ temperature might increase with depth, which would imply an unstable stratification. Another would be that in-situ density would imply there is a very shallow overturning circulation, since flow is mostly along isopycnals, but we know that is not the case.

   (1 mark for each stated contradiction, 1 mark for each associated explanation.)

2. (a) For the banded flow on Jupiter, $U = 100\ \mathrm{m\ s^{-1}}$, $L = 10000\ \mathrm{km} = 10^7\ \mathrm{m}$, and $2\Omega \approx 10^{-3}$ (10 hours for one Jovian day, a bit more than double that of Earth, rounded up), we have $\mathrm{Ro} = 10^2/(10^7 \times 10^{-3}) = 10^{-2}$, and geostrophic balance should hold (could take all the values a bit larger.)

   (b) For gyre flow away from the WBC, $U = 0.01\ \mathrm{m\ s^{-1}}$, $L = 1000\ \mathrm{km} = 10^6\ \mathrm{m}$, and $2\Omega \approx 10^{-4}$, so $\mathrm{Ro} = 10^{-2}/(10^6 \times 10^{-4}) = 10^{-4}$, and geostrophic balance should hold.

   (c) For circulating flow on Venus, $U = 100\ \mathrm{m\ s^{-1}}$, $L = 1000\ \mathrm{km} = 10^7\ \mathrm{m}$, but $2\Omega \approx 10^{-6}$ (rotation rate of 240 days or so, about 200 times slower), we have $\mathrm{Ro} = 10^2/(10^6 \times 10^{-6}) = 10^2$, so geostrophic balance should not hold.

   (d) For a dust devil on Earth, $U = 10\ \mathrm{m\ s^{-1}}$, $L = 10^2\ \mathrm{m}$ and $2\Omega \approx 10^{-4}$ for Earth, we have $\mathrm{Ro} = 10/(10^2 \times 10^{-4}) = 10^3$, so geostrophic balance should not hold.

   (e) For a big toilet, $U = 1\ \mathrm{m\ s^{-1}}$ and $L = 1\ \mathrm{m}$, $\mathrm{Ro} = 10^0/(10^0 \times 10^{-4}) = 10^4$, so rotational effects are negligible (and so the claim about how water flushes the other way in different hemispheres has very little dynamics basis; the geometry of the toilet bowl probably has a stronger influence).

   (1 mark for each sensible estimate, differences of about an order of magnitude is fine. Take 0.5 marks off if choices of scale are not sensible, take another 0.5 mark off if whether geostrophic balance holds is inconsistent with the given estimate of the Rossby number. Give marks if answers are in 10^a or 100 and similar.)

3. Assignment 2 of Spring 21/22, but re-adjusted.

   - negative pressure gradient should point *out* of the eddy (because $-1 > -2$, so the eddy is a high pressure).
   - Northern Hemisphere, so to the right of out would be *clockwise*. This would be *anti*-cyclonic, which has to be the case since we have a high pressure.
   - The wind curl at the South is clockwise so *cyclonic* in the Southern Hemisphere.
   - EQ should be *negative* with Ekman downwelling (convergence in the Ekman flow).
   - The coastal one should be *negative* Ekman convergence and Ekman downwelling, since we are in the Southern Hemisphere and the Ekman flow is to the left and into the coast.
   - The cyclonic wind curl should give an Ekman divergence, so *positive* and Ekman upwelling.

   (1 mark for each part, but only if the justification is correct.)

4. (a) A purely meridional velocity is pointing in the same direction as $\mathbf{\Omega}$, so $2\mathbf{\Omega} \times \mathbf{u}$ is zero by definition.

   (1 mark for the explanation. Accept mathematical demonstration if provided. Give some sympathy marks / partial credit if definition of cross product is mentioned.)

   (b) The vector that is both perpendicular to the zonal velocity and meridionally pointing $\mathbf{\Omega}$ is radial (in or out of the Earth), so has no horizontal component, so there is thus no horizontal Coriolis effect on a horizontal velocity at the equator. This would show up in the vertical momentum equation, but we would probably drop it relative to gravity/buoyancy in most cases.

   (1 mark for the up/down/radial, 1 mark for dropping it relative to buoyancy. Accept mathematical demonstration if provided. Give some sympathy marks / partial credit if definition of cross product is mentioned.)

   (c) The traditional approximation basically drops the terms of the Coriolis effect associated with the vertical (either vertical component, or related to vertical velocity). Mostly this is because the vertical things are argued to be small relative to the horizontal things. This might be dubious when the rotation is weak (e.g., near the equator), when buoyancy is weak (i.e., in weakly stratified cases), and might also be a bit dubious if we are not in the shallow fluid regime, as hydrostatic balance is then not a great approximation (formally the traditional approximation and the shallow approximation are separate, but they are invoked together for consistency reasons).

   These don't happen too often on Earth, but can occur for example when associated with convective events, or strong vertically propagating waves. Such cases are slightly less common in the ocean, but occur in atmosphere, and in astrophysical examples (e.g. some planets and stars are known to have deep convection zones, which is generally unstratified or only very weakly stratified).

   The paper by Gerkema *et al.* (2008) in Review of Geophysics is pretty comprehensive about the traditional approximation. More recent works include those of

   - Colin de Verdière (2012), Journal of Physical Oceanography
   - Tort *et al.* (2016), Journal of Fluid Mechanics
   - Yano (2017), Journal of Fluid Mechanics
   - Zeitlin (2018), Physics of Fluids.

   (1 mark for brief explanation of traditional approximation, 1 mark for weak or unstratified mediums, 1 mark for slowly rotating bodies, 1 mark for invoking a reference somewhere. 1 bonus mark may be given for qualitative answers using maths / dimensional analysis / scaling analysis. Give some sympathy marks if reasoning incomplete but citations used; the citation needs to support the reasoning.)