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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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- As said on the repository, I have tried to honestly use content that is self made, open source or explicitly open for fair use, and citations should be there. If however you are the copyright holder and you want the material taken down, please flag up the issue accordingly and I will happily try and swap out the relevant material.

OCES 2003 :
Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 6: Sea water properties (density)

Outline

- density $\rho = \rho(T, S)$

  $\rightarrow$ equation of state (EOS)
- pressure p

  $\rightarrow$ weight + hydrostatic balance (briefly, more in Lec. 7)

  $\rightarrow$ dynamical consequences
- in-situ vs. potential (vs. neutral) density

  $\rightarrow$ ρ vs. ρ_θ (vs. γ_n)

  $\rightarrow$ example and concept of work done

  $\rightarrow$ dynamical consequences

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Take home: it's **almost never** in-situ temp/dens we care about!

Key terms: EOS, hydrostatic balance, in-situ vs. potential/neutral density

Recap: parcel argument for buoyancy

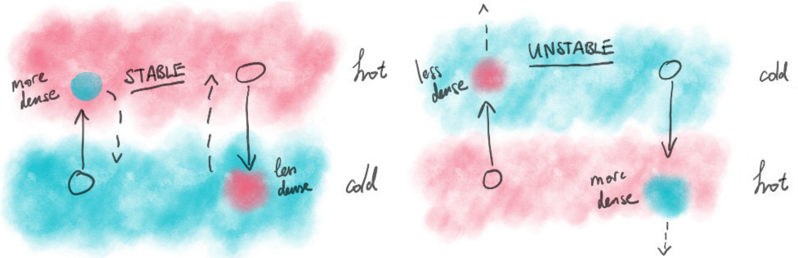


Figure: (Un)stable temperature configurations.

- density (!) ρ (units: kg m^{-3})
- buoyancy $b = -(\delta\rho/\rho_0)g$ (so units of...?)

  $\rightarrow$ how "floaty" something is (e.g. warm water, lighter density, **more** buoyant)

ultimately it's density/buoyancy we care about

Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F_u} + \boldsymbol{D_u} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Ocean density

- 10 m of seawater $\approx$ 1 atm (recall Lec. 1)
- over most of ocean, ρ varies from $\rho_0 = 1026$ kg m^{-3} by no more than **2%** (e.g. Gill, 1982)

  $\rightarrow$ small(!) but **CRUCIAL** variations! (see Lec. 7 + 8, 13 + 14)

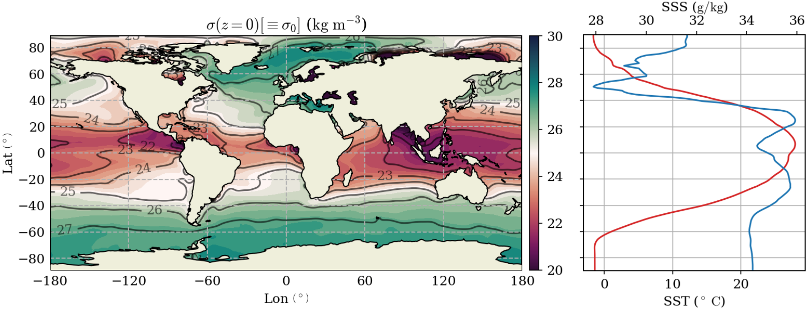
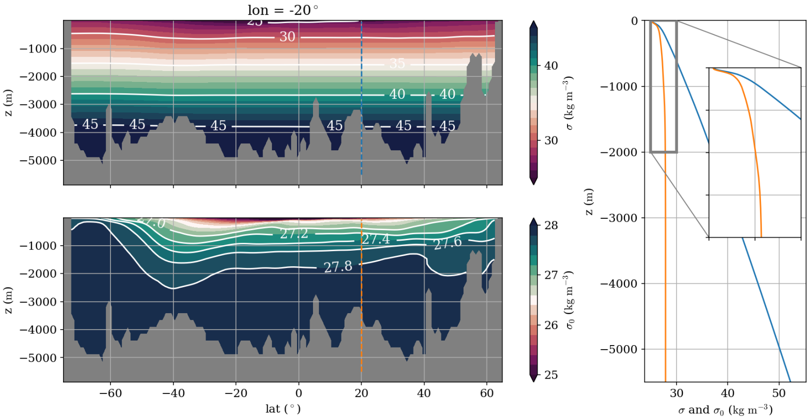


Figure: (left) Density at the surface (in-situ or referenced to sea surface) and (right) zonal averaged SST (red) and SSS (blue). Year-averaged data based on World Ocean Atlas 2013. See `plot_eos.ipynb`

Ocean density (more about this later...)

- isopycnal = lines/surfaces of constant density
- pycnocline = place/region below mixed layer where density gradient is largest (changes fastest)
  $\rightarrow$ cf. isotherm and thermocline (last Lec.)



Equation of State

T and S contribute to density, i.e. $\rho = \rho(T, S)$, and

$$\rho \nearrow \text{ as } T \searrow, \qquad \rho \nearrow \text{ as } S \nearrow$$

Equation of State (EOS) is the actual $\rho = \rho(T, S)$

- $\rho \nearrow$ as $S \nearrow$ so possible example is

$$\rho \sim \beta S, \qquad \beta \geq 0$$

- $\rho \nearrow$ as $T \searrow$ so could be

$$\rho \sim -\alpha T + \beta S, \qquad \alpha, \beta \geq 0$$

$\rightarrow$ can be negative as is...?

Equation of State

A linear EOS of seawater ($\alpha, \beta \geq 0$):

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

- expected behaviour with changing T and S
  $\rightarrow$ relative to reference T_0 and S_0

Equation of State

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LHS and RHS units need to agree!

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LHS and RHS units need to agree!

- α has units $^\circ$C^{-1} (or K^{-1}), thermal coefficient of expansion
- β has units g^{-1} kg, haline coefficient of contraction

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. by hand:

- $\rho_0 = 1$, $T_0 = S_0 = 0$, $\alpha = \beta = 1$,

  $$\rho = 1 - T + S$$

  so if $T = 10$, $S = 2$ then $\rho = \ldots$?

- "sensible" references are (partly taken from Rouquet et al. (2015), *J. Phys. Oceanogr.*)

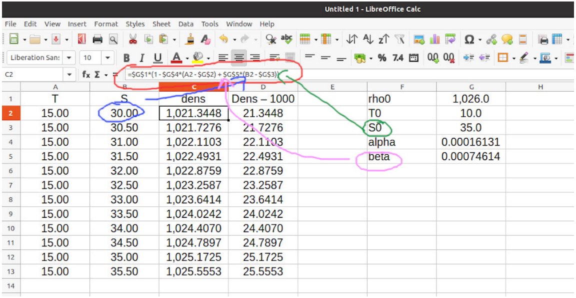
  $$T_0 = 10\,^\circ\mathrm{C}, \quad S_0 = 35\ \mathrm{g\ kg^{-1}}, \quad \rho_0 = 1026\ \mathrm{kg\ m^{-3}}$$

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

How do you use it?

e.g. in Excel



Untitled 1 - LibreOffice Calc

C2 =G1*(1 - G4*(A2 - G2) + G5*(B2 - G3))

	A	B	C	D	E	F	G	H
1	T	S	dens	Dens – 1000		rho0	1,026.0	
2	15.00	30.00	1,021.3448	21.3448		T0	10.0	
3	15.00	30.50	1,021.7276	21.7276		S0	35.0	
4	15.00	31.00	1,022.1103	22.1103		alpha	0.00016131	
5	15.00	31.50	1,022.4931	22.4931		beta	0.00074614	
6	15.00	32.00	1,022.8759	22.8759				
7	15.00	32.50	1,023.2587	23.2587				
8	15.00	33.00	1,023.6414	23.6414				
9	15.00	33.50	1,024.0242	24.0242				
10	15.00	34.00	1,024.4070	24.4070				
11	15.00	34.50	1,024.7897	24.7897				
12	15.00	35.00	1,025.1725	25.1725				
13	15.00	35.50	1,025.5553	25.5553				
14								

Figure: Sample calculations of density using linear EOS in Excel (highly recommend you don't use Excel, because syntax is a bit messy...)

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

What does it look like? (define $\sigma = \rho - 1000$ here)

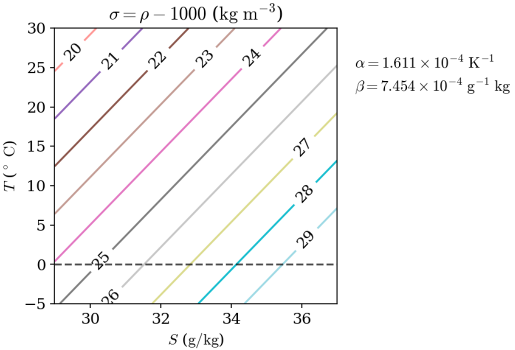
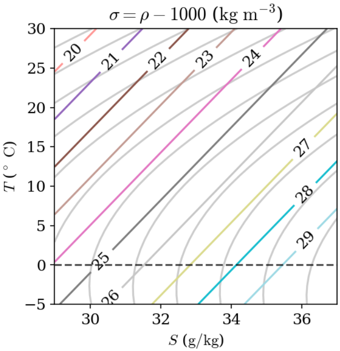


Figure: Linear EOS in *TS* space. See `plot_eos.ipynb`

Linear EOS

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)]$$

Compare with “real” EOS? (TEOS-10 here using Fabien’s 75-term formula)



$\alpha = 1.611 \times 10^{-4}$ K^{-1}
$\beta = 7.454 \times 10^{-4}$ g^{-1} kg

Figure: Linear EOS in *TS* space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

"Mildly" Nonlinear EOS see Geoff Vallis' (2006) book

$$\rho = \rho_0 \left[1 - \alpha \left(T_a + \frac{\lambda_1}{2} T_a^2 \right) + \beta \left(S_a - \frac{\lambda_2}{2} S_a^2 \right) - \nu T_a S_a \right]$$

- $T_a = T - T_0$ and $S_a = S - S_0$, the anomalies

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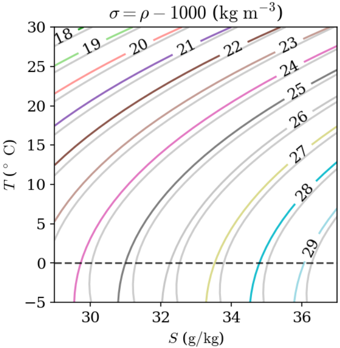


$$\begin{aligned} \alpha &= 1.611 \times 10^{-4}\ \mathrm{K}^{-1} \\ \beta &= 7.454 \times 10^{-4}\ \mathrm{g}^{-1}\ \mathrm{kg} \\ \lambda_1 &= 5.952 \times 10^{-2}\ \mathrm{K}^{-1} \\ \lambda_2 &= 5.491 \times 10^{-4}\ \mathrm{g}^{-1}\ \mathrm{kg} \\ \nu &= 2.370 \times 10^{-6}\ \mathrm{K}^{-1}\ \mathrm{g}^{-1}\ \mathrm{kg} \end{aligned}$$

Figure: Toy nonlinear EOS (no thermobaric effect) in *TS* space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

Nonlinear EOS

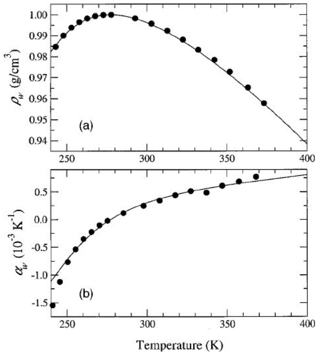


Figure: (top) $\rho = \rho(T)$ for pure water, (bot) $\alpha = \alpha(T)$. From Ashbaugh *et al.* (2002), *J. Chem. Phys.*.

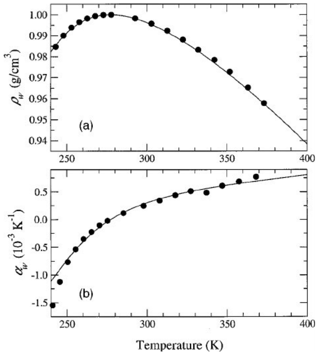
- "real" EOS should be nonlinear

  $\rightarrow$ e.g. water densest around 4° C (so ice floats), i.e. cannot be linear in T

  $\rightarrow$ "real" EOS ongoing research

  (e.g. TEOS-10, works by Trevor McDougall)

Nonlinear EOS



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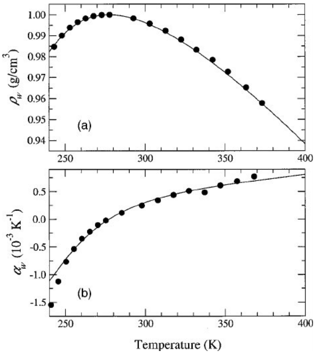
Nonlinear EOS



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- small discrepancies = can ignore?

  $\rightarrow$ **NO**! "small" difference really matter!

  $\rightarrow$ over most of ocean, ρ varies from $\rho_0 = 1026\text{kg m}^{-3}$ by no more than 2% (e.g. Gill, 1982)

"Full" Nonlinear EOS (e.g. Roquet et al. (2015), *J. Phys. Oceanogr.*)

- TEOS-10 standard, polynomial with 75(!) terms chosen(!) to fit with real data
- note the change of sign in gradient close to freezing point $\rightarrow$ remember ice is **less dense** than water normally!

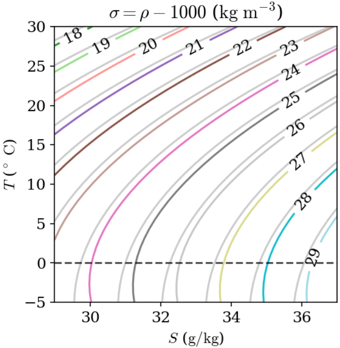

Figure: TEOS10 EOS (at surface) in *TS* space with toy nonlinear EOS as gray contours (same contour levels). See `plot_eos.ipynb`

Pressure + hydrostatic balance (briefly)

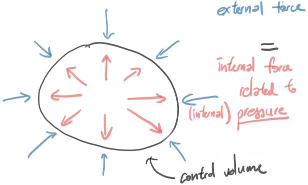

Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume $\Rightarrow$ steady
- steady $\Rightarrow$ in force balance, no net force

- pressure = force per area,

$$p = F/A, \qquad \text{units: N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^6 Pa (Pascals) (see e.g. Wikipedia for others)

$\rightarrow$ cf. millibars (mbar) in atmosphere

$\rightarrow$ lines of constant pressure = isobar

Pressure + hydrostatic balance (briefly)

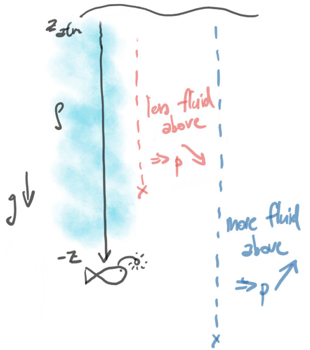

Figure: Schematic of hydrostatic pressure

- hydrostatic approximation: pressure **equal** to weight above when static

  $\rightarrow$ weight is $F = mg$ so for force balance,

$$F = mg = \boxed{g \int_{-z}^{z_{\mathrm{atm}}} \rho \,\mathrm{d}z = p},$$

  with $g \approx 9.81$ m s^{-2}

  $\rightarrow$ if $\rho = \text{const}$ then $p = \rho g z + p_{\mathrm{atm}}$

Concept of work done

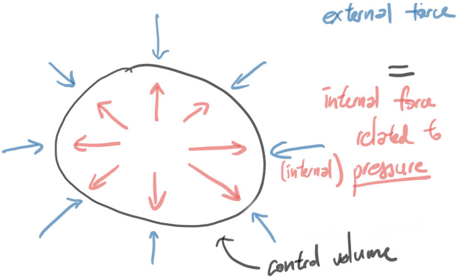

Figure: Working to compress a volume.

Concept of work done

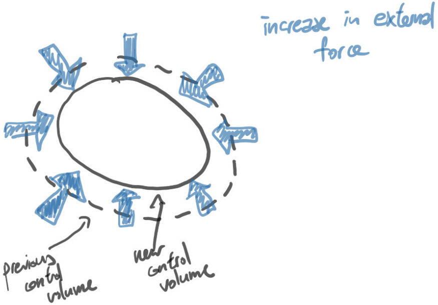

Figure: Working to compress a volume.

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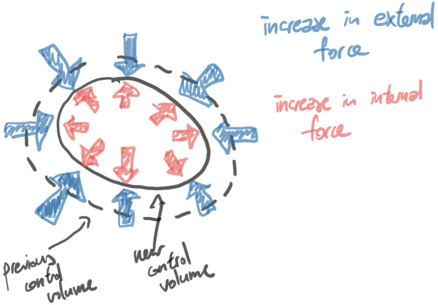

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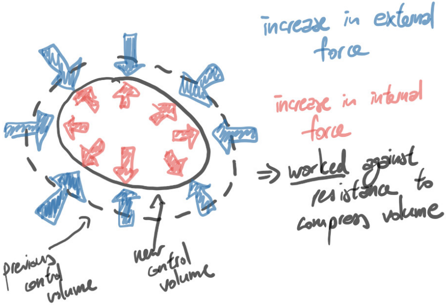


Figure: Working to compress a volume.

- to compress, need to work against something (in this case internal pressure)
- energy has to be put in (cf. $T \nearrow$ in fluid volume)

Work done and potential/neutral densities

Remember this graph from a few slides ago?

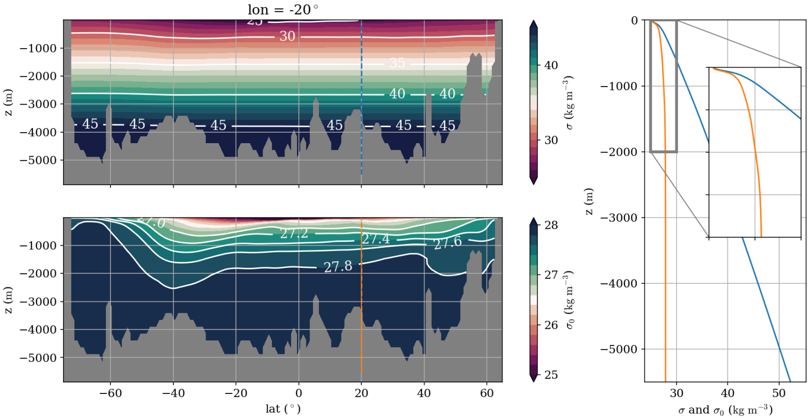


Figure: Meridional section in the Atlantic of (top left) in-situ density and (bot left) potential density referenced to sea level, with the corresponding vertical profiles plotted (right). See `plot_eos.ipynb`

Work done and potential/neutral densities

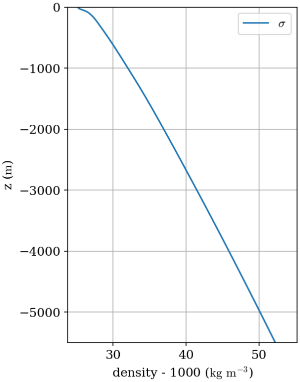


Figure: Vertical profile of in-situ density at the same location as in the previous graph. See `plot_eos.ipynb`

In-situ density $\rho(T, S, p)$

- increases with depth because more weight above (compression), but not necessarily dynamical!

  $\rightarrow$ abyssal increase of ρ mostly from p, but not necessarily from having to do work
- in-situ density profile is telling us there shouldn't be up/down motion, but we know we do have it!

Want something that adjusts for some/all of pressure contribution to density

Potential temperature and density

Potential temperature θ

- the temperature it will have if you take some water and move it to some reference pressure p_0 (usually p_{atm}) without exchange of heat and salt

  $\rightarrow$ without exchange of heat + mass = adiabatic

  $\rightarrow$ reference needed to set base line and **is a choice**

  $\rightarrow$ account for pressure contributions **relative to reference level**

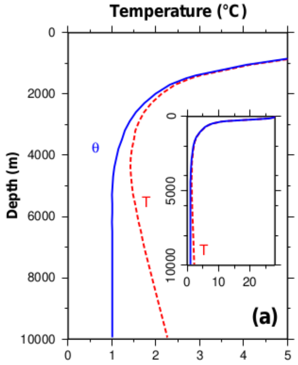


Figure: Vertical profile of in-situ (red) and potential temperature (blue) in the Mariana Trench to highlight the differences. From Talley *et al.* (2011) Fig 4.10(*a*).

According to in-situ temperature, bottom water should be <u>more</u> buoyant (i.e. <u>unstable</u> + overturn)!

Concept of work done

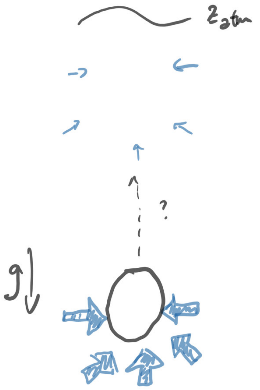
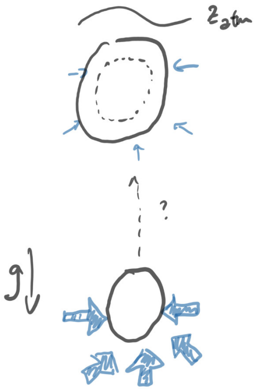


Figure: Volume change from pressure, but work done reversible if adiabatic.

Concept of work done



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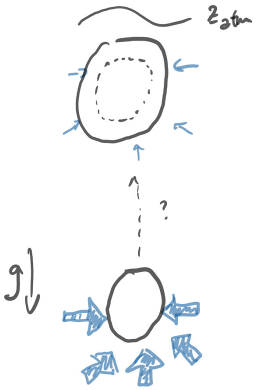


Figure: Volume change from pressure, but work done reversible if adiabatic.

- work is done by the system against the surroundings

Concept of work done

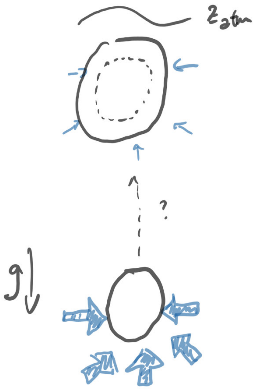


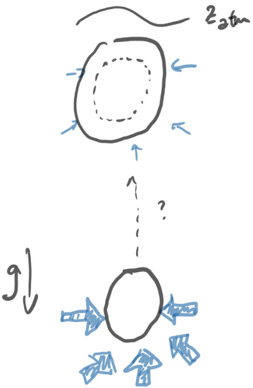
Figure: Volume change from pressure, but work done reversible if adiabatic.

- work is done by the system against the surroundings
- energy lost from system with expansion, change in

  $\rightarrow$ density (from volume since $\rho = m/V$)

  $\rightarrow$ temperature (less energy in system, cooling)

Concept of work done



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- energy lost from system with expansion, change in

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but if *adiabatic*, then this process is **reversible** (no change in **entropy**), but we normally care about **irreversible** processes!

Potential temperature and density

Potential density ρ_θ

- the density calculated using θ in EOS

  $\rightarrow$ reference dependent (from θ)

  $\rightarrow$ shallow region focus? choose sea level (0m) or 1000m

  $\rightarrow$ deeper region? choose 2/3/4000m

  $\rightarrow$ used to identify water masses

- crucial bit is that $\sigma_{0,1,2,3,4}$ has very **small gradients** (i.e. essentially **flat**) in the deep!

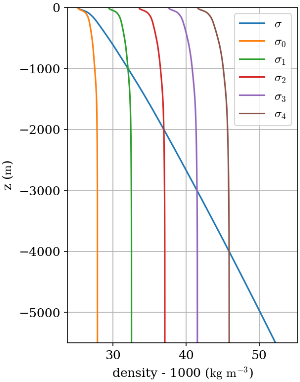

Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Potential temperature and density

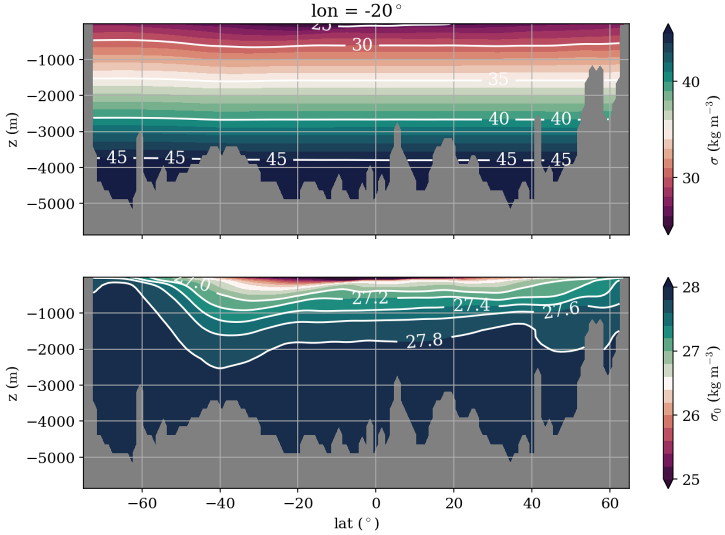

Figure: Meridional section in the Atlantic of (top) in-situ density and (bot) σ_0. See `plot_eos.ipynb`

Potential temperature and density

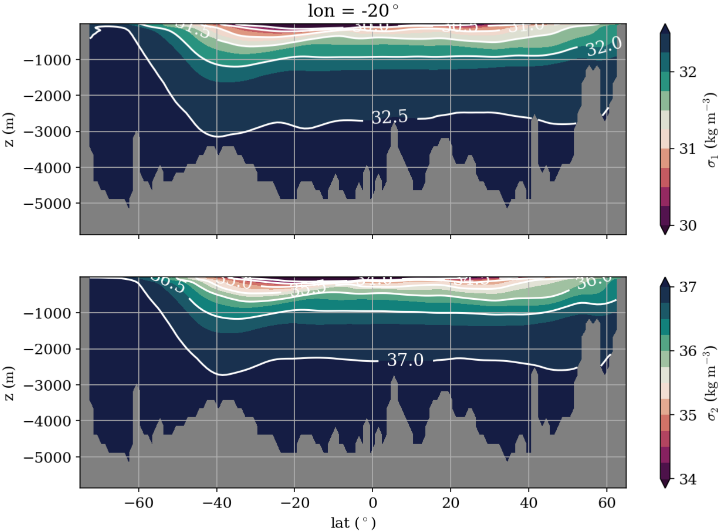

Figure: Meridional section in the Atlantic of (top) σ_1 and (bot) σ_2. See `plot_eos.ipynb`

Potential temperature and density

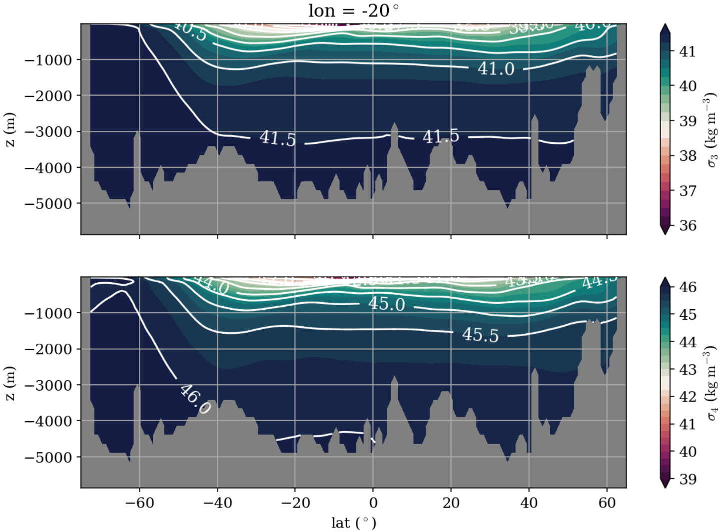

Figure: Meridional section in the Atlantic of (top) σ_3 and (bot) σ_4. See `plot_eos.ipynb`

Neutral density

Potential density needs a reference

- neutral density γ_n in principle does not need a reference

  $\rightarrow$ complications with computing and/or existence of neutral surfaces (e.g. Jackett & McDougall, 1997, *J. Phys. Oceangr.*)

- can be used to identify water masses but restricted to present day ocean (cf. Lec. 5, see lec. 13 + 14)

  $\rightarrow$ e.g. not necessarily suitable for paleoclimate + very long-term climate change applications

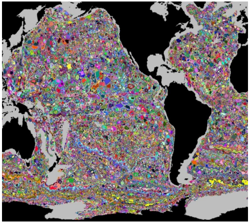

Figure: Topobaric surfaces, an almost neutral surface. From Stanley (2019), *Ocean. Modell.*, Figure 4.

Neutral density sample: Atlantic

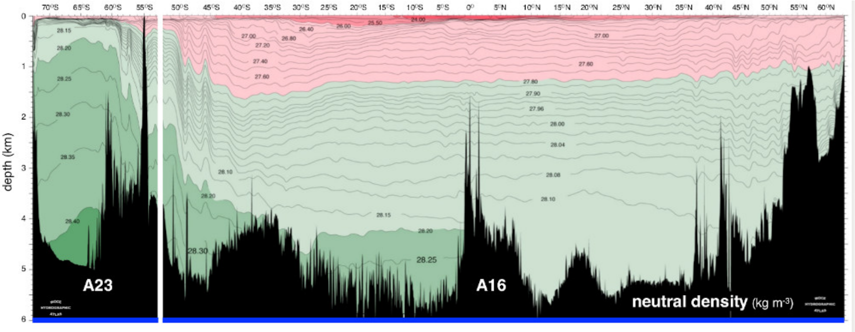

Figure: Neutral density (see later) meridional section in the Atlantic. From Koltermann *et al.* (2011), vol 3 of Hydrographic Atlas of the World Ocean Circulation Experiment (WOCE).

- flow largely along isopycnals (path of less resistance, not needing to do as much work) (revisit in Lec. 13, 14, 17)

  $\rightarrow$ consequence for global meridional overturning circulation (MOC)

Summary

- EOS to relate temperature and salinity to density
- hydrostatic approximation (more next Lec.)

  → weight to balance fluid above (related to density)
- in-situ vs. potential (vs. neutral) density

  → dynamics cares about work done

  → in-situ density σ has substantial pressure contributions

  → potential density σ_θ removes some pressure contribution (reference dependent)

  → neutral density γ_n removes more/all pressure contributions in principle

Take home: it's **almost never** in-situ temp/dens we care about!