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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 17: Dyanmics 3 (instabilities and mechanisms)

Thur 15th Apr

Outlook of the next few lectures

Dynamics important, next two lectures on

- waves (this Lec. + 16, 18) and instabilities (Lec. 17)

  $\rightarrow$ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- how to describe waves (Lec. 15)
- types of waves (Lec. 16)

  $\rightarrow$ consequence + leading to instabilities
- **instabilities** (Lec. 17)

  $\rightarrow$ parcel-type (mechanistic) arguments for instability
- tides (particularly as internal gravity waves) (Lec. 18)

Outline

- static/convective instabilities
  - → largely buoyancy related
  - → Rayleigh-Taylor and double diffusive instabilities
  - → parcel argument revisited (recall Lec. 5)
- shear instabilities
  - → largely flow shear (thus vorticity gradient) related
  - → baroclinic instabilities (MOC, Lec. 13)
  - → Kelvin–Helmholtz (diapynal mixing, Lec. 14)
  - → a unifying picture (?): constructive interference of two vorticity waves

Key terms: instabilities, static, shear, wave interference, action-at-a-distance

Recap: role of waves and instabilities

Role also of baroclinic instability (Lec. 13, see also Lec. 17), important for

- vertical momentum transfer by interfacial form stress
- scale transfer of energy
  → mesoscale eddies, conduit between large-scales and submesoscales
- along-isopycnal mixing and also MOC

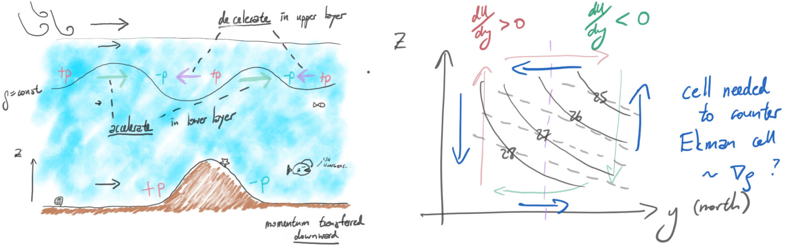


Figure: Schematic of form stress and eddy induced overturning cell in Southern Ocean (see Lec. 14)

Recap: role of waves and instabilities

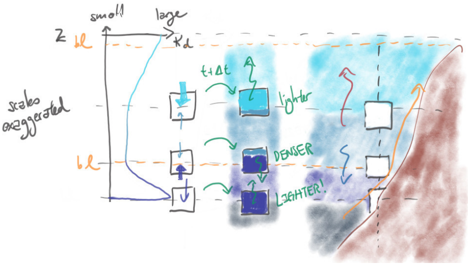


Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

  → ~~broad diffusive~~ boundary intensified upwelling

what causes the bounary intensification of κ_d? dynamics!

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) → internal gravity waves (Lec. 16) → shear instabilities (Lec. 17)

Concept of stability

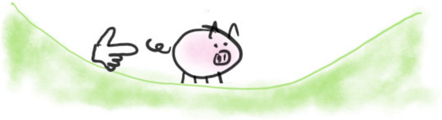


Figure: Pig being prodded (probably don't try this in real life).

- imagine a (spherical) pig is in a valley and you prod it a bit...

Concept of stability

Figure: Pig being prodded (probably don't try this in real life).

- the pig wants to stay "close" (not defined rigourously but can be) to its initial position, stable state (relative to position anyway, you might want to run)

Concept of stability

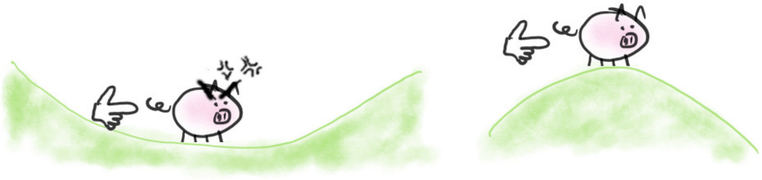


Figure: Pig being prodded (probably don't try this in real life).

- imagine a (point mass spherical?) pig now on a hill and you prod it a bit...

Concept of stability

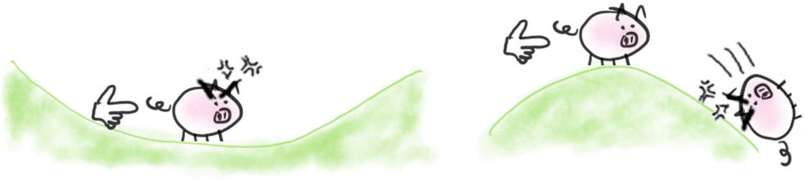
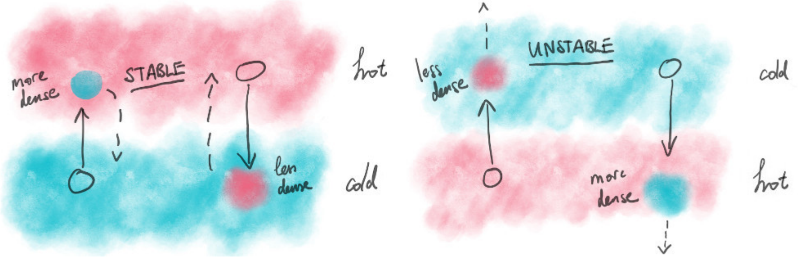


Figure: Pig being prodded (probably don't try this in real life).

- the (spherical) pig rolls down the hill, i.e. **not** staying close to its original position, unstable state

Static instabilities: parcel argument (recall Lec 5)



Essence of parcel argument:

- take a parcel and assume, if moved, conserves (carries) water properties with it

  $\rightarrow$ assume fast time-scales ($\tau_{\text{diff}} \ll \tau_{\text{dyn}}$ i.e. weak mixing)
- consider forces and motion arising from anomalies

  $\rightarrow$ here temperature (or density) anomalies, leading to buoyant forces

Static instabilities: $N^2 > 0$

- recall Brunt–Väisälä or buoyancy frequency

$$N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$$

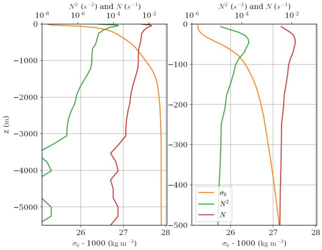
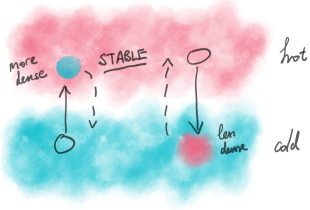


Figure: σ_0 (see Lec. 6) and the associated N^2 and N. See `plot_eos.ipynb`.

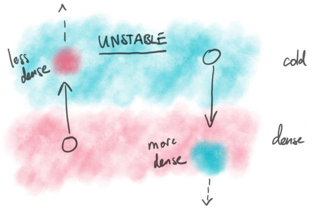
- $N^2 > 0$, stratification is **stable**



- if no damping, particle overshoots and oscillates, i.e. **wave**-like motion

Q. what is the **frequency** of motion?

Static instabilities: $N^2 < 0$



- if $N^2 < 0$ then stratification is **unstable**

  $\rightarrow$ Rayleigh–Taylor instability (or just **static instability**)

  $\rightarrow$ ample videos/pictures online[1]

- instability is **very quick** in the ocean! ($<$ months)

  $\rightarrow$ upper bound **growth rate** by $|N^2|$ (e.g. Haine & Marshall, 1998, *J. Phys. Oceanogr.*)

  $\rightarrow$ overturns to re-establish stable stratification

  $\rightarrow$ large transport (i.e. $\kappa_d \gg 1$ if you want to think of it that way)

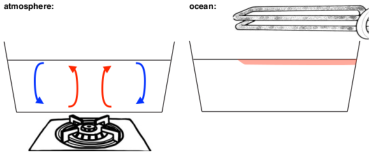
[1] see e.g. Megan Davies Wykes's website `http://www2.eng.cam.ac.uk/~msd38/gallery.html`

Static instabilities: $N^2 < 0$

- deep convection not that easy to get in ocean (see Lec. 14)

  → intense cooling

  → some as part of overflows



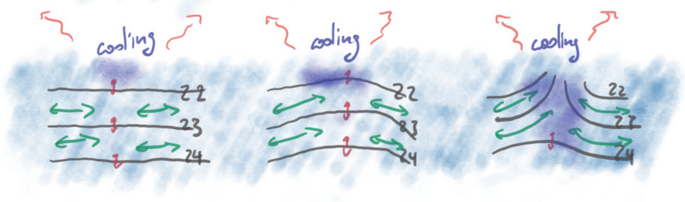


Figure: Watermass transformation by decreasing temperature.

Double diffusive instability

Recall that $\rho = \rho(T, S, p)$ and

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T\right) = \kappa_T \nabla^2 T,$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S\right) = \kappa_S \nabla^2 S$$

- generally $\kappa_T \neq \kappa_S$ (hence double diffusion)

  $\rightarrow$ ocean has $\kappa_T \gg \kappa_S$

  $\rightarrow$ sometimes known as semi-convection

  (astrophysics, e.g. helium and heat in the Sun)
- can lead to non-negligible transport

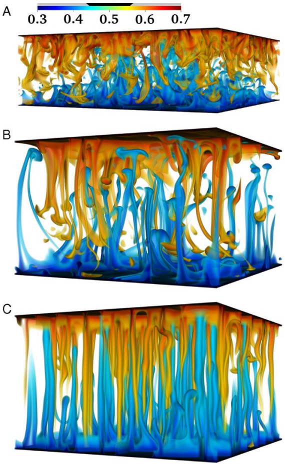


Figure: Simulation in in temperature stable but salt unstable regimes. From Yang, Verzicco & Lohse (2016), *Proc. Nat. Acad. USA*, modified from their Fig. 1.

Double diffusive instability: salt fingering

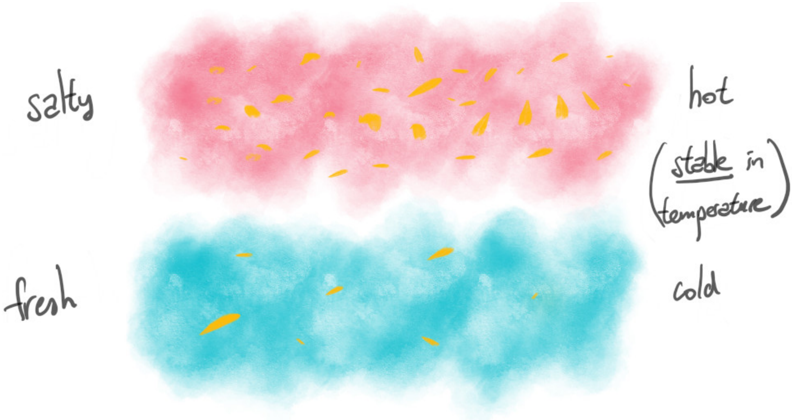
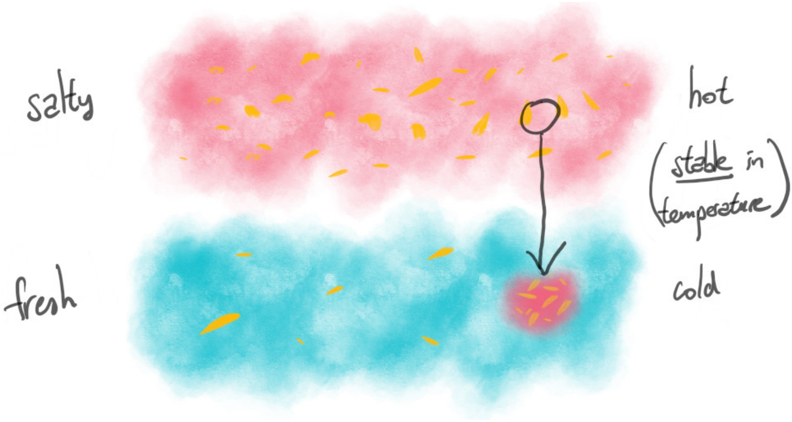


Figure: Salt fingering schematic: stable in temperature gradient but <u>un</u>stable in salinity gradient.

Double diffusive instability: salt fingering



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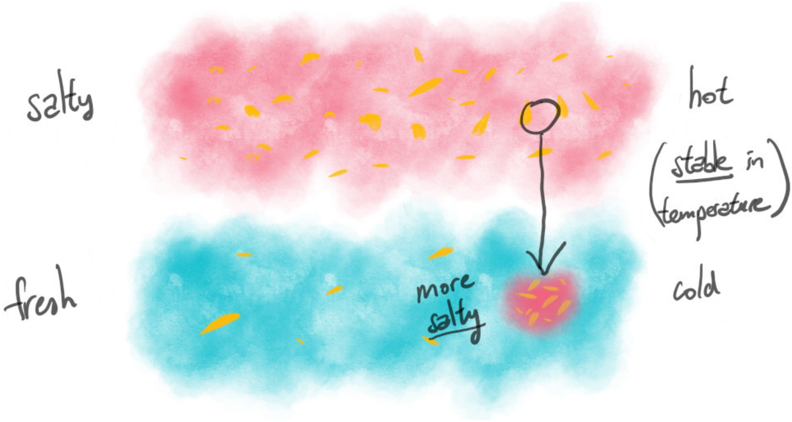

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Double diffusive instability: salt fingering

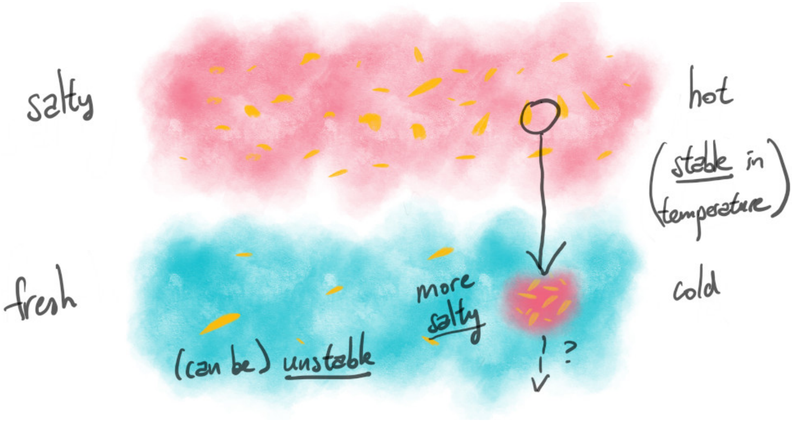


Figure: Salt fingering schematic: stable in temperature gradient but unstable in salinity gradient.

Shear instabilities

Figure: Schematic of mixing by (irreversible) wave breaking leading to e.g. diapycnal mixing.

- growing disturbance, instability
  $\rightarrow$ convective and/or shear (see Lec. 17)
  $\rightarrow$ mixing of material **across** isopycnals after reconnection, leading to diapycnal mixing
- feedback onto MOC (see Lec. 14)

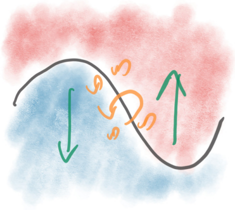


Figure: Velocity shear from waves can lead to mixing.

Shear instabilities

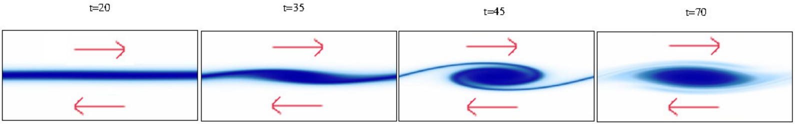


Figure: Roll-up of flow arising from a shear instability (horizontal, no stratification here). Adapted from Mak *et al.* (2017), *Phys. Rev. Fluids*, Fig.1.

- Kelvin–Helmholtz instability

  $\rightarrow$ roll up into billows, stratification usually involved
- Holmboe instability

  $\rightarrow$ in estuaries? (cf. Lec 21 + 22)

Shear instabilities

Baroclinic instability (cf. Lec 13)

- sloping isopycnal surfaces $\Leftrightarrow$ vertical shear flow (from thermal wind)

  $\rightarrow$ wants to flatten isopycnals, important for MOC (cf. Lec 13 + 14)
- mixing by **isopycnal** rather than dipycnal
- Eady + Charney paradigm (see more in Vallis, 2006)

Shear instabilities as vorticity wave resonance

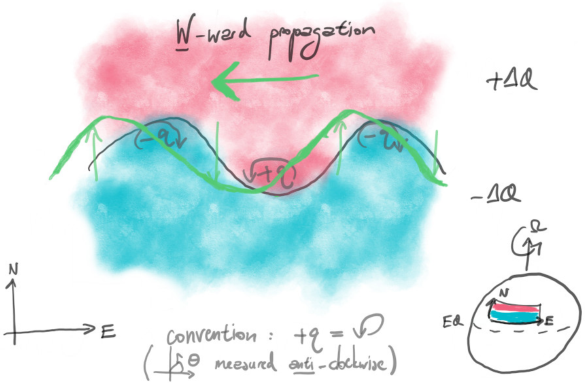


Figure: Parcel argument for Rossby wave propagation. (1) Assumes conservation of vorticity by wave. (2) Vorticity anomalies induces flow. (3) Flow leads to self-advection.

How does / can this transfer to shear flows?

Vorticity wave propagation in background flow

Vorticity wave propagation in background flow

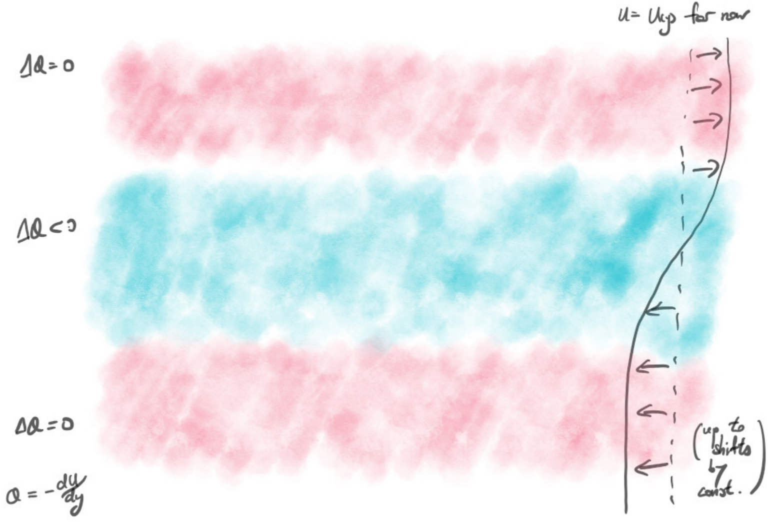
$\Delta Q = 0$

$\Delta Q < 0$

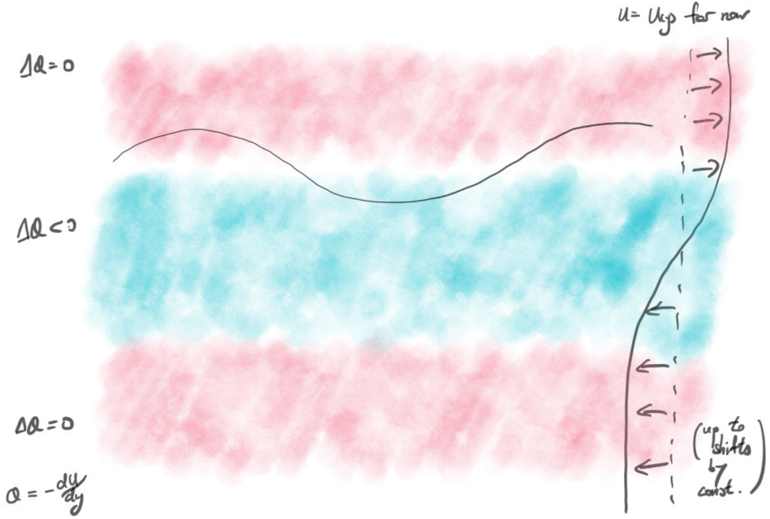
$\Delta Q = 0$

$Q = -\frac{dU}{dy}$

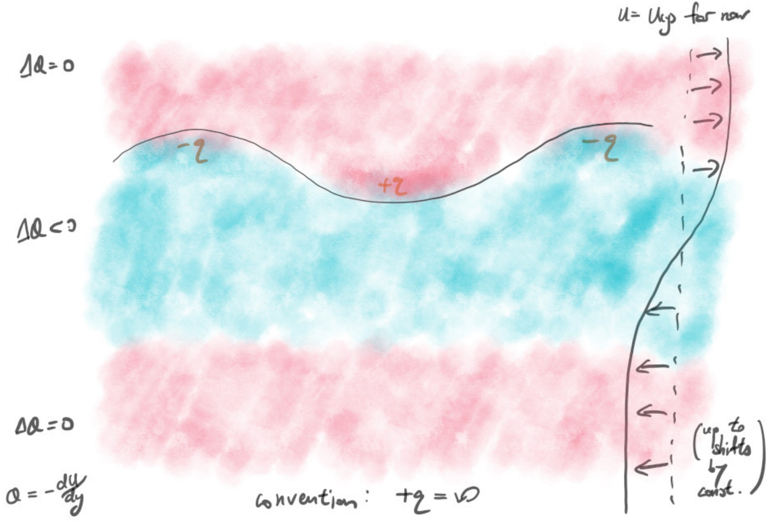
Vorticity wave propagation in background flow



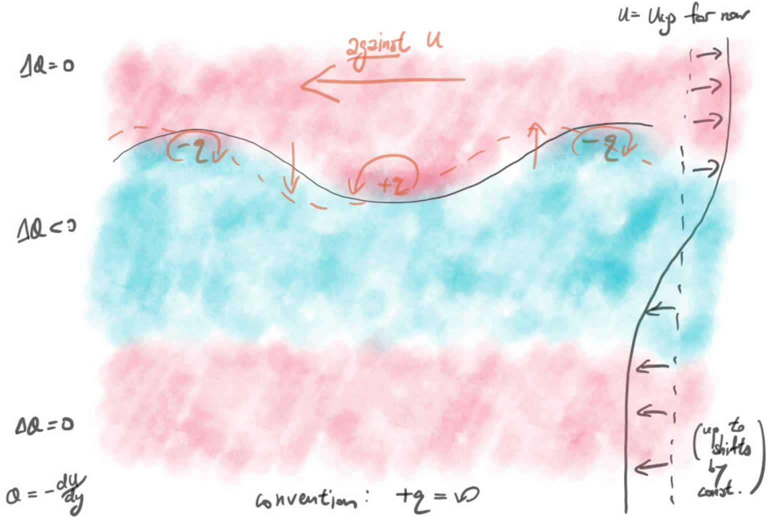
Vorticity wave propagation in background flow



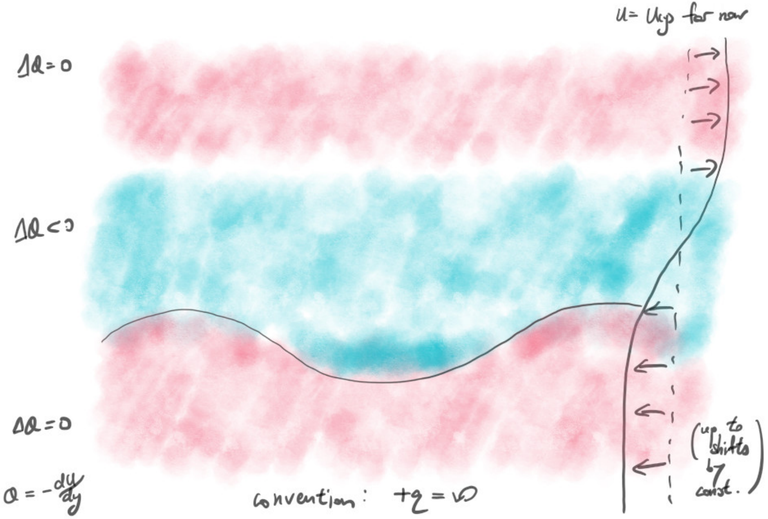
Vorticity wave propagation in background flow



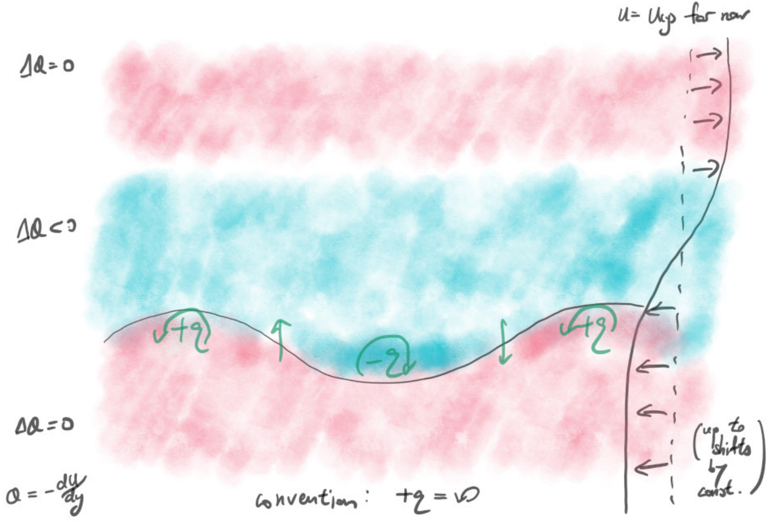
Vorticity wave propagation in background flow



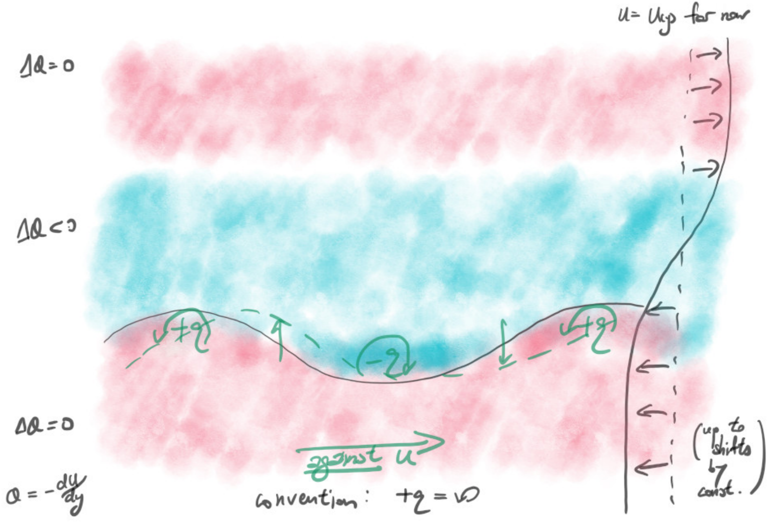
Vorticity wave propagation in background flow



Vorticity wave propagation in background flow



Vorticity wave propagation in background flow



Vorticity wave propagation in background flow

- fundamentally same mechanism

  $\rightarrow$ waves conserve vorticity

  $\rightarrow$ resulting anomalies induce flow

  $\rightarrow$ flow induces propagation
- can think of them as **generalised Rossby waves**

  $\rightarrow$ normal Rossby waves: background vorticity gradient from planetary rotation

  $\rightarrow$ here: background vorticity gradient from **background shear flow**

Vorticity wave propagation in background flow

- fundamentally same mechanism

  → waves conserve vorticity

  → resulting anomalies induce flow

  → flow induces propagation
- can think of them as **generalised Rossby waves**

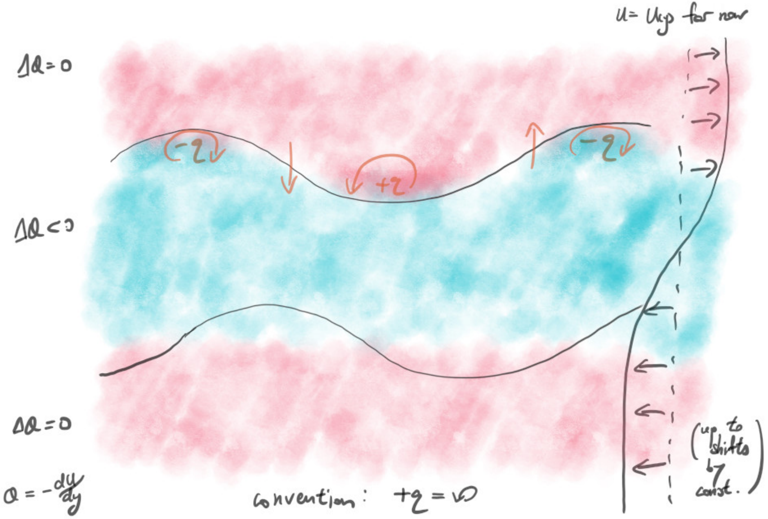
  → normal Rossby waves: background vorticity gradient from planetary rotation

  → here: background vorticity gradient from **background shear flow**

How do these waves play with each other then?

- depends on the phase shift

Stable case (top wave **lags** bottom by $\pi/2$)



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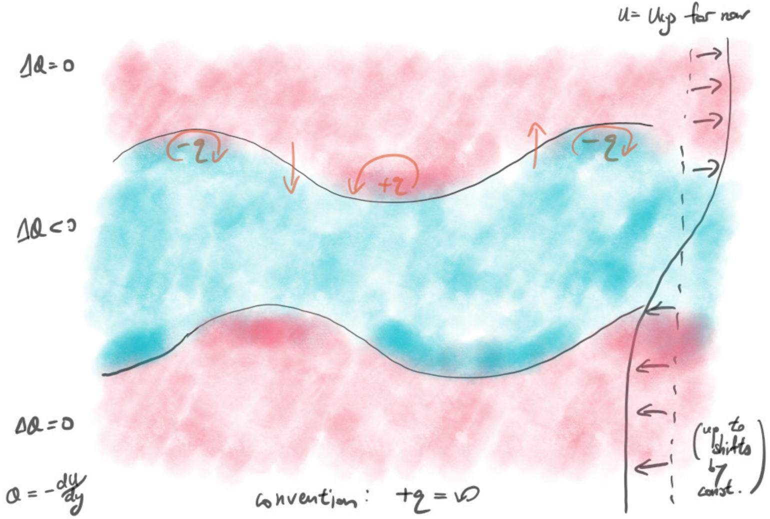

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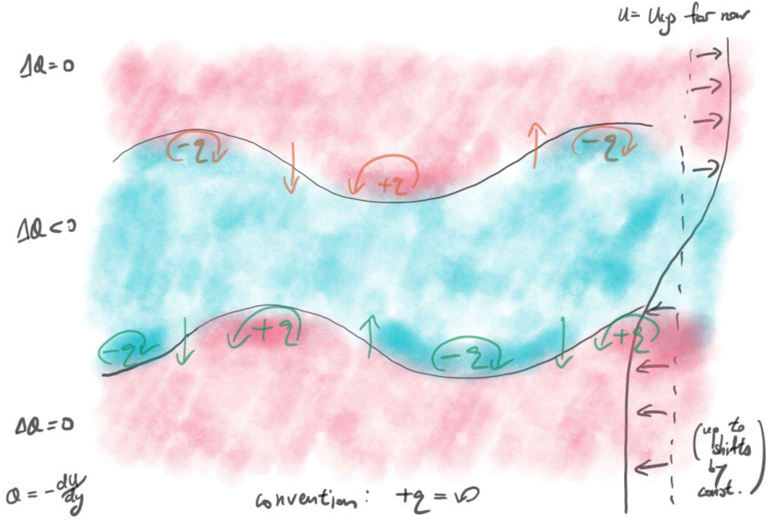

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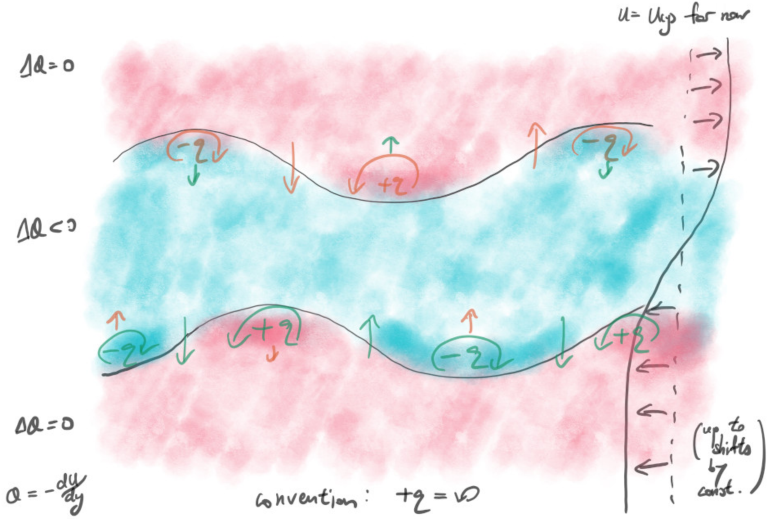

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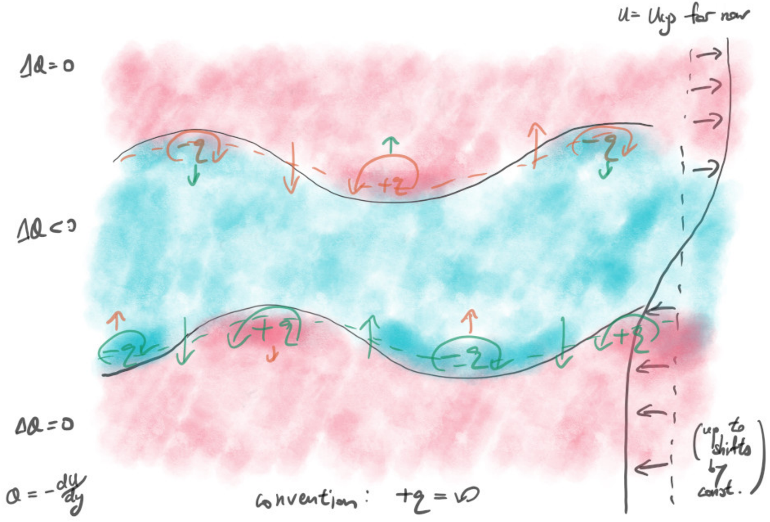

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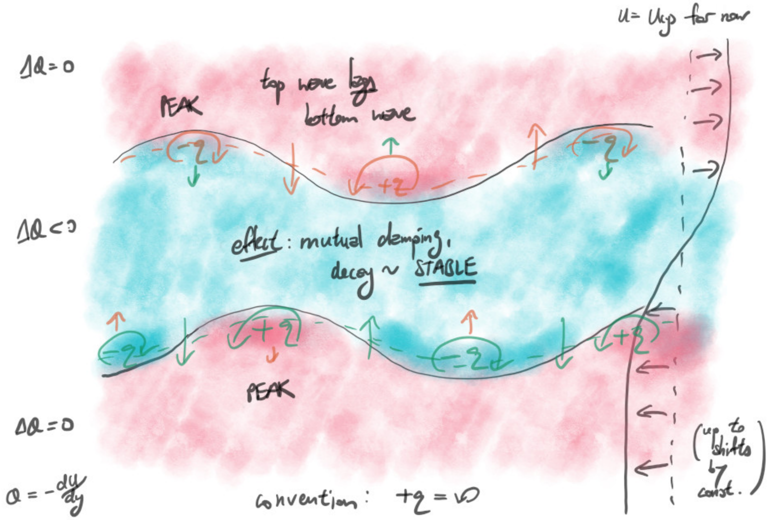
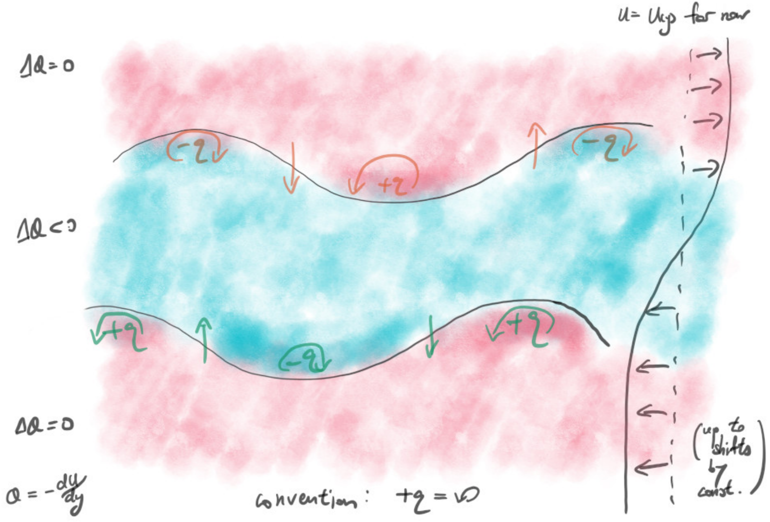
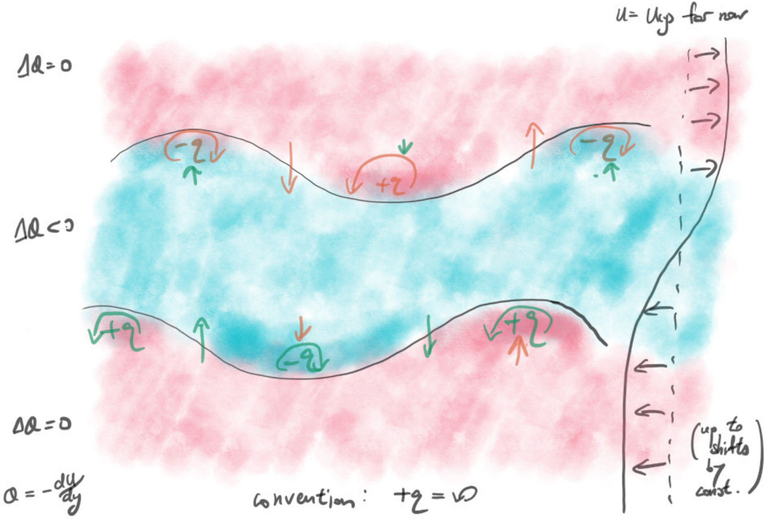


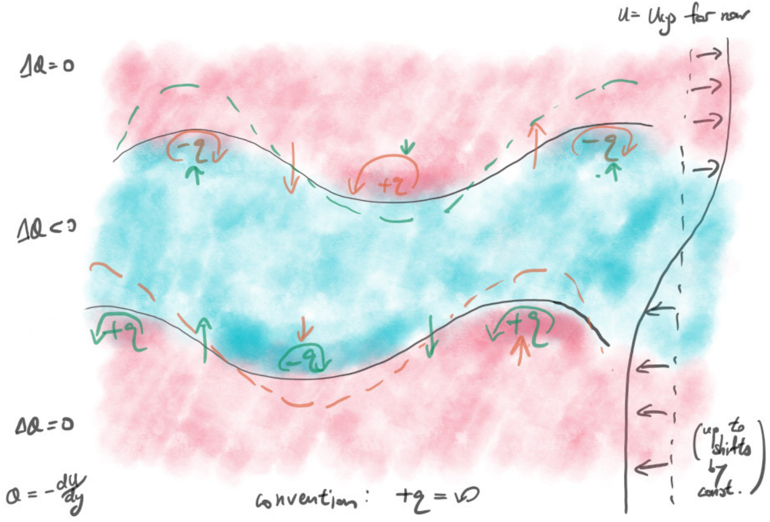
Unstable case (top wave **leads** bottom by $\pi/2$)



Unstable case (top wave **leads** bottom by $\pi/2$)



Unstable case (top wave **leads** bottom by $\pi/2$)



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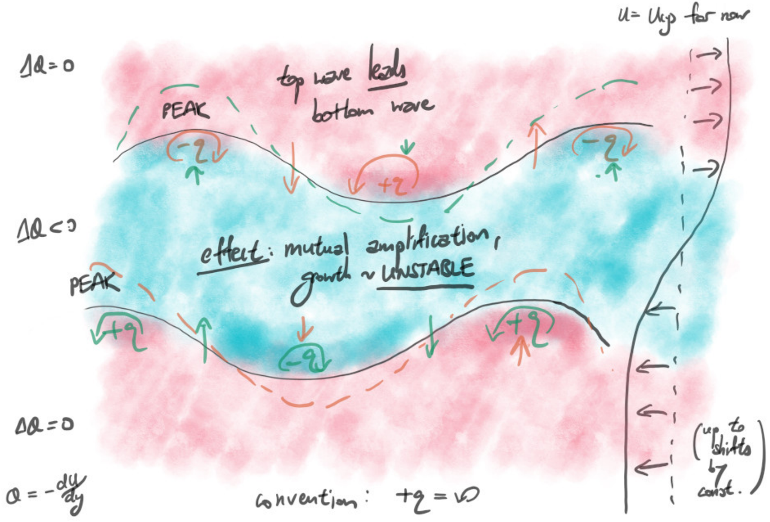
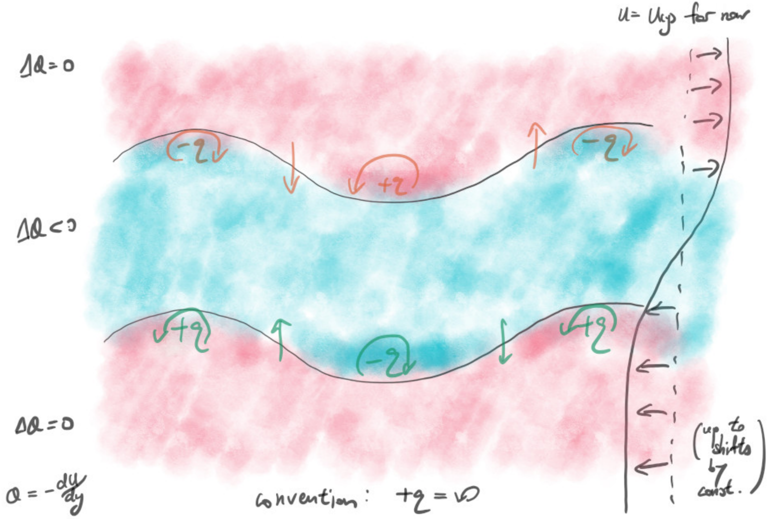


Other cases (waves in phase)



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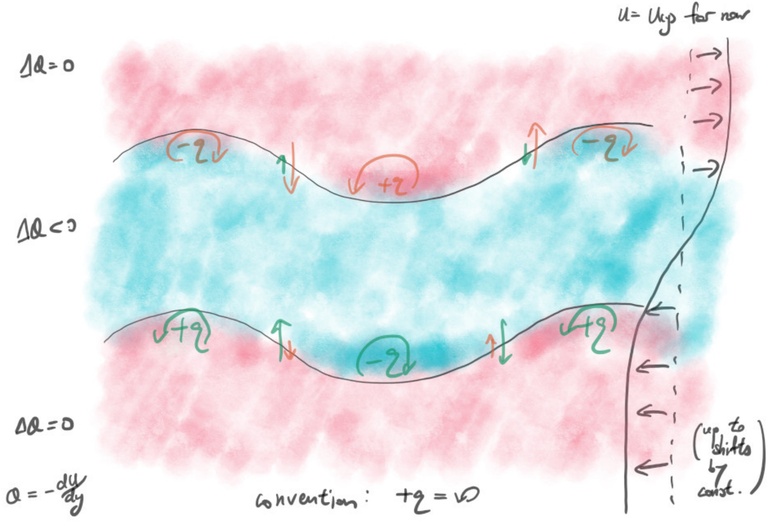

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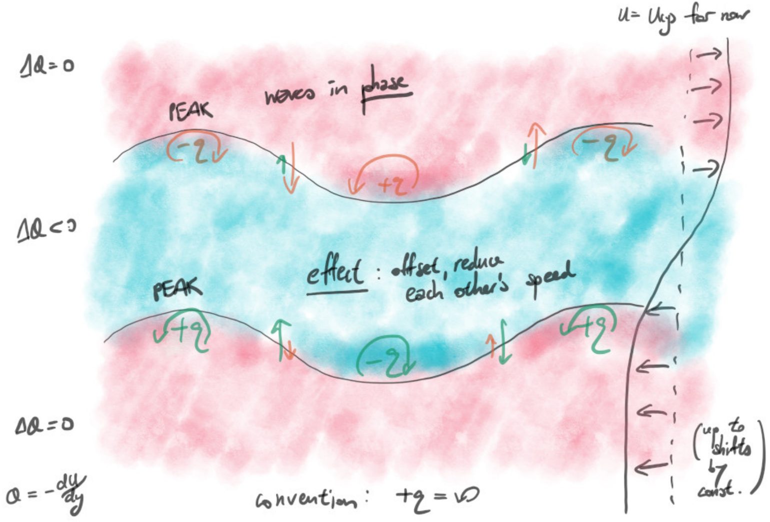
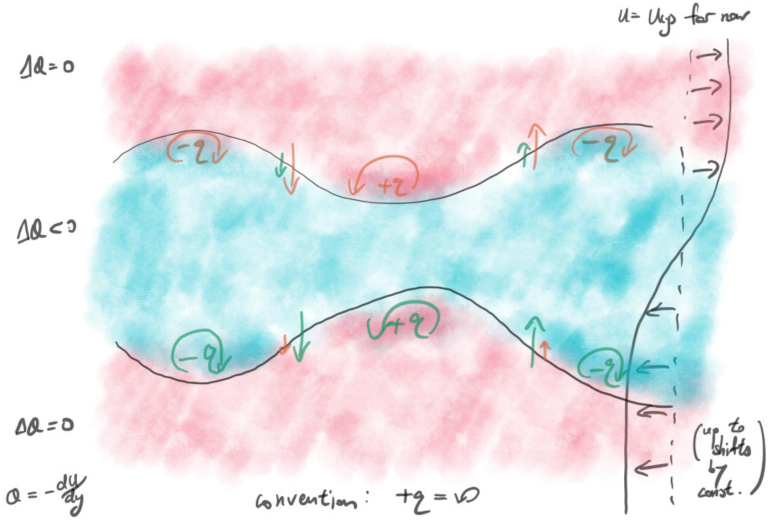
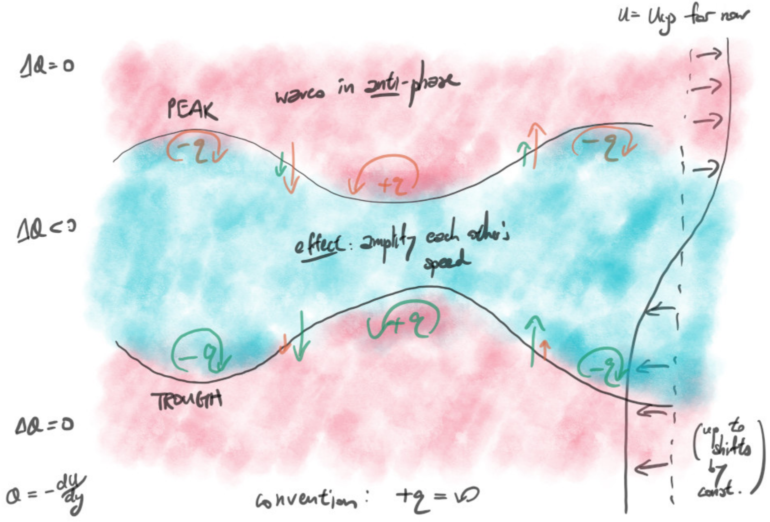


Other cases (waves in **anti**-phase)



Other cases (waves in **anti**-phase)



Summary

- concept of stability and instability

  $\rightarrow$ reservoir of "stuff" (e.g. free energy), can tap into it?

  $\rightarrow$ release of "stuff" leads to vigourous motion
- enhanced mixing (**isopycnal** and **diapycnal**)

  $\rightarrow$ important for MOC, energy/momentum cycles etc.
- some kinematic arguments for instability

  $\rightarrow$ parcel argument for static instability

  $\rightarrow$ constructive interference of waves for shear instability

  (generic?)

- all instabilities here generic, applies to all sorts of systems

  $\rightarrow$ ocean, planetary/stellar atmospheres/interiors, ...