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PHYSICAL OCEANOGRAPHY BY DRAWING PICTURES

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Foreward

When I was first approached to deliver a first undergraduate course on physical oceanography, but that I should refrain from using maths and calculus because of reasons, my first (somewhat knee jerk) reaction was to laugh in the senior professor's face and said "*no, tell the students to suck it up, you can't learn physics things without maths*". The senior, being a more professional and less confrontational type than myself, asked me to sit on it a bit and have a try. Long story short the result is this set of notes, which is to complement a course on descriptive physical oceanography with a focus on the *dynamical processes* that lead to some of the phenomena we observe in the ocean. The bias is on things I can explain by drawing pictures and particularly on *dynamics* (because that's the thing that interests me, and I make no apologies for this focus). Some of my own arguments for and against this kind of descriptive course are as follows:

✓ yes, you actually can learn a lot (about oceanography and the physical arguments) just by drawing pictures!

× no, drawing pictures is only qualitative and gives you very little "teeth" if you are wanting to e.g. make predictions or quantitative statements

- e.g. pictorial/descriptive arguments can tell you why you might have Western intensification, but to get the *Stommel boundary layer* dependence you need some maths (although the pictures might suggest what should be involved)

✓ it does help develop intuition (speaking as a maths person by training, I learnt a lot by writing this course)

? it highlights why to take physical oceanography further you really do need the maths, because intuition can sometimes be misleading (the devil <u>really</u> is in the details)

The selling points (or deficiencies depending on your point of view) of the content here are:

- ocean phenomena motivated, but united by *dynamics*[1]
- skimpy on facts and things to memorise, focus more on *logical deductions* (partly because I am very lazy and hate remembering things)
- broad but relatively little *depth*, though enough concepts to start on the other books in the literature to take it further maybe

[1] Personal gripe: Prior editions of *Descriptive Physical Oceanography* (Pickard & Emery, 5th edn. and before) do not do this, and in my opinion it makes the topics more complicated/disconnected than it needs to be.

This set of notes presents a pictorial/geometric way of looking at ocean dynamics (or, more generally, aspects of *geophysical fluid dynamics*) that might be beneficial for those wanting some exposure to concepts in physical oceanography without necessarily specialising in it (e.g. marine biologists, ocean engineers, ecologists etc.), and those who are already familiar with the maths wanting to see things in another way (e.g. symbol worshippers like myself). The notes are relatively self-contained and don't really assume any background really; relevant concepts in physics and maths will be recalled here but in a very skimpy manner, and only on the concepts that are directly used. Calculus is employed here but the focus is going to be on their *geometric* meaning. Symbol manipulations and calculations are almost non-existent, limited to order of magnitude estimates and working out some signs (e.g. negative times negative is positive); see the exercises for the more involved calculations. The notes here are not meant to be cover everything in physical oceanography, and there are many other better resources out there with a more comprehensive ocean focus (e.g. Talley et al. [2011], Williams and Follows [2011]) and/or with more focus on dynamics (e.g. Vallis [2006]).

As you may have also gathered, the writing style here is deliberately conversational/informal/unprofessional, and the drawings are cartoon-ish/coarse. The former is by choice, the latter is because (*a*) I am artistically challenged, (*b*) I want to highlight that these drawings are things you can (and should) do yourselves to convince yourselves of the logic and arguments behind them, and (*c*) I am lazy (this will be a recurring theme...) Some pictures and animations I did make using Python (via Jupyter notebooks), and the codes will be available on a GitHub repository (again, no promises on clean or Pythonic code, again laziness). Do what you like with the material, just keep it open source and non-commercial. If you find any errors, have suggestions or even want to contribute (!), feel free to open an issue or pull request on the GitHub repository. The document is prepared using the the Tufte-LaTeXclass[2], modified from the files[3] of Jody Klymak (University of Victoria), with additional editing based on the excellent Finite Element Methods course notes[4] of Patrick Farrell (University of Oxford).

[2] `https://github.com/Tufte-LaTeX/tufte-latex/`

[3] `https://github.com/jklymak/Eos314Text`

[4] `https://pefarrell.org/teaching/`

Contents

1 "Big picture"

This section goes through some facts and observations about the ocean, partly to argue why you might care about the ocean (if you need convincing), and partly to highlight the phenomena we will try to explain in the subsequent sections using pictorial arguments. Since the course attached to this set of notes was designed somewhat for people with minimal physics/maths background, there are two subsections at the en of this chapter that will highlight the bare minimum of the concepts that are needed to link the symbols used with the pictures drawn. The convention of symbols and a cheat sheet of sorts is given at the end of this chapter. Taking artistic license for setting the scene for telling a story, and attempting to strike a balance between an attempt to whet one's appetite but also (attempt as best as possible) remain somewhat technically accurate, concepts and terminology will be used here in this chapter, but not elaborated on in detail until later chapters.

Of course as the reader it is absolutely your prerogative to use/skip/reject the propaganda presented here as you see fit. Don't just assume everything is right either! I will have invariably made tpyos, or worst, *thinko*'s (I first saw the term in Vallis [2006]), so you should convince yourself the ideas and arguments actually have some value. If you can spare the time please report any tpyos, thinkos or suggested improvements on the GitHub repository where you got this document as an issue or, if you like, as a pull request.

1.1 Oceanography, and physical oceanography

The two general questions regarding the ocean to me are:

- *what* does the ocean look like?
- *why* does it look like the way it does?

The former encompasses *observations* (Ch. 7), not just for the physical quantities such as temperature, salinity and ocean currents, but also for biogeochemistry or other quantities such as phytoplankton concentration, oxygen content, plastic distribution, ecosystem behaviours, and so forth. The latter involves understanding the processes underlying the physical, but also chemical, biological, ecological processes, to name but a few. The two are not mutually exclusive of each other: that *what* requires the *why* for explanation, the *why* sometimes tells you *what* to expect, but the *what* acts as a constraint on the explanations of the *why*. Oceanographers tend to have a particular focus, on theory, modelling, observations, physics, biology etc., but, fundamentally, most things are intrinsically linked one way or another, and the differing approaches compliment each other.

Physical oceanography focuses on the physics aspect, and in a nutshell (to me anyway) is the study of *how water in the ocean moves around*, i.e. *dynamics*. Making this "simplification" in a sense, we are in a slightly privileged position to know the governing equations for the phenomena (e.g. Ch. 1.4.3), which itself is a modification of Newton's equations in classical mechanics, from which we can in principle derive everything else. Of course we generally can't that in the general case, so we employ a variety of tools, such as approximations, numerical methods, and observation techniques, to help us in this venture.

The general approach here is that we will first highlight the features observed in the ocean to set the scene, but mostly talk about *dynamics*[1], with a focus on *intuition* by going through *pictorial* arguments, to convince you why the arguments might be true (again, the devil is really in the details, but we will not touch on those too much here). Fundamentally, physical oceanography is an *interdisciplinary* science at its core, benefiting from multiple approaches in order to chip away at the overall problem, such as that depicted in Fig. 1.1.

[1] The study of rotating stratified fluid dynamics is generally grouped under *Geophysical Fluid Dynamics* (GFD), which is applicable to studies of planetary atmospheres and/or oceans. It also has extensions to stellar atmospheres and interiors with suitable extensions (e.g. include magnetic effects).

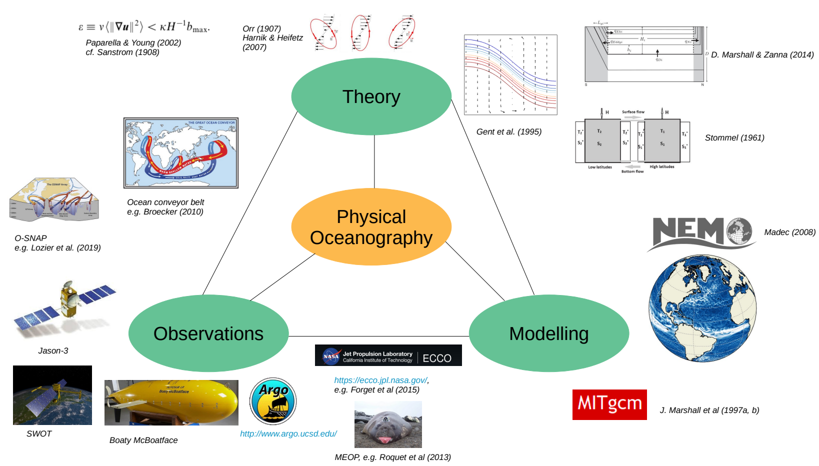

Figure 1.1: A schematic of how I see physical oceanography, like three sides of the triforce (if you know that reference), and all three parts are needed to make a whole (not going to comment which one represents wisdom, courage and power...) There are more developing branches coming out that doesn't quite fit neatly (e.g. data oceanography, though that could be somewhere between observation and modelling), and in reality most people are somewhere in between.

1.1.1 *Motivation: climate*

Here I give my own spin[2] as to why I think the marine environment and particularly why the physics is important. The study of the Earth's *climate* from a holistic point of view requires an understanding of the several "spheres", split for example as the *lithosphere* (solid Earth related things), *biosphere* (living things), *cryosphere* (ice), *atmosphere* (air), *hydrosphere* (the water stuff), and *anthroposphere*

[2] This is just one spin of why you might care about marine environments, and it's not the only one. A useful exercise would be to create your own depending on your interests, e.g. if you want to focus on stable isotopes.

(human things). These are not isolated subsystems of course: there is life in the ocean, ocean interacts with the atmosphere and ice and vice-versa, land boundaries and movements constraints and drives ocean dynamics, and so on. We are going to focus on the hydrosphere.

Fig. 1.2 shows a map of the globe but with an ocean focus rather than the usual land focus. From this point of view it is perhaps more convincing that the oceans actually covers around 70% of the Earth's surface. Two more attributes about the ocean:

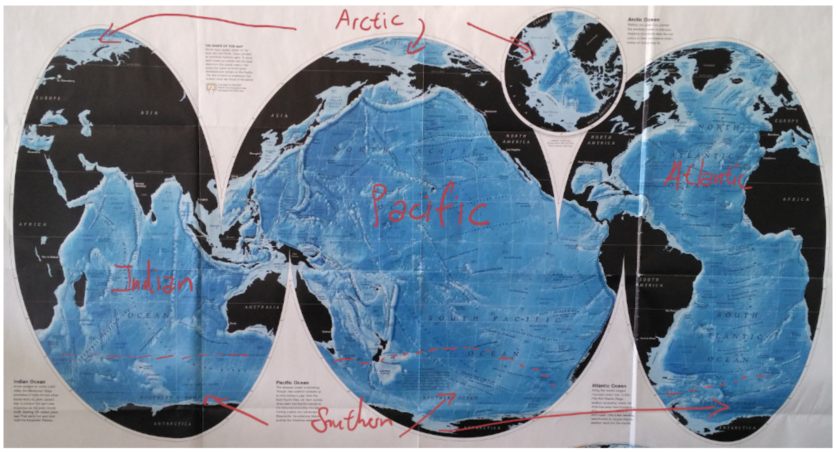

Figure 1.2: Ocean map focusing on the five major oceans by splitting the land. From National Geographic at some point.

1. the ocean holds around 50 times more *carbon*[3] than the atmosphere,

2. the upper 2.5 m holds as much *heat* as the atmosphere.

[3] Some inorganic (carbonates etc.) but largely organic, since most life on Earth is carbon based.

With these two observations, there are multiple points we can make for the marine environment being a central part of the climate system:

- The marine environment is a crucial component of the *carbon cycle*, and we care about that because carbon dioxide in the atmosphere is a *greenhouse gas*, and has consequences for the *energy balance* of the Earth. While we view the ocean largely as a sink for atmospheric carbon particularly for long-term storage (by the *biological* or the *physical pump*). This does not have to be the case and we want to know how the ocean evolves in the future, and what impacts this has for the carbon cycle.

- Related to above, the ocean is an important component of the *energy balance* of the Earth system acting largely as a heat reservoir absorbing a large amount of excess heat (see Fig. 1.3), partly

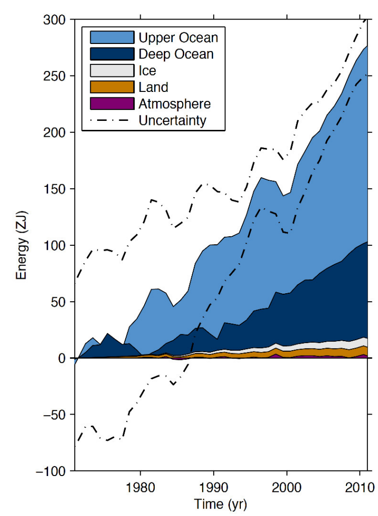

Figure 1.3: Figure 3.1 from the IPCC AR5 WG1 report, showing the destination of the excess energy received by the Earth. Most of it goes into the ocean.

because seawater has a much larger *heat capacity* than air and land (see Ch. 2). The *Meridional Overturning Circulation* (MOC, see Ch. 5) plays an important role in the transport of energy around the globe, and again we want to know how that evolves in time.

- If the ocean is warming because it is absorbing excess energy, then this has consequences for the following:
  - the density *stratification* and the MOC, which feeds back onto the the ability of the ocean in transporting/absorbing excess heat
  - *biogeochemical* cycles and its content, because surface warming is expected to lead to a strengthening of the stratification in the upper ocean, affect nutrient supply from the deep (arising from upwelling of colder, nutrient rich waters at depth), the ability of the water to hold chemicals (outgassing of dissolved oxygen and carbon dioxide because warmer water holds less dissolved gases), and others
  - general rise in *sea level*, since seawater larger than around 4° C expands when it is warmed up[4]
  - effects on the ocean ecology by physical stressors (such as increasing temperature) and/or biogeochemical stressors (such as decrease in nutrient supply, acidification, oxygen depletion[5])
  - consequences for economy via fisheries (food web consequences), shipping (changes in climate affecting the viability of certain route, or opening new routes in e.g. the Arctic), energy, ...

[4] Known as *thermosteric sea level*, sea level arising from thermal expansion. We revisit the point about 4° C in Ch. 2.

[5] Known as *hypoxia* and *anoxia*.

The points made for the importance of the ocean is non-exhaustive, but hopefully that provides a sample of the linkages to motivate the study of the physical aspects to do with marine environments.

1.2 *Oceans*

While the hydrosphere covers all things to do with water, for the purposes here we are going to mostly focus on the largest bodies of salty water (e.g. not touching on rivers and lakes here).

We first focus on **oceans**, which are taken to be the largest bodies of salty water that are bounded by **continental land masses**. These are the *Pacific*, *Atlantic*, *Indian*, *Southern* and *Arctic* ocean, sorted by surface area; these have been marked on the map given in Fig. 1.2. The first thing to highlight is that the oceans tend to be fairly deep, with an average depth of $H = 4000$ m over a significant area; contrast this to *seas* where the average depth is around 1000 m, even if they can get very deep at a few locations (see Ch. 1.3 and Ch. 8). A schematic of oceans vs. non-oceans is given in Fig. 1.4, where we show a slice outlining the regions being referred to.

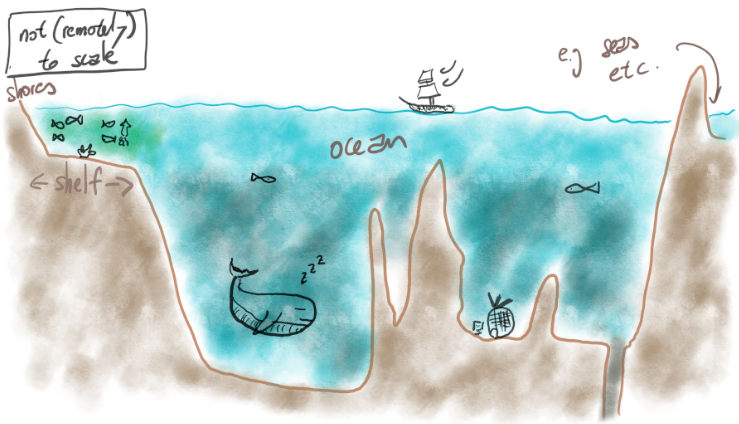

Figure 1.4: Schematic showing the oceans, shelf seas and smaller bodies of water. based on Figure 2.2 of Pickard & Emery (1990), 5th edn.

In the coean there is a massive discrepancy with the horizontal and vertical length scales: we are talking horizontal length-scales of at least $L = 1,000$ km (note the units!), so the **aspect ratio** H/L is small. The smallness of the aspect ratio has important dynamical consequences, which we will revisit in the subsequent discussion relating to dynamics. In some sense large-scale ocean dynamics turns out not to be so dissimilar to large-scale atmospheric dynamics, bar the important difference on lateral boundaries provided by the land[6]. The effects of boundaries lead to phenomena unique to the ocean that we will highlight and explain in due course (Ch. 4 and 5).

[6] Except in the case of the Southern Ocean (Ch. 5.1) where there are open latitudes that are unblocked.

Before we go on to talk a little about the five oceans, going to quickly introduce some terminology. The land features are normally referred to as **topography** and/or **bathymetry**. Technically the former refers to features above land and the latter refers to those features below sea level, but sometimes both terms are used in the ocean community. Some notable *bathymetric features* are labelled in Fig. 1.5. There are somewhat technical (though by no means completely universal) definitions for these features, but I am of the opinion that most of the definitions are not hugely important to the narrative here[7], so I refer the reader to other sources (e.g. Wikipedia, refs). The main point is that there are mountains and hills underneath the ocean that has significant influences on the dynamics, much like the case of the atmosphere, and we will need to take into account of the boundaries when talking about ocean dynamics and circulation.

[7] We will talk about shelves and continental slopes soon, and in Ch. 8.

Most of the following discussions focuses on highlighting dynam-

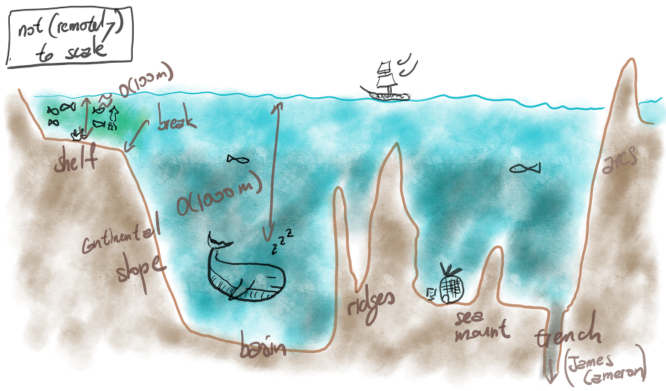

Figure 1.5: Some bathymetric features of note in the ocean. Figure based on Figure 2.2 of Pickard & Emery (1990), 5th edn.

ical features rather than going into details such as how many square km the particular ocean covers or what is the average temperature etc., unless they contribute to the narrative (these are covered in other books and can be searched for in Wikipedia).

1.2.1 *Atlantic*

We are actually going to start first with the Atlantic even if the Pacific is the biggest by surface area coverage. The Atlantic has traditionally received much more attention than the Pacific perhaps for the following reasons: ocean science was first systematically studied in Europe and North America; the Atlantic is perhaps scientifically more interesting because of the role of the *Atlantic Meridional Overturning Circulation*, its links with the global circulation, and its contribution to the weather/climate; the Atlantic is easier to observe and navigate simply because it is narrower than the Pacific. The Atlantic neighbours Europe, Africa, and the Americas, and is connected to the Arctic ocean to the North, the Southern Ocean to the south, and can be seen in Fig. 1.6.

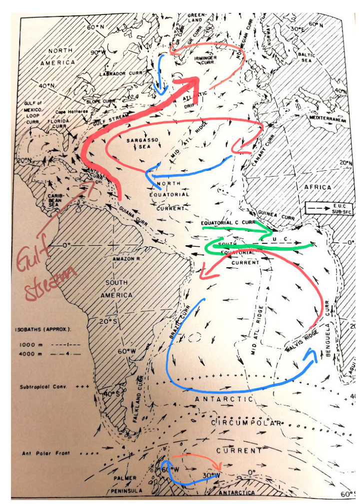

Figure 1.6: A marked up figure of the Atlantic ocean detailing the gyres and the Western Boundary Current. Modified Figure 7.9 from Pickard & Emery (1990), 5th edn.

Also marked onto Fig. 1.6 are some of the surface circulation features of note, such as the *equatorial currents* (the green arrows near the equator), the *gyres* (the recirculating currents), and the *Western Boundary Current* known as the *Gulf stream* (the big red arrow on the America side going from Equator towards the North Pole). The **equatorial currents** are fairly fast east to west flowing currents in the low latitudes ($\pm 20°$ N/S) that are largely driven by the *trade winds*, though there is a equatorial counter current that is normally slightly north of the equator, but goes the other way (against the wind).

The two big **gyres** immediately north and south of the Equator are known as the **subtropical gyres**, and these rotate clockwise and anti-clockwise in the Northern and Southern Hemisphere respectively. The smaller gyres at higher latitudes are the **subpolar gyres**; technically in the Atlantic the subpolar gyre is in the north, since the subpolar gyre in the south belongs to the Southern Ocean. The Atlantic subpolar gyre rotates anti-clockwise[8]. The gyres acts to bring warm equatorial waters to higher latitudes and returning colder waters towards the equator.

[8] In both hemispheres the subtropical gyres rotate *anti-cyclonically* while the subpolar gyres rotate *cyclonically*; see Ch. 3.2 and 4.

The **Western Boundary Current** (WBC) in the Atlantic is called the *Gulf Stream* and is a particularly intense current going north-eastwards from the Gulf of Mexico towards the poles, and thus transporting warm equatorial waters towards the Western Europe. The Gulf Stream has a particularly important role for shipping during the Age of Discovery when the Europeans were colonising America: going from Europe to the Americas, it is normally preferable to sail down to Africa before going across (making use of the intense Westward *trade winds* at the equator), while coming back with goods (loot?) it makes sense to make use of the Gulf Stream to speed up the journey.

The Gulf Stream is a fairly narrow current (typically identified by the warm water it carries around), with an across-stream extent of around 100 km and extends down to around 1000 m depth. The speed of the current decreases with depth but at the surface it can get up to 2.5 m s^{-1}. This may not sound much, especially relative to atmospheric winds, but note that depth-average speeds of mean flows in the ocean are normally measured in cm s^{-1}, so the Gulf stream surface speeds may be two orders of magnitude faster than average ocean flows. The Gulf Stream transports around 30 Sv[9], which is particularly notable given that the current takes up relatively little volume (cf. the *Antarctic Circumpolar Current* in the Southern Ocean).

[9] $1\ \mathrm{Sv} = 10^6\ \mathrm{m}^3\ \mathrm{s}^{-1}$. The unit of Sverdrup is named after the Norwegian oceanographer Harald Sverdrup (1988-1957). For reference, the Amazon river has the world's largest discharge of freshwater into the ocean and the average discharge rate is around 0.2 Sv.

Since seawater is better than air in retaining heat due to water having a larger heat capacity, and that the Gulf Stream transports a significant amount of warm water towards Western Europe, the Gulf Stream has a significant impact on the weather and climate in Western Europe. The trivia to note is that while London is about 10° higher in latitude than New York (around 51° N and 41° N respectively), we might have expected London to be quite a bit colder especially in the winters (because the higher latitudes receives less sunlight). In fact it almost barely ever snows in London, yet New York gets particularly nasty snow storms and the average temperature in the winter months are sub-zero. The warmer Gulf Stream water carries heat with it as it transverses the Atlantic, and releases a portion of its heat to the atmosphere as it reaches the colder higher latitude air, leading to the more temperate climate observed in Western Europe.

The Gulf Stream should in some sense be seen as *averages/means* with a strong instantaneous signal. In reality the current meanders around, evolves over multiple time-scales (seasonally with the winds and solar forcing, longer time-scales with intrinsic variability), and on top of the mean signal there are smaller-scale fluctuations. A snap shot of the Gulf stream is given in Fig. 1.7 showing sea surface temperature. Within the figure we can clearly identify meanders within the main current (as the main body of red), as well as loss of coherency towards the North East (cf. a water hose that is splattering around). The breaking of the jets arises from *instabilities* (cf. Ch. 6), which leads to what is referred to as **eddies**, for the moment to refer to mean closed regions of re-circulation (e.g. the blobs of green and red to the south and north of the mean current respectively)[10]. These eddies, which are effectively the ocean analogue of atmospheric high and low pressures, trap water from its location of generation and carry the water with it as it moves around, and can themselves interact with each other and lead to additional phenomena (e.g. merging, further breaking, forcing of the mean state).

[10] There is some argument as to what really should be called 'eddies'. To quote Ryan Abernathey (or at least that's where I first heard it described this way), should we treat *eddy* as a 'noun' (the circular-esque blobs) or a 'verb' (the fluctuations about the mean)? I would say this distinction is not as clear as it should be when the term 'eddy' is used in the community. I personally take the latter view (the 'verb' includes the 'noun' but not vice-versa).

So what drives the gyres and the WBCs? Why is it *Western* and not *Eastern* boundary currents? Given the WBC it seems there is a net transport at the surface to the poles, so where is the return flow? What leads to eddies and what do/can they do? Answers to these questions are sketched out in Ch. 3, 4, 5, 6 and 8.

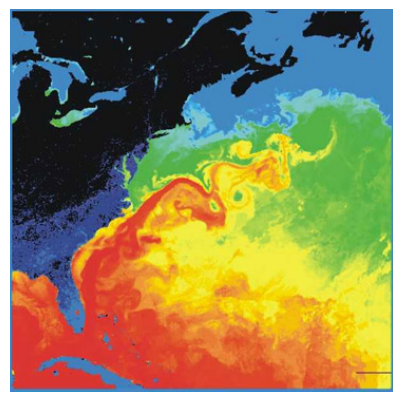

Figure 1.7: NASA observation of the Gulf stream, showing the sea surface temperature (warm waters in red and cooler waters in blue). Image taken from Wikipedia.

1.2.2 *Pacific*

The Pacific is the largest ocean in the world and covers around 46% of the Earth's surface; historically this has meant it was difficult to chart and navigate. The Pacific mainly neighbours the Southern Ocean, but is also connected to the Indian ocean via the Indonesian Archipelago, and to an even lesser extent the Arctic ocean via the Bering strait to the north; the Bering strait is very shallow and water transport between the two oceans is limited.

The surface circulation features are highlighted in Fig. 1.8, and largely similar to the Atlantic ocean, in that there are equatorial currents, subtropical gyres at lower latitudes, subpolar gyres at higher latitude in Northern Hemisphere (the Southern one belongs to the Southern Ocean), and the **Kuroshio** WBC off the coast of Japan. One aspect we do highlight is the **Eastern Boundary Upwelling System** (EBUS) associated with the *Peru current*[11]; the analogous one in the Northern Hemisphere in the Pacific is called the *California current*. The difference with WBCs are that EBUS currents tend to be relatively shallow, slower flowing, and take colder water from the poles towards the Equator. These currents are associated with

[11] Sometimes the *Hulmboldt current*, after the Prussian naturalist Alexander von Humboldt (1769-1859).

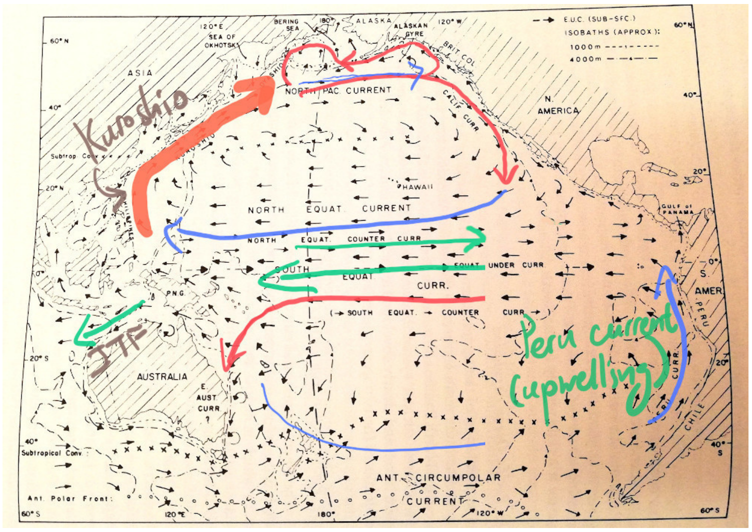

Figure 1.8: A marked up figure of the Pacific ocean detailing the gyres, the Western Boundary Current and the Peru current, which is part of a Eastern Boundary Upwelling System. Modified Figure 7.31 from Pickard & Emery (1990), 5th edn.

Ekman upwelling (Ch. 3.3.3 and 8), bringing cold nutrient-rich waters up towards the surface, and thus have important consequences for marine ecology and fisheries.

In the Pacific a particularly interesting climate phenomenon called the *El-Niño Southern Oscillation* (ENSO)[12] occurs in the tropical Pacific. This phenomenon is fundamentally one of the atmospheric-ocean coupled system[13]. El-Niño events occurs between two to seven years, generally seen with the reversal of the trade winds in the Pacific and an increased warming of the sea surface temperature in the region bordering the East Pacific. Associated with El-Niño events are increased precipitation (which can lead to severe landslides) and reduction in fishery yields (deepening of the *thermocline* in the Eastern Pacific; Ch. 2.2.1), while the Western Pacific gets much less precipitation, which can lead to drought-like conditions. El-Niño also appears to have an influence on global weather patterns, and has been claimed to have contributed to refs:

- human sacrificial practices in the Aztecs
- the French Revolution (1789-1799)
- the spread of Christianity in Qing dynasty China (associated with the Great North China Famine in 1876-1879)
- the Arab spring (2010-2012)

These theories are quite entertaining to hear about and there are perhaps some plausibility to thems, but of course *correlation does not imply causation*, and I will leave it at that. This document does not touch on a mechanistic rationalisation of the ENSO phenomenon

[12] *El Niño* literally means *the boy* in Spanish, a reference to the Christ child, since El Niño events tend to happen around Christmas time.

[13] Via the *Bjerknes feedback*, named after the Norwegian-American meteorologist Jacob Bjerknes (1897-1975). Not to be confused with his father the Norweigan meteorologist Vilheim Bjerknes (1862-1951), who is known for his contributions to weather forecasting.

(it's generally agreed *waves* are important in the mechanism; more references in Ch. 6.1).

1.2.3 *Indian*

The Indian ocean borders mainly the Southern Ocean, and has connections with the Pacific and the Atlantic through the leakages from the *Aghulas current* (which is suggested to be the world's largest WBC with transports of around 70 Sv). There is only really a subtropical gyre in the Indian ocean because of land boundaries, and that associated subpolar gyre belongs to the Southern Ocean. Note that there are equatorial currents and smaller gyres towards the coast of Indian and Persian Gulf, which reversing seasonally because of changes in the wind forcing from the seasonally varying *monsoon winds* (Ch. 3.3 and 8).

One interesting aspect to note is that the Atlantic is seen to be saltier than the other oceans (e.g. look forward to Fig. 2.9), and part of this is attributed to the leakage of the Aghulas current, leading to a transfer of warm, salty water through the Southern tip of Africa into the Atlantic (see e.g. Fig. 1.10). This is perhaps interesting if you think about it: the Aghulas current is a WBC so the flow goes to the East, and there is the neighbouring *Antractic Circumpolar Current* which is also going to the East, so the scenario is stacked against Westward transfer, yet it exists. As can be seen from Fig. 1.10, these *Aghulas rings* can maintain its coherency and travel quite far into the Atlantic basin. This route of transfer is known as the *warm route* (as opposed to the longer *cold route* going eastward all the way round the globe to reach the Atlantic), and is a component with the global MOC (more in Ch. 5).

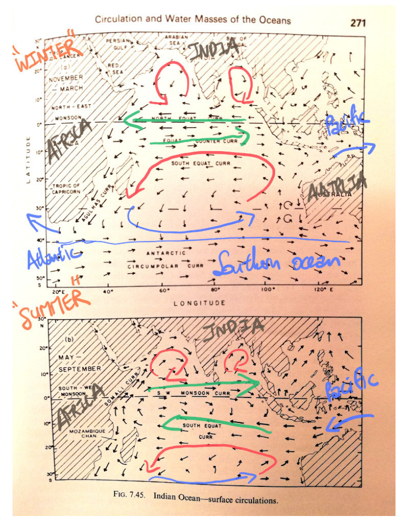

Figure 1.9: A marked up figure of the Indian ocean detailing the gyres and the equatorial currents in Summer and Winter. Modified Figure 7.45 from Pickard & Emery (1990), 5th edn.

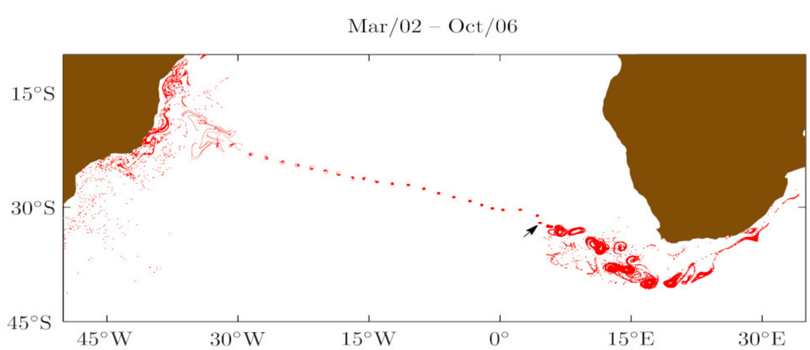

Figure 1.10: Tracking of Aghulas eddies. Image from Yan Wang (HKUST).

1.2.4 *Arctic*

The Arctic ocean is one of the smaller oceans (some people call it a margin sea to the Atlantic), and neighbours the Atlantic, and marginally connected to the Pacific through the Bering strait between North America and Asia.

The main thing in the Arctic is of course the presence of *sea ice*, which has thermodynamic as well as mechanical consequences for the dynamics. When sea ice forms because it is cold, the ice rejects the salt (*brine rejection*), leading to an increase of salinity in the region below the ice, which may have dynamical consequences (cf. *double diffusion* in Ch. 6.2). The presence of ice provides some shielding of the ocean from direct wind forcing, which reduces the momentum transfer to the water column by the atmosphere, but also provides an extra surface for the already flowing water underneath to rub against[14]. On the other hand, when ice breaks up or melts, the ocean is exposed to the cold temperatures and the winds in the atmosphere, leading to a loss of ocean heat and thus buoyancy *loss* (i.e., water gets colder and more dense), as well as momentum transfer into the ocean leading to stronger currents. The seasonal changes in the wind as well as the ice cover leads to interesting dynamics in the Arctic region as well as circulation features such as the *Beaufort gyre*.

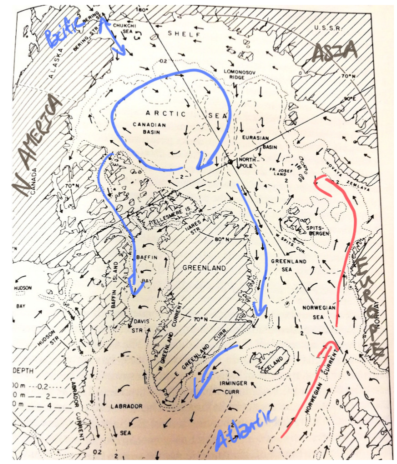

Figure 1.11: A marked up figure of the Arctic ocean detailing the Beaufort gyre and some of the currents. Modified Figure 7.26 from Pickard & Emery (1990), 5th edn.

[14] Referred to as the *ice governor mechanism*, ref.

There has been recent talk that, with the decrease in sea ice cover in the Arctic, new shipping routes could open ref. The existing shipping routes that use the Bering strait to go between the Atlantic and the Pacific go along the coast of Russia, and the route is somewhat hazardous. If however the sea ice cover decreases enough there could be some incentive to just go straight through the Arctic. Although just because we could doesn't mean we should: while this is of course sound from an economic point of view, the associated consequences of loss of sea ice cover presumably will be catastrophic, that any gain will probably seem insignificant in comparison...

1.2.5 *Southern Ocean*

Last but certainly not least is the Southern Ocean, which is connected to all the major oceans except the Arctic. The boundary of the Southern Ocean is somewhat ill-defined, but is usually done by *fronts* separating regions with different *watermass properties*, i.e. somewhat sharp boundaries that separates water types with different temperature, salinity, or other *tracer* properties (see Ch. 2 and 5.2.1). The Southern Ocean experiences some of the strongest wind forcing in the world, and is influenced somewhat by the fact that there is sea and land ice extruding from Antarctica. Since the Southern Ocean is connected to all the major ocean basins, it is sometimes

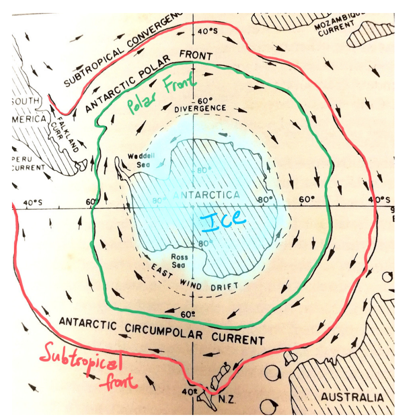

Figure 1.12: A marked up figure of the Southern Ocean, highlighting two of the fronts that roughly denote boundaries with significant difference in watermass properties. Modified Figure 7.45 from Pickard & Emery (1990), 5th edn.

regarded as the center of the global ocean, and plays a central role in the global MOC (more in Ch. 5). For example, the subpolar gyres exist in the Southern Ocean (e.g. the *Ross* and the *Weddell gyre*), and these interact with the abyssal water forming regions responsible for producing the densest waters that fill the deep and abyssal regions of the ocean. It is an ongoing question as to who the interplay between wind, ice, thermodynamic forcing affect the intrinsic dynamics in the Southern Ocean, and its subsequent impacts on the global MOC.

Probably the most significant difference between the Southern Ocean and other oceans is the presence of *open latitudes* where there is no north-south land boundaries. The dynamical balances are noticeably different to the gyres, and the resulting dynamics within the Southern Ocean actually bears resemblance to atmospheric dynamics (see Ch. 3 and 5). Partly because of the open latitudes the Southern Ocean possesses the largest current in the world, the *Antarctic Circumpolar Current* (ACC), with a transport of around 130 Sv. While the current is not as intense as the Gulf stream, it is much larger in terms of cross-stream and vertical extent, leading to the much larger transport. The ACC travels through a choke point at the *Drake passage* (between the tip of South America and Antarctica), then lurches northward and joins the *Brazil current* (a weaker WBC coming off South America), before proceeding around the globe whilst being steered by the bathymetry (such as the *Kerguelen plateau*, between Africa and Australia, closer to Africa). The ACC, being a strong current and subject to strong wind forcing, is a very dynamic and turbulent current, possessing surface *waves* that can have very large amplitudes[15] as well as being susceptible to lots of *instabilities*, which in turn has consequences for things like *air-sea exchanges*, *momentum/energy transfers* and, in turn, the global MOC. We will revisit these in Ch. 5 and 6.

[15] We are talking wave heights measured in meters; see for example `https://youtu.be/WQUXbkAdZhg` one of these surface waves battering a naval ship.

1.3 Not oceans

1.3.1 Some terminology

In contrast to oceans, the smaller bodies of water are more ambiguous to define (because there always seems to be exceptions to the rule), so for the purposes here are simply to be referred to as "not oceans", to include *seas*, *shelf regions*, *estuaries*, *lakes* etc. What is fairly unambiguous is the regions separating the ocean that are on average fairly deep, to the shallower non-ocean regions that have an average depth of 1000 m or less, which are the **continental slopes**. While Fig. 1.13 draws these slopes as very steep, you have to remember the aspect ratio is massively exaggerated, the *gradients* (the slopes;

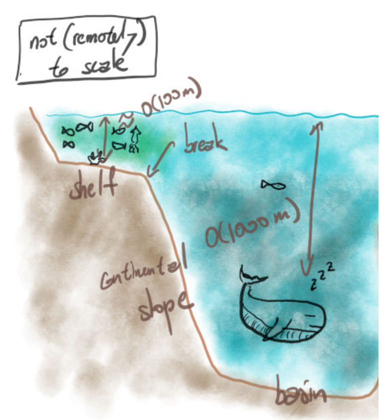

Figure 1.13: Zoomed in version of Fig. 1.4, focusing on the shelf regions. Based on Figure 2.2 of Pickard & Emery (1990), 5th edn.

Ch. 1.5.2) are actually numerically small, at around $1/20$ (or around $3°$ is you want think of angles). While we might expect that the aspect ratio H/L is smaller in not-oceans because H is smaller, this is in fact not true since the horizontal extent L of these non-ocean regions are significantly smaller. The aspect ratio is still small but no longer as small, which has some consequences for the dynamics (see Ch. 8 for more).

Seas are smaller bodies of salty water that typically have a connection with the ocean and are partially enclosed by land (**semi-enclosed seas**), but of course exist, such as the *Black sea*, which may be regarded as essentially covered by land (so an **inland sea**). **Shelf Seas** are seas over shelf regions, which are particularly important for biogeochemistry and ocean ecology, due to the presence of nutrients such as from river runoff and/or *Ekman upwelling* (Ch. 3.3.3), and the fact that the region is relatively shallow and light penetration provides the necessary ingredients for *primary production*. A statistic to note is that while shelf seas cover around 8% of the global area, they account for around 15 to 20% of the ocean's primary productivity, and is a particular reason why all major fisheries in the world are in the shelf sea regions.

Figure 1.14: Picture taken near the Dead Sea (visit in 2014).

Fun trivia: the following are not 'seas' even though they have 'sea' in their name (they are technically lakes because they lie on land):

- *Sea of Galilee* (of the biblical fame) in northern Israel near the border of Jordan, which is a freshwater lake and is sometimes referred to as *Lake Tiberias*
- *Dead Sea* (of the cosmetics fame?) between Israel and Jordan, which is a salt lake (near the lowest point on land, see Fig. 1.14)
- *Caspian Sea* (of the caviar fame) north of Iran and south-east of Ukraine, which is also a salty lake

Figure 1.15: (left) Estuary in the Amazon; from Science Photo Library. (right) Pearl River Delta showing Humen bridge; from Wikipedia, user Tung Wu.

Estuaries are usually defined to be areas around the river mouths (e.g. Fig. 1.15) where there is influence from both *freshwater runoff* from the rivers and the salty water from the seas/oceans. These regions tend be very shallow and the effects of *tides* are very prominent, leading to *tidal excursions* that result in a particularly notable signal in the salinity (more precisely the boundary between freshwater with low salinity and water with high salinity). Notably these regions are generally near settlements, and thus experience forcing from human activity such as nitrogen loading, pollutants, heavy metal input, soil erosion, and so forth, which puts pressure on the ecological activity that exists in these regions. There are several ways to classify these based on the geometry as well as the watermass properties, which will be visited in Ch. 8.

Just a few more. **Fjords** is one of the famous Scandinavian exports to the English language. These are inlets with steep sides carved by *glaciers*, so fjords are normally only found in higher latitudes. Fjord waters are usually salty, for example, the many fjords that are dotted around the coast of Norway, although there are ones in the US Great Lakes that are not. Much like the use of the term 'seas', the use of 'fjords' is not entirely uniform either, and the Scandinavian use of the term is much more liberal. **Lakes** as mentioned above are bodies of water that are on land, and most of these are freshwater ones (e.g. the famous one in Scotland given in Fig. 1.16) but again with some famous exceptions already noted above.

There are other terms such as *lagoons*, *gulf*, *coves* etc., and we can go on all day, but perhaps lets go to some case studies instead of throwing words around... Below we give some case studies relating to not-oceans to highlight some features of interest in these smaller bodies of water, paying particular attention to the physical aspects. The list is non-exhaustive and clearly biased (e.g. I've ignored *corals* partly because I don't know much about it).

Figure 1.16: What lake am I? Pictures originally from Royal Caribbean website and TopPNG.

1.3.2 *Case study 1: Mediterranean Sea*

The Med Sea (I'm going to be lazy) is the big body of water between Europe and Africa, and is connected to the Atlantic via the Strait of Gibraltar (a **strait** is a narrow passage of water connecting two bodies of water, e.g. Bering Strait for Atlantic and Pacific, Strait of Malacca for South China Sea and Indian ocean). The Med Sea has traditionally be a very important area for travellers, warmongers, and merchants, enabling transport between Europe, Africa and to the Middle East, as well as providing an important source of food for the civilisations around the area.

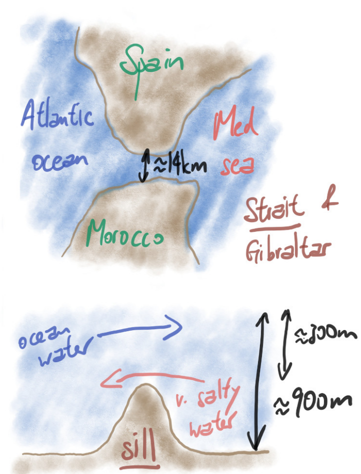

Figure 1.17: Schematic of the Med sea and its connection with the Atlantic via the Strait of Gibraltar. Schematic demonstrating the sill and resulting constraints on water exchanges.

The Med Sea has an average depth of around 1500 m but can get up to several thousand meters deep. However note that the connection with the Atlantic is choked at the Strait of Gibraltar, horizontally because the strait is only 14 km wide, and vertically because there is a **sill** that protrudes from the ocean bottom, leading to a **sill depth** (the depth between ocean surface to the top of the sill) of around 300 m. This limiting factor in the exchange results in the Med Sea waters having a particular signature (the *watermass property*). The Med Sea, being located just north of the equator, falls within the region of the *subtropical high* on the edge of the *Hadley cell*[16]. What this means is that this is a region of regular high atmospheric pressure, which suppresses atmospheric convection, the formation of clouds, and thus limits precipitation. Given this area is also warm, the Med Sea typically experiences strong *evaporation*, leading to water that is generally warm, but also very salty.

[16] A component of the atmospheric overturning circulation. More in Ch. 3.3

Occasionally there will be a cold burst of continental air coming from Europe, which leads to a cooling of the Med Sea water. Now, slightly cooler water and very salty water is very dense (Ch. 2.3) has the capability to sink, and the salty water starts filling up the bottom of the Med Sea. While the water may circulate around within the Med Sea, the fact that there is a sill at the Strait of Gibraltar means this water really has nowhere else to go[17], and piles up behind the sill. The presence of the sill allows for the relatively warm and very salty water characteristic of the Med Sea to build up, until it spills over into the Atlantic in the form of *overflows* (think under water waterfalls). Relative to the less salty and cooler water of the Atlantic, the Med Sea water is more dense, and ends up contributing somewhat to the *North Atlantic Deep Water* that sits at the mid-depths of the Atlantic[18].

[17] The Med Sea, being shielded by land, means tidal effects are weak, and the associated internal waves and diapycnal mixing is expected to be weak too; see Ch. 5 and 6.

[18] The deepest and densest water however originates from the Antarctic and is very cold. Part of the reason might be that the overflows out of the Med Sea leads to significant mixing (Ch. 3.4 and 5) so the resulting water is not as dense.

1.3.3 *Case study 2: Labrador Sea and Weddell sea*

Given we just talked about the Atlantic and *North Atlantic Deep Water* we first say a few things about the Lab Sea (still going to be lazy). The Lab Sea is just south west of Greenland, and given the high latitude location experiences very cold atmospheric temperatures, which is a prime site for forming cold and dense water. Recall from the text in Ch. 1.2.1 that, from the surface observations of the currents, mass seems to be converging polewards, so for mass conservation reasons the water has to return somehow even though there is no notable evidence for this at the surface. Well if it doesn't return southward at the surface it could do it at depth, and it turns out the Lab Sea is one of the areas that contributes to sinking of waters that subsequently flow back to the south (fueling the *Deep Western Boundary Current* below the Atlantic WBC). A schematic of the circulation is given in Fig. 1.18.

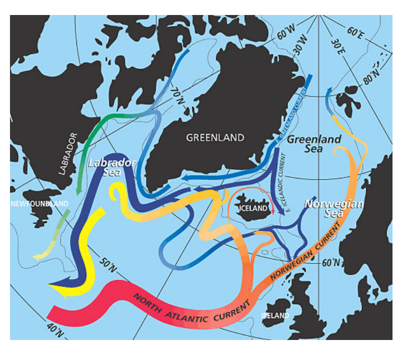

Figure 1.18: Schematic of North Atlantic Circulation. Taken from Wikipedia, image from Jack Cook at WHOI.

The Lab Sea contributes to the deep water formation via intense cooling of the water particularly in Northern Hemisphere winter, leading to *deep convection* that transports water over large vertical depths, which is notable since *convection* due to unstable density gradients in the ocean is usually shallow (Ch. 2). One thing which is beyond the discussion here is that the deep convection in the Lab Sea is in combination with interesting effects such as *cabbeling*, arising from the *nonlinear equation of state* (Ch. 2.3), where mixing of two different water parcels actually leads to a water parcel that has a higher density than the simple addition of the individual parcels' densities together[19].

[19] cf. Metaphorically, cabbeling leads to $1+2>3$ in terms of density.

Something analogous happens in the Weddell Sea in the Southern Ocean. Water reaching the Antarctic is cold, but the Antarctic atmosphere is colder still. The intense cooling of the water again leads to formation of cold dense water. In a process similar to the Med Sea, there is a sill holding the water back, which eventually overflows and fills the abyssal ocean with the densest waters in the ocean, the *Antarctic Bottom Water*. Note that of course the water doesn't just carry temperature and salinity around with it, but also chemical tracers and in particular dissolved carbon, so the sinking of these cold waters into the ocean abyss contributes to storage of carbon. Questions then arise as to if and how this abyssal water gets upwelled (some of this in Ch. 5), how this part of the MOC might evolve, and what consequences this could have for the global carbon cycle.

1.3.4 *Case study 3: Black Sea*

Changing gears a bit, we go back to around the Med Sea region to visit the Black Sea. There are several competing theories as to why the Black Sea is 'Black' in the first place. One rationalisation I will take here for the sake of the story is to do with the fact that the Black Sea might have been a somewhat treacherous place to navigate, and shipwrecks used to occur in the region quite a bit before modern day. The particular interest here is that these shipwrecks are well-preserved without decay, covered in black sludges. The deeper parts of the Black Sea seems devoid of life, and the water smells very strongly of sulphur, which in Christianity is suggestive of the description of hell with its fire and brimstone (brimstone is another name for sulphur).

The above observations are now known to be because of **anoxia**, referring to the absence of oxygen content in the water, and that the physics contributes significantly to this phenomenon. The Black Sea is fed by rivers around it, but also from the Med Sea through the shallow connection at The Bosphorus (or sometimes the Strait of Istanbul). The discrepancy in density between the freshwater from the rivers and the salty water from the Med Sea leads to the salty water sinking and filling up the bottom parts of the Black Sea, while the fresh part stays near the surface, as in the schematic in Fig. 1.19.

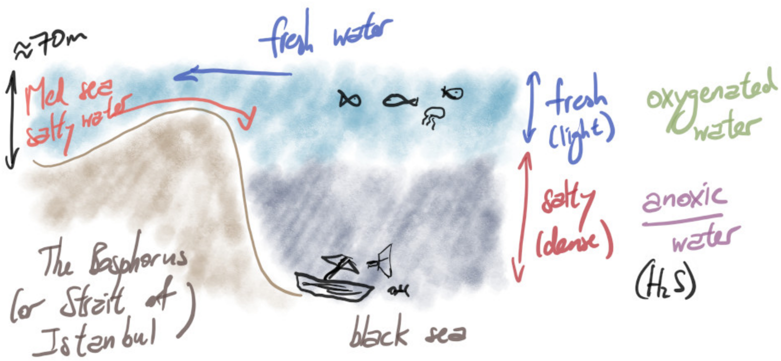

Figure 1.19: Black sea schematics water property schematic.

What this leads to is a strong vertical density gradient that is very difficult to erode, inhibiting mixing and thus vertical transfer of stuff (including oxygen). Any oxygen that was in the deeper parts is used up, which ends up forcing the organisms that use oxygen for *aerobic respiration* to go up the water column (assuming it is not in their nature to want to die by suffocation), consistent with the deep parts of the Black Sea being seemingly devoid of life. However of course that's just visible life. It turns out what remains behind are the micro-organisms that undergo *anaerobic respiration*, such as hydrogen sulphide producing bacteria, which explains the bad smell[20]. The lack of oxygen content means *oxidation* of material is limited, which allows wreckage to remain somewhat intact over longer periods of time[21].

[20] When eggs rot some of the proteins break down into hydrogen sulphide, associated with that rotten egg smell.

[21] Rescue missions in 2018 AD have uncovered an almost intact Greek merchant vessel dating back to around 400 BC.

1.3.5 *Case study 4: South China Sea*

As a final example, the South China Sea is one of the regions that seems to have everything thrown in. It is the largest marginal sea in the region, with the main connections to the oceans via the Luzon, Taiwan and Midoro strait (to the Pacific ocean) and the Malacca strait (to the Indian ocean); see Fig. 1.20. Because of the geographical set up, the region is one of the busiest areas for shipping activity for transport of goods between Asia and the rest of the world.

The region includes shelf regions, as well as deep regions (going down to around 4000 m), so coastal, shelf and ocean dynamics interact and feedback onto each other. The region, though relatively

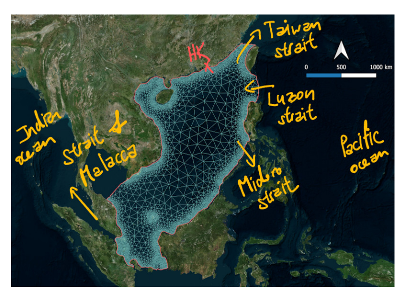

Figure 1.20: Marked up version of the South China Sea, with a triangulated mesh over it (for use with Finite Element models). Mesh and original diagram from Chinmayee Mallick.

small in terms of surface, has been claimed to account for around a third of the world's biodiversity. Fisheries thrive in the area but there are concerns that human activity are stressing the marine ecology in the region (such as nutrient loading, shipping activity, pollution, and over-fishing), leading to various action in the surrounding countries, which contributes to the political tension in the region.

On a regional circulation point of view the Luzon strait connection is probably the dominant factor since it has a sill depth of around 1000 m, while the other straits are shallow in comparison. There is thus the possibility of interaction between the ocean and regional circulation. In additional to the oceanic connections, the region in addition is strongly affected by Asian monsoon winds (Ch. 3.3), which leads to a seasonally varying driving by the regional circulation.

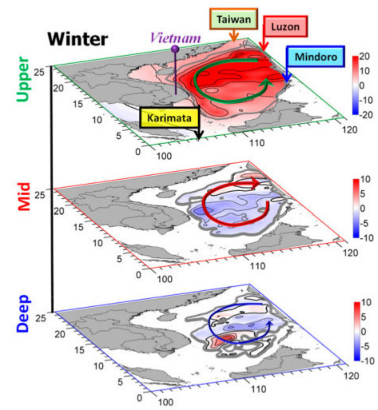

Figure 1.21: Schematic of the sandwich circulation in the South China Sea using numerical simulation data. Figure modified from Fig. 1 of Gan et al. [2016].

One interesting aspect that is being revisited recently is the regional circulation patterns seems to display a 'sandwich' pattern in the vertical, i.e. a anti-clockwise–clockwise–anti-clockwise flow (or cyclonic–anti-cyclonic-cyclonic pattern, see Ch. 3.2). The understanding is that the surface layer is wind-driven (it can't really be anything else...), while the bottom layer is probably from Pacific flow intrusion. The middle layer on the other hand is a bit mysterious because, dynamically speaking, such a vertical configuration might be expected to be *baroclinically unstable* (Ch. 6), which would erode the middle layer, but it seems to be persistent. It is an ongoing question on what controls the South China Sea circulation, how it interacts with the other ocean and/or coastal components, and what consequences this can have in a changing climate.

1.4 *Concepts in Newtonian mechanics, and governing equations for ocean dynamics*

Hopefully the above narrative has made a case for studying the physical dynamics present in the ocean. Ocean dynamics is motion on length-scales that **Newtonian mechanics**[22] works well. Here a brief overview is provided, introducing the concepts and form of the equations that is used for describing ocean dynamics. While we won't do very much with the actual equation itself, it is there for reference as we will refer to them particularly in Chap. 2 and 3.

[22] After English scientist Issac Newton (1642-1726?), widely recognised as one the most influential scientists of all time; his achievements are too many to fit into this margin (fake internet glory points for those who get this obscure reference). His formulation of mechanics was only recently superseded by Einstein's theory of relativity; we are not going to use relativity to formulate ocean dynamics, that's just overkill...

1.4.1 *Forces and Newton's laws*

Lets do the book keeping definitions first. **Newton's laws of motion** are given by:

1. a body at rest or in *steady* motion in a straight line remains at rest or in steady motion unless there is a *net* force acting on it

Note: we should be dealing with non-accelerating *inertial frames of reference* really, because if not we have to add in *fictitious forces*. We are actually going to live with the fictitious force in the form of the *Coriolis effect*, but that's for Ch. 3.2.

2. rate of change of *momentum* of a body over time is directly proportional to the *net* force applied, and in the same direction as the *net* force

3. for every action there is an equal and opposite reaction

Lets put this into pictorially before going into a bit of maths. Intuitively, things move when **forces** act on a body. For no particular good reason we will take to the body of interest to be a pig[23].

[23] Spherical or point mass pig if you like. It's a pig because it's easy to draw.

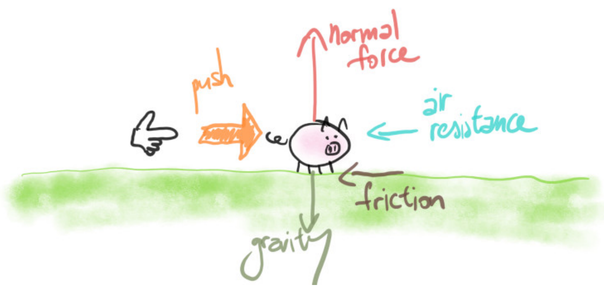

Figure 1.22: Schematic of forces acting on a pig. While it is customary in classical physics textbooks to assume we are in a vaccum, this would of course be against animal rights, so we do have air resistance if the pig is moving.

Assuming the pig is moving in steady state (i.e. moving but not accelerating, so in the regime of 1$^{\text{st}}$ law) to the right, there are various forces acting on this pig, namely:

- Earth's gravitational force pulling the pig down, balanced by a *normal force*[24] arising from the ground pushing back (3$^{\text{rd}}$ law), otherwise you might expect the pig to sink into the ground (1$^{\text{st}}$ + 2$^{\text{nd}}$ law)

- a phantom pointy finger pushing the pig to the right, balanced by *friction* from the ground and *air resistance* from the air resisting the motion of the pig (1$^{\text{st}}$ + 2$^{\text{nd}}$ law)

[24] Normal here refers to being normal or perpendicular (at right angles) to the surface.

Overall there are no net forces in the horizontal *and* the vertical, and the pig moves to the right along its merry way (or maybe not because it is being pushed by a phantom pointy finger). One thing to note in relation to the 3$^{\text{rd}}$ law is that, while from the point of view of the pig the ground and atmosphere is resisting its motion, an equivalent view to take is the ground and atmosphere's point of view, where the pig is pushing against them. There are some subtleties involving points of view here, which we will revisit in Ch. 3.1.1, 3.2 and 3.4.

We note first that the magnitude of friction and air resistance depends on the speed of the pig[25]. Now, instantaneously the phantom finger decides to increase/decrease the force magnitude accordingly (but still pointing to the right), friction and air resistance is still the same (since the speed hasn't changed), so there is an imbalance of the

[25] The magnitude and direction depends on the *velocity*, but more on that later.

forces, and so there will be a net change in *momentum*. Assuming the pig has mass m, (linear) **momentum** is defined as

$$\boldsymbol{p} = m\boldsymbol{u}, \tag{1.1}$$

where $\boldsymbol{u}$ is the **velocity**. Both $\boldsymbol{p}$ and $\boldsymbol{u}$ are *vectors*, i.e. something with a direction and a magnitude, to indicate where the pig is going. Since m is fixed in this case, change in momentum really means change in the velocity, so what we have is an acceleration/deceleration (2nd law). The acceleration $\boldsymbol{a}$ is given by the famous equation

$$\boldsymbol{F} = m\boldsymbol{a} = m\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}. \tag{1.2}$$

Note that acceleration and forces are also vectors (hence the boldface), because they all have a magnitude and a direction.

Since the velocity and speed is going to change, friction and air resistance will change, leading to further changes in the acceleration/deceleration, until the forces balance again, after which the pig will be moving on its steady merry way (1st law).

To close this section, note that, in SI units,

- mass usually has units kg while velocity has units $\mathrm{m\ s^{-1}}$
- acceleration has units $\mathrm{m\ s^{-2}}$ (think of the $\mathrm{d}/\mathrm{d}t$ as bringing down a factor of s^{-1})
- the unit of force is called a *Newton*, and since $\boldsymbol{F} = m\boldsymbol{a}$, $1\ \mathrm{N} = 1\ \mathrm{kg\ m\ s^{-2}}$

1.4.2 *Forces acting on the ocean*

Newtonian mechanics basically involves considering all the forces acting on a body or, in the ocean's case, some fluid, at some time t, and calculate the net force. The fluid will then move, being at some new place, and we repeat the process. Thus we have to consider what kind of forces are acting on the ocean and see what it does to the fluid.

One distinction that will be made here is **mechanical** and **thermodynamic** forcing, to be visited in detail in Ch. 2 and 3. The former is forcing that affects *momentum* and is the one described by Newtonian mechanics, and the latter affects the thermodynamic variables, which affects the density but in turn affects momentum. A sample of these are shown in Fig. 1.23. The main types of external forcing on the ocean we will be concerned with are:

- heating of the ocean by (incoming) *shortwave radiation* provided by the sun, and usually lumped in with this is (outgoing) *longwave radiation* by the ocean leading to a cooling, affecting temperature and is thus a thermodynamic forcing (Ch. 2.2.1)

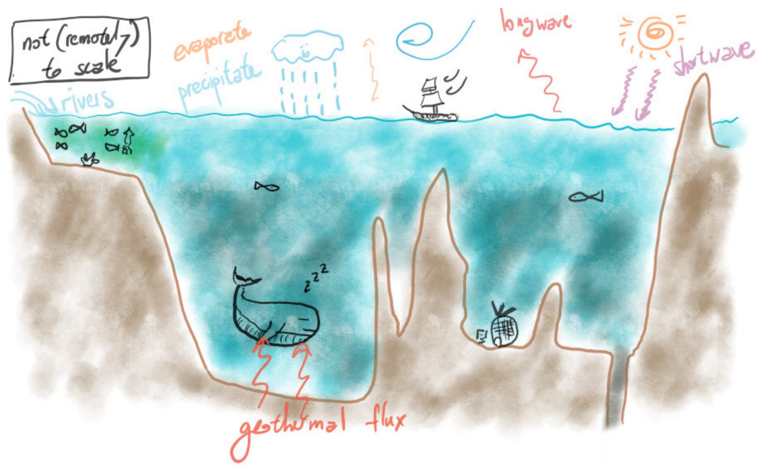

Figure 1.23: Figure based on Figure 2.2 of Pickard & Emery (1990), 5th edn., with some typical forcing drawn on.

- *freshwater forcing*, either as river runoff or *precipitation* (rain, snow, hail, etc.), leading to a decrease in salinity, and *evaporation* of sea water (via heating from e.g. shortwave radiation), leading to an increase in salinity, and thus are thermodynamic forcings (Ch. 2.2.2)
- gravity as a mechanical force affecting the fluids momentum, principally arising from the Earth's attraction of the fluid leading to buoyancy forces, but noting buoyancy depends on the thermodynamic variables (Ch. 3.1.1)
- *wind* forcing involving the atmosphere, as a mechanical forcing, transferring *momentum* from the atmosphere into the ocean (Ch. 3.3)

Note I've deliberately omitted the Coriolis effect because is not a 'true' force (not that it doesn't mean it is unimportant), so be justified in Ch. 3.2. There are other types of forcing that are usually regarded as weak (e.g. *geothermal flux* from the solid Earth, leading to an increase in temperature of deep water, *stirring* by fish moving around in the ocean) and/or rare (e.g. *tectonic movements* with the solid inputting momentum into the ocean), which may be important in certain moments in space and/or time. We won't talk about those in detail here.

1.4.3 *Equations of motion*

The evolution of momentum and the thermodynamic variables could be described in words or, more concisely and in a less cumbersome way, by equations. Taking u, v, w as the **zonal** (east-west), **meridional** and vertical velocities respectively, we denot $\boldsymbol{u} = (u, v)$ as the horizontal velocity and $\boldsymbol{u}_3 = (u, v, w)$ as the full velocity. One form of the equation that describes ocean dynamics is given by[26]

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F}_{\boldsymbol{u}} + \boldsymbol{D}_{\boldsymbol{u}}, \tag{1.3a}$$

$$\frac{\partial p}{\partial z} = -\rho g, \tag{1.3b}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0, \tag{1.3c}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T, \tag{1.3d}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S, \tag{1.3e}$$

$$\rho = \rho(T, S, p), \tag{1.3f}$$

[26] The *Boussinesq* and *hydrostatic* approximation has been used but we are not going to elaborate on those really. See Vallis [2006] for further reference.

where F and D (in bold and normal format) denote the forcing and dissipations respectively, and the precise meaning of the symbols will be elaborated on in Ch. 2 to 3. The equations are, respectively:

(a) the *horizontal* momentum equation within a rotating frame of reference, to include the *Coriolis effect* $2\boldsymbol{\Omega} \times \boldsymbol{u}$ (see Ch. 3.2), with mechanical forcing and dissipation

Note Eq. (1.3a) is Eq. (1.2) in disguise as $m\boldsymbol{a} = \boldsymbol{F}$. I didn't put the Coriolis effect in with the forces, even though sometimes it is referred to as the Coriolis force. This is related to the margin note at the beginning of Ch. 1.4.1.

(b) the *vertical* momentum equation, which actually reduces to *hydrostatic balance* as shown here from the approximations

(c) the *continuity* equation, corresponding to *mass conservation*

(d) an equation for *temperature* with thermodynamic forcing and dissipation

(e) an equation for *salinity* with thermodynamic forcing and dissipation

(f) the *equation of state* to get the density ρ (Ch. 2.3)

The thermodynamical and mechanical aspects are intrinsically linked. The fluid moves around because it is mechanically forced, as described by the momentum equation. The movement of the fluid transports temperature and salinity around. Additionally, the thermodynamic forcing affects temperature and salinity. However, changes in the temperature and salinity affect the density via the equation of state, and through hydrostatic balance affects the pressure, which in turn leads to pressure gradients driving the flow, and so on.

While this all sounds a very complicated, the goal here is to try and delineate this seemingly tangled ball of mess and make some sense of it. While we will focus on simplifications of the larger problem in order to gain some understanding, the thing one should always bear in mind is that everything is intrinsically linked. While the obvious (and to me somewhat cheap) comment/dig/criticism is that the simpler pictures do not represent the real world, in some ways that was never the intention and it is missing the point. The value is in what you learn from them, not from whether they describe the world down to the finest detail (great if they do of course).

If you are familiar the vector calculus and understand enough about what the equations are showing above, then you probably don't need the following section. For those who are not familiar, want to see a bit more detail, or want to see some propaganda, do continue onto the next section.

1.5 *A hand wavy introduction to vector calculus*

While there are basically no calculations involved in this document beyond working out orders of magnitude and the sign of something, vector calculus concepts will be used, simply because in my opinion it is the natural language to express physics related details in a concise format. The following is a very hand wavy tour of the relevant concepts in vector calculus, focusing on the geometric meaning of the symbols. Looking forward, we will use vectors and scalars generally, dot products to talk a bit about wave propagation (Ch. 6.1), cross product to talk about the Coriolis effect (Ch. 3.2.2), vector calculus generally, divergence when talking about Ekman pumping/suction (Ch. 3.3.3), and curl when talking about wind stress curl and vorticity (Ch. 3.3.3).

1.5.1 *Vectors and scalars*

As we have already seen in the discussion of Newton's laws, usually we are dealing with something that has a magnitude as well as a direction, i.e. **vectors**. **Scalars** on the other hand are just numbers. In this text vectors will be boldfaced[27], and scalars are always undecorated. Here are some examples:

- gravitational acceleration $\boldsymbol{g}$ towards center of mass (and perpendicular to the *geoid*; see Ch. 3.1.1) is a vector, with magnitude $g = |\boldsymbol{g}|$, a scalar
- a fluid parcel travels with some velocity $\boldsymbol{u}$ (a vector), which has speed $|\boldsymbol{u}|$ (a scalar) in some direction
- pressure $p = p(x, y, z, t)$ is a *scalar field* but $\nabla p(x, y, z, t)$ is a *vector field*[28] (the gradient operator ∇ will be talked about soon)

While you can do the usual elementary operations with scalars, for vectors it is more limited:

The antagonist in the first *Despicable Me* film is called *Vector* (was Victor, the son of the banker), because he is "committing crimes with both direction and magnitude".

[27] Sometimes you see vectors with underlines, or arrows on top of them, however it is also not uncommon in theoretical physics or maths literature to see them not decorated at all.

[28] A scalar/vector *field* in this context is just a function that associated scalars/vectors to different points in space. A 1d scalar field (e.g. $y = f(x)$) draws a *curve*, a 2d scalar field draws a *surface*, and so on.

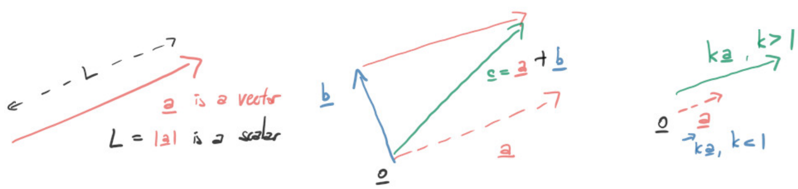

Figure 1.24: Schematic of elementary vector operations.

- add/subtract vectors to give another vector, $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{c}$, equivalent to joining vectors end to end
- multiply/divide vectors by scalars to get a vector, $k\boldsymbol{a}$, equivalent to stretching/squeezing a vector

!!! YOU DO NOT MULTIPLY/DIVIDE A VECTOR BY A VECTOR!!!

There are two other operations to do with vectors we will touch on, but first we note that we can represent a vector in terms of a *basis*[29]. The **standard basis** in 3d is simply

$$\boldsymbol{e}_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{e}_y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

[29] Think of the basis as the lego bricks and any vector can be built by some choice of lego blocks. Just like lego blocks you can build the same vector using different kinds of blocks, i.e. the basis is not unique, but the standard basis is one very convenient choice that we will stick with. Here vectors are put in columns or rows and just chosen depending on whichever form is convenient for writing (we do not distinguish vectors and *co-vectors* or 1-*forms*).

An example of how to use this is as follows for a 2d example is given in Fig. 1.25. We will only use the standard basis here.

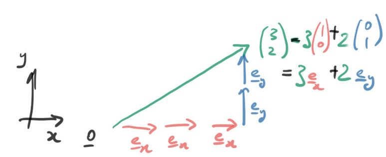

Figure 1.25: Example in 2d representing a vector in the standard basis.

The **modulus** of a vector is then $|a|$ of $\boldsymbol{a}=(a_1,a_2,a_3)$ is then given by $|\boldsymbol{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$ (just Pythagoras' theorem). This is equivalent to taking the *length* of a vector, which takes a vector and returns a scalar, since length itself is a magnitude and has no direction.

The **dot product** or the **scalar product** between $\boldsymbol{a}$ and $\boldsymbol{b}=(b_1,b_2,b_3)$ is defined as

$$\boldsymbol{a}\cdot\boldsymbol{b}=a_1b_1+a_2b_2+a_3b_3, \tag{1.4}$$

which tales two vectors and returns a scalar. We claim without proof that

$$\boldsymbol{a}\cdot\boldsymbol{b}=|\boldsymbol{a}||\boldsymbol{b}|\cos\theta, \tag{1.5}$$

where θ is the angle it takes to take $\boldsymbol{a}$ to overlap $\boldsymbol{b}$, that anti-clockwise orientation as positive as per convention. Thus if $\boldsymbol{a}$ and $\boldsymbol{b}$ are **perpendicular**, then $\boldsymbol{a}\cdot\boldsymbol{b}=0$, and vice-versa.

Finally, the **cross product** or the **vector product** between $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as

$$\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{c}=\begin{pmatrix} a_2b_3-a_3b_2 \\ a_3b_1-a_1b_3 \\ a_1b_2-a_2b_3 \end{pmatrix}, \tag{1.6}$$

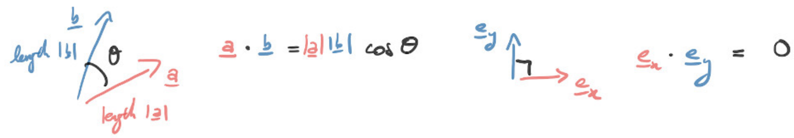

Figure 1.26: Schematic of dot product.

so taking two vectors and returning a vector. By itself equation (1.6) might look a bit meaningless, but geometrically what this is doing is returning a vector $\boldsymbol{c}$ that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$, i.e. $\boldsymbol{a} \cdot \boldsymbol{c} = \boldsymbol{b} \cdot \boldsymbol{c} = 0$. This is done by the *right hand screw* convention as demonstrated in Fig. 1.27. In that example, you want to "twist" $\boldsymbol{e}_x$ into $\boldsymbol{e}_y$, which requires your right hand to be pointing *up* when you twist, i.e. in the direction of $\boldsymbol{e}_z$.

If on the other hand you want to find $\boldsymbol{e}_y$ into $\boldsymbol{e}_x$, then you follow the same logic and conclude that your right hand needs to be pointing down, i.e., in the direction of $-\boldsymbol{e}_z$. This observation is consistent with the *anti-symmetric* property of the cross product, i.e.

$$\boldsymbol{a} \times \boldsymbol{b} = -\boldsymbol{b} \times \boldsymbol{a}.$$

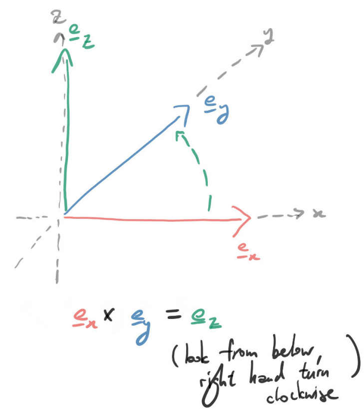

Figure 1.27: Schematic of the cross product, following the right hand screw convention.

1.5.2 *Calculus: derivatives and integrals*

For whatever reason calculus seems to freak a lot of people out (because there are so many rules and tricks to remember and it feels like mathematical gymnastics for the sake of doing it?) At the intuition level and for our purposes it is just **gradients** (i.e. *rate of change*) and **integrals** (i.e. *sums*). If we ever want to talk about how things *change* in, for example:

- how the temperature changes in some material as heat is applied,
- how the rate of reaction changes depending on the concentration of chemicals,
- how the value of Dogecoin (Fig. 1.28) has changed over time on the market,
- how the ocean/atmosphere moves around as forces are applied,

Figure 1.28: Such market, how trade, to the moon, wow.

then calculus is a tool (probably the tool) we probably want to use to describe the statements in a concise and quantitative way.

Lets start with **gradients**, which for **linear** (straight) things is defined as

$$m = \frac{\Delta y}{\Delta x},$$

i.e., how much do you go up/down (say) when you move along horizontally; see Fig. 1.29. If $m > 0$ then we have positive slope, and we go up as we move towards positive x. If $m < 0$ then we have negative slope and we go down as we move towards positive x.

That's all well and good, but generally functions are not linear (as in Fig. 1.29) then what? The beauty of calculus is that it is generally (!) mathematically sensible and well-defined *if we just treat the nonlinear curves as if it is linear by zooming in sufficiently*. This is illustrated in Fig. 1.30. Initially Δx is large and the approximation of the gradient at a point on the curve by a straight line is terrible. So we decrease Δx (i.e. the more we zoom in), and the approximation gets better and better. So why don't we take Δx going to zero? Calculus tells you this limit can be well-defined and controlled accordingly, and it is generally meaningful to talk about

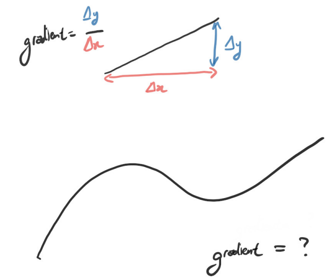

Figure 1.29: Gradient of a straight line is well-defined. But how to define gradient of a nonlinear curve?

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x} \to \frac{\mathrm{d}y}{\mathrm{d}x}.$$

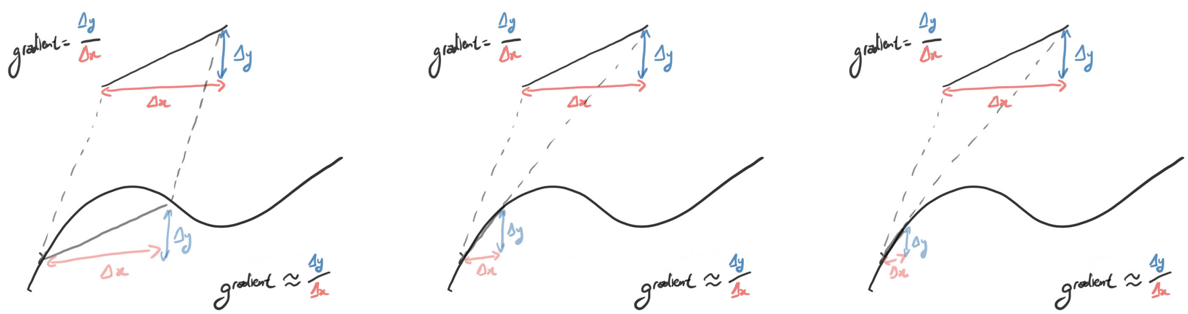

Figure 1.30: Idea behind the derivative, approximating the gradient by taking increasingly smaller increments in Δx such that the linear approximation works for sufficiently small Δx.

And that's it really (at least from a practical point of view)! Derivatives are basically gradients, nothing more. Some examples:

- assuming $p = p(z)$ only, $\mathrm{d}p/\mathrm{d}z$ would be the change of pressure with respect to depth
- assuming $[CO_2]$ is a function of temperature T only for whatever reason, then $\mathrm{d}[CO_2]/\mathrm{d}T$ would be the change of CO_2 concentration with water temperature T, which might be expected to be negative (why?)

The above assumes functions of one dimension, so no ambiguity in talking about the **total derivative** $\mathrm{d}/\mathrm{d}(\text{stuff})$. For a function with multiple dependencies we might talk about the **partial derivative** $\partial/\partial(\text{stuff})$. Instead of a curve and its gradient, the partial derivative can be thought of in 2d as the gradient of a surface in only one of the directions. Some examples:

- for $p = p(x, y, z)$, $\partial p/\partial z$ would be the change of pressure with respect to depth *regarding x and y as fixed*
- for $[CO_2]$ a function of temperature T and p only for whatever reason, then $\partial[CO_2]/\partial T$ would be the change of CO_2 concentration with water temperature T *regarding pressure p as fixed*

We are not going to do any actual calculations that involve taking derivatives as such, but see the Appendix for a collection of rules and examples.

The **integral** $\int$ can be thought of as a sum (hence the stretchy 'S' symbol $\int$) and the opposite of a derivative[30]. Something like

$$\int_{-H}^{0} \rho(z)\ \mathrm{d}z, \qquad \int^{z} \rho(z')\ \mathrm{d}z',$$

means the function (or field) summed (integrated) in the z direction. The first one means sum the function ρ between $z = 0$ and $z = -H$ (a **definite integral**), and you end up with a function that is no longer a function of z (because you summed over it). The latter on the other hands means you are doing a cumulative sum up to some depth z, so the resulting object *is a function of z*; this is sometimes just denoted $\int \rho(z)\ \mathrm{d}z$, which is the **indefinite integral**, the **primitive** or the **anti-derivative**. Further examples:

[30] This one has the grand name of the Fundamental Theorem of Calculus and due to Maria Gaetana Agnesi (1718-1799). Besides being a child prodigy, she was according to the Britannica "*considered to be the first woman in the Western world to have achieved a reputation in mathematics*". She wrote the first textbook discussing both differential and integral calculus and the connection.

There is a dummy variable z' for clarity reasons. Formally when you do $\int \rho\ \mathrm{d}z$ you would integrate out the z-dependence, i.e. the object is no longer a function of z, but then $\int^z \rho(z)\ \mathrm{d}z$ should be thought of as a function of z. To avoid this (hypothetical?) confusion, a dummy variable is used: the resulting object is certainly no longer a function of z'.

- for x denoting the longitudinal direction, $L_x^{-1} \int_0^{L_x} f\ \ \mathrm{d}x$ would mean the **zonal average** of f, and is no longer a function of x
- for V the volume of a region, $V^{-1} \iiint f\ \ \mathrm{d}x\ \ \mathrm{d}y\ \ \mathrm{d}z$ would be the domain-averaged temperature, which would be a function in t possibly

The idea behind integrals again is similar to the derivative, think of it as a generalised way of working out the area under a graph as in Fig. 1.31. If the function is linear this is easy: you end up calculating areas of a trapeziums (triangle plus a rectangle). If the function is not linear (but at least continuous), then you can play the same trick as before, and chop up the graph into increasingly small segments of width Δx, so then again the trapezium approximation gets better and better. Calculus then tells you the $\Delta x \to 0$ can be well-defined and actually works, and there are associated rules for doing integrals. Again, since we are not going to do any actual calculations that involve analytically computing an integral, see the Appendix for a collection of rules and examples.

1.5.3 *Vector calculus: grad, div and curl*

The vector calculus of interest here will mostly involve three operators associated with derivatives, and are used quite a bit through

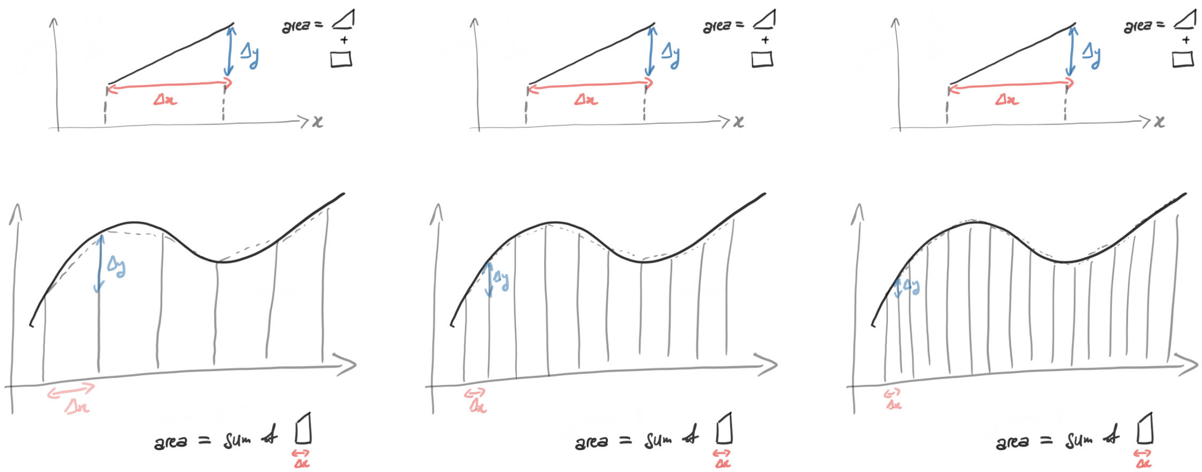

Figure 1.31: Idea behind the integral, approximating the sums by taking increasingly smaller increments in Δx such that the linear approximation works for sufficiently small Δx. The "chopping" is called a *partition*, and in this case we chopped in "x" (the *Riemann integral*). You will also notice you could chop it horizontally, i.e. in "y" = $f(x)$, which turns out to be a more robust definition but needs more complicated machinery; see the *Lebesgue integral*.

the document. There are some integral related ones that do make a showing in geophysical dynamics relatively often (e.g. Green's theorem, divergence theorem and Stokes' theorem) but we won't use them here; see the Appendix for those.

The **gradient** (sometimes just **grad**) operator ∇ ('nabla') takes a scalar field and returns a vector field as

$$\nabla p(x,y,z) = \begin{pmatrix} \partial p/\partial x \\ \partial p/\partial y \\ \partial p/\partial z \end{pmatrix} = \frac{\partial p}{\partial x}\boldsymbol{e}_x + \frac{\partial p}{\partial y}\boldsymbol{e}_y + \frac{\partial p}{\partial z}\boldsymbol{e}_z. \tag{1.7}$$

Again these are just gradients and nothing more (except now it has a direction because it is a vector). The two that we will mostly encounter are $-\nabla p$, the *negative pressure gradient* (i.e. high pressure regions "pushing" stuff into low pressure regions), and $\nabla\rho$, which is the *density gradient*[31].

The **divergence** (sometimes **div**) operator $\nabla\cdot$ takes a vector field and returns a scalar field[32] as

$$\nabla\cdot\boldsymbol{u}(x,y,z) = \nabla\cdot\begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \tag{1.8}$$

Geometrically what div represents is *convergence* or *divergence* of a vector field. Fig. 1.32 shows a 2d schematic, with the dashed lines showing a before control area and solid lines showing an after control area. For converging flow, things are piling in and the control area shrinks, with $\nabla\cdot\boldsymbol{u} < 0$. For diverging flow, things are moving out and control area expands, with $\nabla\cdot\boldsymbol{u} > 0$. For pure rotations

[31] Density is related to $\partial p/\partial z$ via *hydrostatic pressure*, and contribute to *thermal wind* (Ch. 5.1.2)

[32] Recall dot product above.

or uniform translations, the control area simply gets moved around with no change in volume, $\nabla \cdot \boldsymbol{u} = 0$. This is used later particularly in relation to *Ekman pumping/suction* (Ch. 3.3.3); since we have mass should be conserved, if there divergence at the surface, there has to an associated *upwelling* to replenish the mass that are moving out, and vice versa for convergence.

The last one of interest here is the **curl** operator, which returns a vector field from a vector field as[33]

$$\nabla \times \boldsymbol{u}(x, y, z) = \begin{pmatrix} \partial w/\partial y - \partial v/\partial z \\ \partial u/\partial z - \partial w/\partial x \\ \partial v/\partial y - \partial u/\partial x \end{pmatrix}. \tag{1.9}$$

Note that for almost the entire document we will be focusing on the z-component of the curl ζ ('zeta', for z), i.e.

$$\zeta = \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{u}) = \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}. \tag{1.10}$$

These will show up as either the (relative) *vorticity* (the curl of the velocity) or the *wind stress curl* (Ch. 3.3.3).

One way to think of the curl is how much the vector field spins around (how the field "curls" around I suppose); see Fig. 1.33 for a schematic. A uniformly converging, diverging and translating flow has no curl and is *irrotational* (i.e. $\zeta = 0$ here), as the control area does not change its orientation. For a rotation and a shear however it does. By convention anti-clockwise rotation is *positive* curl (because mathematicians measure angles in an anti-clockwise manner), so the examples shown in the figure has negative curl because the control area rotates clockwise.

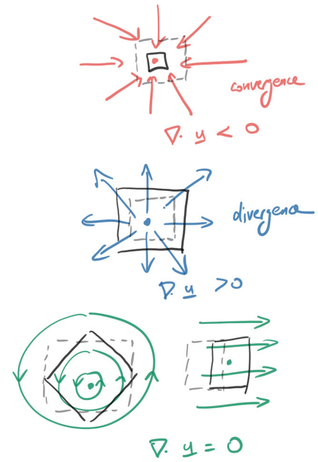

Figure 1.32: 2d schematic for the divergence of a vector field. Dashed lines denote the before control are while the solid lines denote the after control area.

[33] Recall cross product above, and compare the formula with Eq. (1.6).

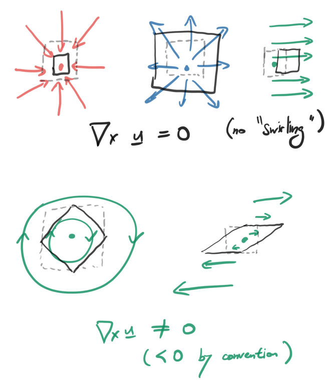

Figure 1.33: 2d schematic for the curl of a vector field. Dashed lines denote the before control are while the solid lines denote the after control area.

1.6 Conventions used here

- x, y, z you can/should think of as east-west, north-south and up-down, respectively called the *zonal*, *meridional* and *vertical* direction
- $x, y, z > 0$ are East, North and up, so $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ points east, north and up
- $\boldsymbol{u}_3 = (u, v, w)$ is a vector field, and u, v, w are scalar fields and the zonal, meridional and vertical velocity, with $u, v, w > 0$ the east, north and upward velocity
- vectors will be **bold** whilst scalars are un-decorated

Summary and further reading

Chapter exercises

1. Provide an estimate of the aspect ratio for the ocean. What about for the atmosphere (take the stratosphere as the vertical limit for the atmosphere if you like)?

2. Show that for $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$, $\boldsymbol{c}$ really is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$ by explicitly computing $\boldsymbol{a} \cdot \boldsymbol{c}$ and $\boldsymbol{b} \cdot \boldsymbol{c}$ using equation (1.6).

3. Fig. 1.34 shows a typical road sign for steep slopes. The sign means you go down 1 units for every 8 units you move horizontally (roughly speaking). With this, compute the magnitude of the gradient, and work out the slope angle relative to the horizontal (give it in degrees). For those mathematically inclined, estimate the angle but don't use a calculator (gradient of 1/10 might be neater to do).

Figure 1.34: Image from HK transport department (`www.td.gov.hk`).

4. If a continental slope has height 3000 m and extends horizontally over 30 km, find the magnitude of gradient.

5. Repeat the above by but give the magnitude and sign of gradient if we are measuring from the coast towards the ocean, and we have a continental slope configuration as in Fig. 1.13.

6. Look up the value of Dogecoin and describe its 2021 price changes in terms of gradients.

7. Give a non-zero vector field that has zero curl and div (drawing or mathematical representation is fine).

8. Is it possible to construct a case where the horizontal winds on Earth are everywhere non-zero? (In vector calculus speak, is it possible to have a 2d vector field on the surface of the sphere that is everywhere non-zero at any instance?)[34] If you can, draw it. If not, convince yourself why not, and see what is the minimum number of zero points you must have on this surface.

[34] Look up the *hairy ball* theorem if you want a hint.

2 Seawater properties and thermodynamic forcing

Key takeaway of this chapter: FROM A DYNAMICAL POINT OF VIEW, IT'S ALMOST NEVER IN-SITU DENSITY YOU CARE ABOUT!

We go on to justify the above claim. In Ch. 1 we made a distinction between the *thermodynamic* and *mechnical* contribution to the fluid dynamics, the former to mean anything that directly affects the *density* of the fluid, and the later to mean anything that drectly affects the *momentum* of the fluid. Of course the two are intimately linked, but we will start first with a discussion of the *thermodynamic* aspects because it is perhaps more intuitive to talk about[1]. Referring to the equations given in Eq. (1.3), the equations of interest are

[1] I hesitate to use the adjective 'easy' because it can get a little mind bending and slightly confusing.

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T\right) = F_T + D_T, \tag{2.1}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S\right) = F_S + D_S, \tag{2.2}$$

$$\rho = \rho(T, S, p). \tag{2.3}$$

Here, the $\boldsymbol{u}_3 \cdot \nabla T$ term represents the *advection* term, i.e. how water moving around carries in this case temperature T around, and we have the forcing as F, while D represents the dissipation and *diffusion* terms (see Ch. 3.4 for more detail about diffusion).

The key quantity that we are aiming to get to is the fluid **density**[2] ρ (with units of kg m^{-3}) or the **buoyancy** $b = -(\rho/\rho_0)g$, where ρ_0 is a reference density and g is the gravitational acceleration (in units of m s^{-2}; see Ch. 3.1.1). The density measures how much 'stuff' there is per volume, and if two blobs of water have identical volume but one is more dense, then the denser one is heavier. An equivalent measure would be by the buoyancy: the heavier blob is less buoyant so has smaller buoyancy. You actually know some of this from intuition already, for example via the thought experiment in Fig. 2.1. On the one hand we have warm water over cold water, which we know will be stable, but on the other hand if we have cold water over warm

[2] For now I am referring to *in-situ density*, but we will get to the subtleties later.

water we expect the water to overturn in the vertical, because cold water is heavier, denser or less buoyant than the warm water. While temperature is used here, for seawater salinity is also important, though ultimately it is actually the density that matters.

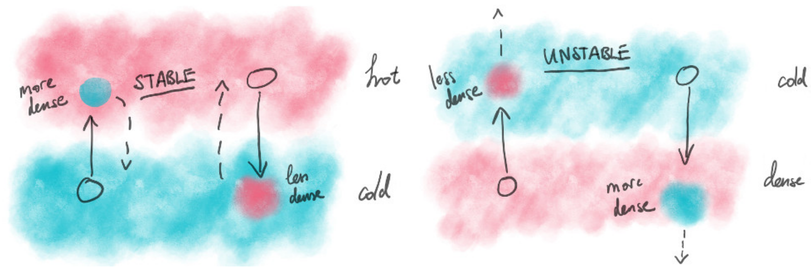

Figure 2.1: Parcel argument for static instability, red and blue denoting warm and cool water respectively. More in Ch. 6.

As can be seen from the thought experiment in Fig. 2.1, density is expected to play an important role in the vertical movement of seawater. It turns out that minor differences in density/buoyancy are also one of the primary drivers for horizontal dynamics in the ocean (through a combination of *hydrostatic* and *geostrophic balance*), which we will visit again in Ch. 3 and 5.

2.1 *Seawater properties*

Since density is argued to be important, we want to know what contributes to density of seawater. Before we do that, just a quick clarification on two physical concepts. **Mass** (usually units of kg) is about how much 'stuff' the body is made of, and is a scalar. **Weight** is force and so a vector, and is the mass multiplied by the magnitude of gravitational acceleration in the direction of the gravitational acceleration, i.e. $\boldsymbol{F} = m\boldsymbol{g}$. Because we are talking about blobs of water, instead of mass we use density, and it is the changes in the density that leads to a difference in the force, and thus an acceleration by Newton's laws.

Temperature T of the water is a measure of how warm or cool something is, and is often measured in degrees Celcius (° C) of Kelvin (K, but no degrees)[3]. One can think of temperature of a blob to be related to the amount of thermal *energy* in the blob, so sometimes we talk about temperature and energy content interchangeably, related to the **heat capacity** (in units of J K^{-1}), which is the energy required to raise the temperature by a certain amount. For practical purposes, we take the heat capacity of seawater to be a constant, and energy Q (in units of Joules, J), mass m, the **specific heat capacity** c (in units of J kg^{-1} K^{-1}, note the per mass bit) and the change in

[3] The 'spacing' of degrees Celcius and Kelvin are exactly the same, except there is a shift so that $0°$ C $= 273.15$ K. 0 K is known as *absolute zero*.

temperature of a material ΔT is given by

$$Q = mc\Delta T, \tag{2.4}$$

where for seawater[4], $c \approx 3850$ J kg^{-1} K^{-1}. This formula is used when we are talking about *heat content* in the ocean, which is roughly the thermal energy in the ocean (cf. Fig. 1.3). To measure temperature, the obvious thing is to use a *thermometer*. Even though a thermometer would provide a gold standard of sorts, we generally can't do that easily if we want a large spatio-temporal coverage of the ocean. There have been suggestions to use for example the *speed of sound* in seawater as a way to get at the temperature: speed of sound depends on the density of water, and differences in density lead to wave *refractions* (Ch. 6.1) therefore measurable changes in the travel time, from which the density and temperature could be inferred for; see more in Ch. 8 (on *acoustic tomography*).

[4] The specific heat capacity for air is much smaller, hence why it takes so much longer to heat up the kettle for that cup of tea than warming the surrounding air, but also the cup of tea keeps its heat much longer.

With regards to density, we expect that increasing T decreases ρ in pure water. However the rate of increase is not necessarily linear above 4° C, and also the density of water actually *decreases* when it is cooled below around 4° C where water is densest[5], and this essentially points to a *nonlinear* dependence of density in water as well as seawater to temperature; see top panel of Fig. 2.2 and note the turning point (the peak of the graph). We say a bit more about this in Ch. 2.3.

[5] Hence why ice floats.

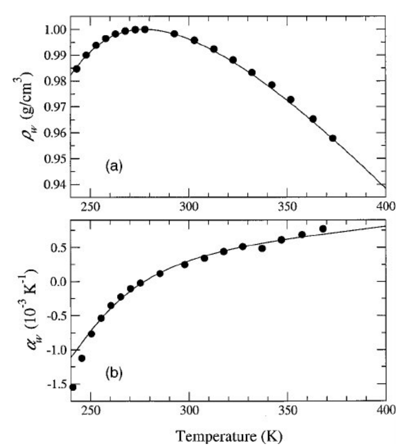

Figure 2.2: (top) density $\rho = \rho(T)$ for pure water, (bot) the coefficient of thermal expansion $\alpha = \alpha(T)$. From Ashbaugh *et al.* (2002), *J. Chem. Phys.*. ref

As already mentioned, one distinguishing feature of seawater is that it is salty[6]. The saltiness derives from the fact there is sodium chloride (NaCl) dissolved in seawater, and salt contributes to the mass and density of the fluid. We denote the **salinity** of seawater by S, which measures how salty or fresh something. The measure of salinity is a concentration and sometimes given by g kg^{-1} (grams of NaCl dissolve in 1 kg of seawater), and note that this is non-dimensional since it is a mass divided by another mass. However you sometimes see salinity given in 'units' of PSU (*practical salinity unit*), because some people think it is weird to not have units tagged on with a measure; the use of PSU is strongly discouraged nowadays.

[6] 'Salt' in chemistry means something slightly more general.

The gold standard of measuring salinity is by measuring the *chlorinity*, i.e. the concentration of chlorine atoms in the water sample. This requires analysing the water sample by, e.g., *titration* against silver nitrate solution to precipitate the chlorine. The resulting processed salinity is known as **absolute salinity** S_A, defined as

$$S_A = 1.80655 \times \text{Chlorinity}. \tag{2.5}$$

As you can probably tell, this is impractical to do on a regular basis (I mean how much silver do you need!?) An alternative is to note that seawater conducts electricity, and the **conductivity** depends on the concentration of salt and hence salinity[7]. Conductivity is relatively speaking much easier to measure, although it is less accurate as strictly speaking the conductivity depends on temperature and pressure as well. Salinity measured this way is usually called **practical salinity** S_p (or just S).

For reference, freshwater tends to have low salinity, on the order of $0.01\ \mathrm{g\ kg^{-1}}$. Seawater tends to have salinity around $35\ \mathrm{g\ kg^{-1}}$, and doesn't vary much (from 33 to 37 or so), though the Med Sea can have salinity up to $40\ \mathrm{g\ kg^{-1}}$. Estuary regions are influenced by rivers as well as oceans so salinities there can vary from 0.5 up to $35\ \mathrm{g\ kg^{-1}}$. For completeness, the Dead Sea has salinities of around $200\ \mathrm{g\ kg^{-1}}$, and that's why they suggest you don't spend too long in there, and definite wash off afterwards, because the saline water dehydrates you quite substantially[8].

There are other things dissolved in water (e.g. chemical *tracers* such as oxygen and dissolved inorganic carbon), but these don't contribute much to the physics so we won't really talk about it (the physics on the other hand is very important for the content). Before we go on, the last major property we will talk about here is **opacity**. Something is *opaque* if light (more accurately, *radiation*; more in a bit) does not travel through the material very well[9]. Air for example is not opaque, because otherwise you would not even be able to see this set of notes. Clouds are opaque because it absorbs/disperses light. Seawater opacity depends on the water condition (e.g. the *turbidity*) but usually it would be regarded as opaque: you usually can't see that deep into the ocean (e.g., Fig. 2.3). This is mostly the reason why you only find *phytoplanktons* above around 200 m depth in the *euphotic zone*, because they need sunlight for *photosynthesis*, and there is not enough light below a certain point.

[7] Fun trivia: Pure water is a very bad electrical conductor because not that many H_2O molecules disassociate into H^+ and OH^-, and free charges are needed to conduct an electric current. In seawater, the dissolved NaCl has their ionic bonds broken by water (because water has a net dipole), resulting in Na^+ and Cl^- charged particles, and therefore a possibility to conduct an electric current.

[8] But is good for cleansing purposes I guess, and it means the muds there are full of minerals, hence it is highly prized from a cosmetics point of view.

[9] Describing things (e.g. a book or someone's speech) say as opaque would then be that something is hard to understand, confusing, or impenetrable, possibly like this set of notes.

Figure 2.3: Picture of the sea. You can't see through it that well so seawater is regarded as opaque. CC0 Public Domain, taken from `phys.org`.

2.2 *Observations and forcing*

The fact that seawater is opaque has important consequences for the ocean. Naturally, a main source of energy on Earth is from the Sun, which is responsible for heating the Earth system and, in turn, driving the winds in the atmosphere that forces the ocean mechanically. However, given sunlight (more precisely, *solar radiation*) doesn't penetrate much into the deeper parts of the ocean we have the set up in Fig. 2.4.

In the atmosphere, because the air allows solar radiation through, the atmosphere may be heated below. Much like boiling a pan of water from below, the fluid at the bottom becomes warmer, less dense than the water above, and overturns, leading to *convection*. Clouds exist because there is convection leading to upward motion of air

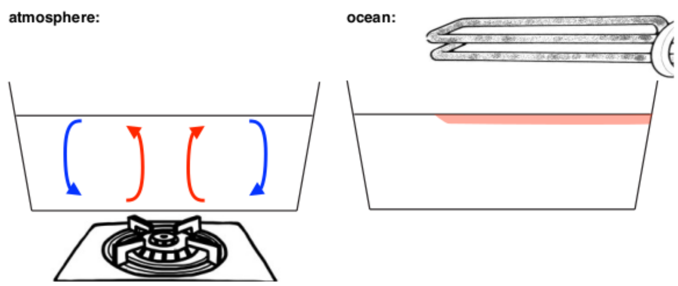

Figure 2.4: Schematic of principal sources of thermal forcing between atmosphere and ocean. Courtesy of David Marshall (Oxford).

carrying moisture up high, so that when the air cools condensation occurs and forms clouds (mostly happening in the *troposphere*). In the ocean, however, since water is opaque, solar radiation gets dispersed easily and the scenario is really like a case where you trying to heat up a pan of water, but from above, which is you may guess is almost impossible[10]! Without some sort of fluid transport the heat basically takes ages to *diffuse* into the interior of the ocean (we are talking about at least thousand year time-scales easily; see Ch. 3.4), and there cannot be significant motion in this kind of set up[11]! Of course despite this inefficient setup, there is in fact a large-scale overturning circulation which can move heat around in the ocean, and we explore some of the reasons for why this is in Ch. 5.

[10] I once tried this when visiting my now wife to boil some dumplings back when she was in her student halls. After 30 mins we gave up and got take-away in the end.

[11] This is related to something now called *Sandstroöm's theorem*, formalised in PaparellaYoung, Nycander.

2.2.1 *Temperature*

To talk about seawater temperature distribution and so on we first make a digression to talk about some physical aspects of **solar radiation**. Energy can generally be transferred in three ways: **conduction**, where a particle hits another particle physically; **convection**, where energy is moved around by a collection of particles; and **radiation**, where energy is transferred by **electromagnetic waves**[12]. The classic examples demonstrating the three is a heater. You could get warmed by the radiator through touching it (conduction, but maybe don't actually do this), by being above the heater and getting the warm air (convection, as heat is transferred through movement of air), or just by sitting slightly away from it (radiation, as **infra-red radiation**). The first two need matter to be present, while radiation does not (hence radiation can travel through space).

[12] These are *photons*, and visible light is a special case. Photons are a bit weird because they are wave-like and particle-like at the same time. See *wave-particle duality* in most *quantum mechanics* books.

The Sun emits all sorts of radiation and these are normally classified depending on their *frequency*/*wavelength*[13]. The types of electromagnetic waves are given in Fig. 2.5. Note that there are many familiar names here, from long radio waves that we use in communication, to microwaves for cooking, infra-red as heat, visible light between 380 to 700 nm (nano-meters, 10^{-9} m), to short and high frequency waves

[13] How often the waves oscillate and how wide two wave crests are apart, which are of course inversely correlated with each other. Measured in Hertz $\mathrm{Hz} = \mathrm{s}^{-1}$ and m respectively. More in Ch. 6.1.

such as ultra-violet (UV) and X-rays.

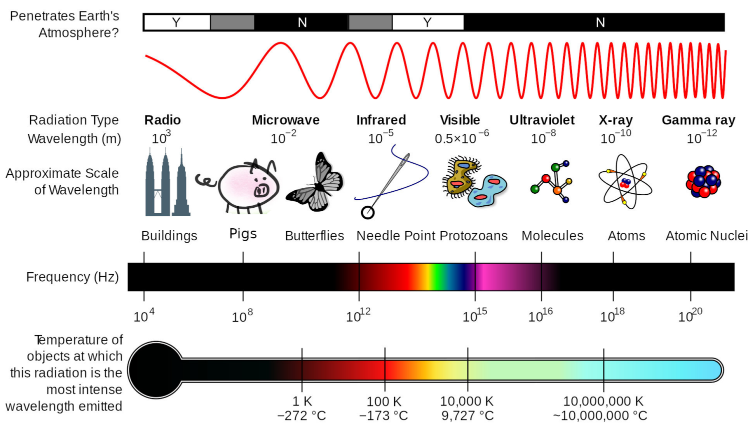

Figure 2.5: The electromagnetic spectrum by wavelength and frequency. Image from Wikipedia, adapted from an image form NASA.

This bit is going to be a bit hand-wavy, but the key take-away is that incoming solar radiation is largely in the form of **shortwave radiation**, energy loss by the ocean is via outgoing **longwave radiation**. If you are eager to get back to explicit talk about the ocean then you can skip the paragraph without losing that much I suppose.

Now, one can imagine that it takes more energy to make the wave oscillate more, so there is more energy in the high frequency waves[14], i.e. the short waves. A body in thermal equilibrium that is hotter emits more radiation, and is able to emit higher frequency radiation; if a cooler body emitted the same type of radiation it would cool down too fast, and thus not be in equilibrium. In fact, with an approximation[15], the kind of radiation and magnitude of radiation a body in thermal equilibrium can emit is uniquely determined by the temperature of the body[16]. For the current day Sun, with a surface temperature of around 6000 K, a good chunk of energy is radiation and thus energy is emitted at the visible and the lower end of the UV spectrum. However, because of the composition of the atmosphere some of the radiation gets reflected or absorbed and then emitted back into space[17], and the main component of solar radiation at the surface of the Earth is *shortwave radiation*. By a similar argument, since the Earth has a temperature, it still emits radiation, but because it is much cooler than the Sun, it emits radiation at a much lower frequency, thus mostly in the form of *longwave radiation*, such as infrared[18].

With that digression, Fig. 2.6 shows a typical profile of **SST** (sea surface temperature) and the incoming shortwave radiation Q_{sr} (in units of W m^{-2}), showing the horizontal distribution overlaid on a map. We also show the *zonally averaged* profile over latitude, defined

[14] This is the *Planck relation*.

[15] The *black body approximation*

[16] This is *Planck's law*, after the German physicist Max Planck (1858-1947).

[17] Look up *absorption spectrum*, and it is mainly ozone (O_3) absorbing the high frequency UV bands, and water vapour absorbing a lot of the remaining incoming solar radiation.

[18] *Greenhouse gases* are then the gases that permit shortwave radiation to pass into Earth, but traps long-wave radiation by absorbing it and re-emitting it back to Earth. 'Fun' trivia: while carbon dioxide CO_2 is talked about so much, the biggest contributor to greenhouse effect on Earth is actually water vapour H_2O. CO_2 is not even that potent a greenhouse gas, but it is a problem because there is so much of it...

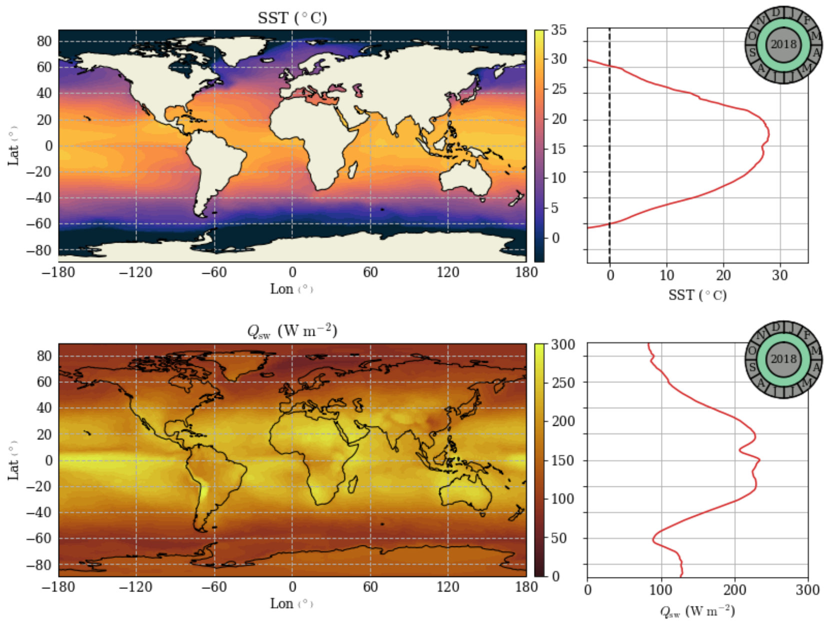

Figure 2.6: Year-averaged (left) and year and zonally averaged (right) sea surface temperature (top) and incoming shortwave radiation (bottom), from the JRA55 dataset (Kobayashi *et al.* 2015, *J. Meteor. Soc. Japan*). See `plot_jra55_sample.ipynb` for code, and `sst_day_avg_2018.mp4` and `rsds_day_avg_2018.mp4` for movies of the analogous daily averaged data through a particular year.

as

$$\overline{f}(y) = \frac{1}{L_x} \int_{0^\circ}^{360^\circ} f(x, y)\ \mathrm{d}x, \tag{2.6}$$

where (x, y) denotes the longitude and latitude, L_x is the circumpolar length, and x here is given in *degrees*[19], i.e. we average stuff in longitude. The data is additionally averaged over a year to show the typical profile. First, note that the SST is largest at the equators at around 30° C and smallest at the high latitudes, dipping below 0°C, which is what you would expect since the equator receives the most heat. Indeed, the $Q_{\rm sr}$ averaged over the year is also largest around the equator and smallest at the poles. In the supplementary movie files `sst_day_avg_2018.mp4` and `rsds_day_avg_2018.mp4`, which shows the daily-averaged data over the year, you can see the **seasonal cycle**, where the peak $Q_{\rm sr}$ starts below the equator, since January is **Austral summer**, i.e. where Southern Hemisphere receives more heating), moves north as the months progress, and is in the Northern Hemisphere by the time we get to **Boreal summer**, i.e. Northern Hemisphere summer, before venturing south again as the months progress. The SST signal follows this behaviour somewhat but the variations are much smaller compared to $Q_{\rm sr}$, and this is again because of seawater has a higher heat capacity (because of the amount of energy that needs to be gained or lost to change the water temperature). The outgoing long wave radiation $Q_{\rm lr}$ is not shown, but is correlated well with the SST evolution (why?)

[19] We should really be using *radians* (180° = 1 rad) for calculations, but we don't do calculations here so stick with degrees.

What about changes of temperature with depth? Fig. 2.7 shows the changes of temperature[20] at fixed longitude and latitude as a function of depth, and red and blue lines denote the (Boreal) summer and winter profiles respectively. There are several features of note:

[20] A cheat and a note here: this is actually *conservative* temperature; more on this later.

- the temperature profile tends to be 'vertical' near the surface particularly in the winter, i.e. the temperature gradient with depth is close to zero, since temperature is barely changing
- in general, there is a region where the where the temperature gradient is large, in a layer between the surface and the deeper parts
- in general, as we get deeper, the gradient decreases again

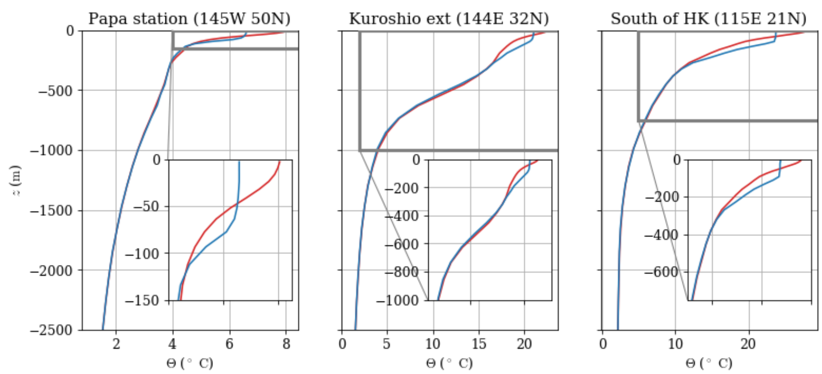

Figure 2.7: Vertical variation of conservative temperature at some designated locations, based on WOA13 data. Red and blue line denote summer and winter climatology. See `plot_WOA13_sample.ipynb`

The exact depths and extent of where the noted features occur at depends on the geographical location[21]. The first observation is of the **mixed layer**, with the winter signal being particularly easy to see. The definitions and measures of the mixed layer varies, but is generally characterised by where the stratification is weak, and is usually located within the top 100 m or so (except in high latitudes during the respective winters). The weak gradients are indicative of strong convective activity (see Ch. 6.2), mixing everything up and eroding the *stratification*[22]. The second roughly denotes the **thermocline**, which is the region where the temperature gradient is the largest in the vertical, i.e. the transition is the fastest, and is located roughly between 200 to 1000 m or so. Below the thermocline is usually regarded as the deeper parts of the ocean, where the temperature gradients are relatively small.

[21] The middle graph shows the Kuroshio, where there is a large gradient in temperature that goes down to around a 1000 m, and this signal is consistent with *thermal wind shear relation* and that the Kuroshio is a WBC (Ch. 5.1.2.)

[22] *Stratification* refers to 'layers'. Although you can think of these as gradients, we usually use stratification when talking about density.

To anticipate the discussion about *watermass properties* when we want to talk about the MOC in Ch. 5.2, Fig. 2.8 shows a **meridional section**[23] of yearly-averaged temperature, in both the Atlantic and Pacific. Temperature is largest at the surface and near the equator, as

[23] Fix a longitude and show data in latitude and depth. *Longitudinal sections* are analogously defined.

expected. There are several features of note, which we will revisit in more detail later:

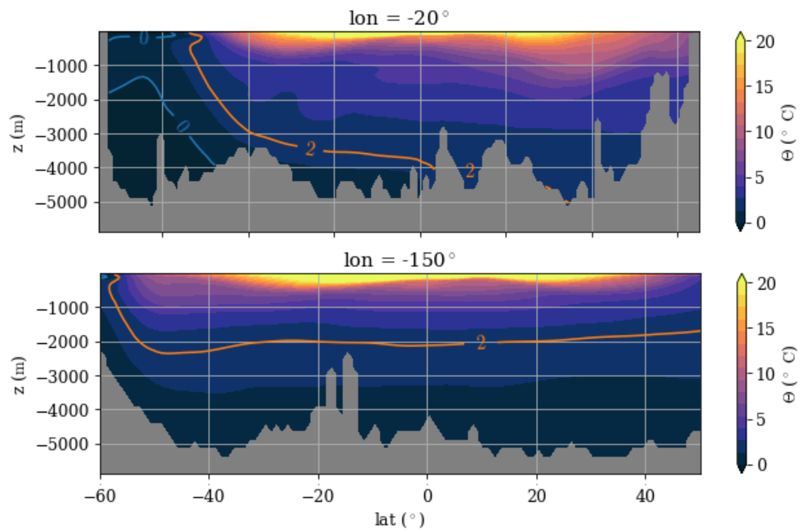

Figure 2.8: Meridional section of yearly-averaged conservative temperature in the Atlantic (top) and Pacific (bot), based on World Ocean Atlas 2013 data. Meridional range chosen to roughly correspond to Talley *et al.* (2011) Fig. 4.11 and 4.12. See `plot_WOA13_sample.ipynb`

- in the Atlantic, there is a hint that the warmer water seems to intrude deeper in the North
- in the Atlantic, the cold water particularly marked by the 2° C and particularly 0° C **isotherm** (lines of constant temperature) at the deep seems to intrude from the South into the basin
- in the Pacific, the aforementioned intrusion of deep cold water seems to be absent (no 0° C isotherm), and the 2° C is much higher in the water column

These are to do with the global MOC and are related to the NADW and AABW watermasses, as well as the pattern of the global MOC; see more in Ch. 5.2. This rather brief discussion is mostly to demonstrate that water formed from various locations could have a signature in observations (Ch. 7), providing a way to identify their origin and how they might change as time goes on.

2.2.2 *Salinity*

Salinity forcing is a bit simpler to talk about, although a caveat here is the total mass of salts in the ocean are effectively conserved, so *salinity forcing* really refers an increase or decrease of salt concentration via *changes in the amount of freshwater content*. Other than that subtlety, incoming shortwave radiation Q_{sr} heats up seawater and causes **evaporation**, but water vapour cannot carry salt with it[24],

[24] Because a solution is needed to break the ionic bonds in salt. No liquid, no keeping ions apart, and salt crystals form again and are left behind. You could try this yourself with saltwater in a pot but be very careful not to over heat the pot!

which leads to an *increase* in salinity. By contrast, **precipitation** such as rain, snow, hail etc. add freshwater into the ocean, thus diluting the solution, and leads to a *decrease* in salinity. The combination is sometimes referred to as **EmP** (evaporation minus precipitation). Besides EmP, **river runoff** and **ice melt** leads to a *decrease* in salinity; correspondingly, growth of *sea ice* actually leads to whats called *brine rejection*, for the same reasons as the note above. It perhaps of interest to note that ocean averaged salinity can change quite drastically on very long time-scales, to do with climate transitions as we go in and out of natural *ice ages*, precisely because ice melt leads to decreases in salinity. Changes in salinity affect the density (and really, density gradients) in the ocean, which has important consequences for the MOC[25].

[25] This is a topic in *paleoclimate*, which we won't touch on here.

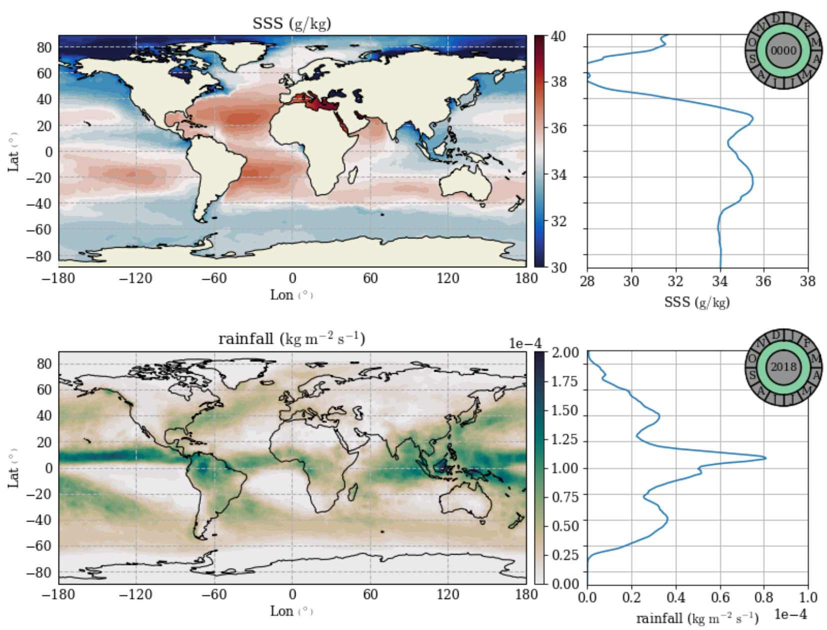

Figure 2.9: Year-averaged (left) and year and zonally averaged (right) sea surface salinity (top) and rainfall (bottom), from the JRA55 dataset (Kobayashi *et al.* 2015, *J. Meteor. Soc. Japan*). See `plot_jra55_sample.ipynb` for code, and `sss_day_avg_2018.mp4` and `prra_day_avg_2018.mp4` for movies of the analogous daily averaged data through a particular year.

There are many fields we can show in relation to salinity but, for brevity, Fig. 2.9 shows the **SSS** (sea surface salinity) and average rainfall (in units of kg m^{-2} s^{-1}), again averaged over the year, and also showing zonal averages. The observation to note here is that SSS is typically large in the surrounding regions around the equator, though not as large generally as the Med Sea. Note however rainfall is also largest at the equatorial regions. This may seem contrary to what we have been talking about but remember it is EmP that ultimately matters. While there the heating is largest at the Equator, leading to large evaporation, large evaporation is also conducive to cloud formation, so precipitation is also largest in the region. For the actual SSS signature, it is EmP that matters, and it turns out EmP is largest in the subtropical regions[26]. One interesting feature to note is that the Atlantic is noticeably more salty than the Pacific, and is largely to do with the MOC[27]; see more in Ch. 5.2.

[26] This is to do with the Hadley cell (mentioned in Ch. 1.3.2) suppressing precipitation but leaving evaporation sizeable. More in Ch. 3.3.

[27] We talked a bit about this in Ch. 1.3.2 and 1.2.3 when talking about Med Sea water overflows and the Aghulas leakage respectively.

In the supplementary movie files `sss_day_avg_2018.mp4` and `prra_day_avg_2018.mp4`, the latter showing the daily-averaged data over the year. You can see the seasonal cycle in the rainfall, but not so much in the SSS. The rainfall not so surprisingly follows Q_{sr} (why?) but the SSS displays smaller variations in the Equator, but is higher in the Arctic, because SSS is influenced by more than just EmP[28] The Atlantic is still saltier than the Pacific in general.

[28] Also I cheated by showing monthly climatology instead of daily-averaged, which is what I had at hand.

In Fig. 2.10 we show the vertical profile of salinity at some specific locations (not the same ones as Fig. 2.7), with red and blue lines denoting the (appropriate) summer and winter profiles. Note that we have similar features as in Fig. 2.7, although the winter signal is not so different to the summer signal. The equivalent of the thermocline for salinity is called the **halocline**. The deeper parts of the ocean again have weaker gradients, except in the Pacific (left column), where salinity is actually higher in the deep than in the intermediate layers. Unlike temperature this is ok: density increases with salinity, so the increase in salinity with depth denotes a stable stratification. The increase salinity as we go up the water column does not denote an unstable stratification however (why?)

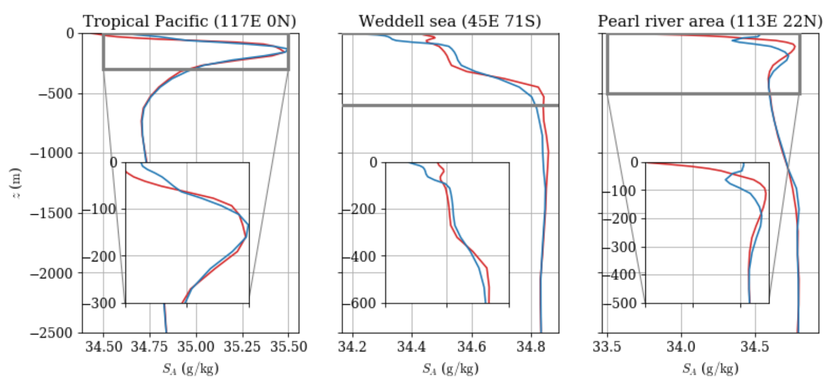

Figure 2.10: Vertical variation of absolute salinity at some designated locations, based on WOA13 data. Red and blue line denote summer and winter climatology. See `plot_WOA13_sample.ipynb`

Fig. 2.11 shows meridional sections of salinity, showing in particular the 35 and 36.4 **isohalines** (lines of constant salinity). As in the discussion relating to Fig. 2.8, we show this here in anticipation of the discussion on watermass properties when we talk about the MOC in Ch. 5.2. Previously we highlighted how there seems to be a warm intrusion at the North of the Atlantic. We see here that this is a warm *and* salty signal, and the North Atlantic water intrusion is much more obvious with the large blob of high-ish salinity at around

1000 m depth, as well as showing the intrusion in the deep into the Southern Ocean between 2000 and 4000 m depth. By comparison, there is a relatively fresher Southern Ocean intrusion into the Atlantic at intermediate depths (the blue tongue just above 1000 m) and also in the abyss. These signals correspond respectively to the NADW, AAIW and AABW watermasses respectively; see Ch. 5.2. In the Pacific there is also a tongue of freshwer seawater, again relating to the AAIW, and in this diagram we note that that salinity increases with depth in the Pacific from around a few hundred meters depth, which is consistent with the vertical profiles in Fig. 2.10(*a*).

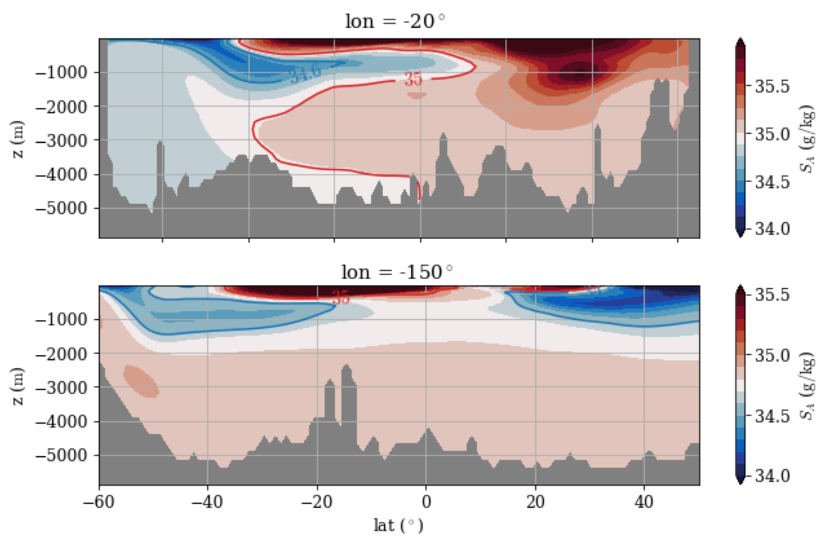

Figure 2.11: Meridional section of yearly-averaged absolute salinity in the Atlantic (top) and Pacific (bot), based on World Ocean Atlas 2013 data. Meridional range chosen to roughly correspond to Talley *et al.* (2011) Fig. 4.11 and 4.12. See `plot_WOA13_sample.ipynb`

2.3 *Density and equation of state (EOS)*

To recap, the density ρ of seawater depends on temperature T, salinity S, and technically on seawater pressure[29] p as well. The exact functional relation between ρ, T, S and p is called the **equation of state** (EOS). Before we go into details relating to the EOS, we motivate the discussion a bit more by talking about some observational details.

Fig. 2.12 shows the yearly-averaged sea surface density (actually showing $\sigma = \rho - 1000$ kg m^{-3} to save on writing so many digits), as well as the yearly and zonally-averaged SST and SSS. A fact to observe is that, while it looks like there is substantial variation, numerically it turns out over most of the ocean, the density[30] varies by no more than around 2% from a reference value of $\rho_0 = 1026$ kg m^{-3}. Of course numerically small doesn't mean it is dynamically small, and these minor differences have significant influences on the dynamics.

[29] But from a dynamical point of view we will want to get rid of the pressure dependence.

[30] I am deliberately being vague about the type of density being used (related to above note). The statement really refers to potential and *not* in-situ density.

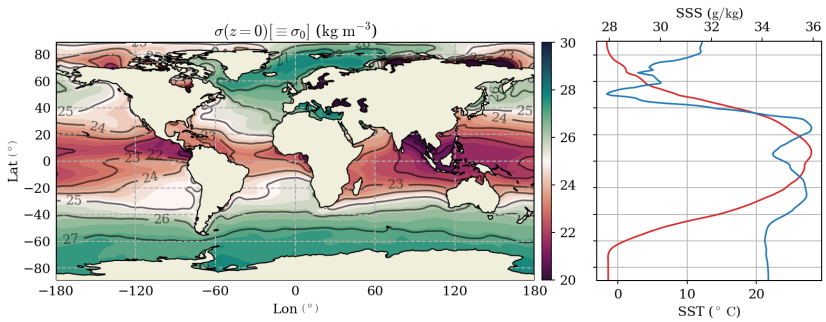

Figure 2.12: (left) Density at the surface (in-situ or referenced to sea surface) and (right) zonal averaged SST (red) and SSS (blue). Year-averaged data based on World Ocean Atlas 2013. See `plot_eos.ipynb`

Thus one point we note is that you want an accurate EOS, otherwise the dynamics and its resulting consequences could be substantially different. The second is that there is competition between T and S contribution to density, as seen in the correlations between the density and the SST and SSS graphs. Broadly speaking increase in T decreases density, whilst increase in S increases density. The more subtle question to ask is the magnitude of control of T and S on density, and where in the globe is one contribution more important than the other[31]

[31] Spoiler: over most the ocean it is temperature that is important contributor to density.

Just some more pieces of important terminology that will be frequently used in this document (more so than the ones relating to temperature and salinity actually). **Isopycnals** refer to the lines of constant density (cf. isotherms, isohalines, isobars in Ch. 3.1.2). The **pycnocline** is the equivalent of thermocline and halocline for density, i.e. where the change in density is largest, and again roughly delineates the upper and lower parts of the ocean (look ahead to Fig. 2.15 if you would like an example now). Two terms that will be increasingly used from Ch. 3.4 onwards is **along-isopycnal** and **diapycnal** (across-isopycnal) motion and mixing. It is the along-isopycnal and diapycnal components that are relevant for dynamics, rather than horizontal and vertical. Roughly, this is because motion across isopycnals requires moving the isopycnals, and this is doing *work* against buoyancy, while you don't need to do that if you go along isopyncals[32]. For example, while the ocean basin interior has relatively flat isopycnals so along-isopycnal and diapyncal happens to be horizontal and vertical, this is not true in the case of the Southern Ocean where the isopycnals are tilted (see Ch. 5.1).

[32] Think walking up stairs compared to walking on flat land. It is very tiring working against gravity! Blame Newton for this.

2.3.1 *Linear EOS*

For the EOS, we note that, since ρ is positively and negatively correlated with T and S respectively over a large part of the ocean, we could try

$$\rho \sim \beta S, \qquad \rho \sim -\alpha T,$$

where we take $\alpha, \beta \geq 0$. The simplest thing to do with would be $\rho \sim -\alpha T + \beta S$, but this can go negative, so we want to add a few more things in. The standard way of doing it is actually

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta(S - S_0)], \qquad \alpha, \beta \geq 0, \tag{2.7}$$

where ρ_0, T_0 and S_0 are references that you choose depending on what you care about (see next paragraph). This is known as a **linear EOS**, because ρ depends on the arguments T and S linearly (notice there is no p dependence here). The coefficients α and β are respectively known as the **coefficient of thermal expansion** (in units of $^\circ\mathrm{C}^{-1}$) and **coefficient of haline contraction** (in units of g^{-1} kg)[33]. The way you use the linear EOS (2.7) is to choose the reference and parameters values, chose a T and S, put the numbers in, and you get a number back that tells you the density. That it! You can do this in a calculator or Excel.

[33] The name is because in material sciences the normal talk is thermal expansion (decrease in density), so for consistency we have haline contraction for talking about *decrease* in density.

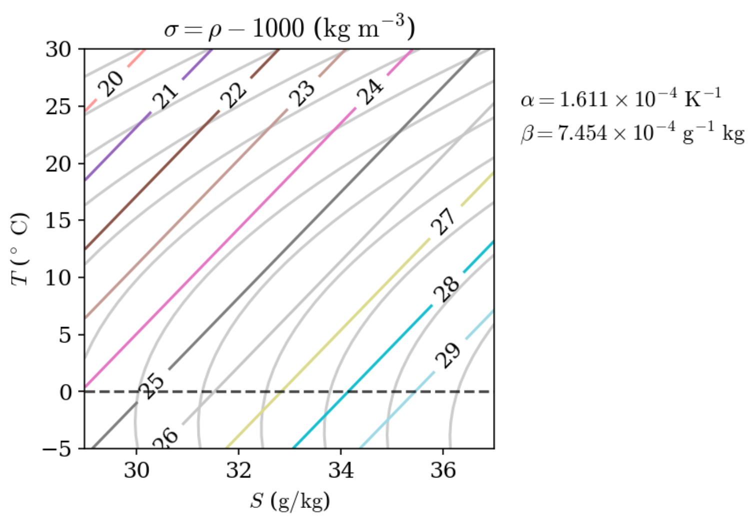

Figure 2.13: Linear EOS in TS space with TEOS10 as gray contours (same contour levels). The reference values used are $T_0 = 10$ and $S_0 = 35$ in the usual units, as used in the NEMO ocean model [Madec, 2008]. See `plot_eos.ipynb`

To fully explore how ρ depends on T and S, I decided I really don't want to use Excel or do calculations one by one on a calculator (people used to!), because ain't nobody got time for learning Excel syntax. A few incantations in your favourite programming language later (I used Python here), the linear EOS over **TS space** is shown in Fig. 2.13; this is plotted on top of the "real" EOS of the ocean[34]. You can convince yourself that if you increase T or S the density is going the right way, so qualitatively speaking the right thing is happening. However, note that the agreement in terms of the sensitivities (the

[34] Of course the world is not nice enough to give us a linear EOS.... By the way, the "real" EOS used here is actually the *TEOS-10* standard, which is a model fitted from data (more later); we actually don't have an analytical form for the real EOS of the ocean, and not for a lack of trying.

contour slopes) is only reasonable at isolated locations, for example around the reference values $T_0 = 10$ and $S_0 = 35$ in the usual units[35], but is pretty bad in the upper left part of TS space. The linear EOS could be regarded as a leading order expansion of the "real" EOS about the reference, so it is maybe a reasonable approximation around reference, but there is no reason it is good away from the reference values (and indeed it isn't). However, if you don't think dynamical phenomena associated with nonlinear EOS is particularly important (e.g. away from regions where *deep convection* occurs, or near places where phase transitions happen, such as ice regions), then linear EOS is probably ok to use.

[35] I'll probably start dropping units unless it is necessary for the prose or the discussion.

Another thing before we move on is that when we choose a reference (T_0, S_0), we also need to choose appropriate values for α and β so the resulting slopes in TS space does not deviate too much from the "real" EOS; this highlights another complication that the "real" α and β themselves are dependent on T and S, and indeed they do! This is perhaps not unexpected: recall that water is densest at around 4° C from the discussion near the beginning of this chapter, and is also seen in the bottom panel of Fig. 2.2.

2.3.2 *Beyond linear EOS*

Although we know that we should not take the α and β parameters to be constant, a possible exercise to consider is whether we could get something reasonably simple but that is still a reasonable approximation to the "real" EOS. Roquet et al 2015 eq 15 suggests something like

$$\rho = \rho_0 \left[1 - \alpha\left(T_a + \frac{\lambda_1}{2}T_a^2\right) + \beta\left(S_a - \frac{\lambda_2}{2}S_a^2\right) - \nu T_a S_a\right], \qquad (2.8)$$

where for cleanliness $T_a = T - T_0$ and $S_a = S - S_0$ are the temperature and salinity *anomalies*. The red terms in (2.8) now highlight the *quadratic nonlinearities*, but otherwise the coefficients α, β, $\lambda_{1,2}$ and ν are constant coefficients[36]. Fig. 2.14 shows how this nonlinear EOS (2.8) compares with the "real" EOS in TS space. With the appropriately chosen coefficients, the general agreement is actually pretty reasonable, except when the temperature is dipping below around 0° C, which is around when ice might start to form.

[36] The new coefficients are known as the *cabbeling coefficients*. I am skipped the *thermobaric coefficients*, which has an extra z term denoting depth dependence. Skipping the units here because this is just a side digression.

As mentioned in passing in a note above, we don't actually have a form of the "real" EOS derived from first principles. What we do have are very good approximations constrained by many seawater samples, which allows a *regression* (fitting) of a model to the data[37]. The "real" EOS is so important that there are UNESCO working groups dealing withe the relevant standards, because the density of fluids around us (air, water and seawater) leads to so many phenom-

[37] Not dissimilar to the *data driven methods* that is the trend these days.

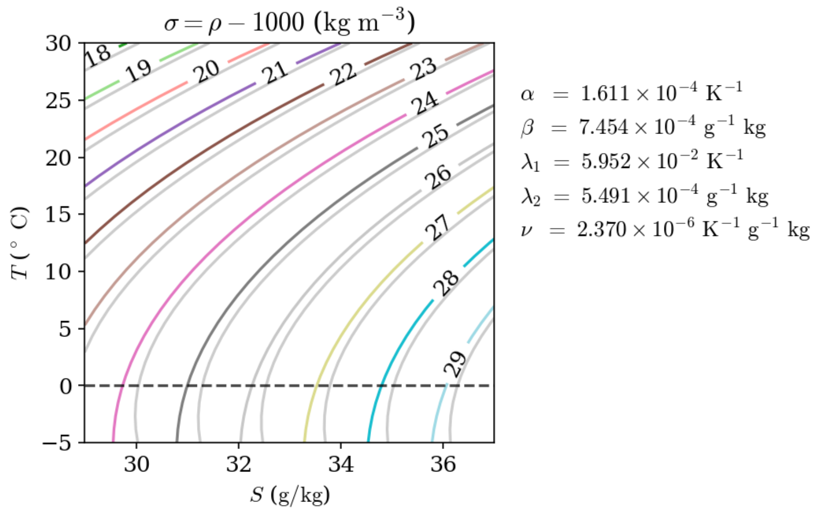

Figure 2.14: Toy nonlinear EOS (no *thermobaric effect*) in TS space with TEOS10 as gray contours (same contour levels). See `plot_eos.ipynb`

ena that directly affects our livelihood. The "real" EOS has evolved over the years, from the EOS-80 standard UNESCO 1983, Bryden 73 which for seawater uses practical salinity and *potential temperature* inputs, to the current TEOS-10 (Thermodynamic Equation of Sea-Water 2010) standard, which for seawater uses absolute salinity and *conservative temperature* as inputs. One important difference is that in the older EOS-80, the thermodynamic quantities are not entirely consistent with each other, while this is fixed in TEOS-10 through taking a Gibbs function[38] approach, such that thermodynamic quantities of interest may be calculated all from the Gibbs function. It is also more sensible to link up conservative temperature needed in TEOS-10 with heat content (cf. Eq. (2.4) and Fig. 1.3), related to subtleties that is beyond the scope here. More on potential and conservative temperature in the next section. For completeness, the TEOS-10 EOS formula used to in Fig. 2.13 and 2.14 is the 75 term polynomial approximation given in Rouqet et al 2015, which is computationally cheaper than evaluating directly from the TEOS-10 Gibbs' free energy (which makes a difference when it is used in a high resolution numerical ocean model).

[38] Or, more formally, the *specific Gibbs free energy*, a fundamental quantity in *statistical mechanics*. Named after the American physical chemist Josiah Willard Gibbs (1839-1903), who made fundamental contributions to thermodynamics, statistical mechanics (he coined this term), and developed modern day vector calculus (independently of Oliver Heaviside).

2.3.3 *The problem with in-situ density*

Right, finally, lets talk about why we mostly don't care about in-situ density if we are considering dynamics, as advertised at the beginning of this chapter, and alluded throughout this chapter where there are repeated undefined references to *potential* / *conservative* temperature and *potential* density. This following discussion was never that intuitive to me (I don't have a very good affinity with *thermodynamics*), but lets give this a go.

Maybe lets start with *how* in-situ density fails in an example before we go onto *why* it fails. Fig. 2.15 shows a meridional section and a vertical profile in the Atlantic for sake of choosing something, and the top panel and the blue line shows **in-situ density** in the form $\sigma = \rho - 1000$, which is the density a water parcel would have if you measured its density *at* the depth where you collect the water sample, with $\rho = \rho(T, S, p)$. The first thing we note that is density is increasing dramatically with depth. From the in-situ density data, we have to conclude there is basically no net up and down transport of water, because the ocean is so stably stratified, so there is a negligible MOC. It would also imply there is no strong transport in the Southern Ocean (from *thermal wind shear relation*, see Ch. 5.1.2).

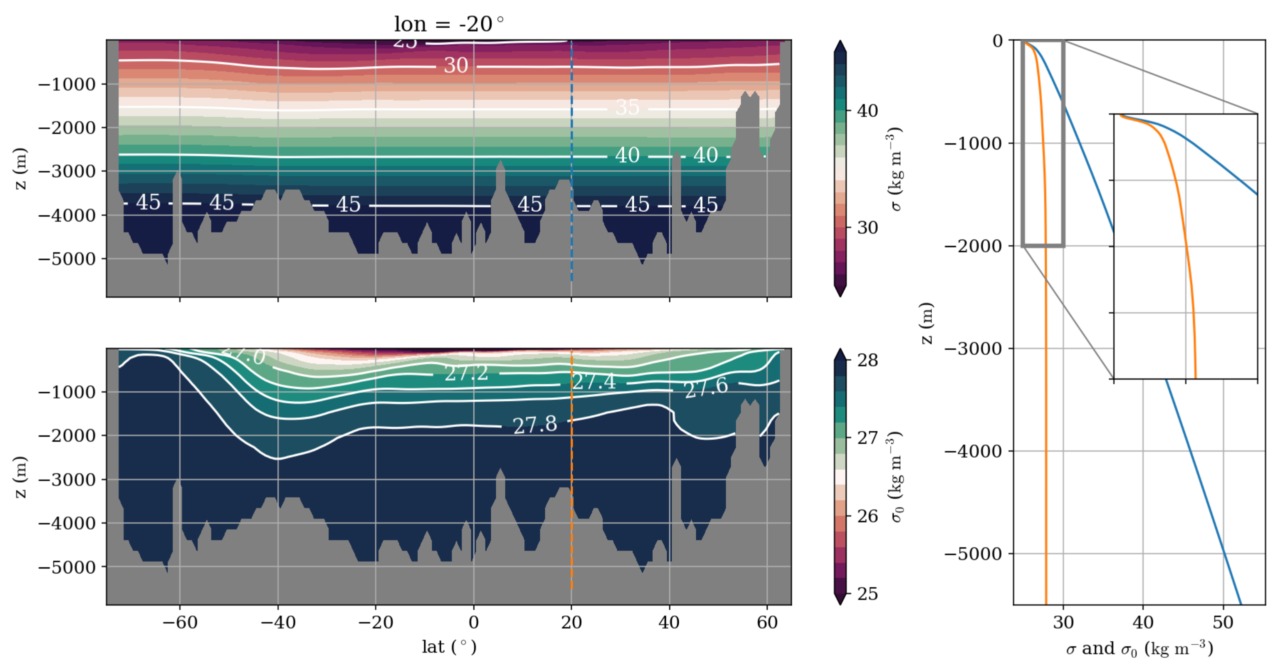

Figure 2.15: Meridional section in the Atlantic of (top left) in-situ density and (bot left) potential density referenced to sea level, with the corresponding vertical profiles plotted (right). See `plot_eos.ipynb`

But we know that is wrong! We know there is up and down transport of water in the ocean, for example in Fig. 2.11 in the salinity signature where there is clear signal of water being dragged down from the surface into the interior in the North and South Atlantic. We know there is a strong ACC taking around 130 Sv of water around the globe, which means we should see tilting isopycnals. Another thing to observe is that below the thermocline and haloclines in Fig. 2.7 and 2.10, the temperature and salinity is relatively constant, but there is a fairly substantial increase in the in-situ density, indicating something else is influencing the density. We do have a MOC, and there are additional signatures in other chemical tracers that serve as a *proxy* for age of watermass last in contact with the atmosphere (e.g. radioactive carbon), but I think this is enough evidence to point to in-situ density being an unsuitable measure for density from a circulation point of view, certainly when the deep ocean is involved. By contrast, the bottom panel and orange line in Fig. 2.15 shows the *potential density* (referenced to the sea surface), and lo and behold! Everything seems to be consistent again, where we have a pycnocline, the deeper parts where the ocean stratification decreases to a constant, tilting of isopycnals in the Southern Ocean, and so forth. So what is this potential density, and what is going on?

You might have guessed, the culprit is pressure p, and potential density is a measure that removes *some* (not all) contributions from pressure to the density. So the complication with seawater, while not as 'squishy' as air, is still *compressible*. Seawater remember is very heavy ($\rho_0 = 1026$ kg m^{-3}), so if you imagine a parcel of water below the sea surface, the pressure pressing down on the parcel would be approximately related the amount of water above the parcel[39], so naturally there is a squashing of the water parcel below the ocean surface.

[39] This is the *hydrostatic approximation*; see Ch. 3.1.2.

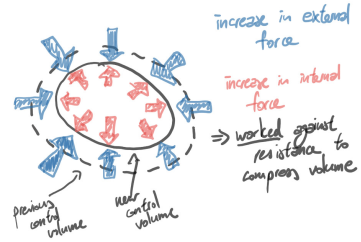

Figure 2.16: Working to compress a volume.

One concept we need to introduce is that of **work done**. Suppose we have a water parcel at some depth such as in Fig. 2.16. The inward and outward pointing forces balance (so there is no net motion), and the equilibrium position is given by the black dashed line. The inward pointing forces would be external pressure, and outward pointing force would be some internal pressure of the fluid which is trying to resist being squashed. Then, to squash the parcel further, we need to increase the external forces (assuming here the outward force is just whatever the parcel needs to do to resist being squashed), but to do this, we need to put **energy** into the system, i.e. by doing work. If we did this, we *added* to the internal energy of the parcel, because we put work into the system, and we also decreased the volume. If we regard temperature and energy as related (e.g. Eq. 2.4), then the parcel should also experience an increase in temperature.

Now contrast this parcel where we put work in, with one where it was hypothetically moved to a deeper part of the ocean without us putting work in or it exchanging mass or heat with the surroundings. Now, the internal energy content is exactly the same, by the assumption of us not having done any work on it. However, the natural external pressure is larger, so this water parcel volume decreases. Because the volume has decreased but internal energy is the same, the temperature as measured of this water parcel (the **in-**

situ temperature) *increases*! Additionally, we have the same amount of 'stuff' but the volume decreases, so the *in-situ* density increases! BUT THERE IS NO CHANGE IN INTERNAL ENERGY, AND NOTHING HAS BEEN 'DONE' TO THE WATER PARCEL SO TO SPEAK!

This effect is particularly noticeable in the in-situ temperature signal when we go deep into the ocean. Fig. 2.17 shows the in-situ temperature T and potential temperature θ profile with depth in the Mariana Trench in the Pacific, which goes beyond 10,000 m depth. Note that the T profile after a certain point *increases* with depth, denoting unstable stratification, and we might expect to see overturns that erodes these unstable gradients, so we shouldn't even be seeing these things in the first place! This kind of configuration surely cannot exist, unless there is an accompanying increasing in salinity (there isn't). By contrast, the use of θ gets rid of this effect, and indicats there is a weak stratification at depth at least in terms of temperature.

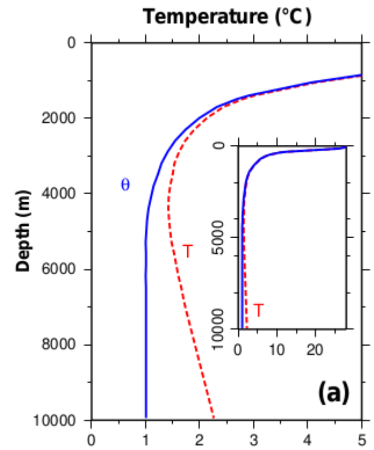

Figure 2.17: Vertical profile of in-situ (red) and potential temperature (blue) in the Mariana Trench to highlight the differences. From Talley *et al.* (2011) Fig 4.10(*a*).

When we are talking about dynamics, we are interested in forces, and forces in particular do work. Thus we need a way to distinguish quantities that are dynamically relevant from the point of view of work done, and get rid of misleading and non-dynamical effects arising from natural pressure, such as those arising when we are considering in-situ temperature and density.

2.3.4 *Potential density and neutral density*

When we are talking about quantities that are potential we need to define a *reference depth* (and therefore a *reference pressure*). **Potential temperature** θ of a water parcel is the temperature this parcel would have if you went to the depth of that water parcel, put it in a plastic bag and seal it, move it the reference depth/pressure, and measure the temperature of that bagged up water parcel has at that depth/pressure after it has relaxed naturally[40]; see schematic in Fig. 2.18. Usually sea surface is used as the reference depth, but in principle other depths maybe used.

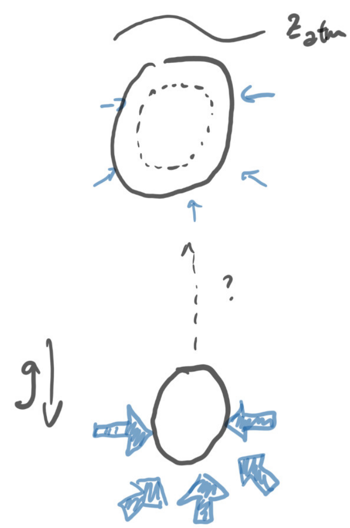

Figure 2.18: Natural relaxation, volume change, but no work done necessarily.

[40] More accurately we consider moving the parcel to some other depth *adiabatically*, i.e., without exchange of mass and heat with the surrounding.

Potential density referenced to the sea surface ρ_θ would be the density computed from the appropriate EOS with the potential temperature θ, and the reference depth and pressure used to compute θ, i.e. if $P_{\rm atm}$ is used to compute θ, then $\rho_\theta = \rho(P_{\rm atm}, \theta, S)$, in contrast to $\rho = \rho(p, T, S)$. Depending on the application, $\rho_{1,2,3,4}$ are also seen if the reference depths (and the related pressures) of 1/2/3/4000 m are used. Examples are seen in Fig. 2.19. The choice of reference makes a difference depending on the ocean depth of focus, as potential density doesn't remove all pressure contributions. Compare this with the linear EOS as a leading order approximation near the reference:

the method is good near the reference but is not as good elsewhere.

The **neutral density** denoted γ^n can be thought of as the continuous analogue of potential density, where the references are all local. Then the idea is that at every point in the ocean over a very small region there is a direction that the parcel can move without experiencing any buoyancy forces, so this is the along-isopyncal direction (or *epineutral* direction). So one would think we can construct these **neutral surfaces** and water parcels have to move along these surfaces. Unfortunately it turns out you can prove these surfaces generally don't exist! Saying that, you can get very good approximations to them, and in practice it is these approximations that are used to compute the 'neutral density'. There are existing algorithms to work these out, and sometimes (approximate forms of) γ^n are used to identify watermasses (see Ch. 5.2.1).

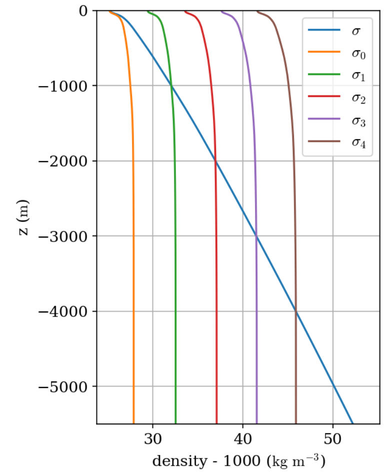

Figure 2.19: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

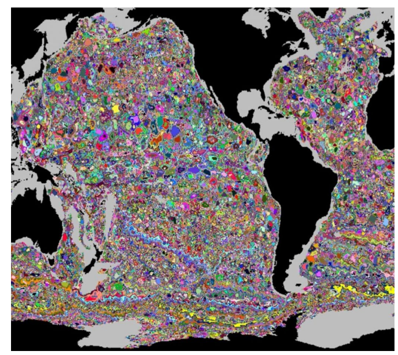

Figure 2.20: Topobaric surfaces, an almost neutral surface. From Stanley (2019), *Ocean. Modell.*, Figure 4.

Summary and further reading

We close this section by noting again that, FROM A DYNAMICAL POINT OF VIEW, IT'S ALMOST NEVER IN-SITU DENSITY YOU CARE ABOUT unless you are really close to the sea surface, otherwise the irrelevant compressibility effects arising from pressure needs to be removed for the resulting data to tell you relevant things.

Science and scientific standards are continually evolving, as more research is carried out, sometimes meaning what we might have treated as gospel truth is, in fact, not, and we can potentially do a bit better. For example, for a long time it was international standard to use practical salinity and it was a good idea to append units to something that was non-dimensional (GET RID OF THE PSU!) Now this is no longer true, as practical salinity is being recommended to be phased out and replaced with absolute salinity; this is not to say we stop measuring salinity by conductivity, but that we use different formulas to compute the resulting salinity[41]. It turns out also potential temperature is a better variable to use than in-situ temperature as it gets rid of pressure effects, but it is not as good compared to **conservative temperature**, which is a more appropriate temperature-like variable to describe ocean heat content McDougall (2003). It is conservative temperature and absolute salinity that are now adopted as the go-to variables as part of the TEOS-10 standard. Neutral density for a long time was touted as the best thing, and increasingly there are alternative ideas and/or better and more efficient approximations coming through (e.g. *topobaric surfaces* from Stanley (2019), *Ocean. Modell.*, in Fig. 2.20).

[41] The better practice really is to record the raw data, from which processed data may be derived, and having a trail leading back to the primary data, which is what is being advocated.

Chapter exercises

1. What are the units of buoyancy $b = -(\rho/\rho_0)g$?

2. Why is the statement "*a pig weighs around 200 kilograms*" not technically correct?

3. Look up the specific heat capacity of air at around room temperature (300 K say) and compare this with the specific heat capacity of water. Is this what you expected? if you put the amount of energy that it takes to warm up 1 kg of seawater by 1 K, how much would 1 kg of air roughly warm up by?

4. For a further challenge, make an estimate of how much volume 1 kg of air occupies, estimate the volume of the atmosphere, work out the amount of energy it takes to raise the whole atmosphere by 1 K (could just do the troposphere for simplicity), and the volume of sea water that could be raised 1 K by this amount of energy. Given this volume and that we know the area the ocean covers, what is the associated depth?

5. Chemistry one: how do you expect conductivity (and hence salinity measurements) to depend on temperature? Justify your answer.

6. Explain why we expect outgoing long-wave radiation Q_{lr} to correlate well with SST patterns.

7. In Fig. 2.10 we see that at certain locations salinity increases as we go up from the bottom towards sea surface. Why does this not necessarily imply an unstable density stratification?

8. In the linear EOS used in Eq. 2.7 as written, how should we interpret the density, as in-situ, potential, or some others? Does it matter? What about for the nonlinear EOS in Eq. 2.7 as written? What about in the same nonlinear EOS Eq. 2.7 but with the thermobaric terms added in (you'll need to look this up in Roquet et al 2015 eq 15)?

9. Make plots of the EOS diagrams yourself and explore the dependencies to parameters accordingly. Use some of the provided code on the GitHub repository as a reference if you like.

3 Mechanical forcing

The last chapter dealt with things that affect T, S and ρ accordingly. Here in this chapter we focus on the mechanical forcing terms that affect the momentum equation, given by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F_u} + \boldsymbol{D_u} \tag{3.1a}$$

$$\frac{\partial p}{\partial z} = -\rho g. \tag{3.1b}$$

We have omitted $\nabla \cdot \boldsymbol{u}_3 = 0$ because that should be thought of as a constraint of mass conservation, i.e. no creation or destruction of volume if the flow is non-divergent with no sources and sinks. Recall that $\boldsymbol{u} \cdot \nabla T$ referred to the advection term for temperature so, similarly, $\boldsymbol{u} \cdot \nabla \boldsymbol{u}$ refers to the self-advection of a fluid parcel and is called the **inertia** term. A major difference here is that $\boldsymbol{u} \cdot \nabla \boldsymbol{u}$ is *nonlinear*, and is a major source of why fluid dynamics generally is so hard/interesting, as this term leads to phenomena associated with *turbulence*. We will not touch on this term too much in this document, as most things we talk about invariably tries to get rid of or approximate this term in some way.

3.1 Gravity and pressure

Given we just talked about the thermodynamic variables, the first thing we talk about is

$$\frac{\partial p}{\partial z} = -\rho g.$$

This actually comes from the vertical component of the approximated momentum equation. The notable term here is g, which leads us to talk about *gravity*. Gravity is important because buoyancy $b = -g\rho/\rho_0$ needs gravity, and buoyancy as argued in Ch. 2 places a strong constraint on the dynamics (because working against gravity is unfavourable in terms of *work*). Gravity is also important for *static instabilities* (Ch. 6.2) and *tides* (Ch. 6.3), so we spend a little bit more time on outlining some related concepts.

3.1.1 *Gravity*

One of the biggest successes of Newton's formulation of mechanics was for explaining how the heavenly bodies moved relative to each other, via **gravity**, but the theory of gravitational attraction between bodies with mass is general. Newton's formulation of gravity has that the force arising from gravitational attraction between two bodies of mass m_1 and m_2 is given by

Recall from Ch. 2 that mass is a scalar and is how much 'stuff' a body possesses. Weight on the other hand is a vector because it is a force, related to gravity and mass.

$$F = G\frac{m_1 m_2}{r^2}, \tag{3.2}$$

where $G = 6.674 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$ is the *gravitational constant*, and r is the distance of separation between the two bodies. If we have multiple bodies we just add up the forces for each pair and proceed, but we will stick with two bodies for the discussion. A schematic of this is given in Fig. 3.1.

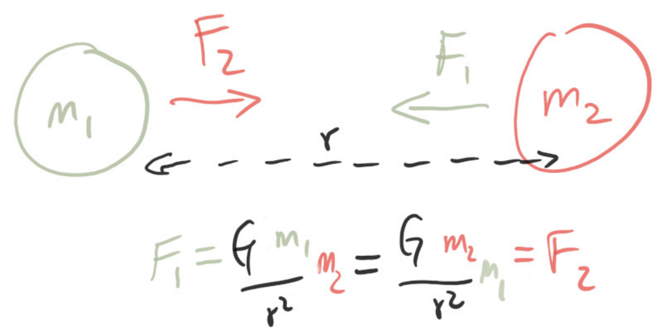

Figure 3.1: Schematic of gravitational attraction for two (supposed to be) spherical masses. If $m_1 \gg m_2$ (e.g. Earth and a pig) then forces on each body are equal, but its effect on one the pig is much larger than it is for the Earth (recall $F = ma$).

Three things to note about gravity are that:

- gravity is purely *attractive* (cf. *magnetism* that can attract and repel)
- the force goes like $1/r^2$, i.e. the force is weak at large distances
- there is an equal and opposite force (Newton's 3rd law): the larger mass attracts the smaller mass, and vice-versa

So where does g, the magnitude of **gravitational acceleration** come from? Taking an example, consider our friendly neighbourhood pig again as in Fig. 3.2. We group the terms in Eq. (3.2) as

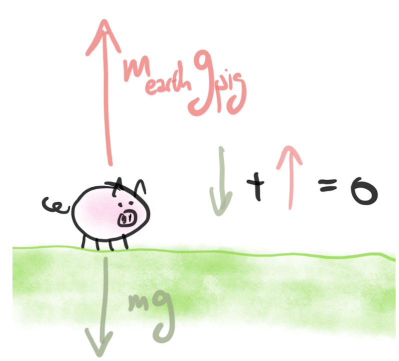

Figure 3.2: Gravity as applied on Earth on the friendly pig.

$$F = m_{\mathrm{pig}}\left(G\frac{m_{\mathrm{earth}}}{r^2_{\mathrm{earth}}}\right) = m_{\mathrm{pig}} g, \tag{3.3}$$

i.e. g is the magnitude of acceleration towards Earth that the Earth causes a body (in this case a pig) to have. If we be a bit cavalier with degrees of accuracy and take $G \approx 6 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}$, $m_{\mathrm{earth}} \approx$

6×10^{24} kg, and $r_{\mathrm{earth}} \approx 6400$ km $\approx 6 \times 10^{6}$ m, then (check the units agree)

$$\begin{aligned} F &= m_{\mathrm{pig}} 6 \times 10^{-11} \frac{6 \times 10^{24}}{(6 \times 10^{6})^2} \\ &= m_{\mathrm{pig}} \frac{6^2}{6^2} \times 10^{-11+24-12} \\ &= m_{\mathrm{pig}} 10 \\ &\equiv m_{\mathrm{pig}} g, \end{aligned}$$

i.e. $g = 10$ m s^{-2}, which is an approximation often used in standard mechanics exercises to keep the numbers nicer (in reality $g \approx 9.81$ m s^{-2}). The weight of the pig on Earth is defined by F. If the pig is on the moon (lets assume for animal rights reasons it is wearing a space suit of negligible mass), then the mass of the pig would still be m_{pig}, but the pig's weight on the moon would be about 1/6 of its value on Earth (because g_{moon} is about a 1/6 smaller).

The schematic drawn in Fig. 3.1 assumes spherical bodies of uniform mass, and the Earth satisfies neither of those conditions! It is not spherical because it is spinning about an axis[1], and it is certainly not a body of uniform mass. An exaggerated version of how the Earth's mass is distributed is given in Fig. 3.3, which shows the variation in the *geoid height*. You can think of where the bulges are to be where there is more mass.

[1] So the mass is 'flung out' near the equator, and is closer to an ellipsoid (see purple dashed line of Fig. 3.4), sometimes modelled as what's called a *Maclaurin spheriod*. After the Scottish mathematician Colin Maclaurin (1698-1746), who was a colleague of Newton. Until 2008 he held the record of being the youngest person to ever hold a professorship.

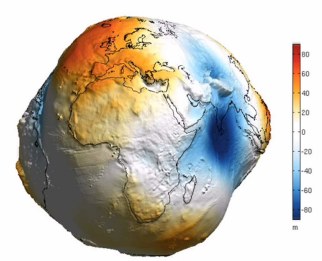

Figure 3.3: The "lumpy potato" Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

To define the geoid height properly we need to say what the **geoid** is first. So I personally always thought I knew what the geoid was, which is *the surface that gravity is perpendicular to everywhere*, i.e. the red line in Fig. 3.4, with the orange vectors (known as *plumb lines*) denoting $\boldsymbol{g}$. So it turns out this is not quite right, and what I just described is what is called an **equipotential surface**. Gravity is what is called a *potential force*, i.e. you can write $\boldsymbol{g} = -\nabla\phi$ for some ϕ (this is called the **geopotential**), and what I just described is a surface of $\phi =$ constant. Immediately you can argue on why the definition I was using doesn't quite work, because "the" geoid implies one, but there are an infinite number of geopotential surfaces depending on the constants I choose for $\phi =$ constant (I can shift the red line in Fig. 3.4 up in height accordingly and I can still get $\boldsymbol{g}$ to be perpendicular to it). So while the geoid is certain an equipotential surface, the actual definition is of the geoid is that *the shape that the ocean surface would take under the influence of the gravity and rotation of Earth alone, if other influences such as winds and tides were absent*. This definition certainly implies there should only be one such surface, but I don't know about you, but I think the one I've been working with is easier to remember...

So the **geoid height** is the signed deviation between the geoid and

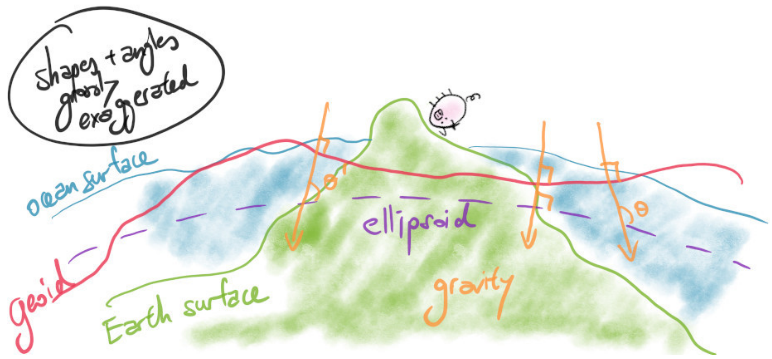

Figure 3.4: Schematic of the ellipsoid and geoid. While gravity is perpendicular to the geoid, it is not perpendicular to the ellipsoid.

the ellipsoid height (the difference between the red and purple line in Fig. 3.4), where the ellipsoid is chosen to be something. There are internationally agreed standards on these (e.g. WGS84) but since we are not going to be talking about *sea level* that much we will leave it as a detail for the reader to chase up. So Fig. 3.3 shows the (magnified) deviations of the Earth from an ellipsoid, arising from the fact that the Earth is not an uniform mass object, with the bulges (the red shading) where there is *more* mass, and dimples where there is *less* mass. Since gravity plays such an important role in the physics on Earth, we really do in fact need very accurate measures for the geoid, and this has only really been possible since the satellite era; we come back to the measure of gravity and the geoid in Ch. 8.

Before we leave this section, we mention some subtleties in relation to defining *sea level*, and even **sea surface height** (SSH). As you can be seen in Fig. 3.4, there are multiple ways to define sea level:

1. sea level above sea floor
2. sea level above/below ellipsoid
3. sea level above/below geoid
4. sea level above/below mean sea level (deviation from average)

and some others (e.g. depending on whether you define 'height' relative to ellipsoid or geoid, which are offset by some angles). Following Gregory et al 2019 Fig. 2, we will define 'vertical' to be relative to the ellipsoid, so 'above' means distance measured in the direction perpendicular to the ellipsoid. With that, the **mean sea level** will be the time-averaged sea level above sea floor, SSH will be the deviations from the mean sea level, and the **dynamic sea level** is the difference between the mean sea level and the geoid. These subtleties are important in the literature to do with sea level rise and so forth, because different authors have been known to use different terminology, and different contributions to sea level rise have different impacts to the different sea levels.

3.1.2 *Pressure and hydrostatic balance*

We encountered *pressure* briefly in Ch. 2.3.3 when we were talking about density and the confusing things it can do to when defining temperature and density. Here we define pressure properly, and highlight the important link between density variations and pressure as well as its consequences for momentum.

If we imagine again we are trying squash a parcel of water (cf. Fig. 2.16), then we have to exert a force which is spread over some area. This is what **pressure** really is, the *magnitude of force exerted per area*, or

$$p = F/A, \tag{3.4}$$

where A is the area and $F = |\boldsymbol{F}|$ is magnitude of the force vector; note pressure is a scalar. Pressure has units N m^{-2}, sometimes in Pa (Pascals), and sometimes in (milli)bar in the atmospheric and oceanic literature[2]. For reference, sea level is sometimes defined as where the pressure is 1 bar = 1000 mbar (millibar) = 10^6 Pa, ocean depth is sometimes measured in bars (e.g. CTDs in Ch. 8), atmospheric weather charts is usually given in units of millibars (see Fig. 3.7 for an example), and atmospheric data is sometimes given in pressure coordinates measured in hPa (*hecto*pascal, 1 hPa = 1 mbar) instead of height coordinates[3]. The lines of constant pressure are called **isobars**.

[2] Pascals is after French scientist Blaise Pascal (1623-1662), who also made contributions to probability theory as well as philosophy and theology. Bar as a unit was introduced by Vilheim Bjerknes (the father of the guy mentioned in Ch. 1.2.2)

[3] Because pressure is more relevant than height dynamically in the atmosphere, similar to how in the ocean we care more about along and across isopycnal directions.

In the ocean, possibly as anticipated from the discussion in Ch. 2, the principal source of pressure comes from the weight of the seawater above a point, because the weight of seawater is so large. A schematic of this pressure due to the water column is given in Fig. 3.5, where the pig in the water bubble is at depth $-z$. If we assume that the pressure experienced by our pig demonstrator is proportional to the amount of water above it, then what this is saying is that, at depth $-z$, the pressure experienced is

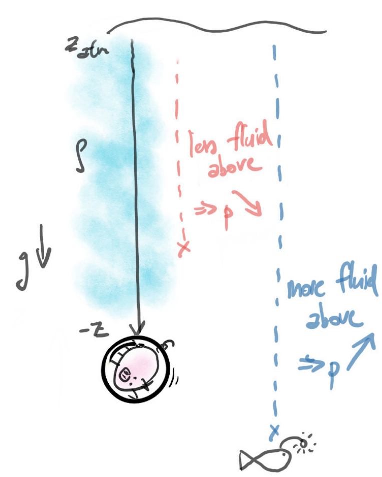

Figure 3.5: Schematic of hydrostatic pressure.

$$p = mg = \left(\int_{-z}^{z_{\rm atm}} \rho(z')\, {\rm d}z' \right) g, \tag{3.5}$$

where the mass of the water column is given by the integral of density from depth $-z$ to the surface. The principal contribution to pressure below sea level really is from this **hydrostatic pressure**, so we are not doing anything that drastic. If ρ = constant then we recover maybe the more familiar form $p = \rho g z + p_{\rm atm}$, with $p_{\rm atm}$ = constant. If we take a derivative of the equation with respect to z, then

$$\begin{aligned}\frac{\partial p}{\partial z} &= \frac{\partial}{\partial z}\left(\int_{-z}^{z_{\mathrm{atm}}} \rho(z')\,\mathrm{d}z'\right) g \\ &= -g\frac{\partial}{\partial z}\left(\int_{z_{\mathrm{atm}}}^{-z} \rho(z')\,\mathrm{d}z'\right) \\ &= -g\rho(-z),\end{aligned}$$

which is called **hydrostatic balance**, and you might recognise this already as Eq. (3.1b) where I've written out the argument of the function ρ explicitly[4]. If we are higher up in the water column, we experience less pressure, and lower down we experience more pressure, as in Fig. 3.5. The upshot is that we have a relation between density and pressure, so if we make the leap that pressure drives fluid motion, then density drives motion.

[4] The minus sign is just because I happen to have chosen to denote my depth as $-z$.

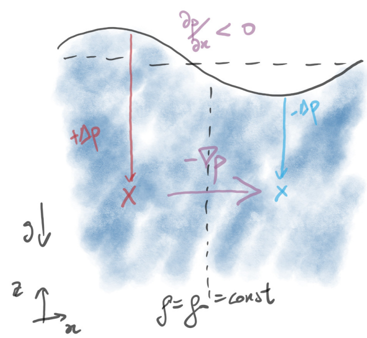

Figure 3.6: Horizontal effect because of hydrostatic pressure (assume ρ = constant and non-rotating for simplicity).

You should immediately object to what I just wrote, because pressure itself has no direction associated with it, and it is **pressure gradients** (and thus density gradients) that drives flows. Hence even though the numerical value of density does not vary much in the ocean, the point is somewhat mute because it is the gradients that matter. Fig. 3.6 shows schematically how pressure gradients drive flows in the ocean. Making the assumption that ρ = constant, the system is non-rotating, and the whole water column moves as one for simplicity, intuitively what we expect is that positive SSH regions will slump via the action of gravity, to fill the water columns that are in deficit, which is only achieved if there is a horizontal movement of the fluid. If we are look from a pressure point of view, then hydrostatic pressure is high where the SSH high, and hydrostatic pressure is low where the SSH is low. Intuitively we expect things at high pressure wants to go to low pressure to even out the pressure differences, so the action or force is pointed towards $-\nabla p$. A force pointing in the direction of the *negative pressure gradient* $-\nabla p$ leads to an acceleration in that direction (since there is a net force), and so the fluid moves from high pressure to low pressure. If the fluid surface is fully slumped (which will happen if there is damping and the system is non-rotating[5]), there is no pressure gradient and therefore no force, so nothing should move, as expected.

[5] Look up *geostrophic adjustments*, which we don't talk about here.

So far so good apart from the extra assumption about rotation. We expect that flow to be in the direction of $-\nabla p$, but is that seen in observations? Fig. 3.7 shows a weather chart and the contours are isobars, and I've taken the liberty and some license in adding the flow in. From the discussion of the geoid, or otherwise, $-\nabla p$ points *across* the isobars (marked in green in figure). So the answer is *no*, it doesn't really work, because most of the flow is *along*-isobars and not across isobars! The same thing turns out to hold true in the ocean

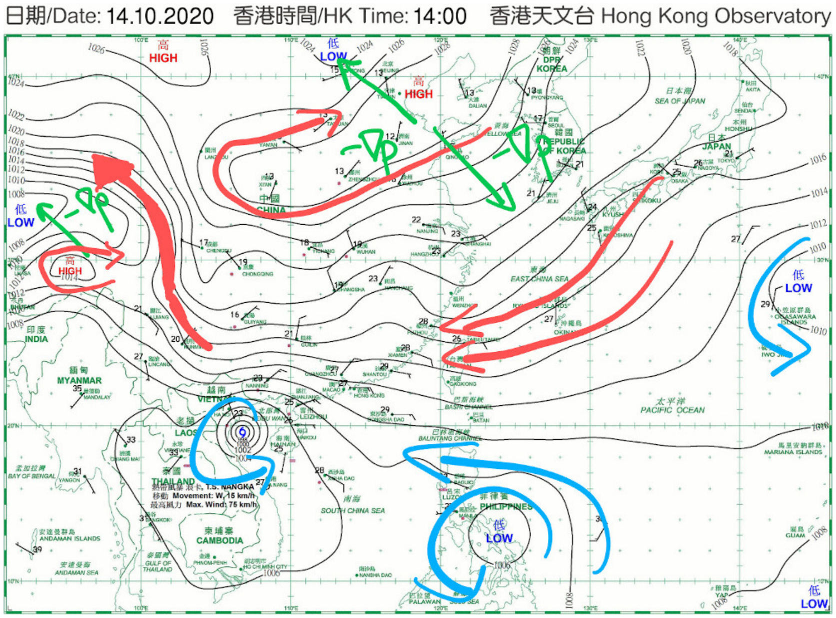

Figure 3.7: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = mbar) and wind directions (the tails of the flags denote where the wind is *coming from*. From HKO.

for larger-scale flow. Fig. 3.8 shows what is called the **mean dynamic topography**, which is just the time-mean SSH. Now from Fig. 3.6 we argued that SSH is related to hydrostatic pressure, so again contours of SSH roughly correspond to the horizontal distribution of isobars in the ocean. The subtropical gyres (the ones adjacent to the equator) we know for example to be rotating anti-clockwise and clockwise in the Northern and Southern Hemisphere respectively (Ch. 1.2). Thus $-\nabla p$ points across contours of SSH, but the flow turns out to be largely along contours of SSH! So what is going on?

If you know some of this material, you will know I am actually cheating quite a bit here. If you look closely in Fig. 3.7, the wind direction is almost but not quite in the direction along the isobars (hence why I said I took some license in drawing the flow on...) We also don't really have such a large coverage of ocean current measurements, so the current I drew on in Fig. 3.8 is largely correct, but really it's done with hindsight through theory that we will go through in detail in the next section. Pressure really does contribute to driving the flow, but *rotation* plays a big role. It turns out the length and time-scales associated with the dynamics matter: on short time- and length-scales, rotational effects are weak, so pressure can dominate (e.g. pipe flows, your water tap, small-scale atmospheric flow), and flow can be in the direction of $-\nabla p$. On longer time- and larger length-scales, rotation effects dominante (e.g. atmospheric *jet streams*, ocean gyres, etc.), and there is a deflection of the flow away from the direction of $-\nabla p$.

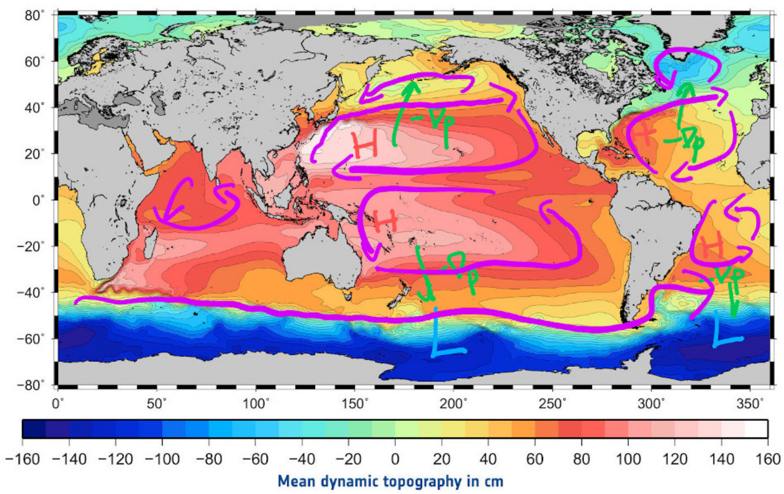

Figure 3.8: Time-mean global SSH (mean dynamic topography), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

For completeness, the **hydrostatic approximation** is the assumption a lot terms are small in the vertical momentum equation for the evolution of the vertical velocity w, such that we can throw away most of the terms except the ones relating to hydrostatic balance. This approximation is ok when the motion has small aspect ratio H/L, which is generally true in the ocean. From a computational point of view what this means is that instead of evolving three three-dimensional fields $\boldsymbol{u}_3 = (u, v, w)$, we only have to evolve $\boldsymbol{u} = (u, v)$, and find w through $\nabla \cdot \boldsymbol{u}_3 = 0$, which is more efficient.

3.2 *Coriolis effect*

The *Coriolis effect* is represented in the momentum equation Eq. (3.1) by

$$2\boldsymbol{\Omega} \times \boldsymbol{u}$$

So for a long time I knew how to use the symbols associated with the *Coriolis effect*[6] and what it does from mathematical point of view, but until I had to teach it I didn't realise how much I don't actually really understand it, and for a good while I couldn't explain it in a manner that I could convince myself. The drawing-a-line-while-rotating-a-piece-of-paper example (Fig. 3.10) was the one that really convinced me what it really is, and I highly recommend you try this out yourself; the Coriolis is probably one of those that you have to convince yourself is a 'real' effect, and other people telling you it is a thing doesn't really work... The Coriolis effect is going to be one of the central concepts that we use regularly until the end of the document, and it would help immensely in my opinion to spend

[6] After the French mathematician and scientist Gaspard-Gustave de Coriolis (1792-1843). Though well known in meteorology, he himself never worked in meteorology.

some time to learn and really understand this not entirely intuitive concept, before we utilise it extensive to do 'fun' (!) things with it[7].

So the Coriolis effect (note I use 'effect' and not 'force') is a *pseudo-force* or a *fictitious force*: it only arises because of the choice of perspective[8]. It turns out the choice is either choose an unwieldy perspective that gets rid of this effect (an *inertial frame*), or choose a more practical perspective but live with the Coriolis effect (an accelerating frame), and we actually choose the latter. If you already don't like the sound of this, I would still urge you to carry on, but for practical reasons, these are the key takeaways of this section:

[7] If it helps, Coriolis effect also applies to atmospheres and generally large rotating bodies of fluid such as Jupiter, so you get extra applications for free!

[8] This is related to the talk about *inertial frames* in Ch. 1.4.1.

- noting that the *work done* (Ch. 2) on a fluid parcel is $\boldsymbol{F} \cdot \boldsymbol{u}$, the Coriolis effect does no work (so it's a "fake" force from the work done point of view);
- if the system is not rotating, or there is no flow, there is no Coriolis effect;
- the force is to the *right* of intended travel in the Northern Hemisphere, and to the *left* in the Southern Hemisphere;
- the Coriolis effect on the (locally) *horizontal* flow is largest in magnitude at the poles, and vanishes at the equator;
- the *geostrophic flow* arising from a balance between pressure gradients and Coriolis effect is along isobars (cf. Fig. 3.7 and 3.8).

  With that, lets dive in...

3.2.1 *Terminology and rationalisation*

Some terminology first. The **rotation axis** is the axis which the Earth rotates around, and is taken to be the (geographical) North Pole; see Fig. 3.9. Then $\boldsymbol{\Omega} = \Omega \boldsymbol{e}_{\hat{z}}$ is in the direction of the North Pole $\boldsymbol{e}_{\hat{z}}$ and Ω is called the **angular frequency** (units: s^{-1}), defined by

$$\Omega = \frac{2\pi}{T}, \tag{3.6}$$

where T is the **period** and the time it takes to complete one rotation, which is $360^\circ = 2\pi$ radians. So Ω is the rotation rate, i.e. the higher the Ω, the faster the body rotates. On Earth, $\Omega =\approx 7.29 \times 10^{-5}\ \mathrm{s}^{-1}$.

One thing we will normally do is to choose a co-ordinate (or reference) that is locally vertical. What this means is that there is generally a mis-alignment of the direction between the local 'depth' co-ordinate $\boldsymbol{e}_z$ and $\boldsymbol{\Omega}$. This implies that the locally non-zero horizontal flow $\boldsymbol{u}$ will feel a different magnitude of the horizontal component of the Coriolis effect $2\Omega \times \boldsymbol{u}$ as the latitude changes. It

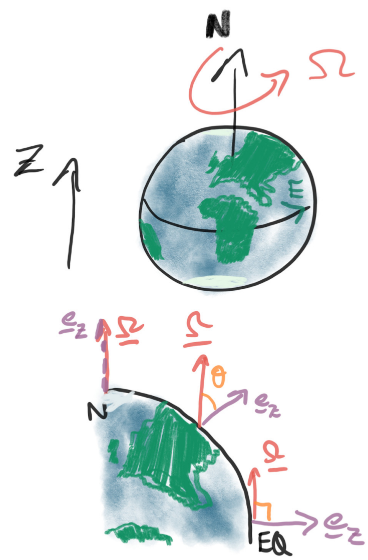

Figure 3.9: Rotation axis and angular frequency Ω. Generally speaking there is a mis-alignment of $\boldsymbol{\Omega}$ and $\boldsymbol{e}_z$ used locally for depth, giving rise to the Coriolis parameter $f = 2\Omega \sin(\text{latitude})$.

is convenient to take care of this mis-alignment by introducing the **Coriolis parameter** f where

$$f = 2\Omega \sin(\text{latitude}), \tag{3.7}$$

so the Coriolis effect in the local co-ordinate system is now given by $f\boldsymbol{e}_z \times \boldsymbol{u}$. Note then, by this definition, if we take (90S, 90N) to be $(-90\circ, +90^\circ)$ or $(-\pi/2, +\pi/2)$, then f is maximally positive at the North Pole, maximally negative at the South Pole, decreases in magnitude with latitude, and is zero at the equator.

So far we talked about rotation and have been alluding to the fact that the Coriolis effect does something to the flow, but have not really explained what it is. The way I convinced myself is to do a little demonstration with a piece of paper and a pen, and try and draw some straight lines while rotating the piece of paper underneath, as in Fig. 3.10. While I am going to describe the experiment and rationalise it, I highly recommend you try this yourself (it really really helped me).

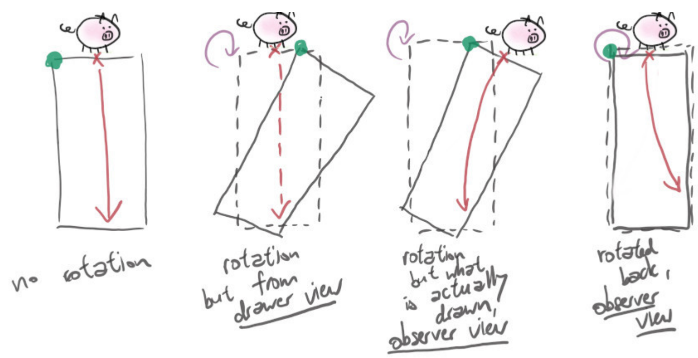

Figure 3.10: Schematic of apparent deflection from Coriolis effect. Here $\boldsymbol{\Omega}$ will be putting in or out of the page (as long as I choose my Ω accordingly).

So here you imagine there is a drawer (you), and an observer (put a plushie there if it helps). So when there is no rotation, you draw a straight line, and you see a straight line, and the observer also sees a straight line, great (1st column). Now, suppose you draw a straight line but rotate the piece of paper: from you the drawer's point of view, you are still drawing a straight line, no problem there (2nd column). However, what the observer (not you) should see is that there is now a deflection on the line drawn (3rd column). If you rotate the piece of paper to where you started from, then both you and the observer see that the line that came out is curved (4th column). The key thing here is that there is no contradiction: the action of line drawing is exactly the same, but different observers end up seeing things differently.

And now you can imagine if it was our pig demonstrator being pushed along the piece of paper (but WITHOUT being affected by the friction of the piece of paper), and the underlying piece of paper was rotating. From our friendly neighbourhood pig's point of view, it is being pushed in a straight line, but from our point of view, the pig's path would trace out a curve. This apparently deflection is purely because we are looking at the action in a different perspective. To reconcile the two different results in the different perspectives, even though the physics should be exactly the same, we either have to add a correction to the forces in the pig's perspective, or we have to add it to our own. It turns out actually the pig's point of view is the 'right' one to take, so we add an extra (but "fake") force to correct the description we see as an observer, which turns out in this case to be the Coriolis effect $f\boldsymbol{e}_z \times \boldsymbol{u}$.

From a looking down on Earth point of view, what we normally want to do is consider the map as *fixed*; for argument sake lets choose the centre to be the zero longitude. So while the Earth is actually rotating in time, at any snapshot in time we recenter it to the zero longitude, so in effect we are rotating the piece of paper back like in the 4th column of Fig. 3.10. From this perspective, we would see the actual trajectory trace out a curve, because we are in the "wrong" (but convenient) perspective and we need the Coriolis effect to compensate us being in the "wrong" (but convenient) perspective.

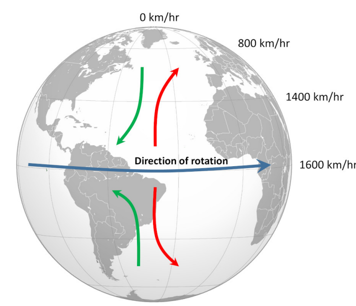

Figure 3.11: Deflection on Earth arising from Coriolis effect, from Vallis (2011).

What this ends up implying is that the deflection is to the *right* in the Northern Hemisphere, and to the *left* in the Southern Hemisphere; see Fig. 3.11. You can see this difference by doing the Fig. 3.10 experiment again, but rotating the piece of paper in a different direction. Also try imagining (or really try) to view the motion from *above*, so like the Northern Hemisphere point of view where you are looking down into the rotation axis, and view the motion from *below*, so like the Southern Hemisphere point of view where you are looking up but in the direction of the rotation axis. If it helps, try tracing a wet finger one of those brown basketballs or something like that while rotating it in a fixed direction, and see how the resulting water mark looks like. Another way to convince yourself that the Coriolis effect changes sign when you go from Northern to Southern Hemisphere is to put your right hand into the thumbs up position. If you are looking *into* the thumb, this is the Northern Hemisphere perspective, and your fingers are curving anti-clockwise from this point view, which is in the direction of Earth's rotation. If you are instead looking *along* the thumb with the little finger closest to your eye, this is the Southern Hemisphere perspective, then the fingers are now curving clockwise from this point of view, and effectively rotation has "reversed", hence the sign change in f.

3.2.2 *Rossby number*

From now on I'm also just going to drop the "deflection in the observer's point of view", because we will be working in this observer's point of view unless otherwise stated. First note that the Coriolis effect is not equally important across time- and length-scales. Going back to our friendly neighbourhood pig in Fig. 3.10, if rotation of the piece of paper is slow and/or the pig moves really fast, then the deflection might be negligible. If the rotation is fast and/or the pig takes its sweet time moving along the paper, then there would be significant deflection[9].

There is a competition between the *advective time-scale*, the time-scale associated with the body's motion, and the *rotation time-scale*, the time-scale associated with rotation of the system, which leads to whether the Coriolis effect is important or not. The measure is given by the **Rossby number**[10]

$$\mathrm{Ro} = \frac{U/L}{f} = \frac{1/T}{f} = \frac{\text{advective time-scale}}{\text{rotation time-scale}}. \tag{3.8}$$

Note that the Rossby number is a *non-dimensional* number. If T large so that $1/T$ small, i.e. motion on long time-scales, or f large, i.e. fast rotation, the $\mathrm{Ro} \ll 1$, Coriolis effect is important and motion is *rotationally constrained*. If $\mathrm{Ro} \approx 1$ then motion is *rotationally influenced*, and if $\mathrm{Ro} \gg 1$ then the Coriolis effect plays a minimal role in the dynamics.

As a numerical example, large-scale motion in Earth's atmosphere in the mid-latitudes (say 50°N) and $\Omega = 2\pi/\text{day}$ gives

$$\mathrm{Ro} = \frac{10\ \mathrm{m\ s^{-1}}/1000\ \mathrm{km}}{2 \times 2\pi\ \mathrm{day}^{-1} \times \sin(50^\circ)} \approx \frac{10^1 \times 10^{-6}}{10^{-4}} = 0.1,$$

so large-scale motion in the atmosphere is rotationally constrained. For the above, just be careful with time and length units, i.e.

$$\mathrm{day} = 3600 \times 24\ \mathrm{s} \quad \Leftrightarrow \quad \mathrm{day}^{-1} = (3600 \times 24)^{-1}\ \mathrm{s}^{-1}.$$

In the ocean Ro depends somewhat on the scale of motion, since the time- and length-scale of the types of motion are usually anti-correlated (e.g. fast time-scale motion is usually small-scale motion, and vice-versa). In the Gulf Stream, we might have (for argument sake)

$$\mathrm{Ro} = \frac{1\ \mathrm{m\ s^{-1}}/1000\ \mathrm{km}}{2 \times 2\pi\ \mathrm{day}^{-1} \times \sin(40^\circ)} \approx 0.01,$$

and the Rossby number is even smaller, because ocean flow tends to be slow, so there is ample time for the Coriolis effect to act. On the

[9] The path of the pig would actually trace out loops if the paper is allowed rotate multiple times, leading to what are called *inertial oscillations*; see Ch. 6.1.

[10] After the Swedish-American meteorologist Carl-Gustav Arvid Rossby (1898-1957), who studied under Vilheim Bjerknes. Rossby made fundamental contributions to modern day meteorology and geophysical fluid dynamics. See later also with *Rossby waves*.

other hand, if we are talking about *sub*mesoscale eddies, we may have

$$\mathrm{Ro} = \frac{0.1\ \mathrm{m\ s}^{-1}/1\ \mathrm{km}}{2 \times 2\pi\ \mathrm{day}^{-1} \times \sin(40^\circ)} \approx 1,$$

so the smaller-scale dynamics are rotationally influenced but by no means rotationally dominant. Going even smaller scales, you can imagine for example *gravity wave* motion will have large Ro, because they are fast as well as small-scale (see Ch. 6.1).

Some other astronomical examples (work these out yourselves maybe):

- Jupiter has these cloud bands associated with very fast jets that go up to $U = O(100\ \mathrm{m\ s}^{-1})$, but $\mathrm{Ro} \ll 1$, because while U is large, L is huge, and Ω on Jupiter is larger than Earth;
- Venus is similar size to Earth and can have very fast winds, but $\mathrm{Ro} \gg 1$, because Ω on Venus is tiny;
- the Solar interior has motion with large L, but Ω of the Sun is not that high, and $\mathrm{Ro} \approx 1$.

3.2.3 *Geostrophic balance*

So the flows in Fig. 3.7 and 3.8 are mostly going to be in the $\mathrm{Ro} \ll 1$ regime, but how does the Coriolis effect end up forcing the flow to be along rather than across isobars then? We recall that the momentum equation Eq. (3.1) (ignoring the forcing and dissipation) is given by (written in terms of f)

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + f \boldsymbol{e}_z \times \boldsymbol{u} = -\frac{1}{\rho_0} \nabla p.$$

The following procedure is called **dimensional analysis** or **non-dimensionalisation**: pull out all the dimensions associated with all the terms, gather everything, and see what you are left with[11]. The reason for doing this is that sometimes there might be multiple parameters in the system (e.g. Coriolis, density, flow speed, etc.), but it's not their individual values that matter, but a combination of their values. If we decide to re-scale as

$$t = Tt^*, \qquad (x, y) = L(x^*, y^*),$$

then we have, by considering the units of various terms,

$$\frac{\partial}{\partial t} \to \frac{1}{T} \frac{\partial}{\partial t*}, \quad \nabla \to \frac{1}{L} \nabla^*, \quad \boldsymbol{u} \to U \boldsymbol{u}^* = \frac{L}{T} \boldsymbol{u}^*, \quad p \to P p^*,$$

where we deliberately didn't do anything to f and ρ_0. Now, writing the equation in terms of the non-dimensional variables with stars and

[11] Think of it as switching units: a 100 kg pig would be 10^5 g, about 220 pounds, 0.1 tonnes, or 5.4 bajillion gazoboks (I made this one up), but it is still a pig with that amount of mass. When non-dimensionalising, you choose the measure so that the mass of the pig is '1', with the understanding that '1' means 100 kg if the choice of mass scaling is chosen to be 100 kg.

collecting factors accordingly, it may be shown that we have

$$\mathrm{Ro}\left(\frac{\partial \boldsymbol{u}^*}{\partial t^*} + \boldsymbol{u}^* \cdot \nabla^* \boldsymbol{u}^*\right) + \boldsymbol{e}_z \times \boldsymbol{u}^* = -\frac{P}{L\rho_0}\nabla^* p^*.$$

Now, in the Ro $\ll 1$ regime, the evolution and nonlinear inertia term is definitely small, so we will look to throw those away. If the scale $P/(L\rho_0)$ is also really small and we throw that away, then we get nothing interesting, because it just says $\boldsymbol{u} = 0$. So what this means is that, in the Ro $\ll 1$ regime, we expect the dominant *balance* between the "forces" (because Coriolis "force" is fake) in the horizontal direction is

$$f\boldsymbol{e}_z \times \boldsymbol{u}_g = -\frac{1}{\rho_0}\nabla p, \tag{3.9}$$

i.e. between the Coriolis effect and the pressure gradients. This is what is called **geostrophic balance**[12], and the flow $\boldsymbol{u}_g$ that satisfies to the geostrophic balance is called the **geostrophic flow**.

[12] For completeness, non-dimensionalisation with the vertical momentum equation is the formal way to obtain hydrostatic balance given by Eq. (3.1b), where the small parameter ends up being the aspect ratio.

A schematic of the implication of geostrophic balance in the Northern Hemisphere is given in Fig. 3.12. First we have our isobars (the red and blue dashed line), chosen so that $-\nabla p$ points up (north) and across isobars. To get 'force' balance (otherwise we have an acceleration), the 'force' arising from the Coriolis effect must be equal and opposite, so pointing down (south) and with a minus sign because it is in the opposite direction. Now, in the Northern Hemisphere, $\boldsymbol{\Omega}$ points out of the page (see the small insert of the globe to convince yourself). The questions to ask which direction is $\boldsymbol{u}_g$ pointing so that $-\boldsymbol{\Omega} \times \boldsymbol{u}_g$ is pointing down (so balancing $-\nabla p$). From the cross product discussion in Ch. 1.5.3, convince yourself $\boldsymbol{u}_g$ points right (east), then $-\boldsymbol{\Omega} \times \boldsymbol{u}_g$ is pointing south, and we have the balance we need.

The upshot is that (1) $\boldsymbol{u}_g$ is parallel to isobars, and (2) in the Northern Hemisphere, $\boldsymbol{u}_g$ goes to the *right* of $-\nabla p$. The same argument could be repeated for the Southern Hemisphere, still with $-\nabla p$ pointing north, and you find that because $\boldsymbol{\Omega}$ is now pointing into the page, the direction of $\boldsymbol{u}_g$ is swapped (i.e. to the west). The geostrophic flow is still along isobars, but now $\boldsymbol{u}_g$ is to the *left* of $-\nabla p$.

An equivalent mathematical way of doing it is to note that, taking (x, y, z) to be zonal-meridional-verical, then the schematic of Fig. 3.12 leads to Eq. (3.9) looking like

$$f\boldsymbol{e}_z \times \boldsymbol{u}_g \sim \boldsymbol{e}_y,$$

where $\boldsymbol{e}_{x,y,z}$ are the vectors pointing east, north and up respectively ($-\nabla p$ points north so is represented by $\boldsymbol{e}_y$). Then, in the Northern

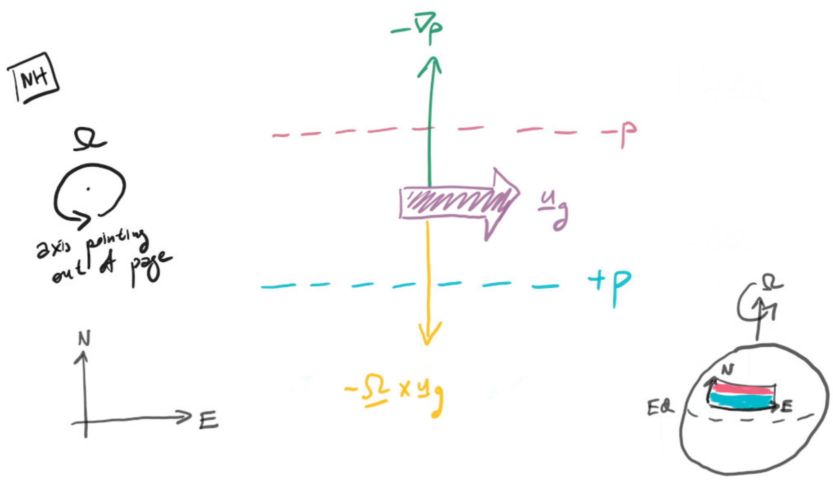

Figure 3.12: Geostrophic balance and resulting geostrophic flow $\boldsymbol{u}_g$ in Northern Hemisphere. Note $\boldsymbol{u}_g$ is **along** (rather than **across**) isobars.

Hemisphere, either by guessing, the cyclic rule associated with the cross product, or otherwise, we have to have $\boldsymbol{u}_g \sim \boldsymbol{e}_x$ since $f > 0$, i.e. pointing east, and to the *right* of $-\nabla p$. On the other hand, if $f < 0$, then by sign considerations, $\boldsymbol{u}_g \sim -\boldsymbol{e}_x$, i.e. pointing west, and to the *left* of $-\nabla p$ that is still pointing North. Try to convince yourself if $-\nabla p$ is pointing in other directions that similar arguments still hold (of course $\boldsymbol{u}_g$ will be pointing in a direction perpendicular to $-\nabla p$ depending on the hemisphere).

Note that at no point in the above discussion on geostrophic balance did we talk about the ocean or atmosphere, merely that the system is rapidly rotating in the sense that $\mathrm{Ro} \ll 1$, and provides an explanation of the observed phenomena in Fig. 3.7 and 3.8 that the flow is largely along isobars or contours of SSH in the ocean. In fact this is one way of how we actually infer for the currents in the ocean (see Ch. 8), although we need another tool that we will visit in Ch. 5.1.2.

To close this section, we talk about how this works for *eddies* (as a noun), as in the schematic given in Fig. 3.13. Here the objective is to rationalise the flow orientation associated with these **geostrophic eddies**, taking the Northern Hemisphere case for concreteness. A geostrophic eddy with a bulge (high SSH) implies it is a high pressure in the eddy, and a low pressure outside. What this means is that $-\nabla p$ points *out* of the eddy, and since $\boldsymbol{u}_g$ points to the right of $-\nabla p$, this implies the eddy is circulating in a clockwise fashion. On the other hand, for a eddy with a depression (low SSH), the pressure is low in the core, $-\nabla p$ points *in* to the eddy, and again because $\boldsymbol{u}_g$ points to the right of the $-\nabla p$, this implies the eddy is circulating in an anti-clockwise fashion.

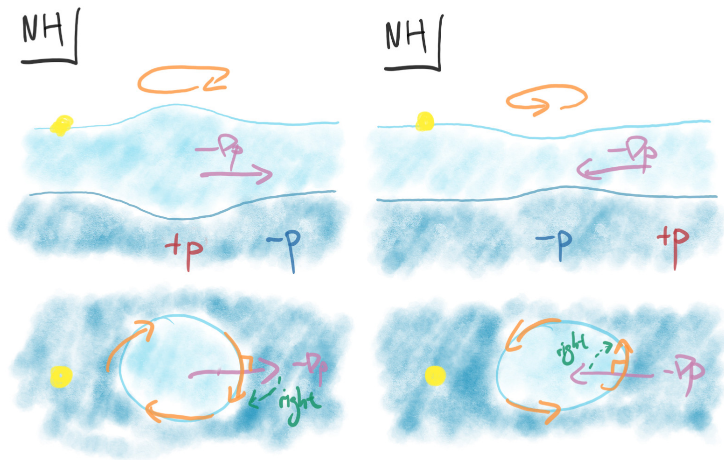

Figure 3.13: Schematic for an (left) anti-cyclonic warm core and (right) cyclonic cold core eddy, in the vertical view as well as a top-down view.

Just a little bit more terminology before we move on. In the Northern Hemisphere, since the Earth's rotation is regarded as rotating in the anti-clockwise sense (about the North Pole), something that is rotating in the same sense as the Earth is denoted **cyclonic**, and **anti-cyclonic** otherwise. In this regard, geostrophic eddies with high SSH are *anti-cyclonic* eddies, while eddies with low SSH are *cyclonic* eddies. It turns out the previous sentence holds true even in the Southern Hemisphere: high SSH, $-\nabla p$ points out, but $\boldsymbol{u}_g$ now to the *left*, so eddy circulation is anti-clockwise, but Earth is regarded to be rotating clockwise in the Southern Hemisphere, so the sense of rotation of the eddy is different to Earth, so we still have an *anti-cyclonic* eddy. The beauty is that this works also for the atmosphere: high pressures systems are **anti-cyclones**, while low pressure systems are **cyclones**, and this statement is true in both hemispheres. From a mathematical point of view, we tend to define the flow as cyclonic if the **vorticity** $\nabla \times \boldsymbol{u}$ has the same sign as f, and anti-cyclonic otherwise. The cyclonic/anti-cyclonic terminology as well as the concept of vorticity will be used with increasing frequency as we progress.

3.3 *Wind forcing*

From the previous section we noted that, in the atmosphere, high pressures systems are anti-cyclonic while low pressure systems are cyclonic. One thing you may know is that high pressure systems are *blocks*, which tend to lead to stable weather, while low pressure systems lead to unstable weather[13]. The reason is that associated with these systems are secondary up and down motion. Cyclones have associated with it upward motion in the eddy, encouraging

[13] Atmospheric *storms* are technically all cyclones.

moisture transport upwards, leading to cloud formation and thus precipitation. Anti-cyclones on the other hand lead to downward motion within the eddy, suppressing cloud formation and thus precipitation. Questions we are going to answer and explain in this section are:

- Are there analogues of up and down motion for ocean geostrophic eddies? (Yes)
- Can we replace $-\nabla p$ above with wind forcing instead? (Yes, and resulting *Ekman transport* is to the right or left of wind depending on f)
- Is there up and down motion associated with the wind forcing? (Yes, *Ekman suction* and *pumping*)

3.3.1 *Observed winds*

Lets start with what the atmopsheric observed winds actually look like first. Fig. 3.14 shows a schematic of the surface wind patterns, with the atmospheric pressure belts marked on. The observed surface winds are generally consistent with the discussion in the previous section, in that while the wind has a component in the direction of $-\nabla p$, it looks like it is mostly along isobars, with the deflection from the $-\nabla p$ in the relevant direction depending on the hemisphere. One reason the observed winds that are expected to be in geostrophic balance are not completely along isobars is because we are talking about *surface* winds, so *friction* (Ch. 3.4.3) arising from wind 'rubbing' against the ground is not entirely negligible (but turns out the effect of friction is consistent to what we see in the wind patterns). The other feature to notice is that the higher latitude winds are more along-isobars; this is consistent with $|f|$ being larger, so the Coriolis effect is relatively speaking stronger even in the presence of friction.

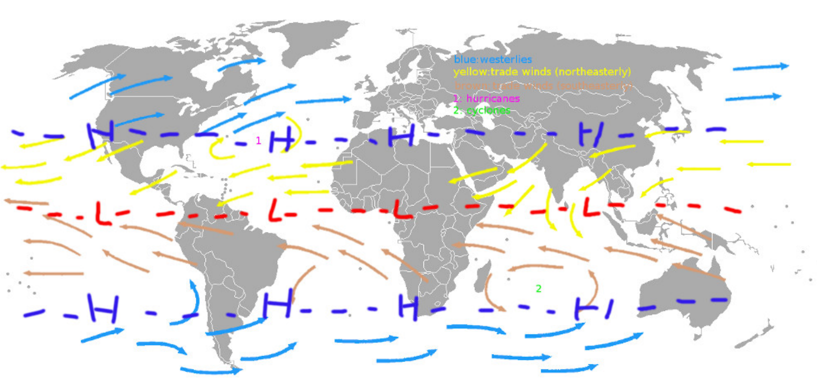

Figure 3.14: Patterns of global surface winds with pressure belts marked on. From `wikimedia.org`.to swap out with year-avg JRA55 data?

First of all why is there the observed atmospheric pressure patterns anyway? The pattern marked on roughly denotes the surface effects of the **Hadley cell**[14]. The Earth receives most heat around the Equator, and heating leads to air moving up vertically at the equator, hence the low pressure at the Equator. Air rises until it is neutrally buoyancy at some height (the *tropopause*), then it has to go somewhere. Since it can't go further up, down or in the zonal direction (why not?), it has to move meridionally away from the Equator. As the air moves higher in latitude, it is cooled because the surrounding temperature is cooler as the higher latitudes receives less heating, and thus it has to sink, leading to a high pressure at the surface. The return flow then manifests as the surface winds. One might have expected the circulation to extend all the way up to the poles with this thermodynamic argument, and it does if the Earth is not rotating. Being very hand-wavy, the argument is that the rotation leads to a deflection as the high altitude air moves to the higher latitudes. The distance the air travels is longer, thus allowing more time for the parcel to cool and sink back down to the surface at a lower latitude. There are more subtleties at play here that we will not need for our purposes so I don't really want to go into it... Anyway, the resulting downwelling region on Earth is around 30° N/S called the **subtropical highs**, where the surface winds are generally weak, and the weather is very stable[15].

[14] After English lawyer and meteorologist George Hadley (1685-1768). In the atmospheric literature there is also the *Ferrell cell*, but that is a can of worms I don't really want to open in this document here...see one of the digressions about *Deacon cells* in Ch. 5.1 for a related can of worms.

[15] It is also called the *horse latitudes*, with one story being that when sailors used to take horses in their ships to travel across the Atlantic, to conserve drinking water, they tend to throw the horses abroad around this region as the ship is barely moving in this region (the winds are weak).

The main bit for our purposes particularly in Ch. 5 and Ch. 6 is the surface wind patterns. The **trade winds** refer to the wind patterns between the subtropical highs, which are generally Equator-ward with a westward component. Between the subtropic highs and to around the edge of the polar regions (around 60° N/S), the winds are **prevailing westerlies**[16], that are predominantly eastward winds. Note that there is a change in the sign of the wind *gradient* as we move up from the Equator towards to Poles; we will come back to this when we talk about the *wind stress curl* later.

[16] In the atmospheric literature it is customary to call the winds by the direction they came *from* (e.g. eastward = westerlies). I will generally *not* take that convention here (I think I understand the rationale behind the terminology but I still don't like it).

For completeness, **monsoons** are seasonally varying winds that particularly affect the coastal areas of certain locations around the world (e.g. South and South-East Asia, sub-Saharan Africa, Central America), and very important for the regional climate with notable consequences for agriculture. The reasoning behind monsoons is to do with the heat capacity of seawater, with the schematic sketched out in Fig. 3.15. In the summer, the land warms up much more than the ocean because the ocean requires much more energy to heat up. This implies a $-\nabla p$ that points into the land, thus driving a wind from the ocean to the land (with appropriate deflections from the Coriolis effect), which carries a lot of moisture with it because the air is warm (so can hold more moisture) and the wind is passing through the ocean. In the winter this situation is reversed, because the ocean holds heat much better than land, and results in a dry cold air blowing away from the land. More on the effects and consequences in Ch. **??**.

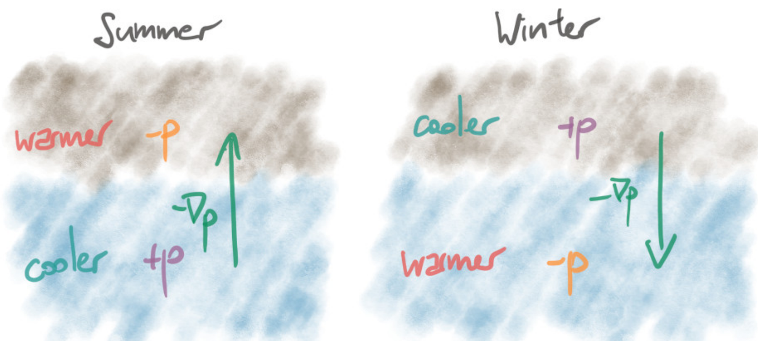

Figure 3.15: Schematic of monsoons, arising from changes in pressure gradients largely governed by heat. Actual wind direction slightly deflected because of Coriolis effect.

3.3.2 *Ekman layer, spiral and transport*

Winds forcing affects the momentum, and in fact wind forcing is the chief source of momentum into the ocean. Normally this is represented as a **wind stress** (units of $\mathrm{N\ m^{-2}}$) in the momentum equation, so we may ask how is the momentum actually being put into the ocean; we leave this until the next section after we talked a bit about *diffusion* and *friction*. Accepting that wind does put momentum in via exerting a force over the top of the ocean, then we might expect the fluid's motion to be along the direction of the force. But we also know that by geostrophic balance the geostrophic flow should end up being perpendicular to the direction of intended motion, i.e. to the right or left of the direction of wind stress depending on hemisphere. So how do we reconcile this?

The answer is actually quite simple: the flow turns! The wind's influence has to have some limited vertical extent (e.g. Fig. 3.16) because the ocean is a fluid and not a solid, so layers of fluid can slide over each other easily, unlike in solids where there will be strong resistance from the material resisting the imposed force (see more in Ch. 3.4). Within the thin region near the ocean surface called the **Ekman layer**[17], geostrophic balance needs to be modified because forcing is not negligible.

[17] We give a better definition of the Ekman layer in Ch. 3.4.2.

What happens then can be seen in Fig. 3.17, and we take Northern Hemisphere for concreteness. Near the surface, the intended flow is in the direction of wind, but because of the deflection from the Coriolis effect the actual flow is at an angle to the intended flow (but not perpendicular to it; think adding vectors associated with the

Figure 3.16: Schematic of Ekman layer (boundary denoted by orange). The stuff underneath the Ekman layer could be regarded as being shielded from the direction influence of the wind.

wind stress and Coriolis effect together). However, as we gradually move deeper, the influence of wind weakens, the flow weakens, but we get closer to geostrophic balance since wind forcing is weaker, so the flow has to turn, in this case to the right of the wind. What results is an **Ekman spiral**[18].

[18] After the Swedish oceanographer Vagn Walfrid Ekman (1874-1954), whose theory was motivated by observations of iceberg motion not being in the direction of the wind after Vilheim Bjerknes got him onto the problem. Under simplifying assumptions one could actually get an analytical form of the Ekman spiral (as Ekman originally did in his doctoral thesis).

The **Ekman transport** is the net transport of the resulting flow, i.e. the depth integral of the flow. While there is some component of the flow near the surface roughly in the direction of the wind, the Ekman transport is essentially perpendicular to the direction to the flow (to the right in Northern Hemisphere). While the flow is largest near the surface, it occupies a relatively small volume, so when it is integrated it is the small flow but over larger volume that dominates the final result.

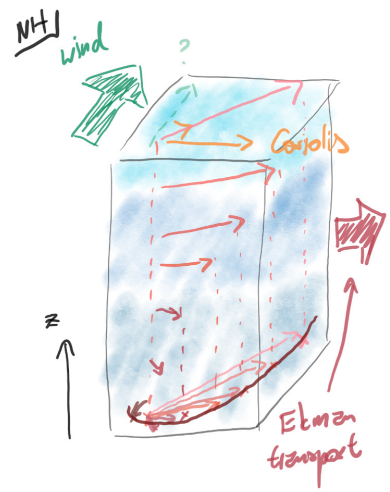

Figure 3.17: Schematic of Ekman spiral over the Ekman layer.

3.3.3 *Ekman pumping and suction*

Ekman dynamics also drive a vertical flow, which has important consequences for coastal dynamics and biogeochemistry (more on impacts in Ch. 8). We illustrate two examples first, which may be useful references to keep in mind before we talk about the general cases of fluid *divergence/convergence* and relating the phenomenon with the *wind stress curl*.

In Fig. 3.18 we assume again we are in the Northern Hemisphere, and we have a surface wind blowing south along a coastline. According to the discussion above, the Ekman transport is to the right and so is off-shore. If we are looking at this from a two-dimensional point of view we immediately have a problem, because we can't continually have an off-short flow as we will run out of water to transport at some point. To maintain *mass conservation*, what we must have is water moving on-shore from the deep parts of the water column to replace the surface water that is being moved off-shore. This in turn implies there has to be an upward movement of water arising from the action of the wind, and is called **Ekman suction**.

Another example is at the Equator where the Coriolis parameter

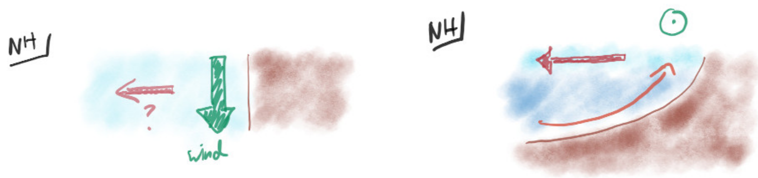

Figure 3.18: Schematic of Ekman suction near the coast. Wind is pointing out of the page.

changes sign, depicted in Fig. 3.19. The trade winds converge at the Equator, and the corresponding Ekman transport leads to a flow *divergence* ($\nabla \cdot \boldsymbol{u}_g > 0$). Again, for mass conservation, this necessarily implies there is a circulation with fluid *convergence* ($\nabla \cdot \boldsymbol{u}_g < 0$) deeper down to replace the surface water diverging, and in turn implies an *upwelling* around the Equatorial region.

Figure 3.19: Schematic of Ekman suction around the Equator. Wind is pointing into the page.

The converse cases where the winds are reversed in the above will lead to a *downwelling*, i.e. **Ekman pumping**. A surface divergence of water leads to Ekman upwelling, while surface convergence of water leads to Ekman downwelling, so we want to know what kind of wind causes divergence and convergence in the flow in the general cases. Lets assume we are away from coasts, we notice that if we have a spatially uniform wind stress then there is no convergence or divergence, because the Ekman transport will be spatially uniform. Therefore we need a *shear* in the wind stress, i.e. a non-zero spatial gradient in the wind stress to get Ekman up or downwelling. Fig. 3.20 shows a schematic of the cases where the meridional shear in the wind stress is negative and positive respectively, assuming we are in the Northern Hemisphere. When the wind stress shear is negative, the wind is forcing harder at the south than at the north. This implies that there is a stronger southward Ekman transport at the south than at the north, so there is a flow divergence, and hence an upwelling. Conversely, for a wind stress shear that is positive, the wind is forcing harder at the north than at the south, the southward Ekman transport is stronger at the

North, implying there is a piling on of water to the south, i.e. a flow convergence, and therefore a Ekman downwelling. In symbols, we take the wind stress to be $\boldsymbol{\tau} = (\tau^x(y), 0)$, then we have for the first case that

$$\frac{\partial \tau^x}{\partial y} < 0 \quad \Rightarrow \quad \frac{\partial v_g}{\partial y} > 0, \tag{3.10}$$

and inequality signs are swapped for the second case.

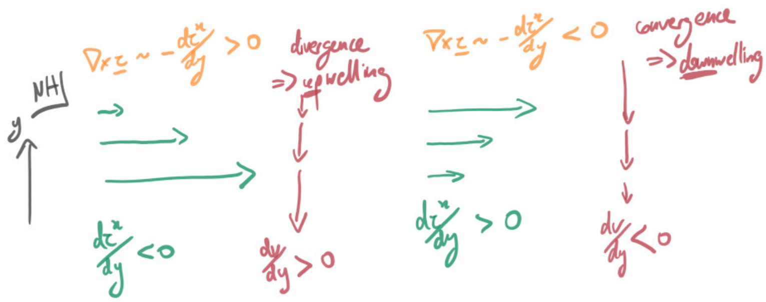

Figure 3.20: Schematic of wind shear (wind stress curl) with Ekman up/downwelling.

If we define the **wind stress curl** as $\nabla \times \boldsymbol{\tau}$, then it can be shown that the relevant component for the horizontal dynamics is the vertical component of the wind stress curl which is given by

$$\boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau}) = \frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}, \tag{3.11}$$

where $\boldsymbol{\tau} = (\tau^x, \tau^y, \tau^z)$. Then we see that the above case in Fig. 3.20 has $\boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau}) = -\partial \tau^x / \partial y$, so that wind stress shear and wind stress curl has a minus sign difference, implying that

$$\boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau}) > 0 \quad \Rightarrow \quad \frac{\partial v_g}{\partial y} > 0, \tag{3.12}$$

and vice-versa. So the divergence of the geostrophic flow has the same sign as the wind stress curl, and hence positive wind stress curl implies an Ekman *up*welling, while negative wind stress curl implies an Ekman *down*welling, at least in the Northern Hemisphere. You can convince yourself that, in the Southern Hemisphere, positive wind stress curl implies an Ekman *down*welling (because the Ekman transport is in the other direction) and vice-versa, which implies a factor of f has to be in play. It turns out one can derive from the equations directly that

$$w_e = \frac{1}{\rho_0 f} \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau}), \tag{3.13}$$

where w_e denotes the Ekman vertical velocity (upwelling in $w_e > 0$ and vice-versa), and the verbal arguments above are consistent with (but is not a proof of) Eq. (3.13).

In Fig. 3.21 we revisit geostrophic eddies. Recalling that the vorticity of the geostrophic flow is defined to be $\nabla \times \boldsymbol{u}_g$ and we are in the Northern Hemisphere, the anti-cyclonic eddy (the one with the high pressure) is rotating clockwise, so the flow has negative[19] vorticity. If we use Eq. (3.13) but with $\boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau})$ replaced by $\omega = \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{u}_g)$, then since $f > 0$, $w_e < 0$, i.e. an associated geostrophic downwelling. Similarly, a cyclonic eddy has $\omega > 0$ in the Northern Hemisphere, so there is an associate geostrophic upwelling. In the Southern Hemisphere, an anti-cyclonic and cyclonic eddy *still* has an associated geostrophic downwelling and upwelling: the signs of ω changes when we go into the Southern Hemisphere, but so does f, and there is a sign cancelling out.

[19] Traditionally in maths vorticity is positive if it is anti-clockwise. The best way to remember is either as a right hand in the thumbs up position (so fingers curl round in an anti-clockwise sense), or that angles are measure in an anti-clockwise fashion.

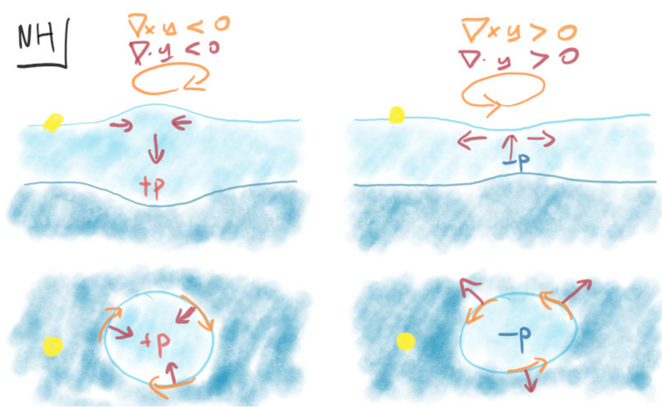

Figure 3.21: Up/downwelling associated with anti-cyclonic (left) and cyclonic (right) eddies (since we are in NH).

There are several ways to rationalise physically why cyclonic and anti-cyclonic eddies (in both ocean and atmosphere) are associated with upwelling and downwelling within the eddy. One way utilises friction at the bottom of the eddy (as 'rubbing against a medium with no motion', e.g. the ground, or a layer of fluid of no motion); see one of the chapter exercises.

3.4 *Diffusion, viscosity and friction*

We have been talking a bit about *friction*, and before focusing a bit more on friction I want to talk about *diffusion* first, which I think is a more fundamental concept.

3.4.1 *Diffusion example: milk in coffee*

Consider adding a bit of milk into say a cup of coffee as in Fig. 3.22. If you don't *stir* the coffee, then you realise it actually takes ages for the coffee to lighten in colour, because the milk just sits there minding its own business, albeit *spreading* but very slowly. On the other hand, stirring the coffee moves the coffee around, which carries the milk around, and you find the rate of spreading is substantially faster.

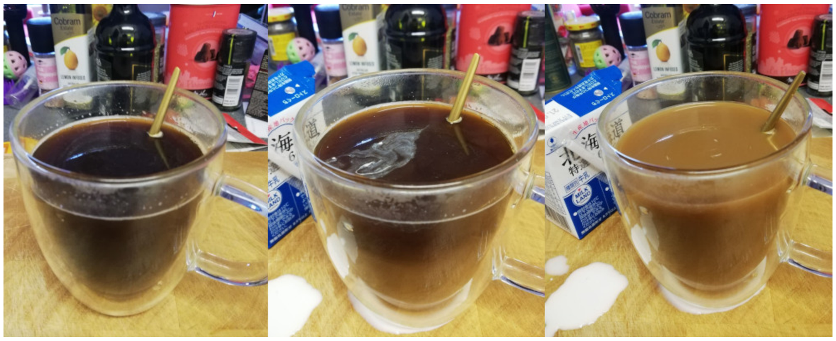

Figure 3.22: Making a mess of coffee + milk...

So here **diffusion** is just going to be referred to as the action that leads to spreading of 'stuff'. The actions above are to be distinguished as *molecular diffusion* and *effective diffusion* respectively, which we say more in due course. Diffusion results in erasing of *gradients*. When you add the milk into the coffee there is a gradient in the milk concentration, and molecular diffusion acts to spread/erase the concentration gradients, albeit really slowly. When you stir it, you involve dynamics that increases the rate of gradient removal, and concoction becomes *well-mixed* sooner, i.e. there are essentially no discernible gradients anymore. There is of course nothing special about milk and coffee, and the arguments work similarly for tracers within a fluid (e.g. chemical concentrations, momentum, heat, salt etc.[20]).

Lets start with **molecular diffusion**. Imagine there are a bunch of particles in a box, so like Fig. 3.23, but imagine even larger numbers (I didn't want to draw too many dots), and we tag the particles by colours. Suppose you start it off so that there is a gradient in the colours, and assume nothing goes out of the box. Particles randomly move around through **Brownian motion**[21], the particles are jiggling about but mixing away from the purely separated configuration. If you leave it long enough (and it can take very long...), and take a look at the box again, what you expect to see is that the colours should be fairly mixed up. Now, this *microscopic* phenomenon has a *macroscopic* effect. If we for example consider one colour as -1 and the other as $+1$, and work out the sum of per horizontal section, initially we have something with a large vertical gradient, but as time goes on and the particles do their dance, the macroscopic gradient gets gradually erased.

[20] Although we may want to distinguish *passive* and *active* tracers, where the former basically moves around the with the flow, but the latter can evolve according to its own equations (e.g. chemicals) or feedback onto the flow (e.g. momentum).

[21] After the Scottish botanist Robert Brown (1773-1858). Describing Brownian motion is actually also one of Einstein's major contributions to science, even if he is mostly remembered for $E = mc^2$.

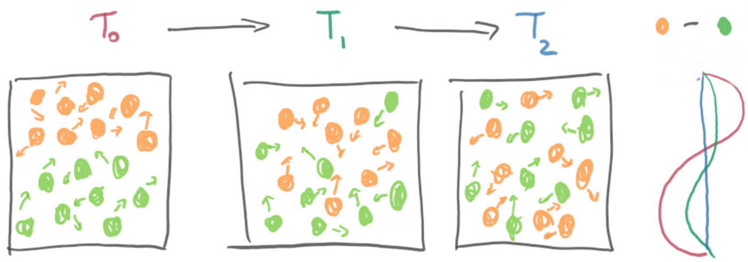

Figure 3.23: Schematic of *microscopic* motion leading to *macroscopic* diffusion. Here it is actual particle types, but can also imagine particles carrying "stuff", bumping into each other and transferring "stuff", and eventually the distribution of "stuff" evens out (but the total is conserved).

The thing I just want to clarify here is that this is not saying there can be no *un*mixing: it is just extremely unlikely. One analogy might be to think about the consequence of the random motion as how many different ways are there to rearrange the different types of particles in this box. In Fig. 3.23 there are eight particles of each colour, and suppose I start it off as a regular array and restrict the action so that the particles can only be swapped around. There are precisely $16!/(8! \times 8!) = 12870$ arrangements, accounting for redundancies and repeated patterns[22]. Now, what you can imagine is that, after you leave the particles doing its thing long enough, they keep swapping positions. When you take a peek into the box, you are sampling one of the large number of possible configurations. But since there is only essentially two configurations where the colours are all on one side, but so many more where the configuration 'looks' mixed, from a probabilistic point of view you are probably going to encounter once of those 'mixed' states. In reality, instead of 16 particles you are going to have about 6×10^{23} particles per mole of 'stuff'[23], and the number of configurations become astronomical! Yet you still only have two configurations where the colours are completely separated, compare to a ridiculous amount of states where it is 'mixed'. So it isn't things cannot *unmix*, it is just extremely unlikely to observe it, so for all intents and purposes we consider this kind of diffusive action *irreversible*.

[22] $n! = n \times (n-1) \times (n-2) \ldots \times 1$ is the *factorial*, and it gets big very quickly.

[23] Look up the *Avogadro constant*.

Staying with molecular diffusion and taking temperature T for concreteness, the mixing rate depends on the magnitude of the tracer gradients ∇T, as well as a (molecular) **diffusivity**, normally denoted κ ('kappa', units of $\mathrm{m^2\ s^{-1}}$). This definition is analogous for salt, and we distinguish the diffusivities by κ_T and κ_S respectively[24] Without going into too much detail, for the fluid itself there are internal stresses that lead to spreading of momentum, but taking various simplifying assumptions this spreading can also be represented as a diffusion, except the diffusivity has a different name and is called the **viscosity**, denoted ν or $\mu = \rho\nu$ (I will use ν, 'nu'). Viscosity measures how *sticky* something is: cement has very high viscosity, but air and

[24] Different tracers need not have the same diffusivities.

material	κ_T	κ_S	ν
seawater	10^{-7}	10^{-9}	10^{-6}
air	10^{-5}	—	10^{-5}
honey	10^{-6}	—	10^{-2}
lava	10^{-7} (!)	—	depends, 10^{0}?
steel	10^{1}	—	big (!?)

Table 3.1: Table of molecular diffusivity/viscosity values at some control conditions. All numerical entries have units of $\mathrm{m}^2\ \mathrm{s}^{-1}$.

sea water not so much. Diffusion tends to show up in equation form as

$$\kappa_T \nabla^2 T, \qquad \nu \nabla^2 \boldsymbol{u}, \tag{3.14}$$

where $\nabla^2 = \nabla \cdot \nabla$ is the *Laplacian operator*, which is a scalar operator consisting of second derivatives. If the diffusivity is spatially varying or more diffuses in different directions different, then they may be written as

$$\nabla \cdot (\kappa \nabla T), \qquad \nabla \cdot (\mathbf{K} \nabla T), \tag{3.15}$$

where $\mathbf{K}$ denotes a *tensor* (not going to say more about that).

The molecular diffusivities or viscosities of various tracers are material dependent and *we know what they are* (e.g. from lab experiments or by inference; see example later). Table 3.1 shows some representative values taken from various places. The exact values tend to depend on temperature and pressure, and only order of magnitudes have been given. So the diffusivities and viscosities for seawater and air are small. The viscosity of honey can actually be measured in the lab, and is lower when it is warm. The viscosities for lava is inferred for by measuring the flow rate (see later for a seawater example). Steel is technically a solid but of course if we are to be awkward, we could argue that on long time-scales everything flows in some way, so we could in principle assign a viscosity[25].

[25] As they say in *rheoloical sciences*, "*everything flows*".

One concept we use to highlight why molecular diffusivity is usually not the relevant one we care about when we are dealing with large-scale dynamics is the **diffusion time**. Since the diffusivity has units of $\mathrm{m}^2\ \mathrm{s}^{-1}$, we can define a time-scale as

$$t = \frac{L^2}{\kappa}, \tag{3.16}$$

where we interpret t as the time it takes for 'stuff' to diffuse a length L if the diffusivity was κ. This is one way to infer for diffusivity of lava for example: if you know how long the lava took to travel some distance, and removing say the contributions due to other forces[26] such as gravity, then you can get back out a diffusivity.

[26] This is actually much harder than I am making it out to be...

For our purposes, lets say we are interested in knowing how long it takes for some heat (as measured by temperature) to diffuse vertically by about 100 m (to roughly below the mixed layer say). Then, if we were to use the molecular diffusivity κ_T value in Table 3.1, then

$$t_m = \frac{100^2}{10^{-7}} = 10^{11}\ \mathrm{s} \approx 3000\ \text{years!}$$

There are various reasons why we know the associated time-scale is substantially faster than the 3000 years we just computed (e.g. chemical tracers, pollutants, etc.), so there is something amiss here. The reason is of course we are not taking into account the effect of *stirring* at all when we are dealing with molecular diffusion. Just like our milk can mix and spread much faster when stirring is involved as in Fig. 3.22, the ocean dynamics is generally seen to lead to an increase in an **effective diffusivity** κ_e (sometimes *turbulent diffusivity* or *eddy diffusivity*). In general $\kappa_e \gg \kappa_m$, where κ_m is the molecular diffusivity, and thus results in $t_e \ll t_m$, i.e. the effective diffusion time is much smaller than the molecular diffusion time. If we take $\kappa_e = 10^{-2}\ \mathrm{m}^2\ \mathrm{s}^{-1}$, then repeating the calculation above leads to

$$t_m = \frac{100^2}{10^{-2}} = 10^{6}\ \mathrm{s} \approx 10\ \text{days},$$

which is substantially faster and probably closer to what we might expect.

The main problem and, really, one of the perpetual problems not just in physical oceanography but generally in fluid dynamics, is that we have no idea what this effective diffusivity should be! We can get some evidence from measurements (Ch. 8) or make some educated guesses with approximated theories, but fundamentally we don't really know what it should be from first principles. The effective diffusion is entirely dynamics dependent because of the stirring aspect, and it is context dependent[27]. In the open ocean, it is generally accepted that, away from *boundary layers* (see Ch. 3.4.2) we may have

[27] The dynamical stirring does not necessarily have to be completely diffusive either! Not opening this giant can of worms here...

$$\kappa_{e,z} = 10^{-4}\ \text{to}\ 10^{-5}\ \mathrm{m}^2\ \mathrm{s}^{-1}, \qquad \kappa_{e,h} = 10^{-1}\ \text{to}\ 10^{3}\ \mathrm{m}^2\ \mathrm{s}^{-1},$$

where the effective vertical (or diapycnal) diffusivity $\kappa_{e,z}$ is much smaller than the horizontal (or along-isopycnal, or neutral) diffusivity because the ocean is density stratified, so motion in one direction is severely inhibited. Again, these depend on dynamics. Additionally, why this is saying is that it is actually very difficult to move water up and down, so there are questions as to how the MOC functions. While we may know how water goes down, how does it come back up again? The *abyssal upwelling* problem is briefly touched up on now when we talk a bit about *boundary layers*, and in more detail in Ch. 5.2.4.

3.4.2 Non-dimensional numbers and boundary layers

Just like the Rossby number (Ch. 3.2.2) defines the relative importance of the Coriolis effect as the ratio of dynamical to rotational time-scales, we can similarly define non-dimensional numbers to measure the importance of diffusion and viscosity relative to the dynamics. Doing analogous non-dimensionlisations but for the tracer equations Eq. (2.1), the **Péclet number**[28] is defined as

$$\mathrm{Pe} = \frac{UL}{\kappa} = \frac{\text{advective transport}}{\text{diffusive transport}}, \tag{3.17}$$

so if $\mathrm{Pe} \ll 1$, diffusion is important at the length and velocity scales concerned. The equivalent to the Péclet number for viscosity has its own name, called the **Reynolds number**[29], and is defined as

$$\mathrm{Re} = \frac{UL}{\nu}. \tag{3.18}$$

Again, if $\mathrm{Re} \ll 1$ then viscosity is important at the length and velocity scales concerned. The expectation is that high Re flow is *turbulent*; if $\mathrm{Re} \gg 1$, the nonlinear inertial terms in the momentum equation Eq. (3.1) are not negligible, and nonlinear effects are important.

When rotation is involved then one can ask about the relative importance of rotation versus viscous effects. This ratio is defined by the **Ekman number**, given by

$$\mathrm{Ek} = \frac{\nu}{fL^2} = \frac{(U/L)/f}{UL/\nu} = \frac{\mathrm{Ro}}{\mathrm{Re}}, \tag{3.19}$$

where $\mathrm{Ek} \gg 1$ means viscous effects dominate. The Ekman number is normally a more fundamental property in rotating fluid dynamics that is used often in geophysics and planetary sciences.

Another way to think about the above numbers is that, given for example ν, this implies a length-scale L_δ below which diffusive effects start dominating over dynamics, i.e. where Re, Pe or $\mathrm{Ek} \approx 1$. For large-scale dynamics, generally we have $\mathrm{Pe} \gg 1$, $\mathrm{Re} \gg 1$, and $\mathrm{Ek} \ll 1$, i.e. viscous effects do not dominate[30]. However, there are locations where the effective diffusivities are higher, and usually these occur near *boundaries* in a thin region, called a **boundary layer**. A way to define the boundary layer is to denote the regions near the boundary where Re, Pe or $\mathrm{Ek} \approx 1$. The boundary layer extent is then given by the appropriate L_δ.

A schematic of the ocean bottom boundary layer is given in Fig. 3.24. Here, if we regard the ground as stationary while the ocean is moving over it, then the flow of the ocean has to go to zero as we approach the bathymetry. Away from the boundary layer, viscous effects are unimportant and the flow does whatever it is doing. As

[28] After the French physicist Jean Claude Eugǹe Péclet (1793-1857). He was Coriolis's brother-in-law.

[29] After the British fluid dynamicist Osborne Reynolds (1842-1912), who made some of the first systematic studies of laminar to turbulent flow transitions.

[30] From a computational point of view this is actually a bad thing, because it means to really represent all the dynamics permitted by the choice of diffusivities and viscosity, we need to a spatial (and temporal) resolution fine enough to resolve down to roughly where these numbers are $O(1)$.

we get into the boundary layer, viscous effects start dominating, leading a stronger diffusion, arising mostly because of the dynamics going on in the region, causing the flow to decrease. From the point of view of the ocean, the ocean is experiencing *friction* provided by the ground, and is losing momentum into the ground. Depending on the bathymetric features, the effective boundary layer may have a larger vertical extent: the ocean may see the ground as more 'rough', and the corresponding friction might be larger. The bottom boundary in this instance is a region of larger diffusivity (and, in particular, *diapycnal diffusivity*) and a place where momentum is lost; we revisit these two important points in Ch. 5.

Figure 3.24: Momentum diffusion and friction. Friction arises because there is relative motion, so it might look like there is momentum loss if we are looking only at the ocean as it's own system. But really what is happening is that the ocean is transferring momentum into the land, and from the ocean + earth system point of view there is no momentum lost.

A similar argument can be made for the Ekman layer (Ch. 3.3.2) and shown schematically in Fig. 3.25. The atmosphere tends to move much faster than the ocean, but there is a region around the atmosphere-ocean interface where diffusive effects are important. The atmospheric flow from this point of view sees the ocean as a drag but, conversely, the ocean sees the atmosphere as a source of momentum.

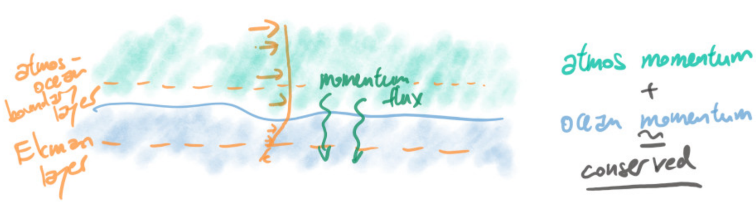

Figure 3.25: Same as above but for atmosphere and land. Atmosphere acts as a **source** of momentum for ocean most of the time (equivalently, the ocean acts as a **sink** of momentum for the atmosphere).

Noting that

$$\mathrm{Ek} = \frac{\nu}{fL^2} \quad \Rightarrow \quad L_{\mathrm{Ek}} = \sqrt{\frac{\nu_e}{f}}, \tag{3.20}$$

if we take $f \sim 10^{-4}\ \mathrm{s}^{-1}$ and $\nu_{e,z} = 10^{-2}\ \mathrm{m}^2\ \mathrm{s}^{-1}$, $L_{\mathrm{Ek}} = 10$ m. However, note that $\nu_{e,z}$ itself maybe a function of depth, so the Ekman layer depth is only an estimate.

3.4.3 *Friction vs. diffusion*

Friction is the resistance to relative motion, so we only have friction if things are moving; contrast this with diffusion where things will spread as long as there is a gradient. Our favourite pig friend in Fig. 1.22 has *ground friction* and *air drag* acting against it when it is in motion, because it is moving relative to the ground and air (which doesn't want to move). Friction can also manifest as internal stresses against relative motion within a body itself. If you ever tried picking a cat up, you will probably get the result in Fig. 3.26. The cat clearly does not want to move, but there is a force acting on it, so the middle part moves up, but since it is connected to the other parts that are not moving, there is an internal stress that is trying to resist the motion. These internal stresses occur generally in solids and fluids, and in fluids the internal stresses manifest as a viscosity for example[31].

[31] As we know cats are liquid, so maybe we should assign viscosities to them.

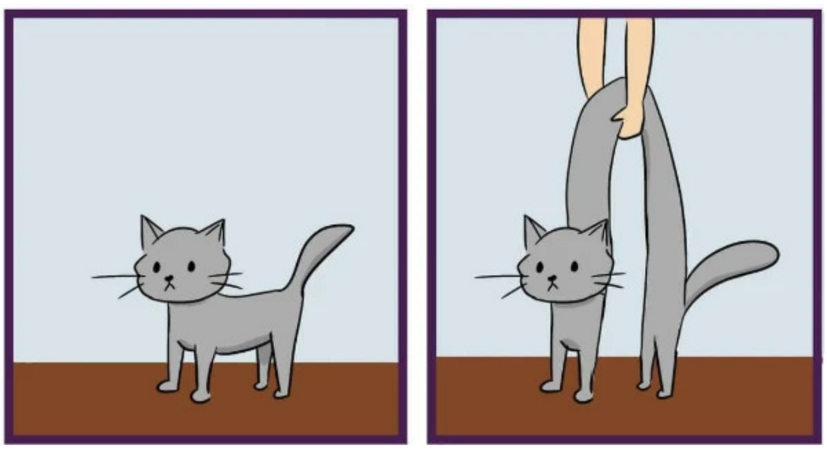

Figure 3.26: Cat physics. Picture from Meowingtons.

There are multiple ways to represent friction. Two examples are

$$-r\boldsymbol{u}, \qquad -C_d|\boldsymbol{u}|\boldsymbol{u}, \tag{3.21}$$

which are called **linear drag** and **quadratic drag** respectively, with some coefficients r and C_d (these have units but omitting them here). These function certainly are zero when there $\boldsymbol{u} = 0$, and act in the opposite direction to $\boldsymbol{u}$. The main point however I want to highlight here is that these functions remove velocity differences (because they are larger for larger flows), which affects momentum (because momentum is mass times velocity). Viscosity (or momentum diffusion) on the other hand does not remove momentum, but redistributes in, i.e. drag can act as a sink of momentum but diffusion cannot.

But you may also ask that, since Newton's laws effectively says we should have momentum conservation, then does drag violate conservation given it removes momentum? The subtlety here is that momentum is conserved in a *closed* system, and the ocean by itself is not a closed system. Consider the cases in Fig. 3.24 and 3.25, where the friction is going to be over the boundary layer. While from the ocean's point of view the ground provides a drag and thus is a **sink** of ocean momentum, from the ground's point of view the ocean is acting as a **source** of momentum. Friction removes relative motion but in such a way that the combined system momentum is conserved. Similarly for the atmosphere and ocean, the atmosphere sees the ocean as a sink, but the ocean sees the atmosphere as a source of momentum. From that point of view there is no contradiction. When we model the ocean however we usually don't model the solid Earth at the same time[32], so we have what looks like a momentum loss out of the ocean, but it's because we don't explicitly account for where it is going.

[32] E.g. when talking about sea level the ocean loading onto land and the land rebounds make a difference, so you do need some land physics there.

Summary and further reading

Changes in density manifest in changes of pressure mostly via hydrostatic balance, and negative pressure gradients are forces and drives flows via its contribution to the momentum equation. A slight complication arises because the Earth is rotating, and we make the standard and convenient decision to view the dynamics in a practical perspective, but one such that we need to include a deflection by the Coriolis effect. Again, the Coriolis effect only arises because of a difference in perspective.

Accepting this slight complication, we argued if the flow is in geostrophic balance (where $\mathrm{Ro} \ll 1$), then the geostrophic flow actually travels to the right of the direction of intended travel in the Northern Hemisphere, and to the left in the Southern Hemisphere (because the Coriolis parameter f changes sign). The same argument works mostly for wind forcing: although we have an Ekman spiral, the Ekman transport is largely in the direction of the geostrophic flow. The Ekman upwelling and downwelling was argued to be related to the wind stress curl and f, and the corresponding geostrophic upwelling and downwelling was argued to be related to the fluid vorticity and f. The concept of diffusion and friction was introduced, highlighting the differences between the know but too small molecular diffusion, and the larger but unknown effective diffusivity that depends crucially on dynamics. While diffusion redistributes momentum, friction removes momentum if we are considering the ocean as its own system.

One of the main things to note is that the boundary layers are regions where most the momentum transfers happen between the ocean and other Earth system components, these regions may also be regarded as disproportionately important regions[33]. The difficulty that we don't touch on here is that, because the boundary layers are relatively shallow, the dynamics are very hard to understand in these regions! There are many tools and tricks that simply don't work in these regions (e.g. geostrophic balance). Lots of small-scale dynamics are at also play, with nonlinear effects and feedbacks all over the place. To understand the boundary layer behaviours and the dependence of eddy diffusivities on dynamics, we need to understand the contributing small-scale dynamics, some of which are introduced somewhat in Ch. 6.

[33] This is in line with the mathematical observation that if you change the boundary condition you are probably going to change everything.

Chapter exercises

1. From the example surrounding Fig. 3.2, doing the calculations and not dropping decimal places everywhere like I did, show that $g \approx 9.81\ \mathrm{m\ s^{-2}}$.

2. From the example surrounding Fig. 3.2, work out g_{moon}.

3. From the example surrounding Fig. 3.2, without working out g_{pig}, do you think this is large or small? Taking $m_{\mathrm{pig}} = 100$ kg, work out g_{pig}.

4. Convince yourself (pictorially or mathematically) that if $\boldsymbol{g} = -\nabla\phi$, then $\boldsymbol{g}$ is perpendicular to the surfaces of $\phi =$ constant.

5. If you are at sea level you experience a pressure of 1000 mb, or 1 atmopsheric pressure (1 atm), i.e. there is one Earth atmosphere's worth of pressure pushing down on you. Work out via hydrostatic balance the pressure you experience just from having 10 m of sea water above you (with no atmosphere above you), assuming for simplicity that $\rho_{\mathrm{sea}} = 1000\ \mathrm{kg\ m^{-3}}$ and $g = 10\ \mathrm{m\ s^{-2}}$. What is the pressure in units of mb if you are 1000 m below the ocean (with no atmosphere above you)?

6. Using the above calculation and assumptions, calculate the largest hydrostatic pressure (in atm) due to just sea water experienced by Captain Nemo's *Nautilus* in Jules Verne's novel. (Wikipedia will help you here; the answer is not $20000 \times 4 \times 1000/10 = 8 \times 10^6$ atm!)

7. Demonstrate mathematically that the Coriolis effect does no work.

8. In the general form the Coriolis effect is given by the term $-\boldsymbol{\Omega} \times \boldsymbol{u}$. Using the properties of the cross product, without calculations (so draw pictures or reason it out), rationalise that, for a horizontal flow parallel/tangent to the Earth's surface at the Equator, there is no horizontal Coriolis effect acting on this flow (hence why $f = 0$ at the equator).

9. Show the above mathematically (probably want to use cylindrical co-ordinates for this).

10. What does the Coriolis parameter f look like for a cylindrical Earth (with rotation axis pointing out of the circle of the cylinder)?

11. Try the Fig. 3.10 experiment but draw the line really slowly, and allow the piece of paper to rotate multiple times, and convince yourself you do get looping motion (cf. *inertial oscillations*).

12. Look up some numbers to work out the Rossby numbers of Jupiter, the Solar interior[34] and Venus. They should be tiny, about one, and large respectively.

[34] Try looking for *Solar tachochline* maybe

13. Comment on whether there is any scientific merit to the claim that "*when you flush the toilet in the Southern Hemisphere, the water flows the other way compared to the Northern Hemisphere because the Coriolis effect is of a different sign*". Back up your claims, and provide some estimates accordingly.

14. Show mathematically that, for a flow in geostrophic balance, i.e. $f\boldsymbol{e}_z \times \boldsymbol{u}_g = -\rho_0^{-1}\nabla p$, $\boldsymbol{u}_g$ is always perpendicular to $-\nabla p$.

15. Normally we hear about storms forming in the ocean and then hitting the south-eastern North America and north-western Europe, but not north-western parts of North America or south-western Europe, why is that?

16. What happens to geostrophic balance when friction is involved? Extend the pictorial arguments in Fig. 3.12 and, assuming friction acts against the direction of $\boldsymbol{u}_g$, show that resulting angle between $\boldsymbol{u}_g$ and $-\nabla p$ has to be strictly less than 90° ($\pi/2$), i.e. the resulting geostrophic flow is tilted into the direction of $-\nabla p$. From this deduce that anti-cyclonic eddies have associated with it divergent flow at the bottom of the eddy, implying a downwelling, and cyclonic eddies have associated with convergent flow, implying an upwelling[35].

[35] Note at no point here have I said anything about the ocean or the atmosphere.

4 Wind-driven gyre theory

Here we give a first example on how to put a good portion of what we learnt so far to provide a dynamical explanation for *gyres* (note the use of <u>a</u> rather than <u>the</u> in this sentence though). While the focus will be on the classical wind-driven gyre theory, see the end of chapter for some complementary descriptions. The wind-driven gyre to me is a particularly good example of a good simple *model*: you learn stuff because the model works, but you *also* learn stuff because the model *doesn't* work 100%!

4.1 Recap: winds and subtropical/subpolar gyres

Recall from Ch. 1 that we have the anti-cyclonic subtropical gyres and cyclonic subpolar gyres in both hemispheres, and associate with these gyres are Western Boundary Currents that tend to flow polewards (at least from surface observations). Recall also from Ch. 3 that, starting from the equator and going towards the poles (again in both hemispheres), we have the trade winds, prevailing Westerlies, and polar winds that lead to an alternating wind pattern over the subtropics, mid-latitudes and polar regions respectively, with the gyre currents in the same direction as the wind forcing. This is not a coincidence, and from intuition (e.g. energetic point of view) it's kind of hard to see how else it could be.

However, the energetic kind of argument does not entirely explain why we should have *Western intensification* and Western Boundary Currents: we could just have a broad flow which might at first sight be more dynamically favourable (e.g. broader current has less shear and should be less *unstable*; see Ch. 6). Here we provide a largely pictorial argument as to why we should have western intensification[1]. The main ingredients and broad logical deduction we will talk about are as follows:

[1] Important note: you get the *why* but not the quantitative aspect of how narrow the Western Boundary Current might want to be, although it does suggest where to look nice, which in my view is one of the strengths of the theory.

1. dissipative/diffusive effects weak away from land boundaries

2. *Sverdrup balance*, relating the wind stress *curl* with the interior flow

3. *mass conservation* implies configuration with essentially two orientations

4. *vorticity balance* sets the orientation and it needs to be on the West, intensification because transition has to occur over the (Stommel) boundary layer length.

4.2 β-plane and toy model set up

For the intended purposes we first start by introducing more formally a concept that we have used previously when talking about the Coriolis effect in Ch. 3 called the β**-plane**.

4.3 Sverdrup balance

such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow

4.4 Vorticity balance

such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow such space very filler many useless not wow

Summary and further reading

1.

5 Meridional Overturning Circulation (MOC)

Really gyres should be here but not...

5.1 Southern Ocean

5.1.1 Overturning

5.1.2 Thermal wind

5.1.3 Form stress

5.2 Global MOC

5.2.1 MOC and watermass identification

5.2.2 Conveyor belt picture (but why we prefer to use MOC)

5.2.3 How does water go down...?

1.3.2

5.2.4 How does water come up...?

6 Dynamics

Hopefully the last chapters on the gyres and the MOC has convinced you somewhat the importance of *small(er)-scale dynamics* in the multi-scale system that is the ocean. Motion at the large-scales can lead to small-scale motion via *instabilities*. *Waves* can be excited by these instabilities or other forms of forcing. To add to the mix, the small-scale motion can interact and lead to feedback onto the large-scale motions, modifying the things that generated/co-exists with the small-scales in the first place. This overall topic of *wave/eddy-mean interaction* is of theoretical interest as a fundamental problem in geophysical fluid dynamics, but also has important practical consequences (e.g. *parameterisation of sub-grid physics* in numerical models used in predicting ocean/atmosphere weather and/or climate). Here we are only going to talk about how the mean (large-scale motion) generates the waves/eddies (small-scale motion), but only marginally touch on the converse aspect (because it's kind of hard!)

6.1 Waves

6.1.1 Concepts

6.1.2 Gravity waves

6.1.3 Inertial waves

6.1.4 Internal waves

6.2 Instabilities

6.2.1 Static instabilities (i.e. density related mostly)

6.2.2 Shear instabilities (i.e. flow related mostly)

The eagle eye among you will of course note that, for large-scale flow, flow and density are of course related through thermal wind shear relation.

6.3 Tides

6.3.1 Astronomical forcing

6.3.2 *Modes*

6.4 *Misc. random extras*

Westward control?

7 Observations techniques

7.1 Recap: what do we actually want?

7.1.1 subsection

7.2 In-situ observations

7.2.1 Fixed stations

7.2.2 Ships

7.2.3 Various bits of equipment

7.2.4 Marine mammals

7.3 Remote observations

7.3.1 Satellites

7.3.2 Radar

7.3.3 Acoustic tomography

8 Dynamics in not-oceans

8.1 What is so different now?

8.1.1 subsection

8.2 Shelf dynamics

8.2.1 Ekman upwelling

8.3 Coastal dynamics

8.3.1 Coastal upwelling

8.3.2 Rip currents

8.4 Some biogeochemistry?

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