8, 15, 17, 27, 28 e 42 cinco primeiros e o limite se ele converge

9.1

8. $\left\{ \frac{n^2}{2n+1} \right\}_{n=1}^{+\infty}$ $\frac{1}{3}, \frac{4}{5}, \frac{9}{7}, \frac{16}{9}, \frac{25}{11},$

L'Hopital

$\lim \frac{x^2}{2x+1} = \lim_{x \to +\infty} \frac{\cancel{2}x}{\cancel{2}} = \lim_{x \to +\infty} x = +\infty$ $\boxed{\text{Diverge}}$

15. $\left\{ (-1)^n \frac{2n^3}{n^3+1} \right\}_{n=1}^{+\infty}$ $(-1)^1 \frac{2}{2}, \frac{16}{9}, -\frac{54}{28},$ $\boxed{\text{Diverge}}$

$\lim_{n \to +\infty} \frac{2n^3}{n^3+1} = \frac{\frac{2\cancel{n^3}}{\cancel{n^3}}}{\frac{\cancel{n^3}}{\cancel{n^3}} + \frac{1}{n^3}} = \frac{2}{1+\frac{1}{n^3}} = \frac{\lim 2}{\lim 1 + \lim \frac{1}{n^3}} = \frac{2}{1+\left(\lim \frac{1}{n}\right)^3} = \frac{2}{1+}$

$$\lim_{n\to+\infty} \frac{\ldots}{n^3+1} = \frac{\ldots}{\frac{\cancel{n^3}}{\cancel{n^3}}+\frac{1}{n^3}} = \frac{\ldots}{1+\frac{1}{n^3}} = \frac{\ldots}{\lim 1 + \lim \frac{1}{n^3}} \quad \frac{\ldots}{1+\left(\lim \frac{1}{n}\right)}$$

17. $\left\{\frac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{+\infty}$ $\quad \frac{6}{2}, \frac{12}{8}, \frac{20}{18}, \frac{30}{32}, \frac{42}{50}$

$$\frac{\frac{(\cancel{n}+1)}{n^{\cancel{2}}} \quad \frac{(\cancel{n}+2)}{n^{\cancel{2}}}}{\frac{2\cancel{n^2}}{\cancel{n^2}}} = \frac{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}{2}$$

$\boxed{\text{Converge}}$

$$\lim_{n\to+\infty} \frac{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}{2} = \frac{1}{2} \lim\left(1+\frac{1}{n}\right) \lim\left(1+\frac{2}{n}\right) = \frac{1}{2}$$

Encontre o termo geral

27

$$\left(1-\frac{1}{2}\right)\cdot\left(\frac{1}{3}-\frac{1}{2}\right)\cdot\left(\frac{1}{3}-\frac{1}{4}\right)\cdot\left(\frac{1}{5}-\frac{1}{4}\right)\cdots$$

$\boxed{a_n = (-1)^{n+1}\left(\frac{1}{n}-\frac{1}{n+1}\right)}$

$\lim_{n\to+\infty} \frac{1}{n} = 0$ portanto $\lim \frac{1}{n} - \frac{1}{n+1} = 0$

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27

$$\left(1-\frac{1}{2}\right)\cdot\left(\frac{1}{3}-\frac{1}{2}\right)\cdot\left(\frac{1}{3}-\frac{1}{4}\right)\cdot\left(\frac{1}{5}-\frac{1}{4}\right)\cdots$$

$$\boxed{a_n=(-1)^{n+1}\left(\frac{1}{n}-\frac{1}{n+1}\right)}$$

$\lim_{n\to+\infty}\frac{1}{n}=0$ portanto $\lim \frac{1}{n}-\frac{1}{n+1}=0$

28. $3, \frac{3}{2}, \frac{3}{2^2}, \frac{3}{2^3} \cdots$ $\boxed{a_n=\frac{3}{2^{n-1}}}$ $\lim_{x\to\infty}\frac{3}{2^{x-1}}=0$

42. $a_1=\sqrt{6}$

$a_2=\sqrt{6+\sqrt{6}}$

$a_3=\sqrt{6+\sqrt{6+\sqrt{6}}}$

$L=\sqrt{6+L}$

$L=3$

$$\boxed{a_{n+1}=\sqrt{6+a_n}}$$

9.2 5, 6, 8, 12, 27

5. verificar se estritamente crescente ou decrescente use $a_{n+1} - a_n$

$(n - 2^n)_{n=1}^{+\infty}$

$n+1 - 2^{n+1} - n + 2^n = 1 - 2^n < 0$

é estritamente decrescente

---

6. $\{n - n^2\}_{n=1}^{+\infty}$

$n+1 - (n+1)^2 - (n - n^2) =$

$n+1 - (2n + n^2 + 1) - (n - n^2)$

$\cancel{n} + \cancel{1} - 2n - \cancel{n^2} - \cancel{1} - \cancel{n} + \cancel{n^2} = -2n$

estritamente decrescente

8. Use $\frac{a_{n+1}}{a_n}$ estritamente crescente ou decrescente

$$\left\{\frac{2^n}{1+2^n}\right\}_{n=1}^{+\infty} \qquad \frac{\frac{2^{n+1}}{1+2^{n+1}}}{\frac{2^n}{1+2^n}} = \frac{\cancel{2}^{n+1}}{1+2^{n+1}} \cdot \frac{1+2^n}{\cancel{2^n}} = \boxed{1+\frac{1}{1+2^{n+1}} > 1}$$

estritamente crescente

---

12. $$\left\{\frac{5^n}{2^{(n^2)}}\right\}_{n=1}^{+\infty} \qquad \frac{\frac{5^{n+1}}{2^{(n+1)^2}}}{\frac{5^n}{2^{n^2}}} = \frac{\cancel{5}^{n+1}}{\cancel{2}^{n^2+2n+1}} \cdot \frac{2^{n^2}}{5^n} = \boxed{\frac{5}{2^{2n+1}} < 1}$$

estritamente decrescente

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12. $\left\{ \frac{5^n}{2^{(n^2)}} \right\}_{n=1}^{+\infty}$

$$\frac{\frac{5^{n+1}}{2^{(n+1)^2}}}{\frac{5^n}{2^{n^2}}} = \frac{5^{n+1}}{2^{n^2+2n+1}} \cdot \frac{2^{n^2}}{5^n} = \frac{5}{2^{2n+1}} <$$

estritamente decrescente

---

27. $a_1 = \sqrt{2}$ $\quad a_{n+1} = \sqrt{2+a_n}$

a) $a_1 = \sqrt{2}$ $\quad a_2 = \sqrt{2+\sqrt{2}}$ $\quad a_3 = \sqrt{2+\sqrt{2+\sqrt{2}}}$

b) $a_2 = \sqrt{2+\sqrt{2}} < \sqrt{2+2} = 2$

c) $a_{n+1}^2 - a_n^2 = \sqrt{2-a_n}^2 - a_n^2 = 2+a_n-a_n^2 = 2+a_n-a_n^2+a_n-a$

$= 2(1+a_n)-a_n(1+a_n) = (2-a_n)(1+a_n)$

d)

9.3 5, 7, 19, 17, 21 e 34

5. Determine se a série converge; se convergir encontre sua soma

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{7}{6^{k-1}}$$

$$\sum_{k=0}^{\infty} (-1)^{k} \frac{7}{6} = \sum_{k=0}^{\infty} 7\left(-\frac{1}{6}\right)^{k} \qquad \frac{a}{1-r} \quad \frac{7}{1+\left(\frac{1}{6}\right)} = \frac{7}{\frac{7}{6}} = \boxed{6}$$

7. $$\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)} \qquad \lim_{n \to \infty} \frac{1}{3} - \frac{1}{(n-3)} = \boxed{\frac{1}{3}}$$

$$\sum_{k=0}^{\infty} \frac{1}{(k+1)(k+2)} = \frac{1}{3} - \frac{1}{n-3}$$

Converge

$\infty \quad (\ldots)^{k}$

14. $\sum_{K=1}^{\infty} 5^{3K} 7^{1-K} = \sum_{K=1}^{\infty} \frac{125^K}{7^{K-1}} = \sum_{K=1}^{\infty} \frac{125^K}{7^K \cdot 7^1} = 7\sum_{K=1}^{\infty} \left(\frac{125}{7}\right)^K$

diverge

$a=1 \quad r=\frac{125}{7}$

$|r| \leq 1$ diverge

$|r| \geq 1$ converge

$r > 1$

17. Uma série infinita converge se a sequencia de seus termos convergir

21. Expresse a dízima periódica como uma fração

21. 0,9999...

$\sum_{K=0}^{\infty} 0{,}9(0{,}1)^K = \frac{a}{1-r} = \frac{0{,}9}{1-0{,}1} = \frac{0{,}9}{0{,}9} = \underline{1}$

34. $\sum_{K=1}^{\infty} \frac{\sqrt{K+1} - \sqrt{K}}{\sqrt{K^2+K}} = 1$

9.4 - 4, 5, 8, 10, 18 e 21

4. Identifique p e determine se a série converge

4. (a) $\sum_{k=1}^{\infty} k^{-4/3} = \sum \frac{1}{k^{4/3}}$

$p = \frac{4}{3} > 1$ convergente

(d) $\sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$ $p = \pi > 1$ convergente

(b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[4]{k}}$ $\sqrt[4]{k} = k^{\frac{1}{4}}$ $p = \frac{1}{4} \leq 1$ divergente

(c) $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k^5}}$ $k^{\frac{5}{3}}$ $p = \frac{5}{3} > 1$ convergente

---

Aplique o teste da divergência e escreva a conclusão

5. (a) $\sum_{k=1}^{\infty} \frac{k^2+k+3}{2k^2+1}$

(d) $\sum_{k=1}^{\infty} \frac{1}{k!}$ nada pode-se afirmar

$\lim_{k \to \infty} \frac{1}{k!} = 0$

(b) $\sum_{k=1}^{\infty} \left(1+\frac{1}{k}\right)^k = e$ divergente

(c) $\sum^{\infty} \cos k\pi$ divergente $\lim_{k \to \infty} \cos k\pi \nexists$

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8. confirme ser aplicável o teste do integral

a) $\sum_{K=1}^{\infty} \frac{K}{1+K^2}$

$$\int_2^{\infty} \frac{x}{1+x^2}\,dx = \lim \int_2^{b} \frac{x}{1+x^2}\,dx = \lim \left[\frac{1}{2}\ln(1+x^2)\right]_2^b$$

Dive

$$= \lim\left(\frac{1}{2}\ln(1+b^2) - \frac{1}{2}\ln 5\right) = \frac{1}{2}\lim \ln(1+b^2) - \frac{1}{2}\ln 5 = +\infty$$

b) $\sum_{K=1}^{\infty} \frac{1}{(4+2K)^{3/2}}$

$$\int_1^{\infty} \frac{1}{(4+2x)^{3/2}}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{1}{(4+2x)^{3/2}}\,dx = \lim \left[\frac{-1}{\sqrt{4+2x}}\right]_1^b$$

$$= \lim_{b\to\infty} \frac{-1}{\sqrt{4+2b}} + \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

Convergente

9.5 - 3, 7, 14, 17, 28, 39 e 48

③ Em cada parte use o teste da comparação para mostrar que a serie converge

$a = \frac{1}{3}$ $r = \frac{1}{3}$

ⓐ $\sum_{K=1}^{\infty} \frac{1}{3^K + 5}$

$\sum_{K=0}^{\infty} \frac{1}{3}\left(\frac{1}{3}\right)^K$ é convergente então $\sum_{K=1}^{\infty} \frac{1}{3^K + 5}$ também é

ⓑ $\sum_{K=1}^{\infty} \frac{5\, sen^2 K}{K!}$

---

⑦ Teste da comparação no limite

$\sum_{K=1}^{\infty} \frac{5}{3^K + 1}$

$\lim \frac{a_K}{b_K} = \lim_{K \to \infty} \frac{\frac{5}{3^K + 1}}{\frac{1}{3^K}} = \lim \frac{\frac{1}{3^K}\left(\frac{5}{1 + \frac{1}{3^K}}\right)}{\frac{1}{3^K}} = \frac{5}{1 + \lim_{K \to \infty} \frac{1}{3^K}} = 5 > 0$

Converge

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14 Teste da razão

$\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$ $\quad \rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k} = \lim \frac{(k+1)\left(\frac{1}{2}\right)^{k+1}}{k\left(\frac{1}{2}\right)^k} = \lim \frac{(k+1)}{k}\left(\frac{1}{2}\right) = \lim\left(1+\frac{1}{k}\right)\left(\frac{1}{2}\right) =$

$= \frac{1}{2}$ $\quad \rho = \frac{1}{2} < 1$ converge

17 Teste da raiz

$\sum_{k=1}^{\infty}\left(\frac{3k+2}{2k-1}\right)^k$ $\quad u_k = \left(\frac{3k+2}{2k-1}\right)^k$

$\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} \sqrt[k]{\left(\frac{3k+2}{2k-1}\right)^k} = \lim_{k\to\infty}\left(\frac{3k+2}{2k-1}\right) =$

$= \lim_{k\to\infty} \frac{k\left(3+\frac{2}{k}\right)}{k\left(2-\frac{1}{k}\right)} = \frac{\left(3+\lim_{k\to\infty}\frac{2}{k}\right)}{\left(2-\lim_{k\to\infty}\frac{1}{k}\right)} = \frac{3}{2}$ $\quad \rho = \frac{3}{2} > 1$

divergente

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2B. $\sum_{k=1}^{\infty} \frac{k!\,10^k}{3^k} =$

$\rho = \lim \frac{u_{k+1}}{u_k}$ $\lim_{k\to\infty} \frac{\frac{(k+1)!\,10^{k+1}}{3^{k+1}}}{\frac{k!\,10^k}{3^k}} = \frac{(k+1)!\,10^{k+1}}{3^{k+1}} \cdot \frac{3^k}{k!\,10^k} = \lim_{k\to\infty} \frac{(k+1)\,10}{3} =$

$= \frac{10}{3} \lim_{k\to\infty} k+1 = \infty$

divergē

3B. $\sum_{k=1}^{\infty} \frac{\ln k}{e^k} =$

$\rho = \lim_{k\to\infty} \sqrt[k]{\frac{\ln k}{e^k}} = \lim_{k\to\infty} \frac{\sqrt[k]{\ln k}}{e} = \frac{1}{e} \lim_{k\to\infty} \sqrt[k]{\ln k} = \frac{1}{e}$

$\rho = \frac{1}{e} < 1$ convergē

4B. $\sum_{k=1}^{\infty} \frac{[\pi(k+1)]^k}{k^{k+1}} =$

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9.6 - 1, 5, 10, 14, 17

① Teste da Série Alternada

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$$

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⑤ $$\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$$

$$e^{-k} = \frac{1}{e^{k}}$$

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(10) Teste da razão para convergência absoluta

$$\sum_{k=1}^{\infty} (-1)^k \frac{k}{5^k} \quad \lim_{k\to\infty} \frac{\frac{k+1}{5^{k+1}}}{\frac{k}{5^k}} = \lim_{k\to\infty} \frac{k+1}{5^{k+1}} \cdot \frac{5^k}{k} = \lim_{k\to\infty} \frac{k+1}{5k}$$

$$\lim_{x\to\infty} \frac{x+1}{5x} = \frac{\frac{d}{dx}(x+1)}{\frac{d}{dx}(5x)} = \lim_{x\to\infty} \frac{1}{5} = \frac{1}{5} \quad \rho = \frac{1}{5} < 1$$

convergente

(11) $$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{4/3}}$$

$$u_k = \left|\frac{(-1)^{k+1}}{k^{4/3}}\right| = \frac{1}{k^{4/3}}$$

convergente

9.8 - 21, 25, 29, 35 e 37

21 $1 + (x-2) + (x-2)^2 + \ldots + (x-2)^k$

converge $|x-2| < 1$

diverge $|x-2| \geq 1$

$$\sum_{k=0}^{\infty} (x-2)^k = \frac{1}{1-(x-2)} = \frac{1}{3-x}$$

---

25 V

29) raio de convergencia

$$\sum_{k=0}^{\infty} \frac{x^k}{k+1}$$

$$\lim_{k\to\infty} \left| \frac{u_{k+1}}{u_k} \right| = \lim \left| \frac{x^{k+1}}{k+2} \cdot \frac{k+1}{x^k} \right| = \lim_{k\to+\infty} \left| \frac{(k+1)x}{k+2} \right| = |x| \left| \lim \frac{k+1}{k+2} \right| = |x|$$

$$\lim_{k\to\infty} \frac{k+1}{k+2} = \lim_{k\to\infty} \frac{1}{1} = \boxed{1}$$

converge se $-1 < x < 1$

não converge se $|x| > 1$