Lista de Revisão: Integrais múltiplas

1. 14.1 - 11, 15 e 32
2. 14.2 - 24, 33 e 43
3. 14.3 - 8, 20 e 30
4. 14.5 - 7 e 38
5. 14.6 - 4, 11 e 13
6. 14.8 - 2, 20 e 42

14.1

(11) $\int_0^{\ln 2} \int_0^1 xy e^{y^2 x} \, dy \, dx$

$\int_0^1 xye^{y^2x} dy \quad \int_0^1 xye^{u} dy \quad \frac{1 x e^{xy^2}}{2x} \Big]_0^1 = \frac{1}{2} e^x - \frac{1}{2}$

$\int_0^{\ln 2} \frac{1}{2} e^x - \frac{1}{2} = \frac{1}{2} e^x - \frac{1}{2} x \Big]_0^{\ln 2} \Rightarrow$

$\int \frac{1}{2} e^x = \frac{1}{2} e^x \qquad \int \frac{1}{2} dx = \frac{1}{2} x$

$\Rightarrow \lim_{x \to 0} = \frac{1}{2} e^0 - \frac{1}{2} 0 = \frac{1}{2}$

$\lim_{x \to \ln 2} = \frac{1}{2} e^{\ln 2} - \frac{1}{2} \ln 2 = 1 - \frac{1}{2} \ln 2$

$1 - \frac{1}{2} \ln 2 - \frac{1}{2} = \boxed{\frac{1 - \ln 2}{2}}$

(15) $\iint_R x\sqrt{1-x^2}\,dA$, $R = \{(x,y) : 0 \le x \le 1,\ 2 \le y \le 3\}$

$\int_0^1 \int_2^3 x\sqrt{1-x^2}\,dy\,dx$

(1) $x\int_2^3 \sqrt{1-x^2}\,dy = xy\sqrt{1-x^2}\Big]_2^3 = 3x\sqrt{1-x^2} - 2x\sqrt{1-x^2} = x\sqrt{1-x^2}$

(2) $\int_0^1 x\sqrt{1-x^2}\,dx$

$\int_0^1 \frac{x^2}{2}\sqrt{u}\,du = -\frac{1}{3}(1-x^2)^{3/2}\Big]_0^1 = -\frac{1}{3}(1-1)^{3/2} - \left(-\frac{1}{3}(1)^{3/2}\right) =$

$= \boxed{\frac{1}{3}}$

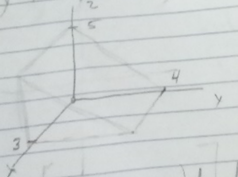
(32) O volume no primeiro octante limitado pelos planos coordenados, o plano $y = 4$ e o plano $(x/3) + (z/5) = 1$

$360 \div 8 = 45$

$x = 1 - \frac{z}{5}$

$\frac{x}{3} + \frac{z}{5} = 1$

$x = 0$, $z = 5$; $z = 0$, $x = 3$



$\int_0^3 \int_0^4 \left(\frac{x}{3} + \frac{z}{5} - 1\right) dy\,dx$

$\int_0^3 \frac{x}{3}y + \frac{z}{5}y - y \Big]_0^4 = \int_0^3 \frac{4x}{3} + \frac{4z}{5} - 4\,dx \Rightarrow \frac{4x^2}{6} + \frac{4z}{5}x - 4x\Big]_0^3$

$\Rightarrow \frac{36}{6} + \frac{12z}{5} - 12 = \frac{12z}{5} -$

data

14.5

7. $\int_0^2 \int_0^{\sqrt{4-x^2}} \left[\int_{-5+x^2+y^2}^{3-x^2-y^2} x \, dz \right] dy \, dx$

$xz \Big]_{-5+x^2+y^2}^{3-x^2-y^2}$

$x(3-x^2-y^2) - x(-5+x^2+y^2)$

$3x - x^3 - y^2x + 5x - x^3 - y^2x$

$-2x^3 + 8x - 2y^2x$

$-2x^3y + 8xy - \frac{2y^3x}{3} \Big]_0^{\sqrt{4-x^2}}$

$-2\sqrt{4-x^2}\,x^3 + 8x\sqrt{4-x^2} - \frac{2(\sqrt{4-x^2})^3 x}{3}$

$2\left(-\frac{1}{5}(\sqrt{-x^2+4})^5 + \frac{4}{3}(\sqrt{-x^2+4})^3\right) - \frac{8}{3}(-x^2+4)^{3/2} + \frac{2}{15}(4-x^2)^{5/2}$

$\left(-\frac{1}{5}(\sqrt{-2^2+4})^3\right) - \frac{8}{3}(-4+4) + \frac{2}{15}(4-4)^{5/2} = \boxed{\frac{128}{15}}$

Use uma integral tupla para deduzir a fórmula do volume do ...ado

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$\boxed{\int_0^{\pi} \int_0^{1} abc \, \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta}$

data

14.6

4. $\left[\int_0^{2\pi} \left[\int_0^{\pi/4} \left[\int_0^{a\sec\phi} \rho^2 \operatorname{sen}\phi \, d\rho\right]_{①} d\phi\right]_{②} d\theta\right]_{③}$ $(a>0)$

① $\int_0^{a\sec\phi} \rho^2 \operatorname{sen}\phi \, d\rho = \frac{\rho^3}{3}\operatorname{sen}\phi \Big]_0^{a\sec\phi} = \frac{a^3\sec\phi^3 \operatorname{sen}\phi}{3}$

② $\frac{a^3}{6}\tan^2(\theta) \Big]_0^{\pi/4} = \frac{a^3}{6}$

③ $\frac{a^3}{6}\theta \Big]_0^{2\pi} = \frac{2\pi a^3}{6} = \boxed{\frac{\pi a^3}{3}}$

Use coordenadas cilíndricas para encontrar o volume do sólido
E: sólido que está dentro da superfície $x^2+y^2=20$ e abaixo da superfície

$z=10$ dz dy dx

$2x^2=20$

$x^2=10$

$x=\sqrt{10}$

$y^2=20-x^2$

$$\iiint z \, dy\,dx\,d\theta = \boxed{\frac{1524}{3}}$$

Coordenadas esféricas: volume
E: sólido limitado acima pela esfera $\rho=4$ e abaixo pelo cone $\phi=\pi/3$, $z=\rho\cos\phi$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4} \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

① $\frac{1}{3}\rho^3 \operatorname{sen}\phi \Big]_0^4 = \frac{1}{3}4^3 \operatorname{sen}\phi = \frac{64}{3}\operatorname{sen}(\phi)$

② $-\frac{64}{3}\cos(\phi)\Big]_0^{\pi/3} = -\frac{64}{3}\cos(\pi/3) - \left(-\frac{64}{3}\cdot 1\right) = \frac{32}{3}$

③ $\frac{32}{3}\theta\Big]_0^{2\pi} = \frac{32\cdot 2\pi}{3} = \boxed{\frac{64\pi}{3}}$

data

14.2

24. $\iint_R x^2\,dA$ R é a região do 1º quadrante compreendida por $xy=1$, $y=x$ e $y=2x$

$y \cdot y = 1$ $2x \cdot y = 1$

$2y = 1$ $3y = 1$

$y = \frac{1}{2}$ $y = \frac{1}{3}$

$\int_0^1 \int_x^{2x} x^2\,dA$

$x \cdot y = 1$

$x \cdot \frac{1}{2} = 1$ $x \cdot \frac{1}{3} =$

$x = 2$

$y = \frac{1}{x}$

33. V ou F explique

$$\int_0^1 \int_{x^2}^{2x} f(x,y)\,dy\,dx = \int_{x^2}^{2x} \int_0^1 f(x,y)\,dx\,dy$$

falsa pois ao fazer $f(x,y)=1$ na primeira resulta um [illegible] e na segunda uma expressão

$$\int_0^1 \int_{x^2}^{2x} 1\,dy\,dx = \int_0^1 2x - x^2\,dx = \frac{2x^2}{2} - \frac{x^3}{3} = \frac{2}{3}$$

$$\int_{x^2}^{2x} \int_0^1 1\,dx\,dy = \int_{x^2}^{2x} x\Big]_0^1 dy = \int_{x^2}^{2x} y\,dy = \frac{y^2}{2}\Big]_{x^2}^{2x} = 2x-$$

data

14.5

7. $\int_0^2\int_0^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x\,dz\,dy\,dx$

$xz\Big|_{-5+x^2+y^2}^{3-x^2-y^2}$ $\quad x(3-x^2-y^2) - x(-5+x^2+y^2)$

$3x - x^3 - y^2x + 5x - x^3 - y^2x$

$-2x^3 + 8x - 2y^2x$

(2) $-2x^3y + 8xy - \frac{2y^3x}{3}\Big]_0^{\sqrt{4-x^2}}$ $\quad -2\sqrt{4-x^2}\,x^3 + 8x\sqrt{4-x^2} - \frac{2(\sqrt{4-x^2})^3 x}{3}$

(3) $= 2\left(-\frac{1}{5}(\sqrt{-x^2+4})^5 + \frac{4}{3}(\sqrt{-x^2+4})^3\right) - \frac{8}{3}(-x^2+4)^{3/2} + \frac{2}{15}(4-x^2$

$= 2\left(-\frac{1}{5}(\sqrt{-2^2+4})^3\right) - \frac{8}{3}(-4+4) + \frac{2}{15}(4-4)^{5/2} = \boxed{\frac{128}{15}}$

38. Use uma integral tripla para deduzir a fórmula do volume elipsoide.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\boxed{V = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} abc\, \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta}$$

20.

30. Calcule a integral iterada convertendo para coordenadas polares

coordenadas polares

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy$$

$$\int_0^{\pi/2}\int_0^1 \cos(r)\,r\,dr\,d\theta = \int_0^{\pi/2} \operatorname{sen}(r)\Big]_0^1 d\theta = \int_0^{\pi/2} \operatorname{sen}(1)\,d\theta$$

$\cos(r)$

$$\operatorname{sen}(1)\cdot\theta\Big]_0^{\pi/2} = \boxed{\operatorname{sen}(1)\cdot\frac{\pi}{2}} =$$

13. coordenadas esféricas Volume

O sólido limitado acima pela esfera $\rho = 4$ e abaixo pelo cone $\phi = \pi/3$ $z = \rho\cos\theta$

$$\int_0^{2\pi}\int_0^{\pi/3}\int_0^4 \rho^2 \operatorname{sen}\phi\,d\rho\,d\phi\,d\theta$$

1. $\frac{1}{3}\rho^3 \operatorname{sen}\phi\Big]_0^4 = \frac{1}{3}4^3 \operatorname{sen}\phi = \frac{64}{3}\operatorname{sen}(\phi)$

2. $-\frac{64}{3}\cos(\phi)\Big]_0^{\pi/3} = -\frac{64}{3}\cos(\pi/3) - \left(-\frac{64}{3}\cdot 1\right) = \frac{32}{3}$

3. $\frac{32}{3}\theta\Big]_0^{2\pi} = \frac{32\cdot 2\pi}{3} = \boxed{\frac{64\pi}{3}}$

14.6

2. Encontre a massa e o centro de gravidade da lâmina

Uma lâmina com densidade $\delta(x,y) = y$, limitada por $y = \operatorname{sen} x$,

$y = 0$, $x = 0$ e $x = \pi$

massa

$$\iint_R \delta(x,y)\, dA \Rightarrow \int_0^{\pi}\int_0^{\operatorname{sen}\pi} y\, dy\, dx =$$

$$x = \frac{M_y}{M} = \frac{0}{0} = 0$$

① $$\frac{y^2}{2}\Big|_0^{\operatorname{sen}\pi} = \frac{(\operatorname{sen}\pi)^2}{2} = 0$$

$$y = \frac{M_x}{M} =$$

② $$\int_0^{\pi} 0\, dx = \boxed{0}$$

$$M_y = \int_0^{\pi}\int_0^{\operatorname{sen}\pi} xy\, dy\, dx = \frac{y^2 x}{0} = \boxed{0}$$

$$M_x = \int_0^{\pi}\int_0^{\operatorname{sen}\pi} y^2\, dy\, dx = \frac{y^3}{3} = \boxed{0}$$

20. Enc. o centroide do sólido

O sólido limitado pelo cilindro parabólico $z = 1 - y^2$ e os planos $x + z = 1$, $x = 0$ e $z = 0$

$0 \leq z \leq 1 - x$

$0 \leq x \leq 1$

$-\sqrt{1-z} \leq y \leq \sqrt{1-z}$

Centroide

$$x_c = \frac{\int x\, dv}{\int dv} = \frac{2/7}{4/5} = \frac{5}{14} \qquad \bar{y} = 0$$

$$\bar{z} = \frac{8/35}{4/5} = \frac{2}{7}$$

$$V = \int_0^1\int_0^{1-x}\int_{-\sqrt{1-z}}^{\sqrt{1-z}} dy\, dz\, dx = \frac{8}{35}$$

$$M = \int_{-2}^{2}\int_0^{1}\int_0^{4-y^2} e\, dz\, dx\, dy = \frac{32e}{3}$$

40. [illegible] exercícios para enc. o centroide do sólido [illegible]

$$V = \int_0^1 \int_0^{1-x} \int_{-\sqrt{1-z}}^{\sqrt{1-z}} dy\,dz\,dx = \frac{8}{35}$$

$$M = \iiint e \, dz\,dx\,dy = \frac{32e}{3}$$

48. Coordenadas esféricas para enc. o centroide do sólido

O sólido limitado acima pela esfera $\rho = 4$ e abaixo pelo cone $\phi = \pi/3$

$\rho = 4$

$\phi = \pi/3$

(ρ, θ, ϕ)

$0 \le \phi \le \pi/3$

$0 \le \rho \le 4$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi} \int_0^{\pi/3} \int_0^4 \rho^2 \operatorname{sen}\phi \, d\rho\, d\phi\, d\theta$$

$$\int_0^{2\pi} d\theta \int_0^{\pi/3} \operatorname{sen}\phi\, d\phi \int_0^4 \rho^2\, d\rho \qquad \frac{\rho^3}{3} = \frac{4^3}{3} = \frac{64}{3}$$

$$\int_0^{2\pi} d\theta \int_0^{\pi/3} \frac{64}{3} \operatorname{sen}\phi\, d\theta = \frac{64}{3} \cos\phi \Big]_0^{\pi/3} d\theta = \frac{32}{3}(2\pi)$$

$$\boxed{V = \frac{64\pi}{3}}$$

45. Volume do sólido

A região ... do cilindro ... pelos planos ...

$$V = \int_{-3/2}^{3/2} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} (y+3)\, dy\, dx = \int_{-3/2}^{3/2} \left(\frac{y^2}{2} + 3y\right)\Big|_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} dx$$

$$V = \int_{-3/2}^{3/2} 6\sqrt{3-4x^2}\, dx$$

$u = 2x \quad 2x = 3\operatorname{sen}\theta$

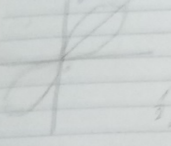
$a = 3 \quad dx = \frac{3}{2}\cos\theta\, d\theta$

$$V = \int_{-3/2}^{3/2} 6\sqrt{9-4x^2}\, dx = \int 6\sqrt{9-9\operatorname{sen}^2\theta}\; \frac{3}{2}\cos\theta\, d\theta = 27\int \cos^2\theta\, d\theta = \int \frac{27}{2}(1+\cos 2\theta)$$

$$V = \frac{27}{2}\left(\theta + \frac{1}{2}\, 2\operatorname{sen}\theta\cos\theta\right)\Big|_{-\pi/2}^{\pi/2} = \boxed{\frac{27\pi}{2}}$$

44.3 Use uma integral dupla em coordenadas polares para calcular a área da região

B. A região compreendida pela rosácea $r = \operatorname{sen} 2\theta$



$$\frac{1}{2}\int_0^{\beta} r^2(\theta)\, d\theta = \frac{1}{2}\int_0^{\pi/2} (\operatorname{sen} 2\theta)^2\, d\theta$$

$$\frac{1}{2}\int_0^{\pi/2} \operatorname{sen}^2(2\theta)\, d\theta = \frac{1}{2}\int_0^{\pi/2} \frac{1-\cos(4\theta)}{2}\, d\theta =$$

$$\frac{1}{4}\int_0^{\pi/2} 1-\cos(4\theta)\, d\theta = \frac{1}{4}\left(\theta - \frac{1}{4}\operatorname{sen}(4\theta)\right)\Big|_0^{\pi/2} =$$

$$\left(\frac{\pi}{2} - 0\right) = \frac{1}{4}\cdot\frac{\pi}{2} = \boxed{\frac{\pi}{8}}$$