1 A V

$f(x,y,z) = \sqrt{1-x^2-y^2}$

$1-x^2-y^2 \geq 0$

$-x^2-y^2 \geq -1 \; (-1)$

$x^2+y^2 \leq 1$

B $y=2$ V

$f(4,2)=2$

$f_x(4,2)=0$

C f

$L(x,y) = (x_0, y_0) + f_x((x,y)(x-x_0) + f_y(x,y_0)(y$

$z = z_0 + a(x-x_0) + b(y-y_0)$

L_D é um plano

d $z = 3x^2 - y^2$ f

$\nabla z = (6x, 2y)$ $x=2$ $y=(-1)$

$\|\nabla z\| = \sqrt{(6 \cdot 2)^2 + (2(-1))^2} = \sqrt{148} \neq 6$

(2) (a) $G(x,y) - f(g(x), h(y)) = \boxed{x^3 e^{x^3(3y+1)}}$

(b) $\lim\limits_{(x,y)\to(0,0)} G(x,y) = 0^3 e^{3 \cdot 0^3 \cdot 0 + 0^3} \to 0\, e^0 \to 0 \cdot 1 \to \boxed{0}$

(c) $\frac{DG}{Dx}(x,y) = x^3 e^{3x^3y + x^3}$

$\frac{D}{Dx} x^3 = 3x^2$

$\frac{D}{Dx} = e^{3x^3y+1} = e^{3x^3y+1} \cdot 9yx^2 =$

$f = e^u \quad u = 3x^3y+1$

$\frac{d}{du}(e^u)\frac{d}{dx}(3x^3y+1) = e^u \cdot 9yx^2 = e^{3x^3y+1} \cdot 9yx^2$

$\frac{d}{dx}(3x^3y+1) = 9yx^2$

$\frac{d}{du}(e^u) = e^u$

$= \boxed{3x^2 e^{3x^3y+1} + e^{3x^3y+1} \cdot 9yx^2 x^3}$

(3) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{x^2+y^2}} = \frac{x^2y^2}{\sqrt{r^2}} = \frac{x^2y^2}{r} = \frac{r^2\cos\theta^2}{}$

$x^2+y^2 \to r^2$

$x = r\cos\theta$
$y = r\,\mathrm{sen}\,\theta$
$r^2 = x^2+y^2$

$= \frac{r^4 \cos^2\theta\, \mathrm{sen}^2\theta}{r} = 0$

$\lim_{(x,y)\to(0,0)} = 0^3 e^{3 \cdot 0^2 \cdot 0 + 0^3} = 0 \cdot e^0 = 0 \cdot 1 = 0$

(c) $\frac{Dz}{Dx}(x,y) = x^3 e^{3x^2 y + x^3}$

$\frac{D}{Dx} x^3 = \ldots$

(4) (a)

$f_x(x,y) = \frac{d}{dx}\left(e^{-(x^2+y^2+2x)}\right) = e^u(-2x-2)$

$= e^{-x^2-y^2-2x}(-2x-2)$

$= e^{-4-1-2\cdot 2}(-2\cdot 2-2)$

$= e^{-9}(-6)$

$=$

$\frac{d}{du} e^u = e^u$

$\frac{d}{dx}(-(x^2+y^2+2x)) = -2x-2$

(b) $\frac{Dz}{DyDxDy} = \frac{D}{Dy}\left(\frac{D}{Dx}\left(\frac{D}{Dy}\right)\right)$

$\frac{D}{Dy}(e^{-(x^2+y^2+2x)}) = -2e^{-x^2-y^2-2x}y$

$\frac{D}{Dx} -2e^{-x^2-y^2-2x}y = -2ey^{-x^2-y^2-2x}(-2x-2)$

$\frac{D}{Dy} -2ey^{-x^2-y^2-2x}(-2x-2) = -2(-2x-2)(e^{-x^2-y^2-2x} - 2e^{-x^2-}$

(c) $\frac{\nabla z(\pi,0)}{\|\nabla z(\pi,0)\|}$

$\nabla z =$

$\frac{D}{Dx} =$

② ⓐ $G(x,y) - f(g(x), h(y)) = \boxed{x^3 e^{x^3(3y+1)}}$

ⓑ $\lim_{(x,y)\to(0,0)} G(x,y) = 0^3 e^{3 \cdot 0^3 \cdot 0 + 0^3} \Rightarrow 0 \cdot e^0 \Rightarrow 0 \cdot 1 = \boxed{0}$

ⓒ $\frac{DG}{Dx}(x,y) = x^3 e^{3x^3y + x^3}$

$\frac{D}{Dx} x^3 = 3x^2$

$\frac{D}{Dx} = e^{3x^3y+1} = e^{3x^3y+1} \cdot 9yx^2 =$

$f = e^u \quad u = 3x^3y + 1$

$\frac{d}{du}(e^u)\frac{d}{dx}(3x^3y+1) = e^u \cdot 9yx^2 = e^{3x^3y+1} \cdot 9yx^2$

$\frac{d}{du}(e^u) = e^u \qquad \frac{d}{dx}(3x^3y+1) = 9yx^2$

$\boxed{= 3x^2 e^{3x^3y+1} + e^{3x^3y+1} \cdot 9yx^2 x^3}$

③ $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{\sqrt{x^2+y^2}} = \frac{x^2y^2}{\sqrt{r^2}} = \frac{x^2y^2}{r} = \frac{r^2 co}{}$

$\to r^2$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$= \frac{r^4 \cos^2\theta \, \mathrm{sen}^2\theta}{r} = 0$