Exercise 6

a) x - free variable

b) $\lambda x.x$ - abstraction

c) $\lambda a.(z)a$ - abstraction

d) $(\lambda a.(z))(a)$ - application

e) $(\lambda m.m)(z)$ - application

f) $\lambda z.(\lambda y.(\lambda x.x)y)z$ - abstraction

g) $(\lambda t.((\lambda t.(\lambda t.t)t)t))(t)$ - application

Problems 1-3

Exercise 2

a)

$$(\lambda x.xy)[\lambda z.z/y] = (\lambda x.x(\lambda z.z))$$

b)

$$(\lambda x.xy)[\lambda z.zx/y] = (\lambda u.u(\lambda z.zx))$$

c)

$$(\lambda f(\lambda x.yx)yx)[fy/x] = (f(\lambda x.yx)y(fy))$$

d)

$$(\lambda f.f(\lambda x.yx)yx)[fy/x] = (\lambda f.f(\lambda x.yx)y(fy))$$

Exercise 3

a)

$$\begin{aligned}(\lambda x.\lambda y.x)yx &= (\lambda z.y)x \\ &= y\end{aligned}$$

b)

$$\begin{aligned}(\lambda f.f(\lambda x.x))(\lambda y.z) &= (\lambda y.z)(\lambda x.x) \\ &= z\end{aligned}$$

c)

$$(\lambda x.\lambda y.yx)(\lambda x.xy) = \lambda z.z(\lambda x.xy)$$

d)

$$\begin{aligned}(\lambda x.xx)((\lambda y.y)(\lambda x.x)) &= (\lambda x.xx)(\lambda x.x) \\ &= (\lambda x.x\lambda x.x)\end{aligned}$$

e)

$$\begin{aligned}(\lambda x.xx)(\lambda y.y)(\lambda x.x) &= (\lambda y.y\lambda y.y)(\lambda x.x) \\ &= (\lambda x.x\lambda x.x)\end{aligned}$$

f)

$$\begin{aligned}(\lambda x.xx)(\lambda x.xx)((\lambda y.y)(\lambda x.x)) &= (\lambda y.yy\lambda y.yy)(\lambda x.x) \\ &= (\lambda x.x\lambda x.x\lambda x.x\lambda x.x)\end{aligned}$$