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Mathematics Anxiety and Mathematics Achievement

Brian F. Sherman and David P. Wither (Post.)
University of Adelaide

This paper is a distillation of the major result from the 1998 Ph.D. thesis of the late David Wither. It details a longitudinal study over five years of the relationship between mathematics anxiety and mathematics achievement. It starts from the already well documented negative correlation between the two, and seeks to establish one of the three hypotheses—that mathematics anxiety causes an impairment of mathematics achievement; that lack of mathematics achievement causes mathematics anxiety; or that there is a third underlying cause of the two.

In 1972 Richardson and Suinn stated of *mathematics anxiety* that it involved "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 551). Implicit in this description is the assumption that mathematical anxiety is a causative agent of some impairment of mathematical functioning. Surprisingly, while there has been a good deal of theoretical surmise about the process by which such an agent would act (Ashcraft & Kirk, 2001; Brush, 1978; Cooper & Robinson, 1991; Hadfield & Maddux, 1988; Richardson & Woolfolk, 1980; Tobias, 1976; Tobias & Weissbrod, 1980), there has been little attempt to verify such causative action by experiment and observation.

This study was a longitudinal investigation of the relationship between mathematics anxiety and mathematics achievement. A moderate but significant negative correlation between the two has been observed by many researchers (Adams & Holcomb, 1986; Betz, 1978; Brush, 1978; Cooper & Robinson, 1991; Cowen, Zax, Klein, Izzo, & Trost, 1963; Dew, Galassi, & Galassi, 1984; Lunneborg, 1964; Resnick, Viehe, & Siegel, 1982; Suinn, Edie, Nicoletti, & Spinelli, 1972; Wigfield & Meece, 1988). However, it should be noted that the observations of Hunsley (1987) using multiple regression, and those of Hadfield and Maddux (1988) using analysis of variance, did not indicate a significant relationship between the two. In particular, the meta-analysis of Hembree (1990), incorporating the results of 151 studies, indicates a consistent negative correlation of 0.3 or more for studies involving school children, and one of 0.25 or more for those involving tertiary students.

The purpose of this study was to attempt to establish that mathematical anxiety does cause a deterioration of mathematical achievement. Indeed, Hembree deduced such a causal relationship, citing the following evidence:

1. Higher achievement consistently accompanies reduction in mathematics anxiety.
2. Treatment can restore the performance of formerly high-anxious students to the performance level associated with low mathematics anxiety.

This study was an attempt to put the conclusion on a sound experimental basis.

To achieve this purpose in this study, the technique of cross-lagged panel analysis (Campbell, 1963; Campbell & Stanley, 1963; Peltz & Andrews, 1964) was used. It was applied to nine pairs of tests of mathematical achievement and mathematical anxiety, applied over five years, to a common group of students.

Further, in light of Rogosa's (1980) criticisms of the technique, Rogosa's own model of structural regression was also applied to provide support for the conclusions to be drawn from the data. The results, however, came as a complete surprise, and confounded our expectations.

Instruments

Mathematics anxiety has been an object of research attention for some time. It is distinct from general anxiety, and, in particular, from test anxiety. Because the context in which mathematics anxiety is measured is much akin to a test, special attention is paid to distinguishing between mathematics anxiety and test anxiety.

There are a number of tests available for determining mathematics anxiety – Suinn et al.'s Mathematics Anxiety Rating Scale (MARS) (1972), and also their Revised Mathematics Anxiety Rating Scale, Fennema and Sherman's Mathematics Anxiety Scale (1976).

Separating test anxiety from mathematics anxiety has not been an easy task. Using the Mathematics Anxiety Rating Scale of Suinn et al. (1972), Rounds and Hendel (1980) used factor analysis to locate two factors, which they labelled *Mathematics Test Anxiety* and *Number Anxiety*, while in 1982 Resnick, Viehe and Segal found three factors, essentially the two above, and a Social Responsibility Anxiety. Hembree's 1990 meta-analysis concluded that mathematics anxiety correlated with, but was separate from, test anxiety, and Kazelskis et al. (2000) found a similar result. Indeed, they warn that, because of the substantial weighting of mathematics test anxiety within it, users of Suinn et al.'s MARS would be well advised to use the individual factors of the measures rather than an omnibus total score.

For the present investigation, a pilot study was undertaken of both a ***mathematics achievement test*** and a ***mathematics anxiety test***. A cluster analysis of the results of the latter separated out those questions which addressed test anxiety as opposed to mathematics anxiety, and items of that kind were removed from the tests used in the main study. In the main study, a separate test for each year level was constructed, using concepts and material appropriate for that year level. The students were asked to respond with one of the answers, "Does not worry me at all", "worries me a little", "worries me a fair amount", "worries me a lot", "worries me an awful lot", to statements such as "Drawing accurate circles" at the year 6 level, or to ones such as "Finding the x-intercept of a graph" at year 10 level. These tests were designed in the same format as the MARS of Suinn et al. (1972), with the content-related part adjusted for the current experience of the students, and those items which were determined by the cluster analysis to be related to test anxiety rather than number anxiety were omitted.

Similar problems arose from the need to gear the tests of mathematics achievement to the students' development and classroom learning. From the New Zealand Council for Educational Research PATMATHS series (Reid, 1974), tests 2A, 2B, 3A and 3B were used with the students when they were in years 6 and 7. (The tests were relabelled as 6A, 6B, 7A and 7B, respectively, for the purposes of this study.)

For the remaining years, however, there were no appropriate PATMATHS tests available. Tests 8A through 10B were prepared in similar format to the PATMATHS tests with the assistance of six practising teachers. Each of these teachers helped with each of the tests, particularly in determining the appropriate content and suitability for both the year level and the time of the year.

Unfortunately, because one of the schools in the study completed their Year 10 well before the end of the year, when the 10B tests were due to be administered, only two-thirds of the students undertook these. Consequently, only the nine pairs of tests 6A to 10A were used in the panel analysis, although all tests were used in establishing the overall correlations between mathematics anxiety and mathematics achievement.

As will be seen in Table 4, the results of these later tests correlated highly with those of the PATMATHS tests. The lowest correlation, 0.47, was between the test 6A and the test 10A, and the correlation between any two adjacent tests, except for tests 9B and 10A, was above 0.7. These correlations, along with the expertise of six teachers, would indicate that the later tests were still measuring the equivalent phenomena for the children who were, of course, developing over the period.

Sample

Observations of a cohort of students were made twice a year over a period of five years as they progressed from Year 6 to Year 10. The students were selected from three schools in suburban Adelaide, in South Australia. The choice of schools was dictated by the need to follow the same students over the five years, so that all three schools needed to have both a primary and secondary component. Consequently, three non-government schools were chosen, as, at the time, the only government schools in South Australia with both primary and secondary components were rural Area Schools, and it was determined that the logistics of the study would not have been feasible with such schools.

At each administration of the tests, the whole relevant year level at each of the schools was tested, with numbers ranging from 156 in the first testing to 289 for the first Year 8 tests. As mentioned above, one of the schools had completed Year 10 before the scheduled time of the tests, so that the students there were unavailable. This testing was not included in the analysis. All nine testing sessions were completed by 66 of the original 156 students. While a number of other conclusions were drawn from the data gathered from all the other students tested, it was only from these 66 that the major conclusions of the study were drawn.

Methods of Analysis

Although cross-lagged panel analysis was first used in the early 1900s, it was formalised by Campbell in 1963. In its basic form, the technique considers two variables X and Y measured at two times t_1 and t_2, giving rise to four observed variables X_1 and X_2 and Y_1 and Y_2. The six correlations between these four variables fall naturally into three pairs:

- the correlations between the same underlying variable measured at different times (called the ***auto-correlations***), r_{X1X2} and r_{Y1Y2};
- the correlations between the two underlying variables measured at the same time (called the ***synchronous correlations***), r_{X1Y1} and r_{X2Y2};
- the two remaining correlations, each between one underlying variable at the earlier time and the other at the later time (called the ***cross-lagged correlations***), r_{X1Y2} and r_{Y1X2}.

The logic underlying cross-lagged panel analysis rests upon the time lag that typically exists when one variable causes another. The argument is that if X causes Y, then the present state of X (X_1) should be more closely related to Y's future state

(Y_2) than Y's present state (Y_1); in other words $r_{X1Y2} > r_{X1Y1}$. Similarly, we would expect to have $r_{X1Y2} > r_{X2Y2}$ and $r_{X1Y2} > r_{X2Y1}$. In fact, in most cases, only this last inequality comparing the two cross-lagged correlations is used.

In 1980, Rogosa examined cross-lagged panel analysis using a structural regression model, and found that many of the conclusions drawn from the method were flawed. He showed that different configurations of the coefficients can be used to produce spurious results for the standard test for causality, which simply compares the cross-lagged correlations.

His structural regression model applied to the basic form of panel analysis (where there are observations of X_1, Y_1, X_2, and Y_2 of two variables X and Y at times t_1 and t_2) essentially considers each of the two later observations as linear functions of the two earlier ones. They are represented by the equations:

$$X_2 = \beta_0 + \beta_1 X_1 + \gamma_2 Y_1$$
$$Y_2 = \gamma_0 + \beta_2 X_1 + \gamma_1 Y_1$$

The postulate that 'X causes Y' implies that Y_1 is not a factor in X_2, but X_1 is a factor in Y_2, and would be represented by $\beta_2 \neq 0$ and $\gamma_2 = 0$. Similarly, the postulate that 'Y causes X' would be represented by $\beta_2 = 0$ and $\gamma_2 \neq 0$, while the postulate that both X and Y are caused by a third agent is expressed by $\beta_2 = 0$ and $\gamma_2 = 0$. The other coefficients, β_0, β_1, γ_0 and γ_1 are irrelevant to the question of causality between X and Y.

The relationship between the coefficients of the model and the correlation coefficient is more easily expressed if the standardised coefficients $\beta_1^* = \beta_1(s_{X1}/s_{X2})$, $\beta_2^* = \beta_2(s_{X1}/s_{Y2})$, $\gamma_1^* = \gamma_1(s_{Y1}/s_{Y2})$ and $\gamma_2^* = \gamma_2(s_{Y1}/s_{X2})$ are used. From the formula defining the correlation coefficient, the relationships can be determined as

$$r_{X1Y2} = \beta_2^* + \gamma_1^* r_{X1Y1},$$
$$r_{X1X2} = \beta_1^* + \gamma_2^* r_{X1Y1},$$
$$r_{Y1Y2} = \beta_2^* r_{X1Y1} + \gamma_1^*,$$
$$r_{X2Y1} = \beta_1^* r_{X1Y1} + \gamma_2^*,$$

and from these the following are derived:

$$\beta_2^* = (r_{X1Y2} - r_{Y1Y2} r_{X1Y1})/(1 - r_{X1Y1}^2)$$
$$\gamma_2^* = (r_{X2Y1} - r_{X1X2} r_{X1Y1})/(1 - r_{X1Y1}^2)$$

Of course, Rogosa's model is subject to the assumptions that the causative effects are linear, and, for the third postulate, that the causal time lag from the third agent is the same for both X and Y. It also raises the difficulty of deciding what coefficients being equal to zero means, as any measurements must allow for some random variation.

To settle this last point, it is observed that the fourth possibility, that $\gamma_2 \neq 0$ and $\beta_2 \neq 0$, contradicts the assumption that one of the above three forms of causation is present. Consequently, the degree of approximation allowed should be enough to exclude this possibility, given the acceptance of the existence of a non-spurious correlation between the two measured effects.

When dealing with Rogosa's model, the symbols "**f**", "p", and "≈" are to be read as "greater than and outside the degree of approximation", "less than and outside the degree of approximation", and "within the degree of approximation", respectively.

It would be expected that the auto-correlations (the correlations between the same effect at different times) would be positive, while both the synchronous and the cross-lagged correlations should all have the same sign, as they are all correlations between the two effects. From the formulae, then, it would be expected that β_2^* and γ_2^* have the same signs as r_{X1Y2} and r_{X2Y1} respectively, although in some instances the sizes of $r_{Y1Y2}r_{X1Y1}$ and $r_{X1X2}r_{X1Y1}$ could change these.

Results

The initial findings in Table 1 show the correlations between mathematics anxiety and mathematics achievement at each testing; all of the correlations were significant to the 0.001 level. While these measurements are not independent of the panel analysis findings, it must be remembered that they were taken over the whole cohort of participating students at each testing, and not just over the 66 who participated in all tests from 6A to 10A.

Table 1
Synchronous Correlations between Mathematics Anxiety and Mathematics Achievement

Level	6A	6B	7A	7B	8A	8B	9A	9B	10A	10B
Correlation	-0.49	-0.43	-0.50	-0.34	-0.23	-0.40	-0.49	-0.51	-0.39	-0.39

That there is a relationship between mathematics anxiety and mathematics achievement is a conclusion amply verified by not only the correlations given above, but in the many studies considered by Hembree in his 1990 meta-analysis. The next step is to try and assign a direction for causation in the relationship. The three hypotheses considered are:

1. Mathematics anxiety causes a lack of mathematics achievement.
2. Lack of mathematics achievement causes mathematics anxiety.
3. Both are caused by some third underlying factor.

In the cross-lagged panel analysis of this study ***mathematics achievement*** is taken to be the variable *Ach*, and ***mathematics anxiety*** the variable *Anx*. Remembering that all these correlations would be expected to be negative, the first hypothesis is expressed as $0 > r_{Ach1Anx2} > r_{Ach2Anx1}$, and the second as $0 > r_{Ach2Anx1} > r_{Ach1Anx2}$. Cross-lagged panel analysis in its simplest form, however, does not take the third hypothesis into account. For that, Rogosa's model, with the assumption that the third factor has the same causal lag for each of anxiety and lack of achievement, is a more appropriate tool.

Table 2 gives the correlations between each of the nine mathematics achievement tests and mathematics anxiety tests, using just the 66 students who completed all of them. Those on the leading diagonal are the synchronous correlations, while those below it are the r_{X1Y2} and those above it the r_{X2Y1}.

Table 2
Correlations between Mathematics Achievement and Mathematics Anxiety

Anxiety Test	Achievement Test								
	Ach1	Ach2	Ach3	Ach4	Ach5	Ach6	Ach7	Ach8	Ach9
Anx1	-0.24	-0.29	-0.24	-0.22	-0.10	-0.17	-0.18	-0.28	-0.24
Anx2	-0.33	-0.41	-0.32	-0.31	-0.21	-0.26	-0.24	-0.39	-0.32
Anx3	-0.50	-0.57	-0.43	-0.49	-0.28	-0.26	-0.33	-0.41	-0.20
Anx4	-0.36	-0.46	-0.38	-0.43	-0.13	-0.25	-0.36	-0.51	-0.25
Anx5	-0.38	-0.49	-0.45	-0.46	-0.23	-0.33	-0.35	-0.40	-0.26
Anx6	-0.38	-0.45	-0.47	-0.39	-0.12	-0.23	-0.35	-0.33	-0.18
Anx7	-0.38	-0.49	-0.34	-0.37	-0.33	-0.41	-0.47	-0.42	-0.39
Anx8	-0.37	-0.45	-0.30	-0.30	-0.31	-0.37	-0.47	-0.39	-0.36
Anx9	-0.41	-0.47	-0.32	-0.32	-0.31	-0.48	-0.54	-0.39	-0.40

There are six inequalities (shaded) which support the first hypothesis, and thirty which support the second. (The six inequalities indicated by the shading are

$$0 > r_{Ach3Anx4} > r_{Ach4Anx3},\ 0 > r_{Ach3Anx8} > r_{Ach8Anx3},\ 0 > r_{Ach4Anx8} > r_{Ach8Anx4},$$
$$0 > r_{Ach5Anx6} > r_{Ach6Anx5},\ 0 > r_{Ach5Anx7} > r_{Ach7Anx5},\ \text{and } 0 > r_{Ach5Anx8} > r_{Ach8Anx5},$$

all of them showing a stronger correlation between earlier anxiety with later achievement than that between earlier achievement and later anxiety.)

There is a real difficulty here of determining the significance of these figures; it is not a matter of simply obtaining the significance of the various correlation coefficients, but of the comparisons of each relevant pair of coefficients. A suitable null hypothesis for each pair is that either coefficient of the pair is as likely as the other to be the larger of the two. However, the problem comes in attempting to combine the probabilities for all thirty six comparisons; the seventy two correlation coefficients are derived from only eighteen separate tests, and so there may be a degree of interdependence between the thirty six comparisons.

If we assume that they are all independent of one another, then the probability that only six out of the thirty six correlation coefficients pairs should satisfy the condition indicating the first hypothesis is one in forty thousand

$$\left(\sum_{i=0}^{6}\binom{36}{i}\left(\frac{1}{2}\right)^{i}\left(\frac{1}{2}\right)^{36-i} = .000028\right)$$

(assuming it is equally likely that each separate one did or did not satisfy the condition).

The eight comparisons of the coefficients in any one column with those in the corresponding row are, however, all taken from distinct tests, and so the eight comparisons should be independent of one another.

The comparisons between the first column and first row, between the second column and the second row, and between the last column and the last row have none favouring the first hypothesis—a chance for each of only one in 256, the smallest possible for eight comparisons. Those between the sixth column and the sixth row, and those between the seventh column and the seventh row have just one favouring the first hypothesis, a chance of one in twenty seven, indicating a significance at the 95% level.

The significance of the results obtained in the whole table should be somewhere between these two (one in 256, and one in 40 000). Hence, cross-lagged panel analysis indicates that the first hypothesis is so unlikely that it is untenable. In other words, using cross-lagged panel analysis, the hypothesis that mathematical anxiety causes a lack of mathematics achievement should be rejected. Perhaps the data might support either the hypothesis that a lack of achievement in mathematics brings about mathematical anxiety or the hypothesis that the two are caused by a third underlying factor.

Rogosa's Structural Regression Model

In order to apply Rogosa's model, the auto-correlations for both mathematical anxiety and mathematical achievement are needed. They are shown in Tables 3 and 4.

Table 3
Mathematical Anxiety Auto-correlations

Anxiety Test	Anxiety Test							
	Anx1	Anx2	Anx3	Anx4	Anx5	Anx6	Anx7	Anx8
Anx2	0.77							
Anx3	0.52	0.65						
Anx4	0.64	0.67	0.77					
Anx5	0.45	0.45	0.63	0.75				
Anx6	0.42	0.41	0.55	0.63	0.76			
Anx7	0.43	0.44	0.51	0.49	0.61	0.58		
Anx8	0.49	0.45	0.47	0.54	0.55	0.52	0.86	
Anx9	0.21	0.20	0.25	0.27	0.34	0.40	0.69	0.69

From these and the correlations between mathematics achievement and mathematics anxiety given in Table 2, the respective β_2* and γ_2* are calculated, and shown in Tables 5 and 6, and they are shown together in Table 7, to make the observation of the conditions for the three hypotheses easier.

Table 4
Mathematical Achievement auto-correlations

Achievement Test	Achievement Test							
	Ach1 Test 6A	Ach2 Test 6B	Ach3 Test 7A	Ach4 Test 7B	Ach5 Test 8A	Ach6 Test 8B	Ach7 Test 9A	Ach8 Test 9B
Ach2	0.77							
Ach3	0.76	0.75						
Ach4	0.76	0.67	0.79					
Ach5	0.72	0.65	0.67	0.71				
Ach6	0.73	0.71	0.78	0.75	0.76			
Ach7	0.64	0.65	0.65	0.60	0.50	0.72		
Ach8	0.58	0.58	0.64	0.63	0.52	0.66	0.72	
Ach9	0.47	0.57	0.66	0.57	0.59	0.71	0.66	0.59

Table 5
Beta$_2$ values of pairings of tests*

	Achievement Test							
	#1	#2	#3	#4	#5	#6	#7	#8
*2	-0.15							
*3	-0.40	-0.36						
*4	-0.22	-0.22	-0.06					
*5	-0.29	-0.37	-0.22	-0.17				
*6	-0.30	-0.34	-0.29	-0.15	0.06			
*7	-0.29	-0.37	-0.15	-0.20	-0.20	-0.29		
*8	-0.27	-0.32	-0.12	-0.08	-0.19	-0.26	-0.08	
*9	-0.38	-0.47	-0.26	-0.25	-0.24	-0.41	-0.28	-0.14

Table 6
$Gamma_2$ values of pairings of tests*

Anxiety Test	Achievement Test							
	#1	#2	#3	#4	#5	#6	#7	#8
*2	-0.11							
*3	-0.06	-0.02						
*4	-0.04	-0.04	-0.18					
*5	0.08	0.07	0.01	0.22				
*6	0.01	0.04	0.09	0.09	-0.16			
*7	-0.03	0.03	-0.06	-0.13	-0.25	-0.19		
*8	-0.15	-0.18	-0.17	-0.29	-0.30	-0.19	-0.10	
*9	-0.13	-0.10	0.10	-0.01	-0.13	-0.02	-0.10	-0.15

Table 7
$Beta_2$, $Gamma_2$* presented together*

Anxiety Test	Achievement Test							
	#1	#2	#3	#4	#5	#6	#7	#8
*2	-0.15, -0.11							
*3	-0.40, -0.06	-0.36, -0.02						
*4	-0.22, -0.04	-0.22, -0.04	-0.06, -0.18					
*5	-0.29, 0.08	-0.37, 0.07	-0.22, 0.01	-0.17, 0.22				
*6	-0.30, 0.01	-0.34, 0.04	-0.29, 0.09	-0.15, 0.09	0.06, -0.16			
*7	-0.29, -0.03	-0.37, 0.03	-0.15, -0.06	-0.20, -0.13	-0.20, -0.25	-0.29, -0.19		
*8	-0.27, -0.15	-0.32, -0.18	-0.12, -0.17	-0.08, -0.29	-0.19, -0.30	-0.26, -0.19	-0.08, -0.10	
*9	-0.38, -0.13	-0.47, -0.10	-0.26, 0.10	-0.25, -0.01	-0.24, -0.13	-0.41, -0.02	-0.28, -0.10	-0.14, -0.15

From these tables it can be observed that all of the β_2* are negative, while all the positive values of γ_2* are relatively low, except for the value 0.22 between the fourth and fifth tests, and all of these positive values are between the earlier anxiety tests (1 to 4) and the later achievement ones (5 to 9). The value of 0.22 might be looked upon as an aberration, in which case the degree of approximation will be set by considering only the other values; or else it will be set at 0.23. Both cases will be considered.

For Rogosa's structural regression model, the first hypothesis (Y causes X) is expressed as $\beta_2^* \approx 0$ and γ_2^* p 0, the second (X causes Y) as β_2^* p 0 and $\beta_2^* \approx 0$, and the third as $\gamma_2^* \approx 0 \approx b_2^*$. As the theory indicates that the three should exhaust all the possibilities, we attempt to set our degree of approximation at the level which excludes any other possible description, which, for this study, is essentially β_2^* p 0 and γ_2^* p 0.

The cell (#4, *5) containing the value of 0.22 for γ_2^* is slightly anomalous, as the next highest value of γ_2^* is 0.10. If that cell is ignored, the degree of approximation would need to be at least 0.21 to ensure that not both β_2^* p 0 and γ_2^* p 0 in any of the cells; if the cell is taken into account, the degree of approximation would have to be at least 0.23.

Table 8 shows which cells satisfy which hypotheses, both using 0.21 as the degree of approximation, and also 0.23, to take the cell containing the value of 0.22 for γ_2^* into consideration. Thus the relevant cell contains the numeral 1 if the values of β_2^* and γ_2^* satisfy the first hypothesis, 2 for the second and 3 for the third; a N is in the cell containing the value of 0.22 for γ_2^* when that cell is not being taken into consideration.

Under both criteria, three have $\beta_2^* \approx 0$ and γ_2^* p 0, supporting the first hypothesis. Using the first condition, twenty three have β_2^* p 0 and $\gamma_2^* \approx 0$, and so support the second hypothesis, while nine have $\beta_2^* \approx 0$ and $\gamma_2^* \approx 0$, supporting the third hypothesis. Under the second condition, twenty support the second hypothesis, while thirteen support the third.

It is difficult to establish an appropriate null hypothesis in this situation, as the acceptance of the existence of a significant correlation requires that one of the three hypotheses must hold. This means that the null hypothesis should incorporate the idea that the sum of the probabilities of the three is one. It is also reasonable that the null hypothesis should be symmetrical, so that the probabilities of the first two hypotheses should be the same; the difficulty comes in assigning a probability to the third hypothesis. If it is set at zero, then each of the other two is one half, and if it is set at one, then each of the others is zero; some figure between zero and one would be more appropriate, with a third being a reasonable selection. These would give the respective probabilities for the three hypotheses as a third, a third, and a third.

Further, as for the cross-lagged analysis, it is first assumed that each comparison is independent of the others. If all three of the configurations determined by the hypotheses were equally likely, that is, that each is one in three, then the chance that one of them would result in at most three out of thirty five is one in nine thousand, while for three out of thirty six is one in twelve thousand.

If we just consider the comparisons between the coefficients in one column and those in the corresponding row, as we did for the cross-lagged analysis, then there are four columns in which the first hypothesis is not supported by any of the comparisons—the first, second, third and last—and the probability for this occurring in a column is $\left(\frac{2}{3}\right)^8 = .039$.

Table 8
Indications of which of the Three Hypotheses are Satisfied by Particular Cells

Anxiety Test	Achievement Test #1	#2	#3	#4	#5	#6	#7	#8
	' ≈ 0' set at '> - 0.21 and < 0.21'							
*2	3							
*3	2	2						
*4	2	2	3					
*5	2	2	2	N				
*6	2	2	2	3	3			
*7	2	2	3	3	1	2		
*8	2	2	3	1	1	2	3	
*9	2	2	2	2	2	2	2	3
	' ≈ 0' set at '> - 0.23 and < 0.23'							
*2	3							
*3	2	2						
*4	3	3	3					
*5	2	2	3	3				
*6	2	2	2	3	3			
*7	2	2	3	3	1	2		
*8	2	2	3	1	1	2	3	
*9	2	2	2	2	2	2	2	3

The significance of the whole table is then between this figure and those given in the previous paragraph. As was the case for the cross-lagged panel analysis, these figures indicate that the first hypothesis is too unlikely to be accepted; in other words, the data indicates that mathematical anxiety does not cause the lack of mathematical achievement with which it is strongly correlated.

The probability calculations for the other results are all such that either hypothesis is possible; the smallest figure is 0.22 for the nine out of thirty five supporting the third hypothesis, under the assumption of equal likelihood for all three hypotheses. Consequently, either the second or the third hypothesis is tenable; the conclusion is that the data do not support the hypothesis that mathematical anxiety causes a lack of mathematical achievement, but that either the lack of mathematical achievement causes mathematical anxiety, or there is a third factor which causes both.

Conclusion

The results were a complete surprise to the authors; the expectation was that the observations would support the hypothesis that mathematics anxiety caused a deterioration of mathematics achievement, particularly as observation of results of intervention would appear to support this hypothesis (Hembree, 1990). While the first hypothesis can be rejected, there is insufficient evidence to show a significant difference between the other two, that is, whether poor mathematics achievement causes mathematics anxiety, or whether there is a third factor causing both.

Several authors have indicated the existence of a third factor; for example, Ashcraft and Kirk (2001) demonstrated that high mathematics anxiety is associated with smaller working memory capacity, and consequently poor performance in mathematics. Adams and Holcomb (1986) identified a construct they called *mathematics efficiency* to explain the correlation; and Hadfield and Maddux (1988) indicated that *cognitive style* (i.e., whether or not the student was field dependent) was an intermediary factor. However, all of these expect that mathematics anxiety is a causative agent of the third factor, rather than the third factor causing both the mathematics anxiety and the poor performance in mathematics.

It is difficult to draw practical conclusions from an essentially negative finding. The improvement of mathematics performance is generally seen as a goal of programmes designed to reduce mathematics anxiety, but the results of this study do not support the conclusion that these programmes would achieve that outcome. Reduction of anxiety is a laudable goal in itself, but, if it does not bring with it an improvement in achievement, is it sufficient to warrant the expenditure of money and effort on such programmes? However, this is a single study, and the sample sizes available for analysis were both small and limited in nature (i.e., because of the necessity to use only students who were present for every one of the nine tests over five years, and because the students were from three similar schools in one city). It should not be looked upon as a call for wholesale changes, but rather as a warning that there may be problems with underlying assumptions. Further investigations are necessary; perhaps a replication of this experiment, to find whether these results can stand. A more direct line of enquiry, such as the measurement of change of mathematics achievement in people undergoing an anxiety reduction programme, might also be a profitable approach to the subject. Nevertheless, this study certainly raises a question mark over the assumption that mathematics anxiety is a cause of poor achievement in mathematics.

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Authors

Brian Sherman, Graduate School of Education, University of Adelaide, S.A. 5005. E-mail: <brian.sherman@adelaide.edu.au>.

David Wither (Post.), Graduate School of Education, University of Adelaide, S.A. 5005.