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Examining the reciprocal relations of mathematics anxiety to quantitative reasoning and number knowledge in Chinese children

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ABSTRACT

This study aimed to examine the longitudinal relations of mathematics anxiety to quantitative reasoning and number knowledge in Chinese children. Three hundred and sixteen 6-year-old Chinese children in Hong Kong participated in two waves of assessments, eight months apart. Cross-lagged panel analyses showed that prior quantitative reasoning and number knowledge predicted lower mathematics anxiety, even after the effects of gender, mothers' educational levels, and general anxiety were taken into account. However, earlier mathematics anxiety did not predict later quantitative reasoning and number knowledge. Our findings were consistent with the Deficit Theory, which postulates that mathematics anxiety comes from poor mathematical competence but not vice versa. We also found a reciprocal association between quantitative reasoning and number knowledge, in which initial quantitative reasoning had a stronger prediction on later number knowledge. Taken together with previous research, this result highlights the importance of quantitative reasoning in children's mathematics learning and its role in mathematics education.

1. Introduction

This study aimed to examine the longitudinal relations of mathematics anxiety to quantitative reasoning and number knowledge in Chinese children. *Mathematics anxiety* refers to "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551). It is considered a trait-level anxiety and is distinct from both state anxiety (Hembree, 1990) and test anxiety (Kazelskis et al., 2001). Across the 65 countries that participated in the 2012 Programme for International Student Assessment (PISA), around one-third (33%) of the 15-year-old students reported a sense of helplessness when they solved mathematical problems (Organization for Economic Co-operation and Development [OECD], 2013). Mathematics does not only generate negative feelings in math-anxious students, but it also creates alternations in physiological responses, such as increased heart rate (Faust, 1992), changes in cortisol secretion (Pletzer et al., 2010; Mattarella-Micke, Mateo, Kozak, Foster, & Beilock, 2011), and neural activations that resemble people who experience physical pain (Lyons & Beilock, 2012).

Research has demonstrated consistent evidence that increased mathematics anxiety is linked to poorer mathematical performance (for reviews see Carey, Hill, Devine, & Szuc, 2016; Dowker, Sarkar, & Looi, 2016; Ma, 1999; Ramirez, Shaw, & Maloney, 2018). Whereas children as young as 6 and 7 years old experienced math-specific anxiety (e.g., Ching, 2017; Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Ramirez, Gunderson, Levine, & Beilock, 2013; Vukovic, Kieffer, Bailey, & Harari, 2013), there has been no consistent connection between mathematics anxiety and mathematical performance in children (Dowker, Bennett, & Smith, 2012; Krinzinger, Kaufmann, & Willmes, 2009). Whether mathematics anxiety predicts poor mathematical competence, or the other way around has also remained contentious. On the basis of three theoretical perspectives (Carey et al., 2016), we aimed to investigate the longitudinal relations of mathematics anxiety to quantitative reasoning and number knowledge in a group of Chinese children using a cross-lagged panel design.

1.1. Mathematics anxiety in children

Most of the research on mathematics anxiety have been conducted among adult samples, and its assessment has predominantly relied on self-reported questionnaires. The earliest measure, to the best of our knowledge, is the one of Dreger and Aiken (1957). Subsequently several widely-known measures, such as the Mathematics Anxiety Research Scale (Richardson & Suinn, 1972) and the Fennema–Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) have been

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developed. These scales and their various adaptations have demonstrated good psychometric properties in terms of reliability and validity in various studies conducted with adult samples (e.g., Hopko, 2003; Levitt & Hutton, 1984; Mulhern & Rae, 1998; Plake & Parker, 1982; Suinn, Edie, Nicoletti, & Spinelli, 1972; Suinn & Winston, 2003). There have been some debates regarding whether mathematics anxiety can also be assessed in similar ways in children because they may not be cognitively sophisticated enough to experience and report how they feel about mathematics (Ashcraft & Krause, 2007; Ganley & McGraw, 2016; Vukovic et al., 2013). However, research has demonstrated that young children are able to understand and describe their feelings of anxiety around mathematical tasks. For example, several cognitive interviews indicated that primary school children showed some good knowledge about what it meant to be nervous, anxious, or tense about mathematics (Ganley & McGraw, 2016; Ramirez et al., 2013; Vukovic et al., 2013). Using pictorial rating scales, such as the Mathematics Attitude and Anxiety Questionnaire (Thomas & Dowker, 2000; Krinzinger et al., 2007; Dowker et al., 2012) and the Children's Attitude to Math Scale (James, 2013), studies have shown evidence that even children as young as 6 years of age experience math-specific anxiety (Aarnos & Perkkilä, 2012; Ramirez et al., 2013, 2016; Thomas & Dowker, 2000).

Over the years, there has been continuous interest in the developmental trend of mathematics anxiety. Based on self-reported assessments, earlier research (e.g., Hembree, 1990; Wigfield & Meece, 1988) has shown that mathematics anxiety is not prevalent among 6th grade students, whereas it peaks at around 9th grade. Other studies, however, have reported evidence with a different pattern. For example, Suinn and Edwards (1982) found that students experienced the highest levels of mathematics anxiety during 7th grade and there was a downward trend from 8th through 12th grade. Chiu and Henry (1990) showed that age differences in mathematics anxiety followed an inverted U-shape pattern with a peak around 6th grade. Gierl and Bisanz (1995) also reported that 6th grade students experienced the highest levels of mathematics anxiety. Evidence for age differences of mathematics anxiety among younger children has also been mixed. Several research that investigated children's mathematics anxiety between 1st and 3rd grade (Krinzinger et al., 2009; Ramirez et al., 2013, 2016; Vukovic et al., 2013) showed a general decrease in mathematics anxiety across school year cohort observed. By contrast, other studies have demonstrated no significant differences across school grades (Grades 4–6: Suinn, Taylor, & Edwards, 1988; Grades 4–5: Yüksel-Şahin, 2008; Grades 3–5: Dowker et al., 2012; Grade 1–3: Ganley & McGraw, 2016; Wu, Barth, Amin, Malcarne, & Menon, 2012; Young, Wu, & Menon, 2012). Taken together, these studies, although many are cross-sectional, suggest that there is not a clear connection between mathematics anxiety and students' age or grade levels.

Whereas mathematics anxiety is not tied to one's age group or school grade, individual differences in mathematics anxiety are typically associated with students' mathematical performance. For example, in 64 out of 65 education systems that participated in PISA 2012, students with higher levels of mathematics anxiety demonstrated lower levels of mathematical performance compared with their peers who were less math-anxious (OECD, 2013). This association between mathematics anxiety and performance has been observed both within and across countries (Foley et al., 2017; Lee, 2009). An issue that has remained relatively equivocal concerns how mathematics anxiety is linked to students' mathematical achievement. In the literature, three theoretical accounts have been proposed to explain the connection. The first perspective refers to the Deficit Theory, which posits that mathematics anxiety is an outcome for poor math-related abilities. The second argues for the debilitating influence of mathematics anxiety on one's performance in mathematical tasks. The third theory postulates that the association of mathematics anxiety and performance is reciprocal.

1.2. The Deficit Theory

The first perspective has been referred to as the Deficit Theory (Carey et al., 2016), which postulates that poor mathematics performance elicits mathematics anxiety (Hembree, 1990). Given that there have been high levels of mathematics anxiety in children and adults with mathematical learning disabilities (e.g., Maloney et al., 2010, 2011; Rubinsten & Tannock, 2010; Passolunghi, 2011), it is believed that mathematics anxiety may stem from a deficit in basic numerical processing. For example, Maloney, Ansari, and Fugelsang (2011, p.14) argued that mathematics anxiety "may result from a basic low-level deficit in numerical processing that compromises the development of higher-level mathematical skills". In this theory, individuals underperform in mathematical tasks because they start with lower levels of numerical or spatial skills, and anxious feelings follow from this underperformance. Maloney (2016) contended that reduced abilities might also heighten individuals' sensitivity to negative social cues in mathematics, which further contributed to increased mathematics anxiety.

Evidence for the numerical/spatial difficulties framework can be gleaned from several studies in which adults with higher levels of mathematics anxiety were slower to complete simple numerical and spatial tasks, such as counting objects (Maloney, Risko, Ansari, & Fugelsang, 2010), deciding which of the two digits were larger numerically (Maloney et al., 2011), and rotating a three-dimensional object mentally (Ferguson, Maloney, Fugelsang, & Risko, 2015; Maloney, Waechter, Risko, & Fugelsang, 2012). Research that employed event-related potential measures also showed that individuals with higher levels of mathematics anxiety had a less precise understanding of numerical magnitudes (Núñez-Peña & Suárez-Pellicioni, 2014). However, the cross-sectional nature of these studies renders the causal link of numerical/spatial skills to mathematics anxiety less definitive. There may be other factors associated with high mathematics anxiety, such as avoidance of mathematical tasks, which contribute to low math-related skills/competence later. Research that follows individuals' developmental trajectory of mathematics anxiety may help clarify the relations between mathematics anxiety and mathematics performance.

There has been a lack of longitudinal studies on mathematics anxiety in the literature, but several have shown some evidence that supports the Deficit Theory. For instance, in the United States, Meece, Wigfield, and Eccles (1990) examined predictors of mathematics anxiety and its association with young adolescents' course enrolment intentions and mathematical performance. They found that earlier perceived mathematics ability was moderately associated with individuals' mathematics anxiety in the following year. However, mathematics anxiety was not measured in the first year of the 2-year study, so their findings cannot rule out the possibility that initial mathematics anxiety also predicted later mathematics ability. More recently in the United Kingdom, Field, Evans, Bloniewski, and Kovas (2019) examined whether the developmental trajectory of mathematical achievements across the transition from primary to secondary education predicted later mathematics anxiety. Consistent with the Deficit Theory, mathematical achievements at age 9 before the transition and changes in mathematical achievements across that transition significantly predicted mathematics anxiety at the age of 18. However, because mathematics anxiety was not measured during the transition period, it does not give any evidence as to whether mathematics anxiety predicted mathematics performance longitudinally. Thus, these longitudinal studies suggest poor mathematics performance may lead to increased mathematics anxiety, but the evidence remains inconclusive because of several methodological limitations.

1.3. The debilitating anxiety model

An alternative theoretical perspective to the Deficit Theory is the Debilitating Anxiety Model, which contends that mathematics anxiety causes low mathematics performance through a momentary reduction in cognitive resources necessary for success in mathematics, such as working memory. Working memory refers to a limited capacity store for keeping information for a brief period of time while performing mental operations on that information simultaneously (Engle, 2002; Miyake & Shah, 1999). When doing a math-related task, for example, working memory allows us to retrieve information relevant to the task (e.g., number facts, rules for addition and multiplication), maintain this information salient over the course of problem solving, and inhibit irrelevant information. Anxious feelings, however, can cause a disturbance to the operations of working memory (Eysenck & Calvo, 1992). When individuals with high levels of mathematics anxiety solve a math problem, they are at the same time trying to find a solution to the problem and dealing with their anxious thoughts and feelings. Because success in mathematics often relies on working memory resources (Ashcraft, Donley, Halas, & Vakali, 1992; LeFevre, DeStefano, Coleman, & Shanahan, 2005; Lemaire, Abdi, & Fayol, 1996), the emotional burden may co-opt the working memory resources that are needed for solving mathematical problems effectively. Ashcraft et al. (1992) have postulated that the association between mathematics anxiety and mathematical performance has a stronger relation to worries that interrupt the functions of cognitive resources than to individuals' actual levels of math competency.

Evidence in support for the Debilitating Anxiety Model comes from studies that examined differences in arithmetic performance between individuals with high and low mathematics anxiety. In particular, people with higher levels of mathematics anxiety responded more slowly and made more errors when they solved basic arithmetic problems, but only for those that involved a carry operation (Ashcraft & Faust, 1994; Ashcraft and Kirk, 2001; Faust et al., 1996). These findings suggest that mathematics anxiety may impair performance through diverting cognitive resources from task-relevant purposes (i.e. the math task) to task-irrelevant aspects (e.g., worry) so that individuals' performance suffers most when successful problem solving demands a large capacity for working memory. Consistent with this evidence, Pletzer et al. (2015) showed a difference in neural efficiency between people who had high versus low levels of mathematics anxiety. In their study, neural efficiency was described as the activation of brain areas relevant for mathematical problem solving and the deactivation of other neurological networks unrelated to mathematics. They found that students with lower levels of mathematics anxiety had higher neural efficiency who showed heightened activation in the dorsolateral prefrontal cortex (an area responsible for attentional control) with reduced activation in the default mode network (an indication of reduced emotional processing). By contrast, whereas students with higher levels of mathematics anxiety also demonstrated increased activation in the dorsolateral prefrontal cortex, their default mode network remained activated during problem solving. Thus, students with lower levels of mathematics anxiety only showed neutral activations relevant to solving the math problems, whereas those with higher levels of mathematics anxiety showed both task-relevant and task-irrelevant activations, thereby exhibiting lower neural efficiency during problem solving.

A few longitudinal studies have been conducted that evaluated whether earlier mathematics anxiety predicted children's mathematical performance later. For example, Ching (2017) followed a group of Chinese children from second to third grade and examined the independent contributions of various factors to mathematical performance. It was found that mathematics anxiety predicted children's performance beyond non-verbal intelligence, working memory, number skills, and general and test anxieties. However, this study did not incorporate mathematical performance in the first wave of assessment, making the comparison between the Deficit Theory and Debilitating Anxiety Model impossible. In another longitudinal study conducted in the United States, Vukovic et al. (2013) showed that mathematics anxiety was negatively associated with calculation skills and mathematical applications in both second and third grade. After controlling for general reading achievement, early numeracy, and working memory, mathematics anxiety made independent contributions to both calculation skills and mathematical applications concurrently. However, second-grade mathematics anxiety did not account for unique variance in both calculation skills and mathematical applications in third grade longitudinally after considering the autoregressive effects. Because mathematics anxiety was measured at the first time point only, it remains unclear whether it was longitudinally predicted by earlier mathematical performance. Taken together, mathematics anxiety seems to predict later mathematical performance when earlier mathematical performance was not controlled (Ching, 2017), but when the autoregressive effects were also considered, the longitudinal predictive power of mathematics anxiety may be substantially diminished (Vukovic et al., 2013).

Individual differences in working memory and different cognitive demands placed by different tasks may reconcile the inconsistent results regarding the prediction of earlier mathematics anxiety for later mathematical performance. Indeed, both studies (Ching, 2017, Vukovic et al., 2013) suggest that mathematics anxiety did not affect all children and all kinds of mathematical performance equally. Ching (2017) found that mathematics anxiety was not linked to performance on simple calculation and easy story problems, whereas it was only associated with poorer performance on mathematical problems that demanded more cognitive resources, consistent with previous studies (e.g., Ashcraft & Faust, 1994; Ashcraft and Kirk, 2001; Faust et al., 1996) and the Debilitating Anxiety Model. There was also an interaction of problem type with individual differences in working memory. Children with higher working memory were more susceptible to the negative influence of mathematics anxiety. Similarly, Vukovic et al. (2013) showed that second-grade mathematics anxiety did not predict third-grade mathematical applications for children with low working memory. These findings may be explained by the "choking effect" proposed by Beilock and Carr (2005), who argue that individuals with higher working memory capacity tend to solve math-related problems with strategies that are highly dependent on working memory to execute. When these individuals experience intrusive thoughts or anxious feelings that co-opt their cognitive supply, they often shift to simpler but less effective problem-solving strategies, thereby diminishing their performance on tasks that demand higher working memory (Beilock & Carr, 2005; Beilock & DeCaro, 2007; Beilock, Rydell, & McConnell, 2007; Ramirez et al., 2013, 2016). Thus, working memory may moderate the strength of predictions of children's earlier mathematics anxiety for subsequent mathematical performance. Taken together, these studies provide some evidence for the Debilitating Anxiety Model in which mathematics anxiety causes low performance through co-opting cognitive resources necessary for solving mathematical tasks.

1.4. The reciprocal theory

The mixture of evidence for each of the two theoretical perspectives suggests that the relation between mathematics anxiety and mathematics performance may operate in both directions. Mathematics anxiety may reduce individuals' engagement with math-related activities. Evidence showed that people with higher mathematics anxiety were more likely to avoid taking courses involving mathematics (for a review see Hembree, 1990), tended to answer maths-related questions quickly but inaccurately so as to escape this anxiety-provoking situation (Ashcraft & Faust, 1994), and demonstrated low levels of cognitive reflection when they were asked to solve mathematics word problems (Morsanyi, Busdraghi, & Primi, 2014). Recent evidence showed that anticipation of mathematics resulted in neural activation associated with pain in people with high mathematics anxiety (Lyons & Beilock, 2012), which may explain their tendency to avoid mathematics. These studies suggest that mathematics anxiety may hamper mathematics performance at the pre-processing level by reducing learning opportunities. This avoidance may cause students to lag further behind in their math-related skills and understanding, which contributes to further anxiety. Thus, the link between mathematics anxiety and mathematical competence can be reciprocal. Research that employed a cross-lagged design is a good approach to evaluate this hypothesis; however, this kind of study on mathematics anxiety remains scarce in the literature.

The longitudinal studies reviewed in the preceding sections share a common limitation that either mathematics anxiety or mathematical performance was only measured at one time point. Studies that incorporated both variables over time longitudinally would provide stronger evidence for the directional link between mathematics anxiety and mathematical performance. Ma and Xu (2004) employed a longitudinal panel design that investigated the causal ordering of mathematics anxiety and mathematics achievement tracking a group of young adolescents (Year 7) in the United States over time. They found that prior mathematics achievement was significantly associated with later high mathematics anxiety for boys across both the junior and senior high school periods, but for girls this association was observed at critical transition points only. Contrary to the Debilitating Anxiety Model, prior mathematics anxiety was not related to mathematics achievement longitudinally. Thus, this evidence suggests that mathematic anxiety is more likely to be caused by low mathematical performance, which is consistent with the Deficit Theory. However, the cross-lagged coefficients were small (range from -0.05 to -0.20) though significant in their study, and the statistical significance of these small effects might partially be attributed to the large sample size ($N = 3116$). These situations call for further replications of their findings. More recently, Aldrup, Klusmann, and Ludtke (2019) conducted a similar longitudinal study with secondary school students in Germany. Their cross-lagged panel model showed that there was a reciprocal association between mathematics anxiety and mathematical achievements over a 2-year period. However, Wang, Rimfield, Shakeshaft, Schofield, and Malanchini (2020), who examined mathematics anxiety among Italian secondary school students, did not observe a bidirectional relation between mathematics anxiety and performance. In favour of the Deficit Theory, they found that mathematics achievement predicted mathematics anxiety longitudinally over a 4-month period, but mathematics anxiety did not predict subsequent mathematics achievement.

To the best of our knowledge, there has been only one study that employed a cross-lagged panel design to address the reciprocal hypothesis in children. Gunderson, Park, Maloney, Beilock, and Levine (2018) evaluated first and second graders' mathematical achievement, motivational frameworks (e.g., entity theories, performance goals), and mathematics anxiety longitudinally over a 6-month period. In contrast to previous studies (Ma & Xu, 2004; Wang et al., 2020), this study showed that the reciprocal association was already present in the first 2 years of formal schooling, with mathematics anxiety and mathematical achievement feeding off one another to generate either a positive or negative cycle. However, the magnitudes of associations were not symmetrical: The effect of initial mathematical achievement on later mathematics anxiety appeared to be more than three times stronger than the effect of initial mathematics anxiety predicting later mathematical achievement. The significant bidirectional relations provide support for the Reciprocal Theory, but the relative magnitudes of the associations are more in favour of the Deficit Theory than the Debilitating Anxiety Model. However, it is difficult to compare the results of Gunderson et al. (2018) whose participants were children with other studies conducted with adolescents (Aldrup et al., 2019; Ma & Xu, 2004; Wang et al., 2020). Given that there has been only one longitudinal study that directly tested the bidirectional relation between mathematics anxiety and mathematics performance in children, more longitudinal research is needed to elucidate this association further.

1.5. The present study

On the basis of this gap in the literature, the main purpose of this study was to test the reciprocal relations of mathematics anxiety to quantitative reasoning and number knowledge in Chinese children. The present study aimed to contribute to the literature in several ways. First, we focused on mathematics anxiety in young children. A recent meta-analytic review (Zhang, Zhao, & Kong, 2019) suggests that the negative link between mathematics anxiety and mathematics performance was stronger and more consistent for adults than children. Whereas some studies showed that the negative relation was already present at this age range (e.g., Ching, 2017; Jameson, 2013; Vukovic et al., 2013; Wu et al., 2012), other evidence indicated that mathematics anxiety of children at the primary school level was not associated with their performance in mathematics (e.g., Dowker et al., 2012; Haase et al., 2012; Wood et al., 2012). In view of the mixed findings in the literature, the present study aimed to test whether the negative relation of mathematics anxiety to mathematical performance already surfaced at a young age. If yes, earlier assessments and interventions that mitigate its deleterious impacts are needed. Second, with a longitudinal design and cross-lagged panel analyses, our study would be one of the few studies that empirically documented whether the bidirectional association was present in children.

Third, our study would test the relation of mathematics anxiety to quantitative reasoning, in addition to number knowledge. Within the Deficit Theory, it has been suggested that numerical and spatial difficulties may predispose individuals to higher mathematics anxiety (Maloney et al., 2011). We argue that children's mathematics anxiety may also result from low levels of quantitative reasoning ability that compromises their mathematical development. Numbers and quantities are not the same. As suggested by Thompson (1993, 165–166), "a person constitutes a quantity by conceiving of a quality of an object in such a way that he or she understands the possibility of measuring it. Quantities, when measured, have numerical value, but we need not measure them or know their measures to reason about them." Indeed, we can use a number to represent a quantity, but quantity is not always measured and represented by a number. For example, if we know that A is taller than B and B is taller than C, then we can base on this information about the relation between quantities to conclude that A is taller than C without the need to know any numerical information. The inverse relation between addition and subtraction is another good example for quantitative reasoning: when we add a quantity and take away the same quantity from a set, the quantity in the set remains unchanged (i.e. $a + b - b = a$) even if we do not know the exact numerical values of "a" and "b". Research has demonstrated that reasoning about quantities has a strong predictive power for children's mathematical achievements beyond general cognitive abilities, such as non-verbal intelligence and working memory (Ching & Nunes, 2017a, 2017b; Nunes et al., 2007, 2012; Stern, 2005; Stocket al., 2009a, 2009b; Stock et al., 2010).

Quantitative reasoning has been defined as "the ability to represent quantitative information and act on the representations to come to conclusions not previously known about the quantities represented or about the relationships between them." (Nunes, Bryant, Evans, & Barros, 2015, p. 178). Imagine a situation in which we show 5 boxes to a boy and tell him that there are 2 sweets inside each box. We then ask him to determine how many sweets there are in total in the boxes. Because the sweets are out of the child's sight, he cannot simply count up the number of sweets. In order to solve the problem successfully, he needs to represent the two-to-one relation between the quantities in some ways, and act on this quantitative representation to come up with a correct answer. For example, the boy may point to each box twice and count up the imagined quantities. In this case, the boy uses counting to solve the problem, but prior to that, he has to be able to reason that the quantitative relation mentioned in the problem is about one-to-many correspondence and then establish the appropriate connection between the quantitative representations (e.g., two-to-one relation) and the use of numerical procedure (e.g., counting) to solve the problem.

Research in developmental psychology showed that children often demonstrated a disconnection between their understanding of quantities and numbers. For example, Piaget (1952), Gréco (1962) and Cowan, Foster, and Al-Zubaidi (1993) showed that children may say that two sets of objects have the same quantity while rejecting the idea that both sets have the same number of objects. Similarly, some children may also use the same number to indicate two sets of objects while asserting that they do not have the same quantity. Hughes (1981) demonstrated that some children who solved a problem that referred to quantities without any problems (e.g., "I put 2 bricks in this box and then I put 1 brick in this box; how many bricks are in the box now?") could be completely puzzled by a similar question about numbers (e.g., "what is 2 plus 1?"). Freeman, Antonucci, and Lewis (2000) demonstrated that most of the 4- and 5-year old children could correctly judge that a puppet had miscounted a set of items because it skipped one item during the counting process. However, even if they recognized the puppet's mistake in counting procedure, they still said that the number of objects in the set is the last number word that the puppet used in counting. Bermejo, Morales, and deOsuna (2004) showed that some of the 4- and 5-year old children accepted the last count word as the cardinal of the set even when a puppet had started to count the array from two. The disconnection between the understanding of numbers and quantities has been demonstrated in other research (e.g., Sarnecka & Gelman, 2008; Sophian, 1988). These studies suggest that it is important to distinguish between children's knowledge about numbers and their knowledge about quantities.

In the present study, we conceptualize "mathematical competence" as a construct that consists of two aspects of competence. The first refers to "number knowledge", which is indicated by children's knowledge of the counting procedures (Geary, Bow-Thomas, & Yao, 1992) and their ability to make magnitude comparison between numbers (Griffin, 1997). The second refers to "quantitative reasoning" (Nunes et al., 2015), which is captured by tasks that require children's understanding of additive composition (i.e. any number/quantity can be seen as the sum of two other numbers/quantities), inverse relation between addition and subtraction (i.e. when we add a number/quantity and take away the same number/quantity from a set, the number/quantity in the set remains unchanged), and multiplicative reasoning (i.e. the ability to make inferences about quantities by establishing one-to-many correspondence between quantities). Because previous studies showed that number knowledge and quantitative reasoning made unique contributions to mathematical achievements (e.g., Nunes, Bryant, Evans, Bell, & Barros, 2012), we were intrigued to examine whether they would also longitudinally predict mathematics anxiety independently and whether mathematics anxiety predicted these two forms of mathematical competence independently. The inclusion of quantitative reasoning as a variable in our study may have important educational implications. If quantitative reasoning turns out to be a unique and strong predictor for mathematics anxiety, then it would point toward the training of quantitative reasoning as one of the key areas in the development of remediation strategies for reducing mathematics anxiety in young children.

Another factor that may influence the strength of the association between mathematics anxiety and mathematics performance is the geographical region in which research was conducted. A recent meta-analytic review (Zhang et al., 2019) showed that geographical region was a moderator for the negative maths anxiety-performance link, which was stronger in studies that involved Asian students compared with non-Asian counterparts. This finding may be attributed to extent to which academic achievement is valued in different cultures. Evidence showed that Chinese parents had higher expectations for their children's mathematical achievement than Euro-American parents (Chen & Stevenson, 1989). Together with predominantly examination-oriented education systems (Kirkpatrick & Zang, 2011; Li, 2009; Lin & Chen, 1995; Tsuneyoshi, 2004; Yu, Chen, Levesque-Bristol, & Vansteenkiste, 2016), students in East Asian counties may be under great pressure of being evaluated mainly based on their academic performance. It may partly explain why the negative maths anxiety-performance link is stronger in Asian students because they may be more likely to internalize the academic-based evaluative criteria and have their self-worth contingent upon their academic performance. In Hong Kong, the focus of early childhood education is to help kindergarteners develop interest in mathematics through play-based activities, such as learning basic mathematical concepts (e.g., counting, ordering, sorting etc.) and identifying properties of objects (e.g., colour, size, shape etc.). When children attend the first year of primary schooling, they receive more formal instruction, such as learning numbers to 20, performing addition and subtraction of numbers within 18, recognizing the concepts of length, distance, 2-D and 3-D shapes, describing relative position of objects and so on. Notably, compared with kindergartens, there are less play-based activities whereas more homework and tests are given to children starting from the first year of primary school. Thus, it is likely that the children in Hong Kong would experience mathematics anxiety at a young age, which may increase over time as they receive formal schooling since the age of 6. Given that both studies that assessed the reciprocal relations between mathematics anxiety and mathematical performance were conducted in the United States, the fourth contribution of the present study was our focus on Chinese children where research of this kind is still in its infancy. On the basis of the Reciprocal Theory, we hypothesize that children's initial levels of quantitative reasoning and number knowledge would have negative associations with subsequent levels of mathematics anxiety. In turn, children's initial levels of mathematics anxiety would also be linked to subsequent levels of number knowledge and quantitative reasoning abilities.

2. Methods

2.1. Participants

Three hundred and sixteen children (157 boys, 159 girls) in three primary schools in Hong Kong participated in two periods of data collection in this study. All children spoke Cantonese as their first language and were at the first year of primary school, with a mean age of 76.72 months (SD = 2.53 months, ranging from 69.7 to 79.1 months), during the first period of data collection. The highest educational levels attained by the mothers of the children in the sample were as follows: Primary and below – 19.8% (n = 63), secondary – 43.7% (n = 138), and post-secondary – 36.5% (n = 115). According to the Hong Kong Population Census (2016), the distribution of educational attainment (highest level attained) was: Primary and below – 20.6%, secondary – 46.2%, and post-secondary – 33.2%. Thus, the distribution of educational levels in our sample was comparable to that of the overall Hong Kong population. The initial sample during the first wave of assessment consisted of 328 children. Missing data (n = 12) were identified because of children's failure to complete individual tasks (n = 11) and because of experimenter error in administering the quantitative reasoning task (n = 1). Thus, these data were not used for analyses. Teachers reported that all the participating children in the final sample (N = 316) had normal intelligence, and did not have any learning difficulties, emotional, and behavioural problems.

2.2. Procedure

This study was approved by the institutional review board at the first author's institution. We recruited the participants through local primary schools in Hong Kong. Teachers of each school sent letters to parents and obtained their consents. During data collection, the children were asked for verbal assent and participated individually with the researcher in a quiet location, which was distant from other children in the primary school. To ensure confidentiality, participants were assured that their responses were anonymous, and the data would be kept in a safe place that could only be accessed by the first author and would not be disclosed to other parties without their permission. Participants received no remuneration for taking part in this study. For each child, the interval between the first and second wave of assessments was between 8 and 9 months, with 8 months being the commonest interval (94%). At both time points, the children were asked to complete the measures of mathematics anxiety and tested for their quantitative reasoning, number knowledge, and reading competence in a 40–60-minute session. Parents were asked to fill in a questionnaire that involved their estimates of children's general anxiety and demographic information.

2.3. Measures

2.3.1. Mathematics anxiety

We assessed mathematics anxiety with a 12-item measure used in previous studies (Ching, 2017; Harari, Vukovic, & Bailey, 2013), which required children to respond to items, such as "I get nervous about making a mistake in math", on a 4-point Likert scale ('always = 4, sometimes = 3, a little bit = 2, never = 1'). A researcher read aloud each question to children in order to minimize reading demands. The maximum possible score was 48. The reliability of this measure was good (Time 1: Cronbach's α = 0.87; Time 2: Cronbach's α = 0.89), which was comparable to another study using the same measure with Chinese children (Cronbach's α = 0.81; Ching, 2017).

2.3.2. General/trait anxiety

Because some studies indicated that mathematics anxiety was associated with general/trait anxiety (e.g., Hembree, 1990), our study incorporated the 6-item Anxiety Problems subscale of the Child Behaviour Checklist for the ages of 6 to 18 (Achenbach, 1991; Achenbach, Dumenci, & Rescorla, 2003) at Time 1 to capture children's general/trait anxiety. This checklist required the parents/guardians of the children to indicate whether the child was currently showing or had showed anxiety-related problems or traits within the previous 6 months. Items were rated on a scale of 1 (not true), 2 (somewhat true), or 3 (very true). The maximum possible score was 18. This measure demonstrated good internal consistency (Cronbach's α = 0.91).

2.3.3. Quantitative reasoning

The quantitative reasoning measure consisted of three subtests, including (a) inverse relation between addition and subtraction, (b) additive composition, and (c) multiplicative reasoning. Previous studies have demonstrated wide variations in children's abilities to reason quantitatively at a young age, and these individual differences had strong and independent predictions of mathematical achievements longitudinally (Ching & Nunes, 2017a; Nunes et al., 2007, 2012; Stern, 2005). The maximum possible score was 30. In the present study, this measure showed good internal consistency (Time 1: Cronbach's α = 0.88; Time 2: Cronbach's α = 0.91). The same versions of the quantitative reasoning measure were given at both time points.

The first subtest of quantitative reasoning measured children's knowledge of the *inverse relation between addition and subtraction*. This task has demonstrated satisfactory reliability and validity in previous studies (e.g., Bryant et al., 1999; Nunes et al., 2015). On the basis of Bryant et al. (1999), the inversion task involved two types of items: (1) objects and (2) pictures. For each type of item, inversion knowledge was assessed in the form $a + b - b$, $a + b - (b - 1)$, or $a + b - (b + 1)$. The latter two kind of questions were included because we wanted to maintain children's attention on the task, otherwise they might always repeat the initial numbers of the problems as the answers. In the "object" task, children saw a row of bricks and counted with the experimenter to determine the number of bricks in the row (i.e. "a"). After that, the row was covered by a cloth so that counting was no longer possible, while the endings of the row were kept in full view to the children. The experimenter then added a number of bricks to the row (i.e. "b") and either subtract the same number ("b") or the number plus or minus 1 (i.e. "b + 1" or "b – 1"). The children were asked how many bricks there were on the table. Because they could not count the bricks, they were more likely to answer this question correctly if they understood the inversion principle. In the "picture" task, children saw a picture of a box and were told that there were a certain number of objects inside (for example, 5 teddy bears). The number of objects was printed on the box so that the children did not forget this information. After that, the children were shown the next picture, which was located to the right of the first picture, to form a sequence. They were told how many objects had been added to the box. Then a third picture were added, and the experimenter told the children how many objects had been removed from the box. Finally, the children were asked how many objects there were in the box. The inversion task consisted of 12 items, six of which will be presented with rows of bricks and six with pictures. There was no feedback for their answers and how they should attempt to solve the problems.

The second subtest of quantitative reasoning measured children's understanding of *additive composition*. This task was adapted from the "Shop Task" used in previous studies (Krebs et al., 2003; Lines & Bryant, 1996; Nunes & Schliemann, 1990; Nunes & Bryant, 2015), which has demonstrated satisfactory reliability and validity. The task started with a warm-up activity in which children were shown fictitious coins of different values one by one (e.g., 1 value, 5 values). After this warm-up, the children were invited to play a "Shop Game". A total of six items were used, in which children were asked to use combinations of 1-value and 5-value coins to buy something that worth 8 values and 9 values, use combinations of 10-value and 1-value coins to pay for a product of 14 values and 18 values, and use combinations of 20-value and 1-value coins to buy something that worth 24 values and 25 values. Before each task, the children were reminded of the value of each coin and then asked to pay the exact amount of money. If the children tried to pay with their 1-value coins, they would not have enough money to buy the products. Thus, they must understand the idea of additive composition in order to do so successfully. There was no feedback for their answers and how they should attempt to solve the problems.

The final subtest of quantitative reasoning measured children's ability to *reason multiplicatively*. One the basis of previous studies (e.g., Frydman & Bryant, 1988; Nunes et al., 2015), a total of 12 problems were included in this subtest. Six of the problems involved objects and six were represented by pictures. One example of the multiplicative problems was: Children were shown a street with 4 houses and told that there were 3 cats living inside each house while the "cats" are not visible to them. The children were asked how many cats lived in that street. One example of the division problems was: "There are 6 biscuits and we want to give 2 biscuits to each child; how many children can get the biscuits?" Children were shown the recipients who would receive the biscuits while the biscuits were out of their sight. There was no feedback for their answers and how they should attempt to solve the problems. In order to solve these multiplicative problems successfully, children have to adapt their counting procedures based on their reasoning about the multiplicative relation between quantities. Using the "how many cats are living in the street?" as an example, they have to realize that 1 house corresponds to 3 cats and then they may point to each "house" thrice and count the imagined cats (because actually they did not see any "cats") to come up with a correct answer. Thus, these children have to establish one-to-many correspondence before using the procedure of counting to solve the problem. As for those who do not reason about the problem situation multiplicatively, it is common that they point to each "house" once only or randomly and then count without considering the relation of quantities in the problem.

2.3.4. Number knowledge

We measured number knowledge with a measure adapted from the Number Knowledge Test (Griffin, 1997) and a procedural counting task.

Table 1
Descriptive statistics and bivariate correlations among study variables (N = 316).

Variables	*M*(*SD*)	Correlations 1	2	3	4	5	6	7	8	9
1. T1 Maths Anxiety	22.22 (8.25)	1								
2. T2 Maths Anxiety	30.03 (9.42)	0.584**	1							
3. T1 Quantitative Reasoning	19.84 (4.02)	−0.286**	−0.305**	1						
4. T2 Quantitative Reasoning	23.14 (4.19)	−0.245**	−0.301**	0.714**	1					
5. T1 Number Knowledge	27.89 (3.64)	−0.256**	−0.278**	0.165**	0.227**	1				
6. T2 Number Knowledge	30.55 (3.97)	−0.250**	−0.270**	0.378**	0.384**	0.480**	1			
7. T1 General Anxiety	9.32 (1.93)	0.116*	0.137*	−0.092	−0.072	−0.023	−0.082	1		
8. T1 Reading	14.59 (3.12)	−0.042	−0.086	0.076	0.084	0.045	0.058	−0.029	1	
9. T2 Reading	16.78 (3.46)	−0.063	−0.096	0.065	0.095	0.048	0.063	−0.038	0.587**	1

Note. T1 = Time 1, T2 = Time 2.
** $p < .001$.
* $p < .05$.

The number knowledge test involved four subtasks presented in the same order for all children, including number sequence knowledge (e.g., "what number comes five numbers after 49?"), relative magnitude (e.g., "which is bigger: 51 or 39?"), numerical distance (e.g., "which number is closer to 7: 4 or 9?"), and differences (e.g., "which difference is bigger: the difference between 20 and 17 or the difference between 25 and 20?"). Each of the four subtasks included practice items and six test items and no feedback was given. The maximum possible score was 24. We measured children's procedural counting abilities with two tasks: oral rote counting and object counting. In oral counting, children were asked to count aloud eight series of number sequences in ascending and descending orders. In object counting, they were asked to count two arrays of geometric shapes (e.g., triangles) and two sets of other objects (e.g., pencils). On any given trials, the objects were the same in appearance. The maximum possible score of the procedural counting task (oral plus object counting) was 12. We combined the procedural counting scores with the number knowledge test scores to form a composite score to indicate children's overall knowledge of numbers (maximum possible score = 36), which demonstrated good internal consistency (Time 1: Cronbach's $\alpha = 0.92$; Time 2: Cronbach's $\alpha = 0.93$). The same versions of the number knowledge measure were given at both time points.

2.3.5. *Chinese word reading*

We used the Chinese word reading task (adapted from Ching & Nunes, 2015) to test the specificity of mathematics anxiety for math-related performance. If mathematics anxiety predicted children's performance in quantitative reasoning and number knowledge much better than in Chinese word reading, it would confirm the specificity and importance of mathematics anxiety in children's mathematics learning. In this task, children were asked to read aloud two-character Chinese words printed on a paper. There was no feedback to their answers. The maximum possible score was 30 and this measure showed good internal consistency (Time 1: Cronbach's $\alpha = 0.94$; Time 2: Cronbach's $\alpha = 0.95$). The same versions of the word reading measure were given at both time points.

2.3.6. *Demographic characteristics*

Parents were asked to report their children's gender and mothers' highest education level at Time 1. Previous research (e.g., Nunes, Bryant, Hallett, Bell, & Evans, 2009) showed that, among different indicators of socioeconomic status, mothers' highest educational level was the best predictor of mathematical achievement. Therefore, we selected it as a proxy variable for socioeconomic status in our study.

3. Results

3.1. *Overview of statistical analyses*

Our analyses began with an inspection of the descriptive statistics and bivariate correlations of the study variables. We then tested the hypothetical cross-lagged model with structural equation modelling using AMOS version 23 (Arbuckle, 2014). The model included reciprocal relations between our key variables (mathematics anxiety, quantitative reasoning, and number knowledge) with covariates (based on the results in our preliminary analyses). To control for error variance, we randomly assigned items into two parcels for each of our three main variables (i.e. mathematics anxiety, quantitative reasoning, and number knowledge). We employed item parcelling because research has shown that parcels result in more reliable estimates of latent variables than individual items (Little, Cunningham, Shahar, & Widaman, 2002). Compared with aggregate scores, using item parcelling produces an increase in estimates of structural parameters and a reduction in residual variances (Coffman & MacCallum, 2005). Another advantage is that the use of parcels gives distributions that approximate normality more closely (Bagozzi & Heatherton, 1994). To evaluate the fit of the models, we relied on three fit indices: root mean square residual (RMSEA ≤ 0.06), comparative fit index (CFI ≥ 0.90), and chi-square divided by degree of freedom (χ^2/df ratio < 2) (Hu & Bentler, 1999; Marsh, Hau, & Wen, 2004; Martens, 2005; Quintana & Maxwell, 1999; Weston & Gore, 2006). We also explored the viability of different models by imposing different constraints.

3.2. *Demographic differences*

Table 1 shows the descriptive statistics and correlations among study variables. Paired-sample *t*-test showed that there were significant increases in mathematics anxiety [$t(315) = -17.11$, $p < .001$], quantitative reasoning [$t(315) = -18.83$, $p < .001$], and number knowledge [$t(315) = -12.15$, $p < .001$] over time. Because demographic characteristics may explain differences in children's levels of mathematics anxiety and other variables, we conducted independent t-tests with children's gender and one-way analyses of variance (ANOVA) with mothers' educational levels as a fixed factor to examine potential demographic differences on each variable. We found no significant gender difference in mathematics anxiety at Time 1, $t(314) = -1.17$, $p = .243$. However, there was a significant gender difference in mathematics anxiety at Time 2, $t(314) = -2.48$, $p = .014$, in which girls had higher mathematics anxiety than boys (girls, $M = 31.33$, $SD = 9.23$; boys, $M = 28.72$, $SD = 9.45$). Therefore, we treated gender as a covariate in our subsequent cross-lagged analyses. There were no significant differences between boys and girls in other variables (all *p*-values were above 0.05).

There were significant differences in mothers' educational levels on mathematics anxiety at both Time 1 [$F(2, 313) = 7.17$, $p < .001$, partial $\eta^2 = 0.044$] and Time 2 [$F(2, 313) = 8.24$, $p < .001$, partial $\eta^2 = 0.050$]. As for Time 1 mathematics anxiety, post hoc comparisons using Bonferroni correction indicated that the mean score for the post-secondary education group ($M = 20.06$, $SD = 8.88$) was significantly lower ($p = .001$) than that for the pre-primary education group ($M = 24.77$, $SD = 8.13$). Children from the post-secondary education group ($M = 27.71$, $SD = 9.42$) also demonstrated significantly lower levels of mathematics anxiety at Time 2 ($p < .001$) than children from the pre-primary education group ($M = 33.60$, $SD = 8.12$). Thus, children with parents who have lower educational levels seem to be more vulnerable to developing mathematics anxiety. Because mathematics anxiety appeared to differ according to the educational levels of children's mothers, we treated it as a covariate in our subsequent cross-lagged analyses. No demographic differences in all other variables were found (all *p*-values were above 0.05).

3.3. Bivariate correlations

We next examined the associations between mathematics anxiety and other continuous variables. Table 1 shows the bivariate correlations among the variables. Several key findings are identified. First, both mathematics anxiety and quantitative reasoning were highly stable across time with large effect sizes ($r = 0.58$, $p < .001$ for mathematics anxiety; $r = 0.71$, $p < .001$ for quantitative reasoning). Number knowledge was moderately stable ($r = 0.48$, $p < .001$) across the two waves of assessments. Second, mathematics anxiety at Time 1 and Time 2 was negatively and significantly associated with quantitative reasoning and number knowledge at both time points with small to moderate effect sizes (rs range from -0.25 to -0.31, $ps < 0.001$), which showed that children with higher levels of mathematics anxiety consistently demonstrated lower levels of quantitative reasoning and number knowledge across time. These results suggest that there is merit in testing the proposed reciprocal relations with cross-lagged analysis. Third, general anxiety reported by children's parents was significantly linked to mathematics anxiety at both time points ($r = 0.12$, $p = .04$ for Time 1; $r = 0.14$, $p = .02$ for Time 2). Because it may contribute to the variance explained by mathematics anxiety, we considered it as a covariate in our cross-lagged analyses. Finally, we found no significant relations of mathematics anxiety to word reading performance at all time points (all *p*-values are above 0.05), which suggests that mathematics anxiety is likely to be specific to math-related performance.

3.4. Testing reciprocal associations

We examined several structural models regarding the longitudinal associations between mathematics anxiety and mathematical competence (i.e. quantitative reasoning and number knowledge). We assessed each of the models by allowing autoregressive effects to be free between adjacent time points to control for stability effects. When testing the structural models, five models were compared with chi-square difference test ($\Delta\chi^2$). In the first model (Model A), we constrained all cross-lagged effects to zero. This model served as the baseline model against which other models were evaluated. Based on the χ^2/df ratio and the RMSEA value, this model did not fit the data well [$\chi^2(61, N = 316) = 146.31$, $p < .001$, χ^2/df ratio = 2.29, CFI = 0.965, RMSEA = 0.064]. In the second model (Model B), we relaxed the constraints imposed in the first model by freeing the lagged effects of mathematics anxiety on quantitative reasoning and number knowledge. This model helped us test whether earlier higher levels of mathematics anxiety were linked to subsequent lower levels of mathematical competence. The χ^2/df ratio and the RMSEA value also indicated that Model B did not fit the data well [$\chi^2(59, N = 316) = 140.32$, $p < .001$, χ^2/df ratio = 2.26, CFI = 0.967, RMSEA = 0.063] and it did not fit better than Model A ($\Delta\chi^2(2) = 5.99$, $p = .05$). In the third model (Model C), we lifted the constraints on the cross-lagged effects of quantitative reasoning and number knowledge on mathematics anxiety and constrained the cross-lagged effects of mathematics anxiety on quantitative reasoning and number knowledge to zero. This model was used to evaluate the alternative theory that earlier lower levels of mathematical competence were linked to subsequent higher levels of mathematic anxiety. The χ^2/df ratio and the RMSEA value indicated that Model C showed a marginally good fit to the data [$\chi^2(59, N = 316) = 131.26$, $p < .001$, χ^2/df ratio = 2.12, CFI = 0.971, RMSEA = 0.060], whereas it fitted better than Model B ($\Delta\chi^2(2) = 9.06$, $p = .01$).

In the fourth model (Model D), we relaxed the constraints in the third model and tested the reciprocal effects of mathematics anxiety and mathematical competence. This reciprocal model also showed a marginally good fit to the data [$\chi^2(57, N = 316) = 126.13$, $p < .001$, χ^2/df ratio = 2.10, CFI = 0.972, RMSEA = 0.059], and it did not fit better than Model C ($\Delta\chi^2(2) = 5.13$, $p = .08$). However, this model showed better fit than Model A ($\Delta\chi^2(4) = 20.18$, $p < .001$) and Model B ($\Delta\chi^2(2) = 14.19$, $p < .001$). In the final model (Model E), in addition to freeing the reciprocal associations among mathematics anxiety, quantitative reasoning, and number knowledge, we also relaxed the constraints between the cross-lagged effects between quantitative reasoning, and number knowledge. That is, we also assessed whether there was a reciprocal relation between quantitative reasoning and number knowledge (all cross-lagged paths were not constrained in this model). The final model showed a good fit to the data [$\chi^2(55, N = 316) = 90.34$, $p < .001$, χ^2/df ratio = 1.56, CFI = 0.986, RMSEA = 0.042] and it fitted better than Model C ($\Delta\chi^2(4) = 40.92$, $p < .001$) and Model D ($\Delta\chi^2(2) = 35.79$, $p < .001$). The standardized estimates of the final model (Fig. 1) are presented in Table 2, and they are interpreted as follows.

First, consistent with our bivariate correlation analyses, all of our main variables (mathematics anxiety, quantitative reasoning, and number knowledge) continued to be stable over time even when other variables were considered simultaneously. Second, gender was the only covariate that remained significant in predicting T2 mathematics anxiety (gender: $\beta = 0.100$, $p = .03$). Third, after considering the effects of covariates, T1 quantitative reasoning significantly predicted lower T2 mathematics anxiety ($\beta = -0.164$, $p = .003$), but T1 mathematics anxiety did not significantly predict T2 quantitative reasoning ($\beta = -0.019$, $p = .70$). Similarly, T1 number knowledge predicted lower T2 mathematics anxiety ($\beta = -0.108$, $p = .04$), but T1 mathematics anxiety did not significantly predict T2 number knowledge ($\beta = -0.031$, $p = .592$). Thus, the results did not support our hypothesis that mathematics anxiety was reciprocally related to quantitative reasoning and number knowledge. The strengths of associations were stronger for the predictions of initial quantitative reasoning and number knowledge on later mathematics anxiety. However, we found a bidirectional link between quantitative reasoning and number knowledge. T1 quantitative reasoning significantly predicted T2 number knowledge ($\beta = 0.328$, $p < .001$), whereas T1 number knowledge significantly also predicted T2 quantitative reasoning ($\beta = 0.105$, $p = .03$). Although both directional relations were statistically significant, the predictive power of quantitative reasoning on number knowledge is three times stronger than that of number knowledge on quantitative reasoning

4. Discussion

This study examined the reciprocal relations of mathematics anxiety to quantitative reasoning and number knowledge in Chinese children. On the basis of the Reciprocal Theory, we hypothesize that mathematics anxiety is bidirectionally associated with mathematical competence. However, inconsistent with our hypothesis, there appeared to be a unidirectional link from prior mathematical competence to subsequent mathematics anxiety. In particular, our findings are in favour of the

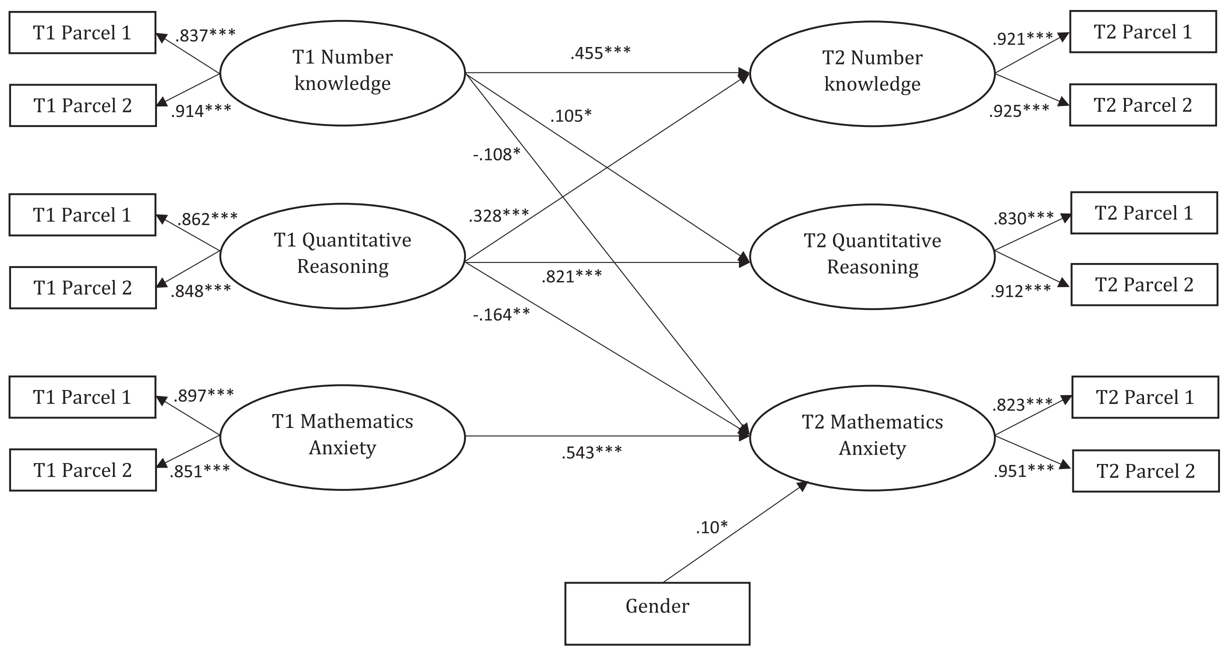

Fig. 1. Structural equation modelling showing longitudinal relations among mathematic anxiety, quantitative reasoning, and number knowledge. Solid lines indicate significant paths (p < .05) and are labelled with standardized coefficients. For simplicity, only significant structural paths are shown. Note. T1 = Time 1, T2 = Time 2; ***p < .001, **p < .01, *p < .05.

Deficit Theory, which postulates that mathematics anxiety stems from poor mathematical competence. Children who start school with lower levels of quantitative reasoning and number knowledge may set the stage for the development of mathematics anxiety. Several key findings are summarized and discussed as follows.

A recent meta-analysis (Zhang et al., 2019) showed that students from senior high schools demonstrated the strongest negative math anxiety-performance association, whereas students from primary school demonstrated the lowest association. Evidence has not been consistent regarding whether the negative link was present in children, with some studies showing that it existed at a young age (e.g., Ching, 2017; Jameson, 2013; Vukovic et al., 2013; Wu et al., 2012), while others showed no significant associations (e.g., Dowker et al., 2012; Haase et al., 2012; Wood et al., 2012). Considering the cultural context (Chinese children in Hong Kong) where the present study was conducted, it seems reasonable to observe a significant correlation between mathematics anxiety and mathematical performance in children of this age. Chinese parents typically had higher expectations for their children's mathematical achievement than Euro-American parents (Chen & Stevenson, 1989). Together in the face of an exam-oriented education systems (Kirkpatrick & Zang, 2011; Li, 2009; Lin & Chen, 1995; Tsuneyoshi, 2004; Yu et al., 2016), it is likely that children at a young age in Hong Kong already experience mathematics anxiety. Indeed, we found that children experienced a significant increase in mathematics anxiety over the 8–9-month period during the first year of primary schooling, and this anxiety was concurrently and longitudinally

Table 2
Standardized estimates of the final model (Model E) (N = 316).

Structural paths	Standardized estimates	Significance levels (*p*-values)
T1 number knowledge → T2 number knowledge	0.455***	< 0.001
T1 quantitative reasoning → T2 quantitative reasoning	0.821***	< 0.001
T1 mathematics anxiety → T2 mathematics anxiety	0.543***	< 0.001
Gender → T2 mathematics anxiety	0.100*	0.03
Mothers' educational levels → T2 mathematics anxiety	−0.080	0.096
General anxiety → T2 mathematics anxiety	0.061	0.103
T1 number knowledge → T2 mathematics anxiety	−0.108*	0.04
T1 quantitative reasoning → T2 mathematics anxiety	−0.164**	0.003
T1 mathematics anxiety → T2 quantitative reasoning	−0.019	0.7
T1 mathematics anxiety → T2 number knowledge	−0.031	0.592
T1 quantitative reasoning → T2 number knowledge	0.328***	< 0.001
T1 number knowledge → T2 quantitative reasoning	0.105*	0.03
Correlations		
T1 number knowledge & T1 quantitative reasoning	0.221**	0.001
T1 quantitative reasoning & T1 mathematics anxiety	−0.332***	< 0.001
T1 number knowledge & T1 mathematics anxiety	−0.308***	< 0.001
General anxiety & T1 mathematics anxiety	0.121*	0.012
Mothers' educational levels & T1 mathematics anxiety	−0.224***	< 0.001

Note. T1 = Time 1, T2 = Time 2.
*** *p* < .001.
** *p* < .01.
* *p* < .05.

associated with two basic forms of mathematical competence, namely number knowledge and quantitative reasoning. Past research that examined longitudinal associations between mathematics anxiety and mathematical performance typically incorporated an overall mathematical achievement test as an indicator for mathematical performance (e.g., Field et al., 2019; Gunderson et al., 2018; Ma & Xu, 2004; Meece et al., 1990). Fewer studies tested the extent to which mathematics anxiety is related to more basic math-related skills. Thus, a novel contribution of the present study was that we have provided evidence for both number knowledge and quantitative reasoning to be negatively associated with mathematics anxiety concurrently and longitudinally.

Within the framework of the Deficit Theory, Maloney et al. (2011) postulate that numerical and spatial difficulties may predispose individuals to higher mathematics anxiety. In the present study, we explore whether the lack of another important aspect of mathematical competence would also be associated with the development of mathematics anxiety. In support of Maloney et al. (2011), our cross-lagged panel model showed that children in the first grade who began the school year with poorer number knowledge had higher levels of mathematics anxiety at the end of the year. Our study further contributes to the literature by showing that low quantitative reasoning is another risk factor for the development of mathematics anxiety in young children. The predictions of number knowledge and quantitative reasoning appeared to be independent from one another because both structural pathways remained significant when another was also present in the model simultaneously. This evidence suggests that mathematics anxiety may not only arise from low levels of numerical processing ability, but it may also stem from weak competence in quantitative reasoning. Thus, it may be important to nurture quantitative reasoning in early childhood education because children who enter primary school with low quantitative reasoning are not only associated with lower overall mathematical achievements in the future (Ching & Nunes, 2017a, 2017b; Nunes et al., 2007, 2012; Stern, 2005), but they are also more likely to experience higher levels of mathematics anxiety.

With respect to the debate about the prospective links between mathematics anxiety and mathematical performance, our findings appeared to be consistent with several longitudinal studies (Field et al., 2019; Meece et al., 1990) that prior mathematical attainment longitudinally predicted subsequent levels of mathematics anxiety. However, our study differed from this research in at least two important ways. First, the participants in their studies were adolescents, whereas ours were children. Second, the present study provided stronger evidence for predictive power of mathematical performance on mathematics anxiety because we also controlled for children's levels of mathematics anxiety at an earlier time point. In the literature, there has been other evidence (e.g., Ching, 2017) that prior mathematics anxiety predicted children's mathematical performance later beyond various control variables, but the autoregressive effect of mathematical performance was often not taken into account. It appears that when earlier mathematical performance is considered, the predictive power of mathematics anxiety is diminished substantially (Vukovic et al., 2013). Taken together, contrary to our expectations and the Reciprocal Theory, we failed to find significant bidirectional relations of mathematics anxiety to quantitative reasoning and number knowledge. Instead, our results are more in favour of the Deficit Theory, which posits that mathematics anxiety results from low mathematical competence. However, the absence of significant links between prior mathematics anxiety and later mathematical performance did not entirely refute the possibility that mathematics anxiety would also lead to decrements in performance. According to Carey et al. (2016), it is possible that the influence of mathematics anxiety on performance is rather immediate and better captured in short-term experimental studies, whereas longer-term prospective research may better serve to detect the effects performance has on future mathematics anxiety.

The significant association between mathematics anxiety and mathematical performance was not explained by general anxiety and gender. We were also able to establish some degree of divergent validity by assessing whether the pattern of findings remained the same if math-related skills were replaced by word reading skills. If mathematics anxiety predicted children's performance in quantitative reasoning and number knowledge much better than in Chinese word reading, it would confirm the specificity and importance of mathematics anxiety in children's mathematics learning. Indeed, we found that there were not significant associations between mathematics anxiety and word reading ability at any time points. Thus, mathematics anxiety is not merely a kind of general anxiety for all academic subjects for children, but it is specifically related to maths-related performance.

Our study also showed that girls experienced more mathematics anxiety than boys, but it only occurred at the second wave of assessment. The effects of gender have not been consistent in the literature, with some studies showing stronger mathematics anxiety in females than in males (e.g., Osborne, 2001; Gunderson et al., 2018), whereas others showed that girls did not experience higher levels of mathematics anxiety than boys (Ma, 1999), or a stronger association in males (Hembree, 1990). One possible account for the gender difference in our study is that girls typically showed higher levels of trait anxiety (McLean & Anderson, 2009), but because general anxiety was controlled for in our analyses, the observed sex differences in mathematics anxiety may stem from other dimensions not investigated by the present study, such as stereotype threat (Maloney, Scharffer, & Beilock, 2013). Our findings also suggest that significant gender differences in mathematics anxiety may not emerge until they are in formal schooling for a longer period of time or simply grow older to become aware of gender-related issues and affected by them.

Although we did not find reciprocal relations of mathematics anxiety to mathematical performance, we demonstrated a reciprocal association between quantitative reasoning and number knowledge. The cross-lagged model showed that children's earlier quantitative reasoning significantly predicted their number knowledge later, whereas their prior number knowledge also significantly predicted subsequent quantitative reasoning ability. Although both directional relations were statistically significant, the predictive power of quantitative reasoning on number knowledge is three times stronger than that of number knowledge on quantitative reasoning. Our finding was consistent with the 'mathematical thinking perspective' and a wealth of studies that quantitative reasoning is a foundation for building up accurate number knowledge (Bryant, 1995; Carpenter & Moser, 1982; Ching & Nunes, b, 2017a; Ginsburg, Klein, & Starkey, 1998; Nunes & Bryant, 1996, 2015; Nunes et al., 2012; Piaget, 1952; Piaget & Inhelder, 1975; Thompson, 1993, 1994; Vergnaud, 1997). According to this perspective, children's success in mathematics learning largely depends on their ability to recognize the connection between their knowledge of numbers and quantities, and understand how the number system works, such as being aware of the logical relations between numbers. For example, when a child was asked "16 and 11, which one is bigger?" and gave a correct answer, does her or his success on this task necessarily reflect an understanding about *why* 16 is bigger than 11? It is possible that the child succeeds simply because s/he knows that the number word "sixteen" comes after "eleven" in the counting sequence, without realizing the fact that 16 actually contains 5 more elements than 11 (i.e. 16 is equal to 11 plus 5 or 16 is composed by 11 and 5). Thus, if the child has mastered the concept of additive composition, s/he is more likely to develop the number knowledge regarding why the number "16" is bigger than another number "11". Similarly, the multiplicative relations in the base ten system (one aspect of quantitative reasoning) are related to the manner the number labels and the place value system operate (one aspect of number knowledge). When we write numbers, the place where the number is located denotes an implicit multiplication, that is, if the number is the last one on the right, it signifies one single unit (i.e. multiplied by 1), the second to the left is multiplied by 10, and the third to the left is multiplied by 100 and so forth. For example, in "245", the number "5" refers to 5 units, whereas the number "2" does not refer to 2 units but 200 units (2 × 100). Thus, multiplicative reasoning may also be conducive for children to develop solid number knowledge. Taken together, one possible reason for the significant prediction of quantitative reasoning on later number knowledge is that quantitative reasoning helps children understand the nature of numbers. The reciprocal finding also suggests that number knowledge may also help children reflect on the logical meanings of numbers, but the relative magnitudes of associations further suggest that quantitative reasoning seems to be the foundation for number knowledge.

A limitation of the present study was its correlational design. Although we observed longitudinal associations among variables even after controlling for prior scores on all measures, gender, general anxiety, and mothers' educational levels, we cannot infer a definite causal relation because it remains possible that unexamined factors may contribute to the associations we found. Working memory, for example, may moderate the link between mathematics anxiety and mathematical performance (Beilock & Carr, 2005; Beilock & DeCaro, 2007; Ching, 2017, Ramirez et al., 2013; Vukovic et al., 2013). Future studies that involve an experimental design and a larger number of cognitive measures may help clarify the temporal connection and independent contribution of each variable to another. The small age range of this study may also limit the generalizability of findings to other age groups. The association between mathematics anxiety and mathematical performance may be even stronger in older children and adolescents because of stronger social comparisons and higher competitiveness in late primary and secondary schools. Longer-term longitudinal studies may also illustrate whether the potential influence of prior mathematics anxiety/mathematical performance in children would extend to adolescence period.

Given that mathematics anxiety is associated with low mathematical competence, remediation strategies that aim to reduce young children's mathematics anxiety may focus on improving their number knowledge and quantitative reasoning. Prior evidence has demonstrated that enhancing students' mathematical skills is effective at reducing mathematics anxiety (e.g., one-on-one cognitive tutoring, Supekar, Iuculano, Chen, & Menon, 2015). The present study suggests that quantitative reasoning is a kind of mathematical competence that merits extra attention as the predictive power of quantitative reasoning on number knowledge is much stronger than that of number knowledge on quantitative reasoning. This evidence suggests that an improvement in quantitative reasoning may contribute to the development of a more solid understanding of numbers. Thus, teachers are encouraged to identify ways to keep mathematics teaching intimately linked to quantities and their logical connections in the real world, which may include the use of representational tools (e.g., Ching & Wu, 2019; Nunes et al., 2009; Nunes, Bryant, Barros, & Sylva, 2012), schema-based instruction in problem solving (e.g., Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004; Jitendra & Hoff, 1996; Marshall, 1995), or guided discussion with children (Bermejo et al., 2004). To deal with anxious feelings more directly, children may learn to reappraise mathematical situations more positively, such as viewing them as a challenge that they can overcome rather than a threat they should escape (Blackwell, Trzesniewski, & Dweck, 2007; Blascovich & Mendes, 2010). It is also believed that such mind-set changing interventions may work even better if they go along with a shift in the mind-set beliefs (e.g., performance goals versus learning goals) of the educational settings and culture in which children learn mathematics (Hooper, Yeager, Haimovitz, Wright, & Murphy, 2016; Murphy & Dweck, 2010).

In conclusion, on the basis of three theoretical perspectives, the present study has provided some evidence regarding the temporal ordering of mathematics anxiety and mathematical competence. In a group of Chinese children at an early age of formal schooling, our findings support the Deficit Model of mathematics anxiety, which postulates that earlier poor mathematical competence elicits mathematic anxiety. Two basic forms of mathematical competence, namely quantitative reasoning and number knowledge were significant longitudinal predictors for mathematics anxiety. By contrast, the cross-lagged model did not show that prior mathematics anxiety was associated with later mathematics performance. Our findings suggest that enhancing children's mathematical competence may be the emphasis when educators attempt to lesson mathematics anxiety in children. In particular, given that quantitative reasoning had a stronger prediction for number knowledge longitudinally, it may deserve more attention and efforts in future research and practice in mathematics education.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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