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How mathematics anxiety affects students' inflexible perseverance in mathematics problem-solving: Examining the mediating role of cognitive reflection

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Background. Too many students persevere in relying upon one (sometimes suboptimal) strategy for solving a wide range of problems, even when they know more efficient strategies. Although many studies have mentioned such phenomena, few studies have examined how emotional factors could affect this type of inflexible perseverance in strategy use.

Aims. To examine whether mathematics anxiety could affect students' inflexible perseverance in strategy use and whether this effect could be mediated by cognitive reflection, which is the ability to engage in deliberate reasoning.

Sample and method. In Study 1, 164 undergraduate students' (18–22 years) mathematics anxiety, cognitive reflection, and performance in overcoming inflexible perseverance were measured by a questionnaire battery. Structural equation models were used to examine the correlations between these variables. In Study 2, 98 undergraduate freshmen (17–18 years) were assigned to two groups, where one group's mathematics anxiety was temporarily induced by task instructions, while the other group served as a control group. Cognitive reflection and inflexible perseverance of the two groups were compared.

Results. Study 1 showed that mathematics anxiety was negatively correlated with students' performance on overcoming inflexible perseverance, while cognitive reflection mediated such an effect. Study 2 showed that compared to the control group, the experimental group showed lower cognitive reflection, which led to lower performance in overcoming inflexible perseverance.

Conclusions. Mathematics anxiety was showed to impair students' ability to engage in deliberate reasoning and was associated with inflexible use of strategies. Alleviating students' mathematics anxiety should be considered when promoting students' strategic flexibility.

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Solving problems flexibly and adaptively – which requires students to know multiple strategies and to shift between strategies based on the problem context so that the problem can be solved in the most efficient way – has been recognized as an important goal in mathematics learning and thinking (Heinze, Star, & Verschaffel, 2009; Heirdsfield & Cooper, 2002; National Research Council & Mathematics Learning Study Committee, 2001; Star & Rittle-Johnson, 2008; Star & Seifert, 2006; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009; Xu *et al.*, 2017). Students who develop mathematical flexibility are more capable of utilizing their procedural knowledge to solve novel problems, and they also show a greater understanding of mathematics concepts (Blöte, Van der Burg, & Klein, 2001; Rittle-Johnson, Star, & Durkin, 2012). Moreover, flexibility is a key character of the development of mathematics expertise (Baroody, 2003; Star & Newton, 2009). However, studies have found that too many students persevere in using a single, sometimes non-optimal, strategy to solve many similar problems, perhaps only switching strategies to a more efficient one when explicitly prompted to do so (Hickendorff, 2018; Newton, Lange, & Booth, 2019; Star & Rittle-Johnson, 2008; Star & Seifert, 2006; Xu *et al.*, 2017).

Many studies have tried to address this tendency – persevering in the use of a suboptimal strategy even when one has knowledge of more efficient alternatives – including examining how instructional interventions (Star, Newton, *et al.*, 2015; Star, Pollack, *et al.*, 2015; Star & Rittle-Johnson, 2008), prior knowledge (Newton *et al.*, 2019; Schneider, Rittle-Johnson, & Star, 2011), and motivational factors (Liu *et al.*, 2018; Wang, Liu, Star, Liu, & Zhen, 2019) could affect students' ability to overcome such inflexibility. However, the role of emotions in the development of mathematical flexibility has been relatively unexplored, despite the fact that emotions are one of the most salient contributors to academic achievement and play an important role in school settings, instructional practice, and students' motivation and cognition (Linnenbrink, 2006; Mega, Ronconi, & De Beni, 2014; Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011; Schutz, Pekrun, & Phye, 2007). Among the many different emotions that impact mathematics learning, mathematics anxiety is considered to play a particularly important role (Ashcraft, 2002; Ashcraft & Krause, 2007). To understand how mathematics anxiety might affect students' strategy use in mathematics problem-solving, the present study examined the relationship between mathematics anxiety and students' inflexible perseverance and the potential mechanism underlying such a relationship.

Mathematics anxiety and inflexible perseverance in problem-solving

Mathematics anxiety is generally defined as a feeling of tension or fear that is provoked by mathematics, mathematics problems, or mathematical situations (Ashcraft, 2002; Legg & Locker, 2009). Mathematics anxiety has received considerable attention in the field of education and psychology because of its potential negative association with mathematics performance and many other cognitive and motivational factors (Ardi *et al.*, 2019; Ashcraft, 2002; Ashcraft & Kirk, 2001; Henschel & Roick, 2017; Meece, Wigfield, & Eccles, 1990; Ramirez, Gunderson, Levine, & Beilock, 2013; Wigfield & Meece, 1988). Students with high mathematics anxiety usually demonstrate both psychological symptoms (e.g., memory difficulties and loss concentration) and physiological symptoms (e.g., nervous voice and sweaty palms) when confronted with mathematics problems or numerical information in school or daily life. These same symptoms occur even when mathematics problems are easy and straightforward.

Through the use of self-reported mathematics anxiety, performance on standardized tests, and students' speed and accuracy in solving basic arithmetic problems, previous studies provide overwhelming evidence for understanding the relationship between mathematics anxiety and students' general mathematics performance (Chang & Beilock, 2016; Dowker, Sarkar, & Looi, 2016). However, it is still unclear how mathematics anxiety could affect more specific mathematics abilities beyond general performance. Specifically, few studies have explored the changes in students' flexible thinking and inflexibility of strategy use that mathematics anxiety could induce (for a review, see Ramirez, Shaw, & Maloney, 2018). Ramirez, Shaw, *et al.* (2018) suggested that mathematics anxiety might affect students' ability to develop various problem-solving strategies and thus might hinder the development of flexibility. However, there is not enough research to draw a meaningful conclusion about such a relationship (Imbo & Vandierendonck, 2007; Ramirez, Chang, Maloney, Levine, & Beilock, 2016). By repeatedly relying on only a few strategies, inflexibility may not only affect students' efficiency in problem-solving, but it may also impede the development of both procedural and conceptual knowledge (De Jong & Ferguson-Hessler, 1996; Ramirez, Shaw, *et al.*, 2018; Star, Newton, *et al.*, 2015). Thus, if the literature on mathematics anxiety could be expanded to include the study of inflexibility, it would provide a better understanding of how mathematics anxiety may affect students' mathematical thinking and learning. In the present study, we focus on students' *inflexible perseverance*[1] in problem-solving – the perseveration in using strategies that were successful in prior tasks but are no longer efficient or appropriate in later tasks (Carr & Steele, 2009). We assume that mathematics anxiety might play an important role in students' inflexible perseverance by contributing to learners' reliance on one strategy to solve mathematics problems.

Previous studies have demonstrated that mathematics anxiety could affect students' strategy use (Ashkenazi & Najjar, 2018; Imbo & Vandierendonck, 2007; Ramirez *et al.*, 2016; Si, Li, Sun, Xu, & Sun, 2016). For example, Imbo and Vandierendonck (2007) found that compared to low-anxious children, high-anxious children used more procedural strategies such as computing and counting rather than a more efficient strategy (i.e., retrieval of mathematics facts) in solving simple addition problems. Ramirez *et al.* (2016) also found that high math-anxious students tend to avoid using more advanced strategies and prefer to use more rudimentary strategies when performing mental arithmetic. Specifically, when students were confronted with the word problem (read by experimenters) 'If you had nine crayons and someone gave you eight more, how many would you have altogether?', high math-anxious students tended to use counting strategy (using a counting procedure such as counting fingers) instead of using retrieval strategy (spontaneously knowing the answer) or decomposition (breaking down the presented addends into simpler numbers). Researchers have suggested that mathematics anxiety might interfere with students' ability to see other available strategies as potential options and discourage them to adaptively switch strategies (Imbo & Vandierendonck, 2007; Ramirez *et al.*, 2016; Si *et al.*, 2016). However, further empirical evidence is needed to address the relationship between mathematics anxiety and the ability to flexibly switch strategies.

Some studies suggested that students who felt anxious about mathematics were less flexible when solving mathematics problems (Carey *et al.*, 2019; Dowker, 2005; Vorensky, 2018). From speaking with experienced mathematics teachers and experts, these studies found that students with mathematics anxiety tend to be less capable of

[1] *Note that we use* inflexible perseverance *and* inflexibility *as synonyms in this study. We also use* overcoming inflexible perseverance *and* flexibility *as their complementary terms.*

using various methods to solve mathematics problems and rely more on rudimentary strategies. Researchers also found that mathematics anxiety negatively correlated with students' ability to generate solutions (e.g., 'To solve the mathematics problem, I'll try a variety of methods and then determine the final method') and their ability to use strategies to overcome cognitive obstacles (e.g., 'I'll draw pictures to help myself to better understand difficult mathematical questions') (Lai, Zhu, Chen, & Li, 2015). These results indicate that math-anxious students tend to use basic and limited strategies to solve mathematics problems.

Another stream of studies indicates that anxiety that is caused by mathematics-related stereotype threat could impair students' performance and their ability to overcome an inappropriate strategy (Bosson, Haymovitz, & Pinel, 2004; Jamieson & Harkins, 2009; Johns, Inzlicht, & Schmader, 2008; Mammarella, Caviola, Giofrè, & Borella, 2018; Spencer, Steele, & Quinn, 1999). For example, Jamieson and Harkins (2009) found that women who were confronted with gender stereotype (i.e., women are worse at mathematics than men) threat rely more on a pre-potent strategy[2] (i.e., using rules and equations) when solving Graduate Record Examinations (GRE) quantitative problems. In this study, there were two types of GRE problems: (1) the 'solve' problem, which required a solution using the pre-potent strategy, and (2) the 'comparison' problem, which required alternative solutions. For the solve problem, the pre-potent strategy led to correct answers, but this strategy does not help to correctly solve the comparison problem. The results showed that women who experienced gender stereotype threat performed worse than unthreatened women on comparison problems because they used more pre-potent strategies (which led to more incorrect answers). This result suggests that mathematics-related anxiety has a direct link to students' use of pre-potent strategies and their ability to switch strategies when the problem context is changed.

In the case of inflexible perseverance, the strategy that has been used to solve all problems within a problem set might be considered as a pre-potent strategy because it relies on the same basic and standard procedure to solve all of the problems and it is more easily activated (Imbo, Duverne, & Lemaire, 2007). Thus, students with mathematics anxiety might rely on such a pre-potent strategy and experience difficulty in overcoming such inflexibility. In addition, flexibly abandoning old strategies and switching to new ones not only require students to know multiple strategies but also require students to notice the change in problem structure and context (Elia, den Heuvel-Panhuizen, & Kolovou, 2009). Indeed, prior research has found that switching strategies required extra cognitive resources even when students were solving the same type of mathematics tasks (Lemaire & Lecacheur, 2010; Luwel, Schillemans, Onghena, & Verschaffel, 2009; Schillemans, Luwel, Ceulemans, Onghena, & Verschaffel, 2012). Moreover, students also have to overcome the impulse of using old strategies and develop or recall a new approach (Lemaire & Lecacheur, 2010; Lim & Morera, 2010). All these processes rely heavily on working memory capacity. Many studies suggest that when solving mathematics problems, math-anxious students tend to worry about possible negative consequences of the problem-solving situation and these worries can compromise working memory capacity (Justicia-Galiano *et al.*, 2016; Maloney & Beilock, 2012; Passolunghi, Caviola, De Agostini, Perin, & Mammarella, 2016; Ramirez *et al.*, 2013). Thus, although math-anxious students might have the knowledge needed to solve the mathematics problems, the

[2] A pre-potent strategy/response *refers to a strategy/response that takes priority over other alternatives. In the field of cognitive psychology and decision-making, it also refers to a strategy/response that is impulsive, habitual, and more easily activated, but sometimes inappropriate (Diamond, Kirkham, & Amso, 2002; Imbo, Duverne, & Lemaire, 2007).*

working memory capacity that is implicated due to their worries could lead to problem-solving failure. Thus, the present study hypothesized that mathematics anxiety might interfere with the process of flexible use of strategies and discourage students from overcoming their inflexible perseverance.

Cognitive reflection as a mediator

If mathematics anxiety can impair students' ability to overcome inflexible perseverance and lead to inflexibility, a follow-up question would consider how such an influence works. As we note above, mathematics anxiety might occupy cognitive resources and increase the cognitive load in problem-solving, leading students to be 'mindless'. In other words, the cognitive resources of math-anxious students may be limited in the ability to pay attention to a switching context or to develop a thorough understanding of the problem context. Such deliberate reasoning is critical for overcoming inflexible perseverance. To evaluate this deliberate reasoning process, the present study connects to the idea of *cognitive reflection*. This refers to the ability to override an inappropriate pre-potent response and to engage in deliberate reflection on the problem that leads to appropriate responses (Frederick, 2005; Kahan, 2012; Toplak, West, & Stanovich, 2014). First introduced by Frederick (2005), the Cognitive Reflection Test (CRT) measures such an ability to overcome an inappropriate pre-potent response. The test includes three items that are intended to lead respondents to make pre-potent responses. One of the sample problems on the CRT is as follows:

> A bat and a ball cost \$1.10 in total. The bat costs \$1 more than the ball. How much does the ball cost?

To answer this problem correctly, one must override the intuitive (pre-potent) response of answering \$0.10 and reflect on the problem to obtain the correct answer: \$0.05. According to dual-process theory (Evans & Stanovich, 2013), type 1 processing usually generates intuitive responses, is rapid and automatic, and demands little cognitive resources. Type 2 processing is relatively slow and deliberate, and requires more cognitive resources. To obtain the correct answer in the bat-and-ball problem, one must override the type 1 processing and activate the type 2 processing. People vary in the likelihood that they will think in a reflective way and override the inappropriate intuitive response during problem-solving, and by far, the most popular measure of such ability is the CRT (Toplak *et al.*, 2014).

Cognitive Reflection Test has been found to be correlated with important cognitive outcomes such as syllogistic reasoning (Toplak, West, & Stanovich, 2011) and numeracy performance (Liberali, Reyna, Furlan, Stein, & Pardo, 2012). Recent studies also found that CRT scores were positively correlated with mathematics reasoning and mathematics performance (Gómez-Chacón, García-Madruga, Vila, Elosúa, & Rodríguez, 2014; Lim & Wagler, 2012; Morsanyi, McCormack, & O'Mahony, 2018; Primi, Donati, Chiesi, & Morsanyi, 2018). Specifically, researchers found that students' CRT scores were negatively correlated with their impulsive disposition when solving mathematics problems (e.g., finding the common denominator when solving the fraction problem '$\frac{3}{4}+\frac{1}{10}+\frac{9}{10}$') (Lim & Morera, 2010). Studies also found that students with higher CRT scores are less likely to be affected by reasoning bias (e.g., the gamblers' fallacy) when solving probabilistic problems (Gómez-Chacón *et al.*, 2014; Primi *et al.*, 2018). These researchers have suggested that cognitive reflection allows students to engage in a more analytical thinking style and to critically examine the first answer that comes to mind when solving mathematics problems. In other words, a student with higher cognitive reflection is more likely to carefully examine the problem context, to evaluate alternatives, and to reflect on their pre-potent response when they confront mathematics problems. Following this logic, cognitive reflection might also play a role in students' flexible use of strategies because it requires students to engage in a kind of deliberate thinking that allows them to notice aspects of the problem-solving context and to overcome the pre-potent response of using old and potentially non-optimal strategies. Therefore, students with high cognitive reflection scores might be more capable of overcoming inflexibility.

With regard to the link between mathematics anxiety and cognitive reflection, studies have found that mathematics anxiety is negatively correlated with CRT scores (Gómez-Chacón *et al.*, 2014; Hoover & Healy, 2017; Morsanyi, Busdraghi, & Primi, 2014). Researchers also found that mathematics anxiety can negatively affect students' performance on the CRT through numeracy and calculation ability (Morsanyi *et al.*, 2014; Primi *et al.*, 2018), both of which are important for successful performance on the CRT (Liberali *et al.*, 2012; Sinayev & Peters, 2015). Therefore, cognitive reflection could be a potential mediator in the relation between mathematics anxiety and the ability to overcome inflexible perseverance.

The present study

The present study aimed to examine the relations between mathematics anxiety, the ability to overcome inflexible perseverance in strategy use, and cognitive reflection. According to the above research, we hypothesized that mathematics anxiety might encourage inflexible perseverance and lead to poor flexibility in problem-solving while cognitive reflection may mediate such an effect. In Study 1, we employed a survey-based investigation to examine whether mathematics anxiety negatively correlated with participants' ability to overcome inflexibility and whether cognitive reflection plays a mediating role in this relation. Inspired by the Water Jar task (a widely used task to measure people's perseverance and rigidity of thoughts, see Luchins & Luchins, 1994; Schultz & Searleman, 2002), we designed four types of mathematics tasks to probe students' ability to overcome their inflexible perseverance (for details, see Method in Study 1). Besides previous studies, in Study 2, we conducted an experiment that included two groups of undergraduate students to test whether situationally induced mathematics anxiety would affect students' cognitive reflection as measured by the CRT and their inflexibility as measured by the mathematics tasks.

STUDY 1

Method

Research design

Study 1 employed a survey-based investigation. The independent variable was mathematics anxiety and the dependent variable was students' performance of overcoming inflexible perseverance, while cognitive reflection served as a mediator. Note that all the measured outcomes were considered as individual traits or abilities since survey-based investigations generally reflect participants' characteristics under normal conditions. We hypothesized that mathematics anxiety would be negatively correlated with students' flexibility and that cognitive reflection would mediate such a relationship.

Participants

Participants were 164 undergraduate students (mean age = 19.67 ± 1.37 years; 49 men and 115 women) from a university in Beijing, China. Participants included students from 15 different majors (including psychology, manager, computer, and physics) and were randomly recruited via flyers posted throughout the campus. All participants provided informed consent and were tested in accordance with the national and international norms of using human participants (*Ethics in Research with Human Participants*, Sales & Folkman, 2000). The present study was approved by the Research Ethics Committee of a university in Beijing.

Procedures

All recruited participants were assigned a day and time to complete the survey based on their individual availability and class schedule. The survey was conducted in a room in the psychology building that had a capacity of 30 people. Before the survey, students were told that all their information and their answers on the questionnaires would be used for research purposes only, and they were encouraged to answer the questions according to their true thoughts. Then, they were asked to complete a pencil-and-paper survey assessing mathematics anxiety, followed by the CRT. After students completed the mathematics anxiety survey and the CRT, we presented the researcher-designed mathematics tasks to assess their performance in overcoming inflexible perseverance. It took about 30 min for students to complete all of the above tasks. After participants completed the study, we followed standard procedures at the university by informing all the students that they could call the psychology health centre for help if they needed psychological services.

Measures

In the present study, the English versions of the mathematics anxiety survey and the CRT were adapted into Chinese. To ensure the fluency and accuracy of this translation, the items were translated by two psychology doctoral students and back-translated by two Chinese–English interpreters. A Chinese professor of psychology then reviewed and modified the items to the current versions to fit the Chinese context.

Mathematics anxiety

The Mathematics Anxiety Scale – Revised (MAS-R: Bai, Wang, Pan, & Frey, 2009) is a modification of the Mathematics Anxiety Scale (MAS: Betz, 1978). The revised scale consists of 14 items, with a six-item positive affect dimension (sample item: I find mathematics interesting) and an eight-item negative affect dimension (sample item: Mathematics makes me feel nervous). Cronbach's alpha for the two-dimension scale was .91, and the scale showed a good model fit with the two-dimension structure (Bai *et al.*, 2009). All items in the mathematics anxiety survey were rated on a 5-point Likert-type scale ranging from 1 (*completely disagree*) to 5 (*completely agree*). In the present study, only the negative affect dimension was used because we focused on the negative effect of mathematics anxiety. Cronbach's alpha for the eight-item dimension was .93 in the present study; the single-dimension construct validity was acceptable, $\chi^2/df = 1.91$, CFI = .976, TLI = .958, RMSEA = .097.

Cognitive Reflection Test

The Cognitive Reflection Test – Expansion (CRT-E: Toplak *et al*., 2014) included seven items, with three items from the original CRT (Frederick, 2005) and four additional items (sample item: If John can drink one barrel of water in 6 days, and Mary can drink one barrel of water in 12 days, how long would it take them to drink one barrel of water together? _____ days [correct answer: 4 days; incorrect answer: 9 days]). The CRT score ranged from 0 to 7 depending on how many items were correctly answered, where a higher score indicated higher cognitive reflection ability. The expanded version was developed by Toplak *et al*. (2014), and it has a high correlation with the original three-item test. The expanded version also has been found to be a strong independent predictor of performance on rational thinking tasks that require type 2 processing (Toplak *et al*., 2014). Note that the CRT is generally considered as a performance measure rather a self-report measure. That is, it is not a questionnaire in which people indicate their attitude or preference. Instead, it is a direct measure of the cognitive reflective processing on the problems (Pennycook, Cheyne, Koehler, & Fugelsang, 2016; Toplak *et al*., 2011).

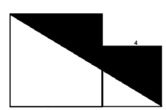
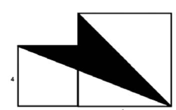
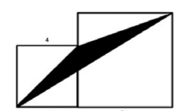
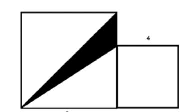
Overcoming inflexible perseverance/flexibility

To measure students' ability to overcome inflexible perseverance, we designed four types of paper-and-pencil mathematics tasks. The four types of tasks drew the mathematics topic areas of mental arithmetic, linear equation solving, solving systems of equation with two variables, and finding the shaded area in two adjacent squares (see Table 1). The mental arithmetic task and shaded area task were adapted from mathematics exercises in a fourth-grade mathematics textbook, while the equation tasks were from a seventh-grade mathematics textbook (Beijing Normal University Press, 2013).

Each task was developed based on the Water Jar task (Luchins & Luchins, 1994; Schultz & Searleman, 2002). In the Water Jar task, people's pre-potent response of using a specific strategy (Strategy A) is induced from performance on the prior items, even though an easier strategy (Strategy B) to complete the last several items is available. Students' inflexible perseverance is reflected by the use of Strategy A in the last items. In the present study, the first three items (i.e., priming items) in each task can be solved by a pre-potent strategy, while the last item (i.e., target item) can be solved either by a pre-potent strategy or by a more efficient one. The target items were created based on previous assessments of strategic flexibility. Each target item could be solved successfully using at least two strategies (Lemaire, Lecacheur, & Farioli, 2000; Xu, Wells, LeFevre, & Imbo, 2014; Xu *et al*., 2017). Examples of the pre-potent strategy and the efficient strategy are presented in Table 2.

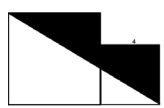
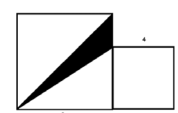
Students were asked to solve all four problems sequentially, from the first one to the last one in each task. Each task was designed so that the first three problems served as priming stimuli that encouraged students to use a specific strategy (i.e., pre-potent strategy). Although the pre-potent strategy is also viable to solve the last problem, there is a more efficient strategy. If students used the pre-potent strategy in the first three problems and switched to a more efficient strategy in the last problem, they were considered as flexible. Unlike the Water Jar task, the present study focused on the

Table 1. Four types of mathematics task in the flexibility test

Task	Priming problems			Target item
Mental arithmetic	$17 \times 12 = ?$	$13 \times 23 = ?$	$29 \times 13 = ?$	$25 \times 32 = ?$
Linear equation	$6(x + 2) - 3(x + 3) = 9$	$5(x - 1) - 2(x + 1) = 2$	$(x + 1) - 2(x - 1) = 1 - 3x$	$7(x - 2) - 3(x - 2) = 16$
Equation system	$\begin{cases} x = y + 3 \\ 3x - 2y = 5 \end{cases}$	$\begin{cases} x - y = 2 \\ 3x + 5y = 14 \end{cases}$	$\begin{cases} x - 2y = 4 \\ 2x + y - 3 = 0 \end{cases}$	$\begin{cases} x - y = 1 \\ 2x + y = 8 \end{cases}$
Shaded area				

Note. The side lengths for each square in the last task are 6 cm and 4 cm, respectively.

Table 2. Pre-potent strategy and efficient strategy in four types of mathematics task

Task	Pre-potent strategy	Efficient strategy
Mental arithmetic	Distribution strategy: e.g., $17 \times 12 = 17 \times 10 + 17 \times 2 = 170 + 17 = 204$	Associative strategy: e.g., $25 \times 32 = 25 \times 4 \times 8 = 100 \times 8 = 800$
Linear equation	Using distribution property: e.g., $6(x + 2) - 3(x + 3) = 9$ $6x + 12 - 3x + 9 = 9$ $3x + 21 = 9$ $3x = -12$ $x = -4$	Combine like items: e.g., $7(x - 2) - 3(x - 2) = 16$ $4(x - 2) = 16$ $x - 2 = 4$ $x = 6$
Equation system	Using substitution to solve for *x*/*y*: e.g., $\begin{cases} x = y + 3 \\ 3x - 2y = 5 \end{cases}$, Substitute $x = y + 3$ in $3x - 2y = 5$ $3(y + 3) - 2y = 5$ $3y + 9 - 2y = 5$ $y = -4$ Substitute $y = -4$ in $x = y + 3$ $x = -1$ Answer: $x = -1$ and $y = -4$	Eliminating *x*/*y*: e.g., $\begin{cases} x - y = 1 \\ 2x + y = 8 \end{cases}$ Add up two equations $3x = 9$ $x = 3$ Substitute $x = 3$ in $x - y = 1$ $3 - y = 1$ $y = 2$ Answer: $x = 3$ and $y = 2$
Shaded area	Subtracting the area of the triangle(s) from the total area: e.g.,  Total area: $6\ \text{cm} \times 6\ \text{cm} + 4\ \text{cm} \times 4\ \text{cm} = 52\ \text{cm}^2$ Area of the triangle: $(6 + 4)\ \text{cm} \times 6\ \text{cm} \times ½ = 30\ \text{cm}^2$ Shaded area: $(52–30)\ \text{cm}^2 = 22\ \text{cm}^2$	Calculating the triangular shaded area: e.g.,  Shaded area: $6\ \text{cm} \times (6–4)\ \text{cm} \times ½ = 6\ \text{cm}^2$.

responses where students successfully overcame the tendency to use the pre-potent strategy on the last problem and switched to a more efficient strategy. This is because students who used the pre-potent strategy to solve the last problem might have done so due to perseverance in strategy use or due to the lack of knowledge of the efficient strategy. To avoid such confusion, we focus only on students who exhibited flexible responses.

The score for overcoming inflexible perseverance could range from 0 to 4. Two different coders coded students' strategies in all four tasks (656 strategies), and the accuracy of the strategy was not considered. The intercoder agreement was 98.6% (648 out of 656). Note that the four tasks were designed to probe students' ability to overcome inflexible perseverance in using the pre-potent strategy. Therefore, only students who showed inflexible perseverance (i.e., who used the pre-potent strategy to solve all items) and who showed flexibility (i.e., who used the pre-potent strategy in priming items and switched to the efficient strategy in the target item) were included in the data analysis. Students who used different/irregular strategies in the first three problems were not included in data analysis because they did not show inflexible perseverance in priming items. In total, eight students (4.8% of the sample) exhibited such an irregular response pattern and they were excluded.

Data analysis

We conducted descriptive analyses for all measures. First, we calculated Pearson's correlations between each main variable. We then built a direct effect model to assess the direct effect of mathematics anxiety on flexible strategy use. Second, we added CRT as the mediator into the above direct path and added predictive paths between these mediators. Statistical analyses were conducted using Mplus 7.0 software (Muthén & Muthén, 2012). We used the following criteria to evaluate the model fit: The CFI and the TLI should be equal to or greater than .90, and the RMSEA should be equal to or less than .08.

Results

Descriptive statistics and correlations

Descriptive statistics and Pearson's correlations are presented in Table 3. Mathematics anxiety, CRT, and flexibility showed significant moderate correlations. Mathematics anxiety was negatively correlated with CRT and flexibility, while the latter two were positively correlated.

Table 3. Means, standard deviations, and correlations among main variables

Variables	$M \pm SD$	1	2	3	4
1. Age	19.65 ± 1.35	1			
2. Gender	–	.118	1		
3. Mathematics anxiety	2.59 ± 1.02	−.047	−.138	1	
4. Cognitive Reflection Test	6.13 ± 1.18	.021	.071	−.173*	1
5. Flexibility	1.79 ± 1.13	−.139	.078	−.171*	.194**

Note. *$p < .05$; **$p < .01$.

Examination of mediating effects

Analysis of the direct effect model

Before we considered the mediating effects, we first examined the direct effect of mathematics anxiety on students' flexibility. The direct model was acceptable, $\chi^2/df = 2.708$, CFI = .955, TLI = .935, RMSEA = .075. This result revealed that mathematics anxiety had a negative and significant effect on flexibility ($\beta = -.184$, $p < .05$).

Analysis of the indirect effect model

Based on the direct effect model, we inserted CRT in the relation between mathematics anxiety and flexibility (see Figure 1). The indirect effect model was also acceptable, $\chi^2/df = 2.99$, CFI = .931, TLI = .906, RMSEA = .088.

Discussion

As expected, we found that mathematics anxiety negatively correlated with cognitive reflection and the ability to overcome inflexible perseverance. Besides, we also found that mathematics anxiety affected students' flexibility through cognitive reflection. In other words, students with higher mathematics anxiety tended to have a lower cognitive reflection, and, as a result, they had stronger inflexible perseverance. Nevertheless, we only observed correlational relations between these variables. To further understand whether mathematics anxiety could cause inflexible perseverance in strategy use, we employed an experiment that induced temporal mathematics anxiety to examine this causal relation.

STUDY 2

Method

Research design

Study 2 employed a single-factor experimental design with induced manipulation of mathematics anxiety (experimental group vs. control group) as the independent variable, students' performance in overcoming inflexible perseverance as the dependent variable,

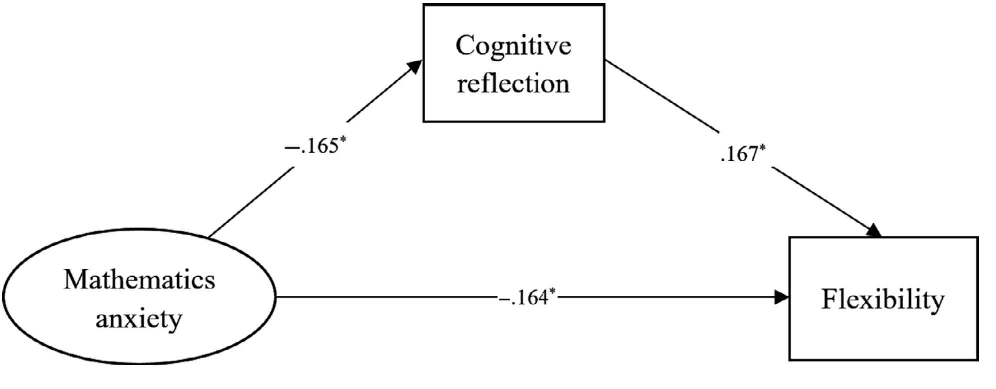


Figure 1. The model of the mediating role of cognitive reflection in the relation between mathematics anxiety and flexibility. *p < .05.

and cognitive reflection as a mediator. The measured outcomes in Study 2 were considered as temporal states since participants' performance or responses were expected to be affected by the experimental conditions.

Participants

Participants were 98 undergraduate freshmen (mean age = 17.88 ± 0.45 years; 26 men and 72 women) from a university in Beijing, China. Two cohorts (cohort 1 and cohort 2) were randomly selected from the School of Education (from 10 cohorts in total, among all students who were enrolled as pre-service teachers with majors in education). All of the cohorts in the School of Education shared a similar average mathematics score on the Chinese College Entrance Examination, and thus, students' average mathematics ability was comparable across cohorts. All participants provided informed consent and were tested following the national and international norms of using human participants. The present study was approved by the Research Ethics Committee of a university in Beijing.

Materials

Students' cognitive reflection and their performance in overcoming inflexible perseverance were measured by the instruments used in Study 1. To induce students' mathematics anxiety, we designed the following task instruction for the experimental group:

> The latest research shows that mathematics has played an important role in people's lives. For example, studies have found that people's early mathematics performance will affect their later academic achievement and their working life (a made-up reference was inserted here). Other studies have shown that mathematical ability often affects one's logical reasoning ability, decision-making, and management ability (another made-up reference was inserted here). It can be seen how important mathematics is to our lives. The following test aims at measuring your mathematics ability. After scoring your test, we will provide feedback about your mathematics ability.

In addition, the problems on the CRT and the researcher-designed mathematics tasks were marked with different point values with the intention of making the packet look like a test and to boost the effectiveness of the task instruction. For the control group, the same instructions were used, except the terms 'math', 'mathematics performance', and 'mathematical ability' which were replaced by 'subjective happiness'. The instruction for the control group was designed to be irrelevant to mathematics anxiety or math, thus serving as a neutral stimulus while insuring that the control group received a similar amount and type of instruction as the experimental group did. For the control group, there were no point values indicated for each problem. The eight-item mathematics anxiety scale in Study 1 was used for manipulation check to examine whether the instruction and context settings were effective for inducing students' mathematics anxiety.

Procedures

The 98 students were assigned to two groups according to their cohort (cohort 1 or cohort 2). One cohort of students received the mathematics anxiety induction as experimental treatment (N = 42), while the other cohort served as the control (N = 56). Both cohorts received a packet that included the CRT and the researcher-designed mathematics tasks. After the experimenters handed out the packet, students were given 2–3 min to read the instructions (which differed based on whether each student was in the experimental or control group, as described above). Students were then given 30 min to complete the CRT and mathematics tasks. After students completed the packet, they received the mathematics anxiety questionnaire as a manipulation check. After participants completed the experiment, the experimenters explained the purpose of the study and told students to call the psychology health centre for help if they needed psychological services.

Results

As we noted above, our researcher-designed tasks aimed at probing students' ability to overcome inflexible perseverance, and as a result, students who used two or more different strategies in the first three problems on each task were not included in data analysis. Thus, we excluded six students (6.12%) from the original sample. Descriptive statistics for each group are presented in Table 4.

We conducted multivariate analyses of variance on students' CRT scores, flexibility, and mathematics anxiety in the manipulation check, with experimental group as the single independence variable. We found that the manipulation group showed a lower CRT score than the control group, $F(1, 92) = 6.46, p < .05, \eta_p^2 = .07$. The manipulation group also showed lower flexibility, $F(1, 92) = 4.50, p < .05, \eta_p^2 = .05$. The difference in the manipulation check of mathematics anxiety was marginally significant, $F(1, 92) = 3.10$, $p = .08$, $\eta_p^2 = .04$, indicating that a higher level of mathematics anxiety was induced by our experimental manipulation.

Examination of mediating effects

Based on the main effect of group on CRT and flexibility, we further examined the mediating role of CRT in the relationship between experimental manipulation and flexibility by using path analysis (see Figure 2). In this analysis, the control group was coded as 0 and served as the reference group, while the manipulation group was coded as 1. We found that compared to the control group, participants in the manipulation group showed a lower cognitive reflection score, which led to lower flexibility.

Discussion

Study 2 found that induced mathematics anxiety could affect students' inflexible perseverance in strategy use and that cognitive reflection played a mediating role in this relation. This result is consistent with Study 1 but provided stronger evidence of the causal

Table 4. Means and standard deviations of main variables in the manipulation group and the control group

Group	Cognitive reflection	Flexibility	Mathematics anxiety
Manipulation (N = 40)	5.23 ± 1.47	1.08 ± 0.89	2.58 ± 0.96
Control (N = 52)	6.00 ± 1.42	1.50 ± 1.00	2.22 ± 1.01

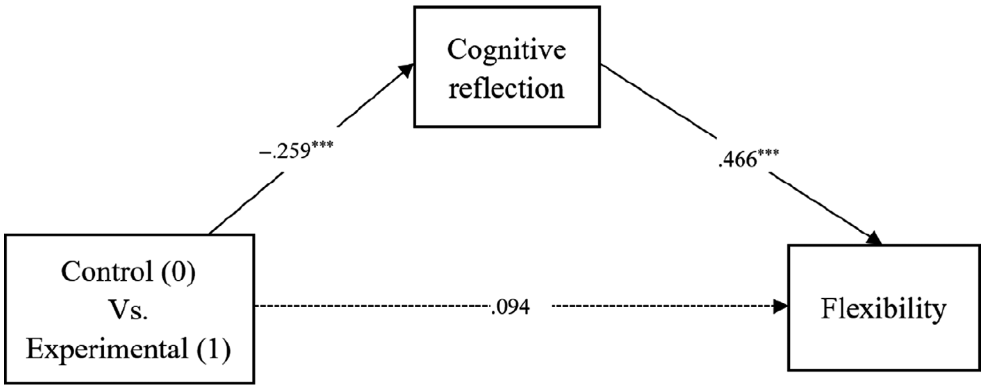


Figure 2. The mediating role of cognitive reflection in the relation between experiment manipulation and flexibility. ***$p < .01$.

relation between mathematics anxiety and students' inflexible perseverance. For two groups of students whose mathematics performance was comparable, the manipulation group was more likely to be inflexibly perseverant in strategy use as compared with the control group. Induced mathematics anxiety also lowered students' analytical thinking and further affected students' flexible strategy use in problem-solving.

GENERAL DISCUSSION

The development of flexible strategic thinking in mathematics has received increasing attention in the field of education and psychology (Liu *et al.*, 2018; Newton *et al.*, 2019; Star & Newton, 2009; Verschaffel *et al.*, 2009; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2011; Wang *et al.*, 2019; Xu *et al.*, 2017). To further understand how emotional and cognitive factors might affect such flexibility, we examined the role of mathematics anxiety and cognitive reflection. In two studies, we showed that mathematics anxiety pressures students towards inflexible perseverance in strategy use in mathematics problem-solving. Moreover, such a relation is mediated by cognitive reflection.

Much of the earlier research on mathematics anxiety has focused on mathematics performance and standardized achievement batteries to examine how mathematics anxiety affects students' mathematics performance and students' speed and accuracy when solving mathematics problems (for a review, see Ramirez, Hooper, Kersting, Ferguson, & Yeager, 2018). The present study expands the literature on mathematics anxiety to include flexibility and provide a further understanding of how mathematics anxiety might affect specific aspects that might affect students' mathematics learning and thinking. Our results are consistent with previous studies suggesting that students with high mathematics anxiety usually experience more difficulty in adapting their strategy use in a different context (Imbo & Vandierendonck, 2007; Ramirez *et al.*, 2016; Si *et al.*, 2016). The present study also highlighted the role of emotional factors in promoting students' flexibility. Previous studies suggested that instructional interventions such as providing worked examples of multiple strategies and encouraging students to compare different strategies could promote students' flexibility (Rittle-Johnson & Star, 2007, 2009; Star & Rittle-Johnson, 2009; Yakes & Star, 2011). However, flexibility gains from these interventions were more limited among certain groups of students, such as students struggling with mathematics (Gersten *et al.*, 2009; Star, Pollack, *et al.*, 2015). One reason that struggling students might be less responsive to flexibility interventions is that they might show a higher level of mathematics anxiety than other students, since poor mathematics performance also contributes to mathematics anxiety (Gunderson, Park, Maloney, Beilock, & Levine, 2018). Students with a high level of mathematics anxiety tend to avoid engaging in mathematics learning, even with additional educational supports (Dew, Galassi, & Galassi, 1984). This might explain why valid interventions only showed a limited effect on struggling students. It would be interesting for future studies to consider the interaction effect between mathematics anxiety and instructional interventions on students' flexibility.

In Study 2, we explored the causal relationship between mathematics anxiety and flexibility and found that even temporally induced mathematics anxiety affects students' flexibility. One could argue that the experimental manipulation would induce test anxiety rather than mathematics anxiety. Indeed, mathematics anxiety has been proven to be highly correlated with test anxiety and it is often difficult to separate the two (Ashcraft, 2002; Hembree, 1990). However, our manipulation provided an instruction page that was intended to lead students to focus on mathematics instead of a test *per se*. The manipulation check showed that the experimental group showed more negative affect about mathematics compared to the control group. Research also suggests that the test context could cause mathematics anxiety that might hinder students' mathematics performance (Boaler, 2014). This is especially the case in China because the Chinese educational system is heavily influenced by the College Entrance Examination. Failure on the examination could have a large impact on students' future academic career and thus tends to produce a substantial level of anxiety. In addition, previous studies suggested that students with high mathematics anxiety tend to set a very high confidence threshold in the choice of using a strategy so that they will not produce any incorrect answers (Imbo & Vandierendonck, 2007; Ramirez *et al.*, 2016). In a test context, students' mathematics anxiety might be severe, suggesting that they would prefer to use a familiar strategy rather than to switch to a new one, which may lead them to demonstrate inflexible perseverance.

We found that cognitive reflection mediated the effect of mathematics anxiety on flexibility in both studies. Cognitive reflection is the ability to override an inappropriate pre-potent response and to engage in deliberate reflection on problems, and it has been shown to be related to many other cognitive outcomes as well as problem-solving (Kahan, 2012; Toplak *et al.*, 2011). However, the process of cognitive reflection is cognitive resource demanding and relies heavily on working memory capacity. According to previous studies, mathematics anxiety tends to impair the function of working memory during the process of problem-solving because emotional regulation and worrisome thoughts occupy an individual's limited cognitive resources (Passolunghi *et al.*, 2016; Ramirez *et al.*, 2013). Thus, students with higher mathematics anxiety might have an impaired working memory capacity which limits their ability to engage in cognitive reflection. As a result, students tend to blindly and repeatedly adopt the same strategy for all the problems in our tasks because they did not reflect on the problem structure and the appropriateness of their strategy use. Our results are in line with previous studies which suggest that regulating mathematics anxiety would help students to release their working memory resources and further improve their mathematics performance (Dowker *et al.*, 2016; Johns *et al.*, 2008; Ramirez & Beilock, 2011).

Our findings also have pedagogical implications for developing students' strategy flexibility. Given the fact that mathematics anxiety is a prevalent issue that occurs in mathematics classes across the world (Dowker *et al.*, 2016), it would be helpful for teachers to recognize this emotional outcome and help students to mitigate their mathematics anxiety. Although mathematics anxiety and poor mathematics performance usually co-occur (Lyons & Beilock, 2011), many studies found that anxious students could perform better in mathematics problem-solving when their anxiety was alleviated (Jamieson, Mendes, Blackstock, & Schmader, 2010; Johns *et al.*, 2008; Park, Ramirez, & Beilock, 2011, 2014; Ramirez & Beilock, 2011). Critically, the techniques employed in these studies aimed to reduce or eliminate the link between mathematics anxiety and poor problem-solving by addressing the anxiety instead of training mathematics skills. When anxiety is regulated or redirected, students often obtain a significant increase in their performance (Maloney & Beilock, 2012). Many treatments, including those that focus on interpreting psychological arousal (Jamieson, Peters, Greenwood, & Altose, 2016), narrative and mindset intervention (Samuel & Warner, 2019; Supekar, Iuculano, Chen, & Menon, 2015), and expressing anxious feelings (Park, Ramirez, & Beilock, 2014), have been developed in the past decades and have proven to be effective for reducing mathematics anxiety (for a review, also see Ramirez, Hooper, *et al.*, 2018). These interventions could serve as a pre-exercise before other interventions that aim to improve students' flexibility.

In addition, researchers have found that *teachers*' mathematics anxiety had a great impact on students' mathematics anxiety and their mathematics achievement (Beilock, Gunderson, Ramirez, & Levine, 2010; Bursal & Paznokas, 2006; Markovits, 2011; Ramirez, Hooper, *et al.*, 2018). These studies indicated teachers' mathematics anxiety might transfer to students by producing negative mathematics-related class experiences (e.g., via witnessing teachers' negative attitudes towards math). Math-anxious teachers also showed less confidence in teaching mathematics and tended to rely on fix-oriented teaching methods (e.g., emphasizing rote memory and following the textbook) (Ramirez, Shaw, *et al.*, 2018). All these factors might further lead to students' mathematics anxiety and their inflexible use of strategies. Thus, teachers need to recognize their own feelings towards mathematics and to regulate their emotions in their teaching practices. Finally, encouraging students to think carefully rather than rush to provide a correct answer might also help students to develop their cognitive reflection ability and strategic flexibility. In the Chinese context, it might be necessary to facilitate a teaching environment where examinations are less emphasized. Schools could also place more attention on students' representation and reflection on the problem rather than their examination scores.

The present study takes an important step in illuminating the relation between mathematics anxiety and strategy flexibility, as well as providing an improved understanding of the potential mechanism of this relationship. However, there are some limitations to this study. First, the participants in the present study were undergraduate students (adults) and we should be careful to generalize the present findings to younger children. Although the majority of mathematics anxiety research has been conducted among college students, recent studies have indicated that the onset of mathematics anxiety occurs among younger children (Ganley & McGraw, 2016; Ramirez *et al.*, 2013). Thus, it is necessary to include younger participants and to examine whether the same mechanisms as found here could be observed. Moreover, previous studies have suggested that the causal relationship between mathematics anxiety and mathematics skills/performance might be different in students of different ages and that the relationship could be reciprocal (Gunderson *et al.*, 2018; Ma & Cartwright, 2003). Therefore, it would be more illuminating if future studies could apply a developmental design to examine the relationship between mathematics anxiety and strategy flexibility.

Second, we observed relatively high CRT scores in the present study. One possible explanation is CRT items are gaining increased attention and some of the students might have seen these items before (Haigh, 2016). Although the expansive version of the CRT (as used here) includes four new items, future studies could adopt an extra survey or interview to examine the potential familiarity effect. In fact, according to Toplak *et al.* (2014), newly added items (CRT4) also showed acceptable reliability and could be used as a substitute for existing items.

Finally, for students who switched their strategy on the fourth problem, they demonstrated both the ability to know multiple strategies and the ability to switch to a more appropriate strategy. It is unclear whether mathematics anxiety affects their knowledge of multiple strategies or their strategy-switching ability, or both. Based on previous studies, strategy flexibility could be defined as a) knowing multiple strategies and b) using the most efficient strategy depending on the context of problems (Star & Newton, 2009; Star & Rittle-Johnson, 2008). By separating these two components of flexibility in future studies, we can examine how mathematics anxiety affects students' flexibility in a more detailed way.

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Conflicts of interest

All authors declare no conflict of interest.

Author contributions

Ronghuan Jiang (Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Writing – original draft; Writing – review & editing) Ru-de Liu (Conceptualization; Resources; Supervision; Writing – review & editing) Jon Star (Supervision; Writing – review & editing) Rui Zhen (Methodology; Writing – review & editing) Jia Wang (Data curation; Resources) Wei Hong (Data curation; Software) Shuyang Jiang (Methodology) Yan Sun (Conceptualization) Xinchen Fu (Data curation).

Data availability statement

The data of the present study are not available for sharing. However, it is available for review purposes (e.g., the reviewer requires to provide the data for further analysis).

References

Ardi, Z., Rangka, I., Ifdil, I., Suranata, K., Azhar, Z., Daharnis, D., . . . Alizamar, A. (2019). Exploring the elementary students learning difficulties risks on mathematics based on students mathematic anxiety, mathematics self-efficacy and value beliefs using Rasch measurement. *Journal of Physics: Conference Series*, *1157*, 32095. https://doi.org/10.1088/1742-6596/1157/3/032095

Ashcraft, M. H. (2002). Math anxiety: Personal, educational, and cognitive consequences. *Current Directions in Psychological Science*, *11*(5), 181–185. https://doi.org/10.1111/1467-8721.00196

Ashcraft, M. H., & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, *130*, 224–237. https://doi.org/10.1037/0096-3445.130.2.224

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*, 243–248. https://doi.org/10.3758/bf03194059

Ashkenazi, S., & Najjar, D. (2018). Non-adaptive strategy selection in adults with high mathematical anxiety. *Scientific Reports*, *8*(1), 10744. https://doi.org/10.1038/s41598-018-27763-w

Bai, H., Wang, L., Pan, W., & Frey, M. (2009). Measuring mathematics anxiety: Psychometric analysis of a bidimensional affective scale. *Journal of Instructional Psychology*, *36*, 185–193.

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 1–33). Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Beijing Normal University Press (2013). 义务教育课程标准实验教科书 数学 *[Compulsory education curriculum standard experimental textbook mathematics]* (Vol. *49*). Beijing, China: Beijing Normal University Press.

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, *107*, 1860–1863. https://doi.org/10.1073/pnas.0910967107

Betz, N. E. (1978). Prevalence, distribution, and correlates of math anxiety in college students. *Journal of Counseling Psychology*, *25*, 441–584. https://doi.org/10.1037/0022-0167.25.5.441

Blöte, A. W., Van der Burg, E., & Klein, A. S. (2001). Students' flexibility in solving two-digit addition and subtraction problems: Instruction effects. *Journal of Educational Psychology*, *93*, 627–638. https://doi.org/10.1037/0022-0663.93.3.627

Boaler, J. (2014). Research suggests that timed tests cause math anxiety. *Teaching Children Mathematics*, *20*, 469–474. https://doi.org/10.5951/teacchilmath.20.8.0469

Bosson, J. K., Haymovitz, E. L., & Pinel, E. C. (2004). When saying and doing diverge: The effects of stereotype threat on self-reported versus non-verbal anxiety. *Journal of Experimental Social Psychology*, *40*, 247–255. https://doi.org/10.1016/S0022-1031(03)00099-4

Bursal, M., & Paznokas, L. (2006). Mathematics anxiety and preservice elementary teachers' confidence to teach mathematics and science. *School Science and Mathematics*, *106*, 173–180. https://doi.org/10.1111/j.1949-8594.2006.tb18073.x

Carey, E., Devine, A., Hill, F., Dowker, A., McLellan, R., & Szucs, D. (2019). *Understanding Mathematics Anxiety: Investigating the experiences of UK primary and secondary school students*, University of Cambridge, UK: Centre for Neuroscience in Education. https://doi.org/10.17863/CAM.37744

Carr, P. B., & Steele, C. M. (2009). Stereotype threat and inflexible perseverance in problem solving. *Journal of Experimental Social Psychology*, *45*, 853–859. https://doi.org/10.1016/j.jesp.2009.03.003

Chang, H., & Beilock, S. L. (2016). The math anxiety-math performance link and its relation to individual and environmental factors: A review of current behavioral and psychophysiological research. *Current Opinion in Behavioral Sciences*, *10*, 33–38. https://doi.org/10.1016/j.cobeha.2016.04.011

De Jong, T., & Ferguson-Hessler, M. G. (1996). Types and qualities of knowledge. *Educational Psychologist*, *31*, 105–113. https://doi.org/10.1207/s15326985ep3102_2

Dew, K. H., Galassi, J. P., & Galassi, M. D. (1984). Math anxiety: Relation with situational test anxiety, performance, physiological arousal, and math avoidance behavior. *Journal of Counseling Psychology*, *31*, 580–583. https://doi.org/10.1037/0022-0167.31.4.580

Diamond, A., Kirkham, N., & Amso, D. (2002). Conditions under which young children can hold two rules in mind and inhibit a prepotent response. *Developmental Psychology*, *38*, 352–362. https://doi.org/10.1037/0012-1649.38.3.352

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. New York, NY: Psychology Press.

Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years? *Frontiers in Psychology*, *7*, 508. https://doi.org/10.3389/fpsyg.2016.00508

Elia, I., den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM*, *41*, 605–618. https://doi.org/10.1007/s11858-009-0184-6

Evans, J. S. B., & Stanovich, K. E. (2013). Dual-process theories of higher cognition: Advancing the debate. *Perspectives on Psychological Science*, *8*, 223–241. https://doi.org/10.1177/1745691612460685

Frederick, S. (2005). Cognitive reflection and decision making. *Journal of Economic Perspectives*, *19*(4), 25–42. https://doi.org/10.1257/089533005775196732

Ganley, C. M., & McGraw, A. L. (2016). The development and validation of a revised version of the math anxiety scale for young children. *Frontiers in Psychology*, *7*, 1181. https://doi.org/10.3389/fpsyg.2016.01181

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools. NCEE 2009–4060*. Washington, DC: National Center for Education Evaluation and Regional Assistance.

Gómez-Chacón, I. M., García-Madruga, J. A., Vila, J. Ó., Elosúa, M. R., & Rodríguez, R. (2014). The dual processes hypothesis in mathematics performance: Beliefs, cognitive reflection, working memory and reasoning. *Learning and Individual Differences*, *29*, 67–73. https://doi.org/10.1016/j.lindif.2013.10.001

Gunderson, E. A., Park, D., Maloney, E. A., Beilock, S. L., & Levine, S. C. (2018). Reciprocal relations among motivational frameworks, math anxiety, and math achievement in early elementary school. *Journal of Cognition and Development*, *19*(1), 21–46. https://doi.org/10.1080/15248372.2017.1421538

Haigh, M. (2016). Has the standard cognitive reflection test become a victim of its own success? *Advances in Cognitive Psychology*, *12*, 145–149. https://doi.org/10.5709/acp-0193-5

Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM*, *41*, 535–540. https://doi.org/10.1007/s11858-009-0214-4

Heirdsfield, A. M., & Cooper, T. J. (2002). Flexibility and inflexibility in accurate mental addition and subtraction: Two case studies. *The Journal of Mathematical Behavior*, *21*(1), 57–74. https://doi.org/10.1016/S0732-3123(02)00103-7

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*(1), 33–46. https://doi.org/10.2307/749455

Henschel, S., & Roick, T. (2017). Relationships of mathematics performance, control and value beliefs with cognitive and affective math anxiety. *Learning and Individual Differences*, *55*, 97–107. https://doi.org/10.1016/j.lindif.2017.03.009

Hickendorff, M. (2018). Dutch sixth graders' use of shortcut strategies in solving multidigit arithmetic problems. *European Journal of Psychology of Education*, *33*, 577–594. https://doi.org/10.1007/s10212-017-0357-6

Hoover, J. D., & Healy, A. F. (2017). Algebraic reasoning and bat-and-ball problem variants: Solving isomorphic algebra first facilitates problem solving later. *Psychonomic Bulletin & Review*, *24*, 1922–1928. https://doi.org/10.3758/s13423-017-1241-8

Imbo, I., Duverne, S., & Lemaire, P. (2007). Working memory, strategy execution, and strategy selection in mental arithmetic. *The Quarterly Journal of Experimental Psychology*, *60*, 1246–1264. https://doi.org/10.1080/17470210600943419

Imbo, I., & Vandierendonck, A. (2007). The development of strategy use in elementary school children: Working memory and individual differences. *Journal of Experimental Child Psychology*, *96*, 284–309. https://doi.org/10.1016/j.jecp.2006.09.001

Jamieson, J. P., & Harkins, S. G. (2009). The effect of stereotype threat on the solving of quantitative GRE problems: A mere effort interpretation. *Personality and Social Psychology Bulletin*, *35*, 1301–1314. https://doi.org/10.1177/0146167209335165

Jamieson, J. P., Mendes, W. B., Blackstock, E., & Schmader, T. (2010). Turning the knots in your stomach into bows: Reappraising arousal improves performance on the GRE. *Journal of Experimental Social Psychology*, *46*(1), 208–212. https://doi.org/10.1016/j.jesp.2009.08.015

Jamieson, J. P., Peters, B. J., Greenwood, E. J., & Altose, A. J. (2016). Reappraising stress arousal improves performance and reduces evaluation anxiety in classroom exam situations. *Social Psychological and Personality Science*, *7*, 579–587. https://doi.org/10.1177/1948550616644656

Johns, M., Inzlicht, M., & Schmader, T. (2008). Stereotype threat and executive resource depletion: Examining the influence of emotion regulation. *Journal of Experimental Psychology: General*, *137*, 691–705. https://doi.org/10.1037/a0013834

Justicia-Galiano, M.-J., Pelegrina, S., Lechuga, M.-T., Gutiérrez-Palma, N., Martín-Puga, E.-M., & Lendínez, C. (2016). Math anxiety and its relationship to inhibitory abilities and perceived emotional intelligence. *Anales De Psicología/Annals of Psychology*, *32*(1), 125–131. https://doi.org/10.6018/analesps.32.1.194891

Kahan, D. M. (2012). Ideology, motivated reasoning, and cognitive reflection: An experimental study. *Judgment and Decision Making*, *8*, 407–424. https://doi.org/10.2139/ssrn.2182588

Lai, Y., Zhu, X., Chen, Y., & Li, Y. (2015). Effects of mathematics anxiety and mathematical metacognition on word problem solving in children with and without mathematical learning difficulties. *PLoS One*, *10*, e0130570. https://doi.org/10.1371/journal.pone.0130570

Legg, A. M., & Locker, L. (2009). Math performance and its relationship to math anxiety and metacognition. *North American Journal of Psychology*, *11*, 471–484.

Lemaire, P., & Lecacheur, M. (2010). Strategy switch costs in arithmetic problem solving. *Memory & Cognition*, *38*, 322–332. https://doi.org/10.3758/MC.38.3.322

Lemaire, P., Lecacheur, M., & Farioli, F. (2000). Children's strategy use in computational estimation. *Canadian Journal of Experimental Psychology/Revue Canadienne De Psychologie Expérimentale*, *54*, 141–148. https://doi.org/10.1037/h0087336

Liberali, J. M., Reyna, V. F., Furlan, S., Stein, L. M., & Pardo, S. T. (2012). Individual differences in numeracy and cognitive reflection, with implications for biases and fallacies in probability judgment. *Journal of Behavioral Decision Making*, *25*, 361–381. https://doi.org/10.1002/bdm.752

Lim, K. H., & Morera, O. (2010). *Addressing impulsive disposition: Using non-proportional problems to overcome overgeneralization of proportionality*. Paper presented at the Proceedings of the 13th Annual Conference Research on Undergraduate Mathematics Education, Raleigh, NC.

Lim, K. H., & Wagler, A. (2012). *Impulsive-analytic disposition in mathematical problem solving: A survey and a mathematics test*. Paper presented at the Mathematics Education Research Group of Australasia, Singapore.

Linnenbrink, E. A. (2006). Emotion research in education: Theoretical and methodological perspectives on the integration of affect, motivation, and cognition. *Educational Psychology Review*, *18*, 307–314. https://doi.org/10.1007/s10648-006-9028-x

Liu, R.-D., Wang, J., Star, J. R., Zhen, R., Jiang, R.-H., & Fu, X.-C. (2018). Turning potential flexibility into flexible performance: Moderating effect of self-efficacy and use of flexible cognition. *Frontiers in Psychology*, *9*, 1–13. https://doi.org/10.3389/fpsyg.2018.00646

Luchins, A. S., & Luchins, E. H. (1994). The water jar experiments and Einstellung effects: II. Gestalt psychology and past experience. *Gestalt Theory*, *4*(16), 205–259.

Luwel, K., Schillemans, V., Onghena, P., & Verschaffel, L. (2009). Does switching between strategies within the same task involve a cost? *British Journal of Psychology*, *100*, 753–771. https://doi.org/10.1348/000712609X402801

Lyons, I. M., & Beilock, S. L. (2011). Mathematics anxiety: Separating the math from the anxiety. *Cerebral Cortex*, *22*, 2102–2110. https://doi.org/10.1093/cercor/bhr289

Ma, X., & Cartwright, F. (2003). A longitudinal analysis of gender differences in affective outcomes in mathematics during middle and high school. *School Effectiveness and School Improvement*, *14*, 413–439. https://doi.org/10.1076/sesi.14.4.413.17155

Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, *16*, 404–406. https://doi.org/10.1016/j.tics.2012.06.008

Mammarella, I. C., Caviola, S., Giofrè, D., & Borella, E. (2018). Separating math from anxiety: The role of inhibitory mechanisms. *Applied Neuropsychology: Child*, *7*, 342–353. https://doi.org/10.1080/21622965.2017.1341836

Markovits, Z. (2011). Beliefs hold by pre-school prospective teachers toward mathematics and its teaching. *Procedia-Social and Behavioral Sciences*, *11*, 117–121. https://doi.org/10.1016/j.sbspro.2011.01.045

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology*, *82*(1), 60–70. https://doi.org/10.1037/0022-0663.82.1.60

Mega, C., Ronconi, L., & De Beni, R. (2014). What makes a good student? How emotions, self-regulated learning, and motivation contribute to academic achievement. *Journal of Educational Psychology*, *106*(1), 121–131. https://doi.org/10.1037/a0033546

Morsanyi, K., Busdraghi, C., & Primi, C. (2014). Mathematical anxiety is linked to reduced cognitive reflection: A potential road from discomfort in the mathematics classroom to susceptibility to biases. *Behavioral and Brain Functions*, *10*(1), 31. https://doi.org/10.1186/1744-9081-10-31

Morsanyi, K., McCormack, T., & O'Mahony, E. (2018). The link between deductive reasoning and mathematics. *Thinking & Reasoning*, *24*, 234–257. https://doi.org/10.1080/13546783.2017.1384760

Muthén, L. K., & Muthén, B. O. (2012). *Mplus user's guide (version 7)*. Los Angeles, CA. Muthén & Muthén.

National Research Council & Mathematics Learning Study Committee (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

Newton, K. J., Lange, K., & Booth, J. L. (2019). Mathematical flexibility: Aspects of a continuum and the role of prior knowledge. *The Journal of Experimental Education*, 1–13. https://doi.org/10.1080/00220973.2019.1586629

Park, D., Ramirez, G., & Beilock, S. L. (2011). *Put your math burden down: Expressive writing for the highly math anxious*. Paper presented at the Annual Meeting of the Midwestern Psychological Association, Chicago, IL.

Park, D., Ramirez, G., & Beilock, S. L. (2014). The role of expressive writing in math anxiety. *Journal of Experimental Psychology: Applied*, *20*, 103–111. https://doi.org/10.1037/xap0000013

Passolunghi, M. C., Caviola, S., De Agostini, R., Perin, C., & Mammarella, I. C. (2016). Mathematics anxiety, working memory, and mathematics performance in secondary-school children. *Frontiers in Psychology*, *7*, 42. https://doi.org/10.3389/fpsyg.2016.00042

Pekrun, R., Goetz, T., Frenzel, A. C., Barchfeld, P., & Perry, R. P. (2011). Measuring emotions in students' learning and performance: The Achievement Emotions Questionnaire (AEQ). *Contemporary Educational Psychology*, *36*(1), 36–48. https://doi.org/10.1016/j.cedpsych.2010.10.002

Pennycook, G., Cheyne, J. A., Koehler, D. J., & Fugelsang, J. A. (2016). Is the cognitive reflection test a measure of both reflection and intuition? *Behavior Research Methods*, *48*(1), 341–348. https://doi.org/10.3758/s13428-015-0576-1

Primi, C., Donati, M. A., Chiesi, F., & Morsanyi, K. (2018). Are there gender differences in cognitive reflection? Invariance and differences related to mathematics. *Thinking & Reasoning*, *24*, 258–279. https://doi.org/10.1080/13546783.2017.1387606

Ramirez, G., & Beilock, S. L. (2011). Writing about testing worries boosts exam performance in the classroom. *Science*, *331*(6014), 211–213. https://doi.org/10.1126/science.1199427

Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2016). On the relationship between math anxiety and math achievement in early elementary school: The role of problem solving strategies. *Journal of Experimental Child Psychology*, *141*, 83–100. https://doi.org/10.1016/j.jecp.2015.07.014

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development*, *14*, 187–202. https://doi.org/10.1080/15248372.2012.664593

Ramirez, G., Hooper, S. Y., Kersting, N. B., Ferguson, R., & Yeager, D. (2018). Teacher math anxiety relates to adolescent students' math achievement. *AERA Open*, *4*(1), 1–13. https://doi.org/10.1177/2332858418756052

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*, 145–164. https://doi.org/10.1080/00461520.2018.1447384

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge?: An experimental study on learning to solve equations. *Journal of Educational Psychology*, *99*, 561–574. https://doi.org/10.1037/0022-0663.99.3.561

Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology*, *101*, 529–544. https://doi.org/10.1037/a0014224

Rittle-Johnson, B., Star, J. R., & Durkin, K. (2012). Developing procedural flexibility: Are novices prepared to learn from comparing procedures? *British Journal of Educational Psychology*, *82*, 436–455. https://doi.org/10.1111/j.2044-8279.2011.02037.x

Sales, B. D., & Folkman, S. E. (2000). *Ethics in research with human participants*, Washington, DC: . American Psychological Association.

Samuel, T. S., & Warner, J. (2019). "I can math!": Reducing math anxiety and increasing math self-efficacy using a mindfulness and growth mindset-based intervention in first-year students. *Community College Journal of Research and Practice*, 1–18. https://doi.org/10.1080/10668926.2019.1666063

Schillemans, V., Luwel, K., Ceulemans, E., Onghena, P., & Verschaffel, L. (2012). The effect of single versus repeated previous strategy use on individuals' subsequent strategy choice. *Psychologica Belgica*, *52*, 307–326. https://doi.org/10.5334/pb-52-4-307

Schneider, M., Rittle-Johnson, B., & Star, J. R. (2011). Relations among conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology*, *47*, 1525–1538. https://doi.org/10.1037/a0024997

Schultz, P. W., & Searleman, A. (2002). Rigidity of thought and behavior: 100 years of research. *Genetic, Social, and General Psychology Monographs*, *128*, 165–207.

Schutz, P. A., Pekrun, R., & Phye, G. D. (2007). *Emotion in education* (Vol. *10*). San Diego, CA: Academic Press.

Si, J., Li, H., Sun, Y., Xu, Y., & Sun, Y. (2016). Age-related differences of individuals' arithmetic strategy utilization with different level of math anxiety. *Frontiers in Psychology*, *7*, 1612. https://doi.org/10.3389/fpsyg.2016.01612

Sinayev, A., & Peters, E. (2015). Cognitive reflection vs. calculation in decision making. *Frontiers in Psychology*, *6*(532), 532. https://doi.org/10.3389/fpsyg.2015.00532

Spencer, S. J., Steele, C. M., & Quinn, D. M. (1999). Stereotype threat and women's math performance. *Journal of Experimental Social Psychology*, *35*(1), 4–28. https://doi.org/10.1006/jesp.1998.1373

Star, J. R., & Newton, K. J. (2009). The nature and development of experts' strategy flexibility for solving equations. *ZDM*, *41*, 557–567. https://doi.org/10.1007/s11858-009-0185-5

Star, J. R., Newton, K., Pollack, C., Kokka, K., Rittle-Johnson, B., & Durkin, K. (2015). Student, teacher, and instructional characteristics related to students' gains in flexibility. *Contemporary Educational Psychology*, *41*, 198–208. https://doi.org/10.1016/j.cedpsych.2015.03.001

Star, J. R., Pollack, C., Durkin, K., Rittle-Johnson, B., Lynch, K., Newton, K., & Gogolen, C. (2015). Learning from comparison in algebra. *Contemporary Educational Psychology*, *40*, 41–54. https://doi.org/10.1016/j.cedpsych.2014.05.005

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, *18*, 565–579. https://doi.org/10.1016/j.learninstruc.2007.09.018

Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*, *102*, 408–426. https://doi.org/10.1016/j.jecp.2008.11.004

Star, J. R., & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology*, *31*, 280–300. https://doi.org/10.1016/j.cedpsych.2005.08.001

Supekar, K., Iuculano, T., Chen, L., & Menon, V. (2015). Remediation of childhood math anxiety and associated neural circuits through cognitive tutoring. *Journal of Neuroscience*, *35*, 12574–12583. https://doi.org/10.1523/JNEUROSCI.0786-15.2015

Toplak, M. E., West, R. F., & Stanovich, K. E. (2011). The Cognitive Reflection Test as a predictor of performance on heuristics-and-biases tasks. *Memory & Cognition*, *39*, 1275–1289. https://doi.org/10.3758/s13421-011-0104-1

Toplak, M. E., West, R. F., & Stanovich, K. E. (2014). Assessing miserly information processing: An expansion of the Cognitive Reflection Test. *Thinking & Reasoning*, *20*, 147–168. https://doi.org/10.1080/13546783.2013.844729

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education*, *24*, 335–359. https://doi.org/10.1007/Bf03174765

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2011). Analyzing and developing strategy flexibility in mathematics education. In J. Elen, E. Stahl, R. Bromme, & G. Clarebout (Eds.), *Links between beliefs and cognitive flexibility* (pp. 175–197). Dordrecht: Springer.

Vorensky, S. B. (2018). *Exploring teachers' self-efficacy about technology use in learning design and student performance in mathematics: A qualitative study about math anxiety*, NJ: Glassboro: (EdD Doctor of Education). Rowan University. Theses and Dissertations. 2528. https://rdw.rowan.edu/etd/2528

Wang, J., Liu, R.-D., Star, J. R., Liu, Y., & Zhen, R. (2019). The moderating effect of regulatory focus in the relationship between potential flexibility and practical flexibility. *Contemporary Educational Psychology*, *56*, 218–227. https://doi.org/10.1016/j.cedpsych.2019.01.013

Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, *80*, 210–216. https://doi.org/10.1037/0022-0663.80.2.210

Xu, C., Wells, E., LeFevre, J. A., & Imbo, I. (2014). Strategic flexibility in computational estimation for Chinese- and Canadian-educated adults. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *40*, 1481–1497. https://doi.org/10.1037/a0037346

Xu, L., Liu, R.-D., Star, J. R., Wang, J., Liu, Y., & Zhen, R. (2017). Measures of potential flexibility and practical flexibility in equation solving. *Frontiers in Psychology*, *8*, 1368. https://doi.org/10.3389/fpsyg.2017.01368

Yakes, C., & Star, J. R. (2011). Using comparison to develop flexibility for teaching algebra. *Journal of Mathematics Teacher Education*, *14*, 175–191. https://doi.org/10.1007/s10857-009-9131-2

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