Apply 2 by Julian D

a)
One way that you could approximate the rate at which the function changes over time is to "connect the dots" between each of the points, and then find the derivative for each of the intervals.

$$f(t) = \begin{cases} \frac{327-318}{10}(t) + 318, 0 \le t < 10 \\ \frac{340-327}{10}(t-10) + 327, 10 \le t < 20 \\ \frac{356-340}{10}(t-20) + 340, 20 \le t < 30 \\ \frac{370-356}{10}(t-30) + 356, 30 \le t < 40 \\ \frac{389-370}{10}(t-40) + 389, 40 \le t \le 50 \end{cases}$$

Which can be simplified to

$$f(t) \begin{cases} 0.9t + 318, 0 \le t < 10 \\ 1.3t + 314, 10 \le t < 20 \\ 1.6t + 308, 20 \le t < 30 \\ 1.4t + 314, 30 \le t < 40 \\ 1.9t + 294, 40 \le t \le 50 \end{cases}$$

Using the constant rule, the power rule and the rule:
$\mathrm{g}(x) = a(x) + b(x), g'(x) = a'(x) + b'(x)$
The (approximate) rate at which the CO_2 levels are increasing by, t years since 1960, are the following rates below, in CO_2 levels (ppm) per year.

$$f'(t) = \begin{cases} 0.9, 0 \le t < 10 \\ 1.3, 10 \le t < 20 \\ 1.6, 20 \le t < 30 \\ 1.4, 30 \le t < 40 \\ 1.9, 40 \le t \le 50 \end{cases}$$

Therefore, the difference between the rate at CO_2 levels were increasing by in 2010 and 1960 is
$2010 - 1960 = 50$
$f'(50) = 1.9$
$1960 - 1960 = 0$
$f'(0) = 0.9$
$1.9 - 0.9 =$
$1\frac{ppm}{yr}$
The rate at which CO_2 enters the atmosphere is $1\frac{ppm}{yr}$ higher in 2010 than in 1960.

b)
Because our function is already defined as a ton of line segments, which (between the curve and the x-axis) are trapezoids, it'll be easy to approximate using a graph. Here's the mathematical derivation first:
The area under a curve can be defined as

$$\int_a^b f(t)dt$$

Since the average value of the function is just the definite integral divided by b-a, it will become

$$\frac{1}{b-a}\int_a^b f(t)dt \to \frac{1}{50}\int_0^{50} f(t)dt$$

We could break this definite integral up into the 10-step intervals that problem asks for, thanks to the following property (where a, b and c is any real constant where $a < b < c$ and d is any real constant value):

$$\frac{1}{d}\int_a^c f(x)dx = \frac{1}{d}\left(\int_a^b f(x)dx + \int_b^c f(x)dx\right)$$

Since that is true for an unlimited amount of partial sums, we can write it in summation notation as (assuming the data points are evenly spaced):

$$\frac{1}{mk}\sum_{n=1}^{k}\int_{m(n-1)}^{mn} f(x)dx$$

Where k is the number of data points and m is the spacing between each of the data points.
Our definite integral becomes

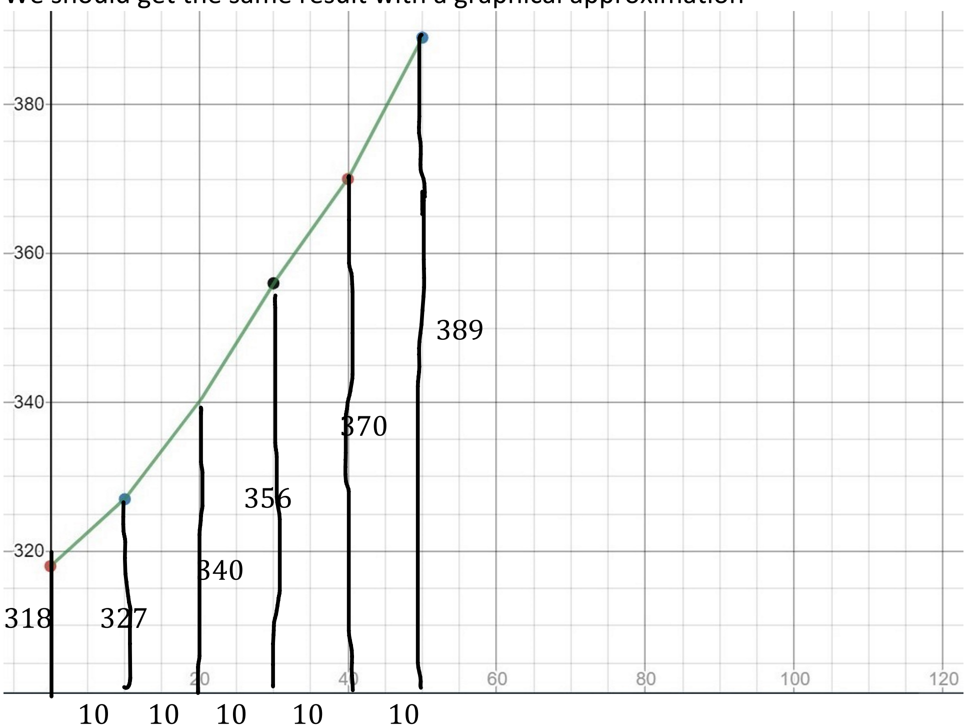
$$\frac{1}{50}\sum_{n=1}^{5}\int_{10(n-1)}^{10n} f(t)dt$$

$$\frac{1}{50}\left(\int_0^{10} f(t)dt + \int_{10}^{20} f(t)dt + \int_{20}^{30} f(t)dt + \int_{30}^{40} f(t)dt + \int_{40}^{50} f(t)dt\right) =$$

$$\frac{1}{50}\left(\left[\frac{0.9t^2}{2} + 318t\right]_0^{10} + \left[\frac{1.3t^2}{2} + 314t\right]_{10}^{20} + \left[\frac{1.6t^2}{2} + 308t\right]_{20}^{30} + \left[\frac{1.4t^2}{2} + 314t\right]_{30}^{40} + \left[\frac{1.9t^2}{2} + 294t\right]_{40}^{50}\right)$$

$$\frac{1}{50}(3225 + 3335 + 3480 + 3630 + 3795) = 349.3\ ppm$$

We should get the same result with a graphical approximation



Since the equation for the area of a trapezoid is $\frac{b_1+b_2}{2}h$ or $\frac{h_1+h_2}{2}b$ area underneath the curve of this graph is

$$5((318 + 327) + (327 + 340) + (340 + 356) + (356 + 370) + (370 + 389)) = 17465$$

To get the average value of this function, we need to divide it by 50.

$$\frac{17465}{50} = 349.3\ ppm$$

And we get the same exact number both ways!

c)

We're given a function $P(t) = 0.01t^2 + 0.95t + 317.5$ as a model for the amount of carbon dioxide in the atmosphere, where t is the number of years after 1960.

$$P'(t) = 0.02t + 0.95$$

by the power rule
$P''(t) = 0.02$
by the power rule

$P'(25) = 0.02(25) + 0.95 = 1.45$
This means that in the year 1985, the rate at which CO_2 was entering the atmosphere was 1.45 $\frac{ppm}{yr}$

$P''(25) = 0.02$
This means that in the year 1985, the rate at which CO_2 was entering the atmosphere was accelerating by 0.02$\frac{ppm}{yr^2}$

d)
The average concentration of CO_2 based off of the $P(t)$ model from the year 1960 to 2010 is

$$\frac{1}{50}\int_0^{50} P(t)dt \Rightarrow \frac{1}{50}\int_0^{50} (0.01t^2 + 0.95t + 317.5)dt \Rightarrow \frac{1}{50}\left[\frac{0.01t^3}{3} + \frac{0.95t^2}{2} + 317.5t\right]_0^{50} =$$

$$\frac{1}{50}\left(\frac{0.01(50)^3}{3} + \frac{0.95(50)^2}{2} + 317.5(50)\right) = 349.58333\ldots$$

That's really close to our approximation!

e)
The average of a function over an interval [a, b] is

$$\frac{1}{b-a}\int_a^b f(x)dx$$

In the context of the problem, this becomes

$$\frac{1}{y}\int_0^y P(t)dt = 380$$

$$\frac{1}{y}\int_0^y (0.01t^2 + 0.95t + 317.5)dt = 380$$

$$\frac{1}{y}\left[\frac{0.01t^3}{3} + \frac{0.95t^2}{2} + 317.5t\right]_0^y = 380$$

$$\frac{1}{y}\left(\frac{0.01y^3}{3} + \frac{0.95y^2}{2} + 317.5y\right) = 380$$

$$\frac{0.01y^2}{3} + \frac{0.95y}{2} + 317.5 = 380$$

Now we have a second degree polynomial in terms of y. We can solve for the zeroes using the quadratic formula!

$$\frac{0.01}{3}y^2 + \frac{0.95}{2}y - 62.5 = 0$$

This is definitely going to have real-valued roots, since it's translated down so much and the second derivative is positive.

$$\frac{-\frac{0.95}{2} \pm \sqrt{\left(\frac{0.95}{2}^2\right) - 4\left(\frac{0.01}{3}\right)(-62.5)}}{2\left(\frac{0.01}{3}\right)} =$$

$$3\frac{-0.475 \pm \sqrt{\frac{0.95^2}{4} + \frac{16\left(\frac{0.625}{3}\right)}{4}}}{0.02} =$$

$$3\frac{-0.475 \pm \frac{\sqrt{0.95^2 + 16\left(\frac{0.625}{3}\right)}}{2}}{0.02} =$$

$$3\left(-\frac{0.475}{0.02} \pm \frac{\sqrt{0.95^2 + \frac{10}{3}}}{0.04}\right) =$$

$$-3\frac{0.475}{0.02} \pm 3\frac{\sqrt{0.95^2 + \frac{10}{3}}}{0.04}$$

We want to find how many years after, so we have to add them together.

$$-3\frac{2(0.475)}{0.04} + 3\frac{\sqrt{0.95^2 + \frac{10}{3}}}{0.04} =$$

$$3\frac{-0.95 + \sqrt{0.95^2 + \frac{10}{3}}}{0.04} =$$

83.1085517553

So, 83.1085517553 years after 1960, or around 1.3 months into the year 2043, the average concentration of CO_2 (based on the current model of CO_2 concentration) from 1960 to 1.3 months into the year 2043 will be 380 ppm.