# Redox Derivations

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## 1 Converting 410/470 intensity ratio to the proportion of oxidized sensor molecules

The $roGFP_{12}$ sensor is genetically encoded into a tissue. The intensity values emitted from the $roGFP_{12}$-containing tissue are recorded after being stimulated 410nm and 470nm light. The ratios between these values describe relative levels of tissue oxidation. For example, a 410/470 ratio of 2.0 indicates that a tissue is more oxidized than a ratio of 1.0. In this brief summary, we algebraically convert the 410/470 intensity ratio into the more biologically-meaningful measure of redox potential ($E$) via the description of ratio of of $roGFP_{12}$ molecules in an oxidized (as opposed to reduced) state ($OxD$).

Assume a fully reduced state. Then, the intensities observed at a wavelength $\lambda$ are equal to the product of $N_T$, the total number of roGFP molecules, and $I_{\lambda,R}$, the intensity of each roGFP molecule at a given wavelength in the reduced state.

$$I_{\lambda,R} = N_T * I_{\lambda,R} \tag{1}$$

The same is true for the fully oxidized state:

$$I_{\lambda,Ox} = N_T * I_{\lambda,Ox} \tag{2}$$

At a redox state between maximally reduced and maximally oxidized, the intensity at a given wavelength is a weighted sum of the molecules found at either discretely oxidized or reduced state. We therefore can rewrite any state in terms of equations (1) and (2)

$$I_\lambda = \frac{N_{Ox}}{N_T} * I_{\lambda,Ox} + \frac{N_{Red}}{N_T} * I_{\lambda,Red} \tag{3}$$

Because all sensor molecules must be in either an oxidized or reduced state, $N_{Red} = N_T - N_{Ox}$. So we can rewrite equation (3) :

$$I_\lambda = \frac{N_{Ox}}{N_T} * I_{\lambda,Ox} + (1 - \frac{N_{Ox}}{N_T}) * I_{\lambda,Red} \tag{4}$$

Using equation (4), consider the intensity ratio of 410 nm / 470 nm:

$$\frac{I_{410}}{I_{470}} = \frac{\frac{N_{Ox}}{N_T} * I_{410,Ox} + (1 - \frac{N_{Ox}}{N_T}) * I_{410,Red}}{\frac{N_{Ox}}{N_T} * I_{470,Ox} + (1 - \frac{N_{Ox}}{N_T}) * I_{470,Red}} =$$

For brevity, let $OxD = \frac{N_{Ox}}{N_T}$. Then cross-multiply:

$$I_{410} * OxD * (I_{470,Ox} + (1 - OxD) * I_{470,Red}) =$$

$$I_{470} * OxD * (I_{410,Ox} + (1 - OxD) * I_{410,Red})$$

Simplify and express $OxD$ in terms of known quantities:

$$OxD = \frac{I_{470}I_{410,R} - I_{410}I_{470,R}}{I_{410}I_{470,Ox} - I_{410}I_{470,R} - I_{470}I_{410,Ox} + I_{470}I_{410,R}} \tag{5}$$

To simplify, let:

$$R_{Red} = \frac{I_{410,R}}{I_{470,R}} \tag{6}$$

$$R_{Ox} = \frac{I_{410,Ox}}{I_{470,Ox}} \tag{7}$$

$$\frac{I_{410}}{I_{470}} = \frac{I_{410}}{I_{470}} \tag{8}$$

$$\delta_{470} = \frac{I_{470_{Ox}}}{I_{470_{Red}}} \tag{9}$$

We can now re-derive the definition of $OxD$ in terms of ratio values.

Step: Re-arrange terms, multiply by $\frac{-1}{-1}$:

$$OxD = \frac{I_{410}I_{470,R} - I_{470}I_{410,R}}{I_{410}I_{470,R} - I_{470}I_{410,R} + I_{470}I_{410,Ox} - I_{470,Ox}I_{410}}$$

Step: Work to factor out $I_{470,R}i_{470}$ from the numerator and denominator write some in terms of ratio values:

$$OxD = \frac{I_{470,R}I_{470}(\frac{I_{410}}{I_{470}} - R_{Red})}{I_{470,R}I_{470}(\frac{I_{410}}{I_{470}} - R_{Red} + \delta_{470}(R_{Ox} - \frac{I_{410}}{I_{470}}))}$$

And simplify:

$$OxD = \frac{\frac{I_{410}}{I_{470}} - R_{Red}}{\frac{I_{410}}{I_{470}} - R_{Red} + \delta_{470}(R_{Ox} - \frac{I_{410}}{I_{470}})} \tag{10}$$

Where $\delta_{470}$ describes the sensor-specific dynamic range $470nm$.

## 2 Analyzing the conversion between intensity ratio and ratio of oxidized molecules

## 3 Converting fraction of oxidized sensors into an electric potential

## 4 Reparameterizing the intensity ratio