One potential goal of this analysis is to reparameterize the independent variable and thereby standardize all measurements. This could be accomplished by changing the independent variable so that it, regardless of other parameters, always ranges from 0 to 1.

To do this with R, we must define an R'' such that $R'' = \frac{R - R_{Min}}{R_{Max} - R_{Min}}$. Then, an R'' value of 1 refers to the ratio value at which $R = R_{Max}$, a value of 0 means $R = R_{Min}$, and a value of 0.5 means that $R - R_{Min}$ is exactly halfway between R_{Min} and R_{Max}.

1 Reparameterize $R \rightarrow R''$

Base truth

$$OxD = \frac{R - R_{Min}}{R - R_{Min} + \delta(R_{Max} - R)} \tag{1}$$

Take the inverse of both sides

$$\frac{1}{OxD} = \frac{R - R_{Min} + \delta(R_{Max} - R)}{R - R_{Min}}$$

Multiply both sides by $R - R_{Min}$

$$\frac{R - R_{Min}}{OxD} = R - R_{Min} + \delta(R_{Max} - R)$$

Add δR_{Min} **to both sides:**

$$\frac{R - R_{Min}}{OxD} + \delta R_{Min} = R - R_{Min} + \delta(R_{Max} - R + R_{Min})$$

Divide both sides by $R_{Max} - R_{Min}$

$$\frac{R - R_{Min}}{OxD(R_{Max} - R_{Min})} + \frac{\delta R_{Min}}{R_{Max} - R_{Min}} =$$

$$\frac{R - R_{Min}}{R_{Max} - R_{Min}} + \frac{\delta(R_{Max} - R + R_{Min})}{R_{Max} - R_{Min}}$$

Let $R'' = \frac{R - R_{Min}}{R_{Max} - R_{Min}}$

$$\frac{R''}{OxD} + \frac{\delta R_{Min}}{R_{Max} - R_{Min}} = R'' + \delta(\frac{R_{Max}}{R_{Max} - R_{Min}} - R'')$$

Subtract $\frac{\delta R_{Min}}{R_{Max} - R_{Min}}$ **from both sides**

$$\frac{R''}{OxD} = R'' + \delta(\frac{R_{Max}}{R_{Max} - R_{Min}} - R'' - \frac{R_{Min}}{R_{Max} - R_{Min}})$$

Simplify right hand side

$$\frac{R''}{OxD} = R'' + \delta(\frac{R_{Max} - R_{Min}}{R_{Max} - R_{Min}} - R'');$$

$$\frac{R''}{OxD} = R'' + \delta(1 - R'')$$

Multiply by OxD, divide to isolate:

$$OxD = \frac{R''}{R'' + \delta(1 - R'')}$$

$$OxD = \frac{R''}{R'' - \delta R'' + \delta} \tag{2}$$