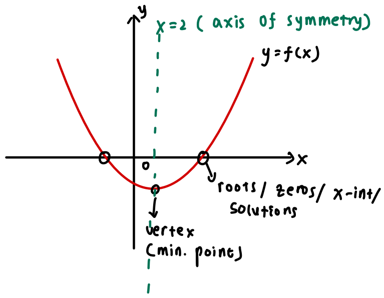
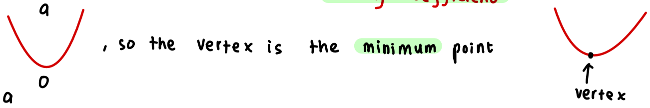
Quadratic Functions

Functions of the form $f(x) = ax^2 + bx + c$ (general form) where a determines the shape of the graph ($a \neq 0$).

called the "leading coefficient"



- If $a > 0$, it will open above, so the vertex is the minimum point



- If $a < 0$, it will open below, so the vertex is the maximum point

Quadratic functions can also be in vertex form : $y = a(x-h)^2 + k$ where (h, k) is the coordinate of the vertex

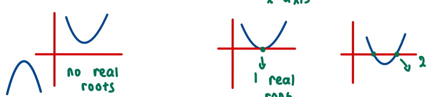
root form : $y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are the zeros / solutions / roots.

What is D (discriminant)? why is it important?

$D = b^2 - 4ac$ in the quadratic function $y = ax^2 + bx + c$.

It is a number to determine the characteristics of quadratic functions.

3 cases of D :



Rational and Irrational roots

- If $D = b^2 - 4ac$ is a perfect square ($D = 1, 4, 9, 16, 25, 36, 49, 64, \dots$), then the roots are rational and distinct.

  $x_1 = \frac{5}{3}$ and $x_2 = 2$ (for example)

2.5 → rational
2.321782 → irrational
2.321321321 → rational

- If $D = b^2 - 4ac$ is NOT a perfect square, then the roots are irrational and conjugate of each other.

  $x_1 = 2 + \sqrt{3}$ and $x_2 = 2 - \sqrt{3}$ (for example)

  conjugate

$x_2 = \frac{-2 + \sqrt{3}}{3}$

$x_1 = \frac{-2 - \sqrt{3}}{3}$

What is the quadratic formula?

It is a formula to find the root(s) / x-intercept of a quadratic function and equation.

$$x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

so,

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$x_{12} = \frac{-2 \pm \sqrt{-1}}{3}$

no real roots

numbers < real, imaginary

When $D < 0$, the inside of the square root will be negative, e.g., $x_{12} = \frac{-2 \pm \sqrt{-4}}{3}$, so the roots will be imaginary because negative numbers don't have square roots.

Note: when factoring quadratics, the left hand side or the right hand side of the equation must $= 0$.

1, 2, 3, 6

$x^2 + x - 6 = 0$

find 2 numbers:
- when added = 1
- when multiplied = -6

$(x+3)(x-2) = 0$

Factorization Exercises:

1) $6x^2 - x - 5 = 0$

$x_1 = 1$, $x_2 = \frac{-5}{6}$

$(x-1)(6x+5) = 0$

2) $12x^2 = 25x$

$12x^2 - 25x = 0$

$x(12x - 25) = 0$

$x = 0$ $\quad 12x - 25 = 0$

$x = \frac{25}{12}$

3) $3x^2 + x = 2$

$x_1 = -1$, $x_2 = \frac{2}{3}$

4) $5x^2 = -21x - 18$

$\underline{6} + \underline{15} = 21$

$\underline{6} \times \underline{15} = 90$

$x = -3$ or $x = -\frac{6}{5}$

5) $-10x^2 - 29x - 10 = 0$

$(2x+5)(5x+2) = 0$

$x = -\frac{5}{2}$ or $x = -\frac{2}{5}$

6) $12m^2 + 34m - 56 = 0$

$2(x+4)(6x-7) = 0$

$x = -4$ or $x = \frac{7}{6}$

7) $x^4 - 2x^3 - 3x^2 = 0$

$x^2(x^2 - 2x - 3) = 0$

$x = -1$ or $x = 3$ or $x = 0$

8) $10z^2 - 43z + 28 = 0$

$(5x-4)(2x-7) = 0$

$x = \frac{4}{5}$ or $x = \frac{7}{2}$

9) $6x^2 - 16x + 8 = 0$

$$\begin{array}{ccc} 2 & -4 & -12 \\ 3 & -2 & -4 \\ & & -16 \end{array}$$

$(2x-4)(3x-2) = 0$

$2(x-2)(3x-2) = 0$

10) $2x^2 - 7x + 6 = 0$ → $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{cc} 1 & -2 = -4 \\ 2 & -3 = -3 \\ & -7 \end{array}$$

$(x-2)(2x-3) = 0$

$x - 2 = 0$ or $2x - 3 = 0$

$x = 2$ $\quad x = 3/2$

$2x^2 - 7x + 6 = 0$

$-4 + -3 = -7$

$-4 \times -3 = 12$

$\frac{(2x-4)(2x-3)}{2}$

$\frac{2(x-2)(2x-3)}{2}$

$= (x-2)(2x-3)$

How to graph quadratic functions? (Remember! $y = f(x)$)

General form

$y = ax^2 + bx + c$

e.g. $y = 2x^2 + 5x - 1$

Steps:

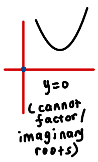
① $a < 0$ or $a > 0$ (is the graph opening above or below?)

$a > 0$

② x-intercept / roots

→ $y = 0$

If cannot factor, check $D = b^2 - 4ac$ (discriminant)



$0 = 2x^2 + 5x - 1$

$x_{1,2} = \frac{-5 \pm \sqrt{33}}{4}$

$x_1 = -2.6$

$x_2 = 0.186$

$(-2.6, 0)$ and $(0.186, 0)$

③ y-intercept → $x = 0$ (if exists)

$y = 2x^2 + 5x$

$2(0) + 5(0)$

$y = 0$

$(0, 0)$

$y = 2x^2 + 5x - 1$

$y = 2(0) + 5(0) - 1$

$y = -1$

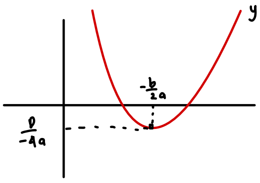
$(0, -1)$

④ vertex coordinates

x (axis of symmetry) $= -\frac{b}{2a}$

y (max or min value) $= \frac{b^2 - 4ac}{-4a} = \frac{D}{-4a}$

⑤ additional points (optional) (any x values you would like to try)



$y = 2x^2 + 5x - 1$

$x = 2 \rightarrow y = 17$

$x = 3 \rightarrow y = 32$

Vertex form

$y = a(x-h)^2 + k$

e.g. $y = 2(x-1)^2 + 5$

Steps:

① $a < 0$ or $a > 0$ (is the graph opening above or below?)

② x-intercept / roots

→ $y = 0$ (need to expand)

If cannot factor, check $D = b^2 - 4ac$ (discriminant) (use square root method)

③ y-intercept → $x = 0$ (if exists)

④ vertex coordinates

$x = h$ and $y = k$

(careful with the signs)

⑤ additional points (optional) (any x values you would like to try)

Root / Factored form

$y = a(x - x_1)(x - x_2)$

e.g. $y = -2(x-2)(x-3)$

Steps:

① $a < 0$ or $a > 0$ (is the graph opening above or below?)

② x-intercept / roots

$x = x_1$ and $x = x_2$

If, for example: $y = (x-1)^2$, then the x-int is $x = 1$ (only one intercept)

③ y-intercept → $x = 0$ (if exists)

④ vertex coordinates (you'll need to expand the brackets)

x (axis of symmetry) $= -\frac{b}{2a}$

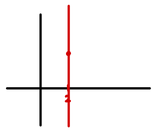
$y = \frac{b^2 - 4ac}{-4a} = \frac{D}{-4a}$

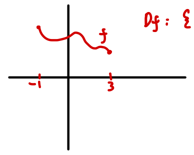
⑤ additional points (optional) (any x values you would like to try)

What is the domain and range of a function?

- The domain of a function is the x-span of the graph / function, written as D_f.
  It is the range of values of x for which $y = f(x)$ is defined.

- The range of a function is the y-span of the graph / function, written as R_f.
  It is the range of values of y for which $\underbrace{y = f(x)}_{\text{the function}}$ is defined.





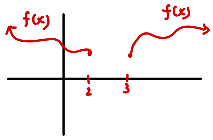
$D_f : \{-1 \le x \le 3\}$

Domain

$x = 2$

$x \le 2 \quad x > 3$

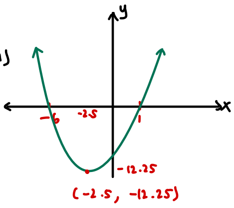
$-1 \le x \le 3$



for example, $y = x^2 + 5x - 6$
$= (x+6)(x-1)$

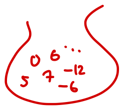
domain:

as you can see, the x will span / go to the left and to the right endlessly.
So, it spans for every x values.
Therefore, $D_f : \{x \in \mathbb{R}\}$ (the domain of x^2+5x-6 is all real numbers.)
in real numbers



range:

as you can see, the vertex is the minimum point with coordinate $(-2.5, -12.25)$.
from the graph, the y-value doesn't go down below the vertex.
the graph / function only goes up endlessly.
so, y must be greater than -12.25.
Therefore, $R_f : \{y \ge -12.25,\ y \in \mathbb{R}\}$ → set
(y is a real number)



numbers < real, imaginary

(the range of x^2+5x-6 is for $y = f(x)$ greater than -12.25).

★ quick way to find R_f (range) from quadratic functions:

Given $y = ax^2+bx+c$, the range depends on a. Is $a<0$ or $a>0$?

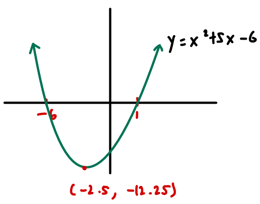
- If $a > 0$  then R_f is $y \ge \frac{D}{-4a}$.
- If $a < 0$  then R_f is $y \le \frac{D}{-4a}$.

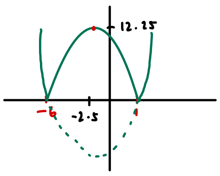
★ Using Geogebra (download app or go to geogebra.org / calculator).

★ Modulus of quadratic functions

The modulus or absolute of a quadratic function is turning all the negative values of $f(x)$ to positive.
If $f(x)$ is already positive, then the value doesn't change.

for example: $y = x^2+5x-6 \rightarrow y = |x^2-5x-6|$



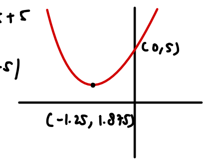


(the graph below the y-axis is reflected or changed the opposite way / upwards.
the values above the x-axis don't change).

the vertex coordinates change from $(-2.5, -12.25)$ to $(-2.5, 12.25)$

(the x-coordinate doesn't change)

for graphs like $y = 2x^2+5x+5$
the graph of $y = |2x^2+5x+5|$ doesn't change.

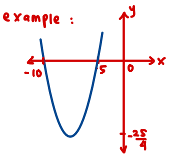


Finding the quadratic function given its graph.

Given vertex and an arbitrary point :

- First, find vertex coordinates (h, k)
- substitute (h,k) to $y = a(x-h)^2 + k$
- find the other arbitrary point on the graph and also substitute it to (x,y) and find the value of a.
- return to the vertex form and substitute (h,k) and the value of a which you found. However, don't substitute the (x, y).

example :



the vertex always lies between (in the middle of) the roots of a quadratic function.

so,

$$\frac{-10+(-5)}{2} = -\frac{15}{2} = h$$

and the y-coordinate (k) is $-\frac{25}{4}$

so, $(h,k) = \left(-\frac{15}{2}, -\frac{25}{4}\right)$

substitute to vertex form:

$$y = a(x-h)^2 + k$$
$$y = a\left(x - \left(-\frac{15}{2}\right)\right)^2 + \left(-\frac{25}{4}\right)$$
$$y = a\left(x + \frac{15}{2}\right)^2 - \frac{25}{4}$$

Then, substitute another arbitrary point on the graph, for example, $(-5, 0) = (x, y)$

$$0 = a\left(-5 + \frac{15}{2}\right)^2 - \frac{25}{4}$$
$$\frac{25}{4} = a\left(\frac{5}{2}\right)^2$$
$$a = 1$$

Then, go back to the vertex form without substituting x and y.

$$y = a(x-h)^2 + k$$
$$y = 1\left(x + \frac{15}{2}\right)^2 - \frac{25}{4}$$
$$y = \left(x + \frac{15}{2}\right)^2 - \frac{25}{4}$$

Given roots / x-intercepts and an arbitrary point

- find the roots / x-intercepts and label them as x_1 and x_2, sometimes quadratic functions can have one root with 2 multiplicities, e.g., $y = (x-1)^2$ with $x_1 = 1$.
  - use the factored / root form of quadratic function: $y = a(x - x_1)(x - x_2)$ and substitute the x_1 and x_2 there. For two-multiplicity root, substitute to $y = a(x - x_1)^2$
  - find the other arbitrary point on the graph and also substitute it to (x,y) and find the value of a.
  - return to the factored form and substitute x_1, x_2, and the value of a which you found. However, don't substitute the (x, y).

note : this form can't be used for quadratics which do not have root(s) or float.

example : using the same graph to the left, we can see that the roots are $x_1 = -10$ and $x_2 = -5$ and $\left(-\frac{15}{2}, -\frac{25}{4}\right)$ is the arbitrary point (x, y)

so,

$$y = a(x - x_1)(x - x_2)$$
$$y = a(x+10)(x+5)$$

Then,

$$-\frac{25}{4} = a\left(-\frac{15}{2} + 10\right)\left(-\frac{15}{2} + 5\right)$$
$$-\frac{25}{4} = a\left(\frac{5}{2}\right)\left(\frac{-5}{2}\right)$$
$$a = 1$$

Then, go back to factored form without substituting x and y.

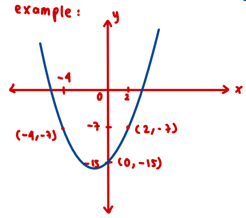
$$y = (x+5)(x+10) = x^2 + 15x + 50.$$

Given 3 arbitrary points :

- find 3 arbitrary points on the graph and label them as (x_1, y_1), (x_2, y_2), and (x_3, y_3)
- using the general form of a quadratic function : $y = ax^2 + bx + c$, substitute (x_1, y_1) to make first equation : $y_1 = ax_1^2 + bx_1 + c$.
- do the same for (x_2, y_2) and (x_3, y_3) and gain 3 equations in terms of a, b, c
- you should obtain a system of 3 equations, try to solve for a, b, and c with the help of substitution and elimination.
- subsitute the values of a, b, c you found to $y = ax^2 + bx + c$

note : this form cannot be used if only 2 points or less are known from the graph.

example :



There are 3 arbitrary points on the graph, $(-4, -7)$, $(0, -15)$, $(2, -7)$

for first point :

$$y = ax^2 + bx + c$$
$$-7 = a(-4)^2 + b(-4) + c$$
$$-7 = 16a - 4b + c \ldots \text{①} \text{ (equation 1)}$$

for second point :

$$-15 = a(0)^2 + b(0) + c$$
$$c = -15 \ldots \text{②} \text{ (equation 2)}$$

for third point :

$$-7 = a(2)^2 + b(2) + c$$
$$-7 = 4a + 2b + c \ldots \text{③} \text{ (equation 3)}$$

substituting $c = -15$ to ① and ③

to ① : $-7 = 16a - 4b - 15$

$$16a - 4b = 8$$
$$4a - b = 2 \ldots \text{④}$$

to ③ : $-7 = 4a + 2b - 15$

$$4a + 2b = 8 \ldots \text{⑤}$$

solving ④ and ⑤ using elimination gives $a = 1$ and $b = 2$

so, $y = 1x^2 + 2x - 15$

$$y = x^2 + 2x - 15$$

Solving quadratic equations using square root method

When the middle term of a quadratic equation is missing : $y = ax^2 + c$ the equation can be solved

Solving Equations with Square Roots

When the middle term is missing (or cancels), the equation can be solved using square roots. **Don't forget $\pm$ when taking the square root of both sides of an equation.**

Ex 1

$$2x^2 - 288 = 0$$
$$2x^2 = 288$$
$$x^2 = 144$$
$$x = \pm 12$$

Ex 2

$$\frac{1}{4}x^2 - 3 = 9$$
$$\frac{1}{4}x^2 = 12$$
$$x^2 = 48$$
$$x = \pm\sqrt{48}$$
$$x = \pm 4\sqrt{3}$$

Equations that look like $a(x-p)^2 + q = 0$ can also be solved with square roots

Ex 3

$$2(2x+1)^2 - 72 = 0$$
$$2(2x+1)^2 = 72$$
$$(2x+1)^2 = 36$$
$$2x + 1 = \pm\sqrt{36}$$
$$2x + 1 = \pm 6$$
$$2x + 1 = 6 \quad \text{or} \quad 2x + 1 = -6$$
$$2x = 5 \quad \text{or} \quad 2x = -7$$
$$x = 2.5 \quad \text{or} \quad x = -3.5$$

TRY – Solve Each Equation

a) $x^2 - 9 = 29$ b) $9b^2 = 64$ c) $25x^2 - 10 = 71$ d) $3n^2 - 6 = 78$ e) $-3(x+1)^2 + 27 = 0$

Exponential Functions

are functions which the variable occurs as part of the index or exponent.

the general form :

$$y = ab^{x}$$

exponent / index

base

a and b are numbers / constants

x must be an exponent

where the y-intercept of the exponential function is the initial value.

examples :

$y = 3^x$, $y = 2^{x-4}$, $y = 6 + 5^{-x}$

for $f(x) = 6 + 2^{-2x}$, find :

a) $f(0)$ $6 + 2^0 = 6 + 1 = 7$

b) $f(3)$ $6 + 2^{-6} = 6 + \frac{1}{2^6}$

c) $f(-x)$ $6 + 2^{-2(-x)} = 6 + 2^{2x}$

d) $f(2x)$ $6 + 2^{-2(2x)} = 6 + 2^{-4x} = 6 + \frac{1}{2^{4x}}$

Determine whether these are exponential functions or not :

a) 7^x yes

b) $10 \cdot 2^{\frac{x}{3}}$ yes

c) x^4 no

d) $-x^6 + 9x$ no

e) $5 - 3^{x-2}$ yes

f) $-2 - 5^{3x}$ yes

Exercise 1

1. Write each expression without brackets and with positive exponents.

a) $4xy^{-3} = 4x \cdot \frac{1}{y^3} = \frac{4x}{y^3}$

b) $\frac{15y^{-3}}{5y} = \frac{3y^{-3}}{y} = 3y^{-4} = \frac{3}{y^4}$

c) $(3x^3y)(5x^{-2}y^4) = 15xy^5$

d) $\frac{24p^{-8}}{16p^{-3}} = \frac{3}{2} \cdot \frac{1}{p^8 \cdot p^{-3}} = \frac{3}{2} \cdot \frac{1}{p^5} = \frac{3}{2p^5}$

e) $\frac{2}{a^{-1/3}} = 2\sqrt[3]{a^1} = 2\sqrt[3]{a} = 2a^{1/3}$

f) $(2x^{-2})^3 = 2^3(x^{-2})^3 = 8x^{-6} = \frac{8}{x^6}$

2) Find the exact value :

a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b) $27^{\frac{4}{3}} = (3^{\cancel{3}})^{\frac{4}{\cancel{3}}} = 3^4 = 81$

c) $\left(\frac{4}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{4}\right)^{3/2} = \left(\frac{3^{\cancel{2}}}{2^{\cancel{2}}}\right)^{3/\cancel{2}} = \left(\left(\frac{3}{2}\right)^{\cancel{2}}\right)^{3/\cancel{2}} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$ //

d) $125^{\frac{1}{3}} - 10^0(64)^{\frac{2}{3}} = (5^{\cancel{3}})^{1/\cancel{3}} - 1(4^{\cancel{3}})^{2/\cancel{3}} = 5^1 - 4^2$
$= 5 - 16$
$= -11$

e) $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = 16$

3. Evaluate for $a = 1$, $b = -2$, $c = 3$

a) $(a^{-2}b^{-4})(a^2b^{-5}) = \left(-\frac{1}{2}\right)^9$

(simplify first)

$a^{-2+2}b^{-4-5} = a^0b^{-9} = b^{-9} = (-2)^{-9}$
$= \left(\frac{-1}{2}\right)^9$
$= \frac{-1}{2^9} = \frac{-1}{512}$

b) $\frac{a^{-1}b^3c^{-2}}{abc} = \frac{4}{27}$

$\frac{b^2}{a^2\ c^3} = \frac{(-2)^2}{1^2\ 3^3} = \frac{4}{27}$

c) $\frac{a^{-1} + b^{-1}}{c^{-1} + c^{-2}} = \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{c} + \frac{1}{c^2}} = \frac{1 + \frac{1}{-2}}{\frac{1}{3} + \frac{1}{9}}$

$= \frac{\frac{1}{2}}{\frac{3}{9} + \frac{1}{9}} = \frac{1}{2} \div \frac{4}{9}$
$= \frac{1}{2} \times \frac{9}{4} = \frac{9}{8}$

Exponent Laws

Multiplication / Product $a^x \cdot a^y = a^{x+y}$

Division / Quotient $\frac{a^x}{a^y} = a^{x-y}$

Power / Brackets $(a^x)^y = a^{xy}$

Zero exponent $a^0 = 1$ as long as $a \neq 0$ (0^0 is undefined)

Negative powers $a^{-x} = \frac{1}{a^x}$ (minus turns into $\frac{1}{\ldots}$)

$(ab)^x = a^xb^x$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$\left(\frac{a}{b}\right)^{-x} = \frac{b^x}{a^x}$

Fractional Exponent $a^{\frac{x}{y}} = \sqrt[y]{a^x}$ ($a^{\frac{5}{6}} = \sqrt[6]{a^5}$)

4. Simplify the expressions (write positive exponents)

a) $\frac{x^5y^{-1}}{x^2y^{-4}} = x^3y^3$

b) $\frac{-a^{-2}(b^{-1})^2}{b^3(-a^4)^2} = \frac{-a^{-2}b^{-2}}{b^3a^8} = \frac{-1}{b^5a^{10}}$

c) $\left(\frac{5x^3}{2y^4}\right)^{-3} = \frac{8y^{12}}{125x^9}$

$= \left(\frac{2y^4}{5x^3}\right)^3 = \frac{8y^{12}}{125x^9}$

d) $(4m^2n)^{-1} \times 2mn^5 = \frac{n^4}{2m}$

$= \frac{1}{4m^2n} \cdot 2mn^5$

$= \frac{2mn^5}{4m^2n} = \frac{n^4}{2m}$

e) $\left(\frac{-8x^8y^5}{24x^2y}\right)\left(\frac{15x^3y^9}{18xy^5}\right)^2 = \left(\frac{-x^8y^5}{3x^2y}\right)\left(\frac{5x^3y^9}{6xy^5}\right)^2 = \left(\frac{-x^6y^4}{3}\right)\left(\frac{5x^2y^4}{36}\right)^2$

$= \left(\frac{-x^6y^4}{3}\right)\left(\frac{25x^4y^8}{36}\right) = \frac{-x^6y^4}{3} \cdot \frac{25x^4y^8}{36} = \frac{-25x^{10}y^{12}}{108}$

f) $\frac{3x^2y^0z^{-4}}{(2xyz)^3}$

$\frac{3x^2z^{-4}}{8x^3y^3z^3} = \frac{3}{8xy^3z^7}$

5. Convert each to the base indicated

a) 32^x to base 2

$(2^5)^x = 2^{5x}$ → exponent

↳ base

b) 81^{x-2} to base 3

3^{4x-8}

c) $\frac{1}{64^{2x}}$ to base 4

$\frac{1}{4^{6x}} = 4^{-6x}$

d) $\left(\frac{1}{16}\right)^{x+1}$ to base 2 $\frac{1}{2^{4x+4}} = 2^{-4x-4}$

e) $\left(\frac{25}{49}\right)^{3x}$ to base $\frac{5}{7}$ $\left(\frac{5}{7}\right)^{6x}$

f) $\left(\frac{27}{64}\right)^{x+2}$ to base $\frac{4}{3}$ $\frac{4^{-3x-6}}{3^{-3x-6}} = \left(\frac{4}{3}\right)^{-3x-6}$

6. Convert to the base indicated

a) $2 \cdot 4^x$ to base 2 $2 \cdot 2^{2x} = 2^{2x+1}$

b) $9 \cdot 27^{x-1}$ to base 3 $3^2 \cdot 3^{3x-3} = 3^{3x-1}$

c) $\frac{1}{4}\left(\frac{1}{16}\right)^{4-x}$ to base 4 $= \frac{1}{4} \cdot \left(\frac{1}{4^2}\right)^{4-x}$

$= \frac{1}{4}\left(\frac{1}{4}\right)^{8-2x} = \frac{1}{4^{9-2x}} = 4^{-9+2x}$

7. $(4x^{-3}y^5)^2 = \frac{16y^{10}}{x^6}$

8. $\frac{(2a^2b)^{-3}}{(ab^2)^{-4}}$

$= \frac{(ab^2)^4}{(2a^2b)^3} = \frac{a^4b^8}{8a^6b^3} = \frac{b^5}{8a^2}$

9) $(36x^{-4})^{-1/2} = \frac{x^2}{6}$

10) $\frac{(64p^2q^{-2/3})^{-1/2}}{(p^5q^{10})^{-1/5}} =$

$= \frac{(p^5q^{10})^{1/5}}{(8^2p^2q^{-2/3})^{1/2}} = \frac{\cancel{p}q^2}{8\cancel{p}q^{-1/3}} = \frac{q^{2+\frac{1}{3}}}{8} = \frac{q^{7/3}}{8}$

Exercise 2

Simplify

1) $(x^{-2}x^{-3})^4 = x^{-8}x^{-12}$
$= x^{-20} = \frac{1}{x^{20}}$

2) $(x^4)^{-3}2x^4 = x^{-12}.2x^4$
$= 2x^{-8} = \frac{2}{x^8}$

3) $(n^3)^3 2n^{-1} = n^9.2n^{-1} = 2n^8$

4) $(2v)^2.2v^2 = 4v^2.2v^2$
$= 8v^4$

5) $\frac{2x^2y^4.4x^2y^4.3x}{3x^{-3}y^2} = 8x^8y^6$

6) $\frac{2y^3.3xy^3}{3x^2y^4} = \frac{2y^2}{x}$

7) $\frac{x^3y^3.x^3}{4x^2} = \frac{x^4y^3}{4}$

8) $\frac{3x^2y^2}{2x^{-1}.4yx^2} = \frac{3xy}{8}$

9) $\frac{x}{(2x^0)^2} = \frac{x}{4}$

10) $\frac{2m^{-4}}{(2m^{-4})^3} = \frac{2m^{-4}}{8m^{-12}}$
$= \frac{m^8}{4}$

11) $\frac{(2m^2)^{-1}}{m^2} = \frac{2^{-1}m^{-2}}{m^2} = 2^{-1}m^{-4}$
$= \frac{1}{2m^4}$

12) $\frac{2x^3}{(x^{-1})^3} = 2x^6$

13) $\underbrace{(a^{-3}b^{-3})^0}_{1} = 1$

14) $x^4y^3.\underbrace{(2y^2)^0}_{1} = x^4y^3$

15) $ba^4.(2ba^4)^{-3} = \frac{1}{8b^2a^8}$

16) $(2x^0y^2)^{-3}.2yx^3 = \frac{x^3}{4y^5}$

17) $\frac{2k^3.k^2}{k^{-3}} = 2k^8$

18) $\frac{(x^{-3})^4.x^4}{2x^{-3}} = \frac{1}{2x^5}$

19) $\frac{(2x)^{-4}}{x^{-1}.x} = \frac{1}{16x^4}$

20) $\frac{(2x^3z^2)^3}{x^3y^4z^2.x^{-4}z^3} = \frac{8x^{10}z}{y^4}$

21) $\frac{(2pm^{-1}q^0)^{-4}.2m^{-1}p^3}{2pq^3} = \frac{2^{-4}p^{-4}m^4.\cancel{2}m^{-1}p^3}{\cancel{2}pq^3} = \frac{2^{-4}p^{-1}m^3}{pq^3} = \frac{2^{-4}p^{-2}m^3}{q^3} = \frac{m^3}{2^4p^2q^3} = \frac{m^3}{16p^2q^3}$ //

22) $\frac{(2hj^2k^{-2}.h^{-4}j^{-1}k^4)^0}{2.h^{-3}j^{-4}k^{-2}} = \frac{h^3j^4k^2}{2}$

Exercise 3 (Albert)

1. $\frac{(3^2)^{-4}}{3^8} = \frac{3^{-8}}{3^8} = \frac{1}{3^{16}}$

2. $x^5x^{-6}.x = x^0 = 1$

3. $(2y^4)^{-2} = \frac{1}{(2y^4)^2} = \frac{1}{4y^8}$

4. Simplify $\left(\frac{4x^{-3}y^2}{3x}\right)^{-2}.2y^2$

$\left(\frac{3x}{4x^{-3}y^2}\right)^2.2y^2$

$\frac{9x^2}{\cancel{16}x^{-6}\cancel{y^4}}.\cancel{2y^2} = \frac{9x^2}{8x^{-6}y^2} = \frac{9x^8}{8y^2}$

5.
a) $x^{\frac{1}{6}} = \sqrt[6]{a}$
b) $\sqrt{2x} = (2x)^{\frac{1}{2}}$
c) $20^{\frac{1}{4}} = \sqrt[4]{20}$

6. $y = \frac{1}{2}\left(\frac{5}{4}\right)^x$ → exponent
$r > 1$ growth
$r < 1$ decay
a
$r = 1.25$
$y = ar^x$ → ratio
initial number/value
growth
$r = 1.25$ means increases by 25%
formula : $r-1 = 1.25 - 1 = 0.25 = 25\%$

7. Exponential equation

$\left(\frac{1}{4}\right)^{5x} = 32^2$

$(2^{-2})^{5x} = (2^5)^2$

$2^{-10x} = 2^{10}$

$-10x = 10$

$x = -1$ //

8. Exponential Function

$y = 20.(2)^x$

$a = 20$ (initial number)

$r = 2$ (ratio) $r-1 = 1 = 100\%$

increase double → exponential growth

c,d)

$r-1 = 20\%$

$r-1 = 0.2$

$r = 1.2$

$a = 100$

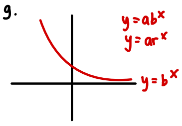
$r = 2$ 100 ⟶ 200 – 400 – 800 – 1600

increases by 100%

$r = 2$

$r-1 = 1 = 100\%$

9.



exponential decay

$b < 1$, $b > 0$

$0.333 = \frac{1}{3}$

0.625

10. $y = 3(1.3)^x$ → growth (graph here)

11. $y = 2.3^x$

$a = 2$ → initial value (y-int) (0,2)

$b = r = 3$

(graph here)

12. $\left(\frac{1}{2}\right)^{-2x+4} = 16^x$

$(2^{-1})^{-2x+4} = 2^{4x}$

$2^{2x-4} = 2^{4x}$

$2x - 4 = 4x$

$-4 = 2x$

$x = -2$ //

13. $y = 2(0.3)^x$

a → y-intercept (0,2)

$r = b = 0.3$ → exponential decay

The amount decreases by 70%, not 30%

exponential growth amount increase : $r-1$ %

decay decrease . $1-r$ %

14. $V = P(1-r)^t$

similar to $y = ab^x$

$V = 21000(1-0.16)^4$

$V = 21000(0.84)^4$

$V = 10,455.30$

15. $a = 117,000$ (1925)

decay: 2% /per year for 90 years

$y = ab^x$
↓
final population

IF:
$r = b = 0.2$
decay 98% per year

$y = 117,000(1-0.02)^{90}$

$y = 117,000(0.98)^{90}$

$y = 18990$

16. $y = 400(1.2)^x$

exponential growth

changes by 20% each year

Exercise 4

1. Simplify

a) $49^{x-1} \cdot 7^{2x-3}$

$= (7^2)^{x-1} \cdot 7^{2x-3}$

$= 7^{2x-2} \cdot 7^{2x-3}$

$= 7^{4x-5}$

b) $216^x \div (1296^{5x-4} \cdot 36^{x+5})$

$= 6^{3x} \div ((6^4)^{5x-4} \cdot (6^2)^{x+5})$

$= \dfrac{6^{3x}}{6^{20x-16} \cdot 6^{2x+10}} = \dfrac{6^{3x}}{6^{22x-6}} = 6^{3x-22x+6}$

$= 6^{-19x+6}$

c) $64^{3x} \cdot 128^{x-1} \div (32^{2x+3} \div 8^{4x-1})$

$\dfrac{(2^6)^{3x} \cdot (2^7)^{x-1}}{\left(\dfrac{(2^5)^{2x+3}}{(2^3)^{4x-1}}\right)} = \dfrac{2^{18x} \cdot 2^{7x-7}}{\dfrac{2^{10x+15}}{2^{12x-3}}} = \dfrac{2^{25x-7}}{2^{-2x+18}} = 2^{25x-7+2x-18}$

$= 2^{27x-25}$

2. Solve for x

a) $x^{\frac{1}{2}} = 5$

$(x^{\frac{1}{2}})^2 = 5^2 \rightarrow x = 25$

b) $x^{-\frac{1}{2}} = 5$

$(x^{-\frac{1}{2}})^{-2} = 5^{-2}$

$x = \frac{1}{25}$

c) $x^{\frac{1}{3}} = -5$

$(x^{\frac{1}{3}})^3 = (-5)^3$

$x = -125$

d) $4x^{-\frac{2}{3}} = 16$

$x^{-\frac{2}{3}} = 4$

$(x^{-\frac{2}{3}})^{-\frac{3}{2}} = 4^{-3/2}$

$x = \dfrac{1}{4^{3/2}} = \dfrac{1}{(2^2)^{3/2}} = \dfrac{1}{8}$

3. Solve for x:

a) $2^x = 16\sqrt{2}$

$2^x = 2^4 \cdot 2^{\frac{1}{2}}$

$2^x = 2^{\frac{9}{2}}$

$x = 9/2$

b) $2^{-x} = 64$ $\quad x = -6$

c) $9^{3x+1} = 27^{3x}$ $\quad x = \frac{2}{3}$

4. Solve for x

a) $\left(\frac{4}{7}\right)^{5x} = \left(\frac{64}{343}\right)^{2x-1}$ $\quad x = 3$

b) $\left(\frac{125}{216}\right)^{-\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

$\left(\frac{216}{125}\right)^{\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

$\left(\frac{6^3}{5^3}\right)^{\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

$\left(\frac{6}{5}\right)^{\frac{3x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

$\frac{3x}{2} = 3x+2$

$3x = 6x+4$

$-4 = 3x$

$x = -\frac{4}{3}$

c) $49\left(\frac{7}{12}\right)^{2x} = 144$

$\left(\frac{7}{12}\right)^{2x} = \frac{144}{49}$

$\left(\frac{7}{12}\right)^{2x} = \frac{12^2}{7^2}$

$\left(\frac{7}{12}\right)^{2x} = \left(\frac{12}{7}\right)^2$

$\left(\frac{7}{12}\right)^{2x} = \left(\frac{7}{12}\right)^{-2}$

$2x = -2$

$x = -1$

d) $\left(\frac{9}{4}\right)^{x+3} = \left(\frac{8}{27}\right)^{-5}$ $\quad x = \frac{9}{2}$

e) $2^{x-1} = 128^x \cdot 2^x$

$2^{x-1} = 2^{7x} \cdot 2^x$

f) $2(6)^{2x} - 74 \cdot 6^x + 72 = 0$

Hint: write as quadratic equation with 6^x as the variable

$2(6^x)^2 - 74 \cdot 6^x + 72 = 0$

let $a = 6^x$

$2a^2 - 74a + 72 = 0$

$a^2 - 37a + 36 = 0$

$(a-36)(a-1) = 0$

$a = 36$ or $a = 1$

$6^x = 36$ $\quad 6^x = 1$

$6^x = 6^2$ $\quad 6^x = 6^0$

$x = 2$ $\quad x = 0$

Exercise 5

1. $2m^2 \cdot 2m^3 = 4m^5$
2. $m^4 \; 2m^{-3} = 2m$
3. $4r^{-3} \cdot 2r^2 = 8r^{-1} = \frac{8}{r}$
4. $4n^4 \cdot 2n^{-3} = 8n$
5. $2k^4 \cdot 4k = 8k^5$
6. $2x^3y^{-3} \cdot 2x^{-1}y^3 = 4x^2$
7. $2y^2 \cdot 3x = 6xy^2$
8. $4v^3 \; vu^2 = 4v^4u^2$
9. $4a^3b^2 \cdot 3a^{-4}b^{-3} \quad \frac{12}{ab}$
10. $x^2y^{-4} \cdot x^3y^2 \quad \frac{x^5}{y^2}$
11. $(x^2)^0 = 1$
12. $(2x^2)^{-4} = \frac{1}{16x^8}$
13. $(4r^0)^4 = 256$
14. $(4a^3)^2 = 16a^6$
15. $(3k^4)^4 = 81k^{16}$
16. $(4xy)^{-1} = \frac{1}{4xy}$
17. $(2b^4)^{-1} = \frac{1}{2b^4}$
18. $(x^2y^{-1})^2 = \frac{x^4}{y^2}$
19. $(2x^4y^{-3})^{-1} = \frac{y^3}{2x^4}$
20. $(3m)^{-2} = \frac{1}{9m^2}$
21. $\frac{r^2}{2r^3} = \frac{1}{2r}$
22. $\frac{x^{-1}}{4x^4} = \frac{1}{4x^5}$
23. $\frac{3n^4}{3n^3} = n$
24. $\frac{m^4}{2m^4} = \frac{1}{2}$
25. $\frac{3m^{-4}}{m^3} = \frac{3}{m^7}$
26. $\frac{2x^4y^{-4}z^{-3}}{3x^2y^{-3}z^4} = \frac{2x^2}{3yz^7}$
27. $\frac{4x^0y^{-2}z^3}{4x} = \frac{z^3}{xy^2}$
28. $\frac{2h^3j^{-3}k^4}{3jk} = \frac{2h^3k^3}{3j^4}$
29. $\frac{4m^4n^3p^3}{3m^2n^2p^4} = \frac{4m^2n}{3p}$
30. $\frac{3x^3y^{-1}z^{-1}}{x^{-4}y^0z^0} = \frac{3x^7}{yz}$

Exercise 6

1. $4^{2x-1} = 8^{x-3}$
2. $4^{x+3} = 2^{x^2} \cdot 2^x$
3. $3^{6x-\frac{1}{2}} = 9\sqrt{3}$
4. $\sqrt{9^{x^2-2x}} = 27$
5. $(2^{x-1})^{\frac{1}{3}} \cdot 2^{-6+3x} = 2^{-x-2}$
6. $216 \cdot 36^{x^2-2x} = 216^x$
7. $5^{2x+3} \cdot 125^{x+2} = \frac{1}{(3^2+4^2)^{\frac{1}{2}}}$
8. $2^{2x+5} \cdot 3^{2x+5} = 216$
9. $\frac{(5^x)^x \cdot 5^{1-x}}{125} = \sqrt{(3^2-2^3)^5}$
10. $\frac{4^{\frac{1}{2}x}(2^x)^x}{2^5} = \frac{4^{\frac{4x+1}{2}}}{4^{-2}}$
11. $\frac{(3^x)^x \cdot (3^{-3})^{(x+1)}}{\frac{1}{27} \cdot 3^{-\frac{x}{2}}} = (2^2-1^2)^{\frac{3x}{2}} \cdot 81^3$
12. $2 \cdot 3^{2x+1} + 3^x = 1$
13. $\frac{2^{2x-3}}{2^{x-4}} + \frac{7}{2^{-\frac{x}{2}}} = 4$
14. $3^{2-x} - 82 \cdot 3^{-\frac{x}{2}} = -9$
15. $2 \cdot 2^x - \frac{2^{(x+1)} \, 5^{\frac{1}{3}}}{40^{\frac{1}{3}}} = (2^{\frac{1}{2}x} + 2^x)^0$
16. $4^{3x-3} - 9 \cdot 2^{2x-3} + 8 = 0$
17. $(x^2-9x+19)^{2x+3} = (x^2-9x+19)^{x-1}$
18. $(x^2-4)^{2x-5} = (11-2x)^{2x-5}$
19. $(x^2-3)^{2x^2-7x-4} = 1$

Exercise 6 answers:

1. $4^{2x-1} = 8^{x-3}$

$2^{2(2x-1)} = 2^{3(x-3)}$

$4x-2 = 3x-9$

$x = -7$

2. $4^{x+3} = 2^{x^2} \cdot 2^x$

$2^{2(x+3)} = 2^{x^2+x}$

$2x+6 = x^2+x$

$0 = x^2+x-2x-6$

$0 = x^2-x-6$

$0 = (x-3)(x+2)$

$x=3$ or $x=-2$

3. $3^{6x-\frac{1}{2}} = 9\sqrt{3}$

$3^{6x-\frac{1}{2}} = 3^2 \cdot 3^{\frac{1}{2}}$

$3^{6x-\frac{1}{2}} = 3^{\frac{5}{2}}$

$6x - \frac{1}{2} = \frac{5}{2}$

$6x = \frac{6}{2}$

$6x = 3$

$x = \frac{1}{2}$

4. $\sqrt{9^{x^2-2x}} = 27$

$[(3^{\cancel{2}})^{x^2-2x}]^{\frac{1}{\cancel{2}}} = 3^3$

$3^{x^2-2x} = 3^3$

$x^2-2x-3 = 0$

$(x-3)(x+1) = 0$

$x=3$ or $x=-1$

OR

$9^{x^2-2x} = (3^3)^2$

$3^{2x^2-4x} = 3^6$

$2x^2-4x-6=0$

$x=3$ or $x=-1$

5. $(2^{x-1})^{\frac{1}{3}} \cdot 2^{-6+3x} = 2^{-x-2}$

$2^{\frac{1}{3}x-\frac{1}{3}-6+3x} = 2^{-x-2}$

$2^{\frac{10}{3}x-\frac{19}{3}} = 2^{-x-2}$

$\frac{10}{3}x - \frac{19}{3} = -x-2$

$\frac{13}{3}x = \frac{19}{3} - 2$

$\frac{13}{3}x = \frac{19}{3} - \frac{6}{3}$

$\frac{13}{3}x = \frac{13}{3}$

6. $216 \cdot 36^{x^2-2x} = 216^x$

$6^3 \cdot (6^2)^{x^2-2x} = (6^3)^x$

$6^3 \cdot 6^{2x^2-4x} = 6^{3x}$

$6^{2x^2-4x+3} = 6^{3x}$

$2x^2-4x+3 = 3x$

$2x^2-7x+3 = 0$

$(2x-1)(x-3) = 0$

$x = \frac{1}{2}$ or $x=3$

7. $5^{2x+3} \cdot 125^{x+2} = \frac{1}{(3^2+4^2)^{\frac{1}{2}}}$

$5^{2x+3} \cdot 5^{3(x+2)} = \frac{1}{5}$

$5^{2x+3+3x+6} = 5^{-1}$

$5^{5x+9} = 5^{-1}$

$5x+9 = -1$

$5x = -10$

$x = -2$

8. $2^{2x+5} \cdot 3^{2x+5} = 216$

$a^c \cdot b^c = (ab)^c$

$(2 \cdot 3)^{2x+5} = 6^3$

$6^{2x+5} = 6^3$

$2x = -2$

$x = -1$

9. $\frac{(5^x)^x \cdot 5^{1-x}}{125} = \sqrt{(3^2-2^3)^5}$

$\frac{5^{x^2} \cdot 5^{1-x}}{5^3} = \sqrt{(9-8)^5}$

$5^{x^2+1-x-3} = 1$

$5^{x^2-x-2} = 5^0$

$x^2-x-2 = 0$

$(x-2)(x+1) = 0$

$x=2$ or $x=-1$

10. $\frac{4^{\frac{1}{2}x}(2^x)^x}{2^5} = \frac{4^{\frac{4x+1}{2}}}{4^{-2}}$

$(2^2)^{\frac{x}{2}} \cdot 2^{x^2} \cdot 2^{-5} = (2^2)^{\frac{4x+1}{2}} \cdot 4^2$

$2^x \cdot 2^{x^2} \cdot 2^{-5} = 2^{4x+1} \cdot 2^4$

$2^{x^2+x-5} = 2^{4x+5}$

$x^2+x-5 = 4x+5$

$x^2+x-4x-10 = 0$

$x^2-3x-10 = 0$

$(x-5)(x+2) = 0$

$x=5$ or $x=-2$

11. $\frac{(3^x)^x \cdot (3^{-3})^{(x+1)}}{\frac{1}{27} \cdot 3^{-\frac{x}{2}}} = (2^2-1^2)^{\frac{3x}{2}} \cdot 81^3$

$\frac{3^{x^2} \cdot 3^{-3x-3}}{3^{-3} \cdot 3^{-\frac{x}{2}}} = 3^{\frac{3x}{2}} \cdot (3^4)^3$

$\frac{3^{x^2-3x-3}}{3^{-3-\frac{x}{2}}} = 3^{\frac{3x}{2}+12}$

$3^{x^2-3x-\cancel{3}-(-\cancel{3}-\frac{x}{2})} = 3^{\frac{3x}{2}+12}$

$3^{x^2-3x+\frac{x}{2}} = 3^{\frac{3x}{2}+12}$

$x^2-3x+\frac{x}{2} = \frac{3x}{2}+12$

$2x^2-6x+x = 3x+24$

$2x^2-8x-24 = 0$

$x^2-4x-12 = 0$

$(x-6)(x+2) = 0$

$x=6$ or $x=-2$

12. $2 \cdot 3^{2x} \cdot 3 + 3^x = 1$

$6 \cdot 3^{2x} + 3^x = 1$

$6 \cdot (3^x)^2 + 3^x - 1 = 0$

Let $3^x = a$

$6a^2 + a - 1 = 0$

$(2a+1)(3a-1) = 0$

$a = -\frac{1}{2}$ or $a = \frac{1}{3}$

$3^x = -\frac{1}{2}$ (neglected because exponent can't result in a negative number)

$3^x = 3^{-1}$

$x = -1$ //

Transforming Exponential Functions ($y = ab^{x \pm h} \pm k$)

The exponential function $y = ab^{x \pm h} \pm k$ is a combination of transformation of standard exponent function $y = b^x$.

$y = b^x \xrightarrow{}$ $y = ab^x \xrightarrow{}$ $y = ab^{x \pm h} \xrightarrow{}$ $y = ab^{x \pm h} \pm k$

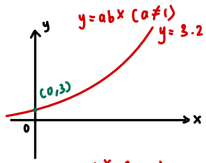
- multiply the graph by a factor a
- Shift the graph to the:
  - left if $x+h$
  - right if $x-h$
  - by h units
- Shift the graph:
  - up if $+k$
  - down if $-k$
  - by k units

General Form: $y = ab^x$

If $b > 1$ exponential growth

$y = ab^x$ $(a \neq 1)$

$y = 3 \cdot 2^x$

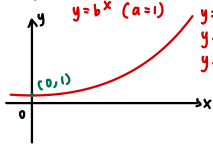


$y = b^x$ $(a = 1)$

$y = 2^x = 1 \cdot 2^x$

$y = 3^x = 1 \cdot 3^x$

$y = 1.25^x = 1 \cdot 1.25^x$

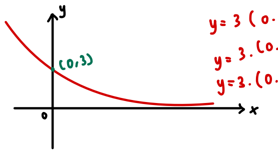


If $b < 1$ (exponential decay)

$y = 3(0.64)^x$

$y = 3 \cdot (0.725)^x$

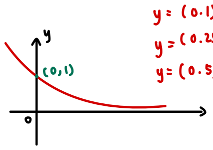
$y = 3 \cdot (0.725)^x$



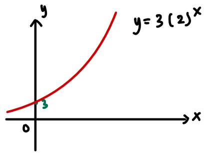
$y = (0.1)^x$

$y = (0.25)^x$

$y = (0.5)^x$

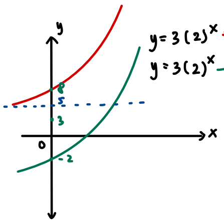


$y = 3(2)^x$

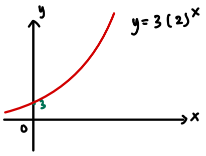


$y = 3(2)^x + 5$

$y = 3(2)^x - 5$



$y = 3(2)^x$

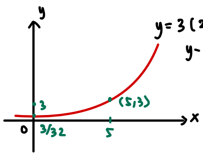


$y = 3(2)^{x-5}$

y-int : $x = 0$

$y = 3 \cdot 2^{-5}$

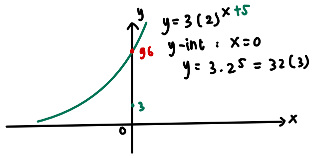
$y = \frac{3}{2^5} = \frac{3}{32}$



$y = 3(2)^{x+5}$

y-int : $x = 0$

$y = 3 \cdot 2^5 = 32(3) = 96$



Graph : $y = 5 \cdot (\frac{1}{2})^{x-3} + 2$

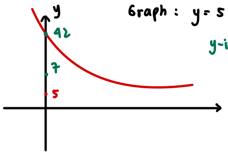
y-int : $x = 0$

$y = 5(\frac{1}{2})^{0-3} + 2$

$y = 5(\frac{1}{2})^{-3} + 2$

$y = 5 \cdot (2)^3 + 2$

$y = 40 + 2 = 42$



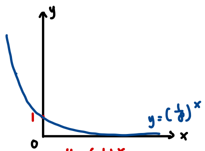
Finding Domain, Range, and Horizontal Asymptote of Graph

- The domain (Df) of $y = ab^x$ is $x \in R$ (all real numbers)
  range (Rf) of $y = ab^x$ is $y > 0$ with horizontal asymptote $y = 0$
- The range (Rf) of $y = ab^x + k$ is $y > k$ with horizontal asymptote $y = k$
  $y = ab^x - k$ is $y > -k$ with horizontal asymptote $y = -k$
- The range of $y = ab^{x \pm h}$ is just $y > 0$ with horizontal asymptote $y = 0$

Exercise: Find the range and the horizontal asymptote:

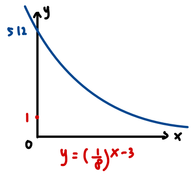
1) $y = 2^x$ Rf: $y > 0$ Df: $x \in R$; horizontal asymp: $y = 0$
2) $y = 2^{x+1}$ Rf: $y > 0$ horizontal asymp: $y = 0$
3) $y = 2^{x-1}$ Rf: $y > 0$ horizontal asymp: $y = 0$
4) $y = 2^x + 2$ Rf: $y > 2$ horizontal asymp: $y = 2$
5) $y = 2^x - 2$ Rf: $y > -2$ h.a: $y = 2$
6) $y = 3^x$ Rf: $y > 0$ h.a: $y = 0$
7) $y = (\frac{1}{8})^{x-3} + 2$ Rf: $y > 2$ h.a: $y = 2$
8) $y = 0.25^{x+3} + 8$ Rf: $y > 8$ h.a: $y = 8$
9) $y = 125^{x-10} + 9$ Rf: $y > 9$ h.a: $y = 9$
10) $y = 2.5^x - 3$ Rf: $y > -3$ h.a: $y = -3$

Graph of No. 7 $y = (\frac{1}{8})^{x-3} + 2$



$y = (\frac{1}{8})^x$

y-int : $x=0$

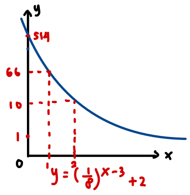


$y = (\frac{1}{8})^{x-3}$

y-int : $x=0$

$y = (\frac{1}{8})^{0-3} = (\frac{1}{8})^{-3}$

$= (8)^3 = 512$



$y = (\frac{1}{8})^{x-3} + 2$

$x=0$ $y = 514$

$x=1$ $y = (\frac{1}{8})^{1-3} + 2$

$y = (\frac{1}{8})^{-2} + 2$

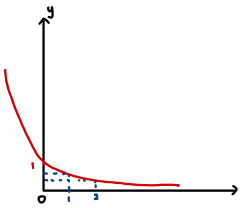
$y = 8^2 + 2 = 66$

$x=2$ $y = (\frac{1}{8})^{2-3} + 2$

$y = (\frac{1}{8})^{-1} + 2$

$y = 8 + 2 = 10$

8) $y = 0.25^{x+3} + 8$

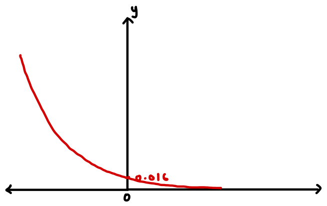


$y = 0.25^x$

$x=1$ $y = 0.25$

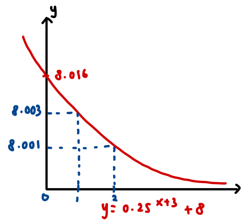
$x=2$ $y = 0.0625$

$x=3$ $y = 0.015625$



$y = 0.25^{x+3}$

y-int : $x=0$ $y = 0.25^3$



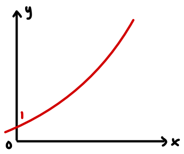
$y = 0.25^{x+3} + 8$

$x=0$ $y = 8.016$

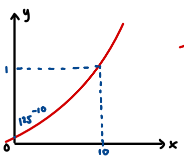
$x=1$ $y = 0.25^4 + 8 = 8.003$

$x=2$ $y = 0.25^5 + 8 = 8.001$

9) $y = 125^{x-10} + 9$



$y = 125^x$



$y = 125^{x-10}$

$x=0 \rightarrow y = \frac{1}{125^{10}}$

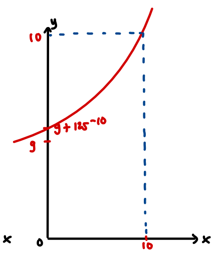
$x=1 \rightarrow y = \frac{1}{125^9}$

$x=10 \rightarrow y = 125^{10-10}$

$y = 1$

$x=12 \rightarrow y = 125^{12-10}$

$y = 125^2 = \ldots$



$y = 125^{x-10} + 9$

$x=10 \rightarrow y = 125^{10-10} + 9$

$y = 9 + 1 = 10$

Exponential Word Problems (Simple and Compound Interest)

(Exponential Growth and Decay)

(Solve for an unknown - initial, final amount and interest rate for compound interest problems - for different computing periods (months, annually, weekly, daily)