Part 1: Scarcity and Choice Exercises

ECON 50 Stanford University

1. For each of the following utility functions and PPFs, use the **Lagrange** method to solve for the optimal consumption bundle. Set up a Lagrangian and take all three first-order conditions, then solve. Then, it will be sufficient to just use the **tangency condition** and the constraint condition to solve for these problems. Finally, plot the PPF, show the optimal consumption bundle, and sketch the indifference curve passing through that bundle.

   a. Utility function: $u(x_1, x_2) = (x_1^{-2} - x_2^{-2})^{-1}$
      PPF equation: $3x_1 + 5x_2 = 360$

   b. Utility function: $u(x_1, x_2) = \sqrt[3]{x_1} x_2^{2/3}$
      PPF equation: $x_1 + 2x_2 = 140$

   c. Utility function: $5x_1 + 2x_2$
      PPF equation: $x_1^2 + x_2^2 = 120$

   d. Utility function: $81x_1^2 + x_2^2$
      PPF equation: $4x^2 + x_2^2 = 9$

   e. Utility function: $8x_1^2 + 2x_2$
      PPF equation: $x_1^2 + x_2^2 = 1$

   f. Utility function: $x_1 x_2$
      PPF equation: $x_1 + x_2 = 200$

   g. Utility function: $12 \ln x_1 + 5x_2$
      PPF equation: $x_1 + 3x_2 = 25$

2. a. Julian spends all of his food budget on pizza and milk. Julian's utility function is

   $$u(x_1, x_2) = x_1 x_2$$

   where x_1 is the units (quantity) of pizza and x_2 is the quantity of milk (measured in glass). Julian's monthly budget is \$120. The production function is defined as

   $$x_1 = f_1(L_1) = 3L_1$$
   $$x_2 = f_2(L_2) = 2L_2$$

   Find Julian's optimal consumption bundle using the Lagrange method. What is the maximum number of utility Julian can achieve?

   b. Julian now wants to find the minimum amount of money that he needs to spend to reach a utility level of 3750. PPF and the utility function are the same as before. Find the necessary bundle to reach a utility level of 3750.

3. a. The satiation or bliss point of a utility function **(must)/(must not)/(may or may not)** lie in the production possibilities set.

   b. The gravitational pull argument holds that if the MRS>MRT, the economic agent can do better by moving to the **(right)/(left)** along the PPF.

   c. The gravitational pull argument holds that if the MRS<MRT, the economic agent can do better by moving to the **(right)/(left)** along the PPF.

   d. The value of the MRS of an indifference curve **(must)/(must not)/(may or may not)** be defined. Further, The value of MRS **(must)/(must not)/(may or may not)** be **(positive)/(negative)**.

e. In the case of perfect complements with strictly positive prices, the optimal bundle **(must)/(must not)/(may or may not)** be located at the 90 degree kink.

4. Suppose Roger can produce rice box (good 1) and ramen bowl (good 2) according to the production functions:

$$x_1 = f_1(L_1) = 8\sqrt{L_1}$$
$$x_2 = f_2(L_2) = 4\sqrt{L_2}$$

He devotes 36 hours of labor to producing these two goods.

a. Derive the MRT and draw the PPF with clearly labeled axes.

b. Suppose Roger's preferences were given by the utility function

$$u(x_1, x_2) = \min\{2x_1, 6x_2\}$$

What is the MRS of this utility function? What is the MRS at the endpoints of his PPF?

c. Find Roger's optimal bundle. Draw his PPF, show his optimal bundle, and his indifference curve passing through that optimal bundle.

5. Michelle loves eating blueberry pies and chocolate tarts. Her preferences over these two goods are **strictly monotonic** and **strictly convex**. Michelle is choosing between the following bundles:

Bundle	Blueberry Pies	Chocolate Tarts
A	2	3
B	3	4
C	3	1
D	4	3
E	1	4
F	3	2

(a) List the bundles from her most preferred to her least preferred. If there is any potential case where there is no enough information to rank the bundles, indicate it/them.

(b) Suppose Michelle tells you she's indifferent between bundles E and F. Using this information, sketch a possible indifference map showing the indifference curves passing through each of the six bundles.