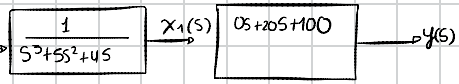
Tarea 2 Vide 1 Julian Moreno 2020100S114

Ejemplo 12.1 Control Systems.

$$G(s)=\frac{20(s+5)}{(s+1)(s+4)}=\begin{cases}\text{Overshoot } 05\%\ 1.5\% \\ t_s=0.745\text{seg}\end{cases}$$



$$\frac{X_1(S)}{U(S)}=\frac{1}{S^3+5S^2+4S} \rightarrow (S^3+5S^2+4S)X_1(S)=U(S) \quad \text{con} \quad x_1=x_1,\ x_2=\dot{x}_1,\ x_3=\dot{x}_1,\ \dot{x}_3=\ddot{x}_1$$

$$\ddot{x}_1+5\ddot{x}_1+4\dot{x}_1=u$$

$$\dot{x}_3=-5x_3-4x_2+u \quad (1)$$

$$Y(S)=b_2S^2+b_1S+b_0(X_1(S))$$

$$=(0S^2+20S+100)X_1(X_1(S)$$

$$Y(S)=(20S+100)X_1(S)$$

$$y=10x_2+100x_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&-4&-5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$y=\begin{bmatrix}100&20&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$\dot{x}_3=-4x_2-5x_3+u$$

$$\rightarrow(-k_3x_3-k_2x_2-k_1x_1+5)$$

$$\dot{x}_3=-k_1x_1-(4+k_2)x_2-(5+k_3)x_3+5$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-k_2&-(4+k_2)&-(5+k_3)\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}5$$

$$\text{Overshoot}=e^{\left(-\pi\zeta/\sqrt{1-\zeta^2}\right)}\times 100$$

$$\ln(0.095)=\ln\left(e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}\right)$$

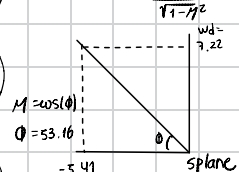
$$-2.3539=-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$-2,3539\left(\sqrt{1-\zeta^2}\right)^2=(-\zeta\pi)^2$$

$$5,5407(1-\zeta^2)=\zeta^2\pi^2$$

$$5.5407-5.5407\zeta^2=\zeta^2\pi^2$$

$$\zeta^2=\frac{5.5407}{\pi^2+5.5407}=0.5496$$



$$\zeta=\sigma/\omega d \quad 0$$

$$\phi=\cos^{-1}(0.5496)=53.16^\circ$$

$$t_s=\frac{4}{\sigma} \quad 0.11=\frac{4}{\sigma}$$

$$\sigma=5.41$$

$$\det(sI-(A-BK))=S^3+(5+k_3)S^2+(4+k_2)S+k_1=0$$

$$(S+5.4-j7.2)(S+5.4+j7.2)(S+5.1)=T$$

$$T=S^3+(5.9)S^2+136.225+413.83=0$$

$$S^3+(5+k_3)S^2+(4+k_2)S+k_1=S^3+15.95S^2+136.22S+413.83$$

$$(5+k_3)S^2=15.95S^2 \qquad (4+k_2)S=136.22S \qquad k_1=413.83$$

$$5+k_3=15.95 \qquad 4+k_2=136.22$$

$$k_3=10.9 \qquad k_2=132.22$$

$$\sigma=\zeta\omega n$$

$$5.41=0.5496\,\omega n$$

$$\omega n=9.02$$

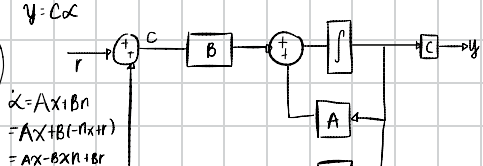
$$\tan\theta=\frac{\omega d}{5.41}$$

$$\omega d=\tan(53.16)\,5.41=7.22$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-4138&-132.22&-15.9\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}T$$

$$\dot{X}=AX+BX$$

$$Y=CX$$

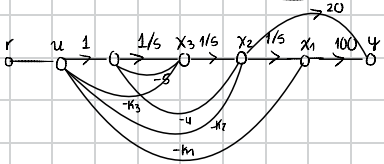


$$\dot{X}=AX_1Bn$$

$$=AX+B(-nx+r)$$

$$=Ax-BXn+Br$$

$$\dot{X}=(A-Bn)+Br)$$



$$Y=\begin{bmatrix}100&20&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} \qquad T(S)=\frac{20S+100}{S^3+15.95^2+136.22S+413.8}$$