Diagrama de Flujo de señal

Julian Santiago M   20201005114

① $G(s) = \dfrac{4}{s^3+2s^2+s+3}$

$\dfrac{x(s)}{u(s)} = \dddot{x} + 2\ddot{x} + \dot{x} + 3x$

$u(s) = (s^3+2s^2+s+3)\,x(s)$

$u = \dddot{x} + 2\ddot{x} + \dot{x} + 3x$

$\dot{x}_3 + 2x_3 + x_2 + 3x_1$

$\dot{x}_3 = -2x_3 - x_2 - 3x_1 - 4u$

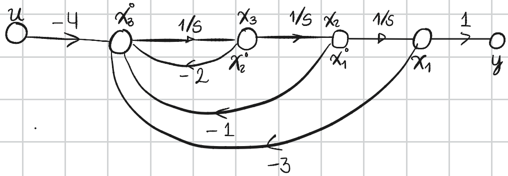
$x_1 = x$

$x_2 = \dot{x}_1 = \dot{x}$

$x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{x}$

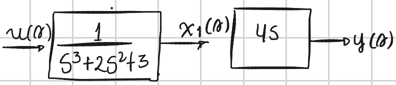
$\cancel{x_4} = \dot{x}_3 = \ddot{x}_2 = \dddot{x}_1 = \dddot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} u$$



$$x_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

② $G(s) = \dfrac{4s}{s^3+2s^2+3}$



$y = 4s(x_1)$

$4\dot{x}$

$y = 4x_2$

$$y = \begin{bmatrix} 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

BONO!

$\dfrac{x_1(s)}{u(s)} = \dfrac{1}{s^3+2s^2+3}$

$(x_1(s))(s^3+2s^2+3) = u(s)$

$\mathcal{L}^{-1}(s^3x_1 + 2s^2x_1 + 3x_1) = u(s)$

$\dddot{x} + 2\ddot{x} + 3x = u$

$\dddot{x} = -2\ddot{x} - 3x + u$

$\dot{x}_3 = -2x_3 - 3x_1 + u$

$x_1 = x$

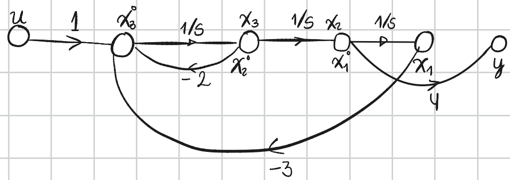
$x_2 = \dot{x}_1 = \dot{x}$

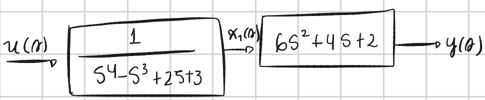
$x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{x}$

$\cancel{x_4} = \dot{x}_3 = \ddot{x}_2 = \dddot{x}_1 = \dddot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

③ $G(s) = \dfrac{6s^2+4s+2}{s^4 - s^3 + 2s + 3}$





$\dfrac{x_1(s)}{u(s)} = s^4 - s^3 + 2s + 3$

$(s^4 - s^3 + 2s + 3)\,x_1(s) = u($

$\mathcal{L}^{-1}$

$\ddddot{x}_1 - \dddot{x}_1 + 2\dot{x}_1 + 3x_1 = u(s)$

$\ddddot{x}_1 = \dddot{x}_1 - 2\dot{x}_1 - 3x_1 - u$

$\dot{x}_4 = x_4 - 2x_2 - 3x_1 - u$

$x_1 = x$

$x_2 = \dot{x}_1 = \dot{x}$

$x_3 = \dot{x}_2 = \ddot{x}_1 = \ddot{x}$

$x_4 = \dot{x}_3 = \ddot{x}_2 = \dddot{x}_1 = \dddot{x}$

$\cancel{x_5} = \dot{x}_4 = \ddddot{x}$

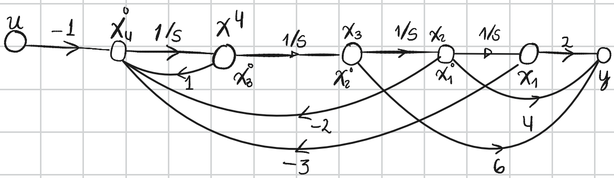
$\dfrac{y}{x_1(s)} = 6s^2 + 4s + 2$

$y(s) = (6s^2x_1 + 4sx_1 + 2x_1)\mathcal{L}^{-1}$

$y = 6\ddot{x} + 4\dot{x} + 2x$

$y = 6x_3 + 4x_2 + 2x_1$

$$y = \begin{bmatrix} 2 & 4 & 6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

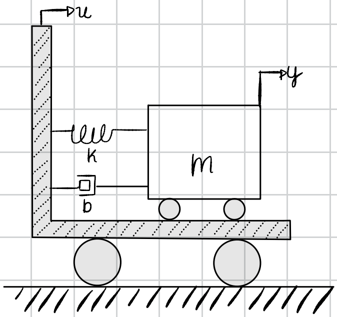


$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} u$$

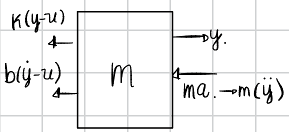
Punto 1 Parcial   Julian Santiago Moreno Cucaita.   20201005114

Ejemplo 3.3 Libro Ogata 5^{ta} edición

Sistema masa-resorte amortiguador



2^{da} ley newton $ma=\sum f$



$ma=-b(\dot{y}-u)-k(y-u)$

$m\ddot{y}=-b\dot{y}+bu-ky+ku$

Espacio de estados:

$\ddot{y}=\frac{-b\dot{y}}{m}+\frac{b\dot{u}}{m}-\frac{ky}{m}+\frac{ku}{m}$

$\ddot{y}=-\frac{b\dot{y}}{m}-\frac{ky}{m}+\frac{b\dot{u}}{m}+\frac{ku}{m}$

$q_1=y$

$q_2=\dot{y}=\dot{q}_1$

$\cancel{q_3}=\ddot{y}=\dot{q}_2$

$\dot{q}_2=\frac{-bq_2}{m}-\frac{kq_1}{m}$

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix}+\begin{bmatrix}\frac{b}{m}\\ \left[\frac{k}{m}-\left(\frac{b}{m}\right)^2\right]\end{bmatrix}u$$

Salida $y=[1\ 0]\begin{bmatrix}q_1\\ q_2\end{bmatrix}$

ecuación 2.35 $\beta_{n-1}=b_{n-1}-a_1\beta_{n-2}-\dots-a_{n-2}\beta_1-a_{n-1}\beta_0$

Por ecuacio diferencial.

$\ddot{y}=\frac{b}{m}\dot{y}+\frac{k}{m}y=\frac{b}{m}\dot{u}+\frac{k}{m}u$

$\frac{b}{m}=a_1$ $\frac{k}{m}=b_1$ $c_1=0$ $a_2=\frac{b}{m}$ $b_2=\frac{k}{m}$ $c_2=0$

$\beta_0=c_1=0$

$\beta_1=a_2-a_1\cancel{\beta_0}^0=\frac{b}{m}$

$\beta_2=b_2-a_1\beta_1-b_1\cancel{\beta_0}^0=\frac{k}{m}-\left(\frac{b}{m}\right)^2$

de 2.34

$x_1=y-\cancel{\beta_0 u}^0=y$

$x_2=\dot{y}-\cancel{\beta_0\dot{u}}^0-\beta_1 u=\dot{x}_1-\beta_1 u=\dot{q}_1-\left(\frac{b}{m}\right)u$

de 2.36

$\dot{q}_1=q_2+\beta_1 u=q_2 \quad =\frac{b}{m}u$

$\dot{q}_2=-b_1q_1-a_1q_2+\beta_2u=-\frac{k}{m}q_1-\frac{b}{m}q_2+\left[\frac{k}{m}-\left(\frac{b}{m}\right)^2\right]u$

$\dot{q}_2=-\frac{k}{m}q_1-\frac{b}{m}q_2+\left[\frac{k}{m}-\left(\frac{b}{m}\right)^2\right]u$

Diagrama de bloques.

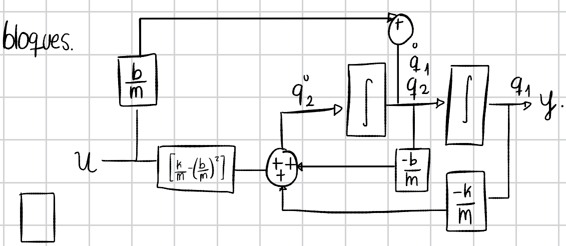
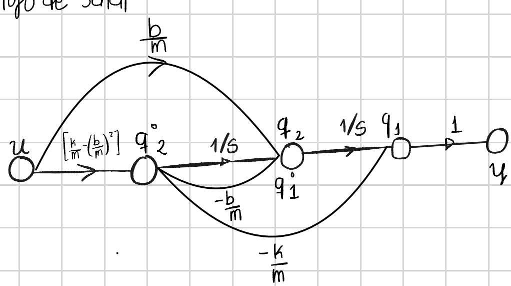


Diagrama de Flujo de señal



ejercicio 2 Ogata → A.3.9.

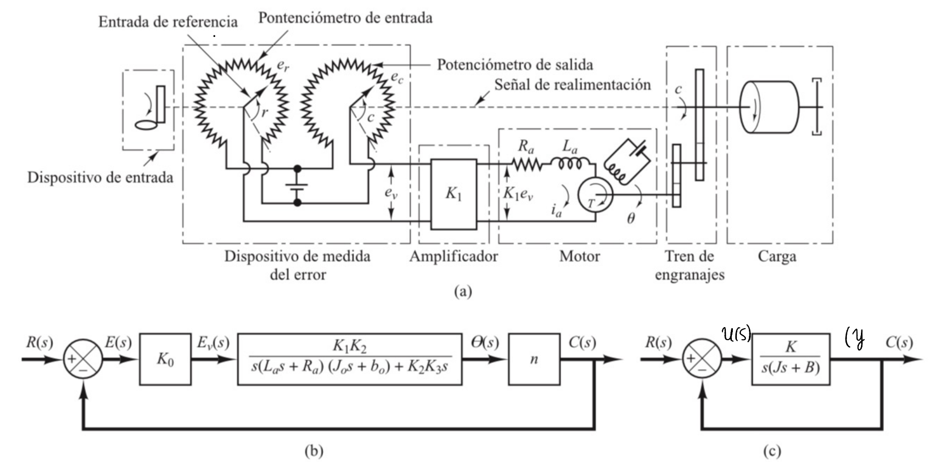


Figura 3-29. (a) Diagrama esquemático de un servo sistema; (b) diagrama de bloques del sistema; (c) diagrama de bloques simplificado.

Ecuación diferencial para el circuito de Inducido:

Función de transferencia del diagrama de bloque simplificado.

$$G(s) = \frac{K}{Js^2+Bs}$$

$$\frac{C(s)}{u(s)} = \frac{K}{Js^2+Bs} = \frac{y}{u}$$

$$y(Js^2+Bs) = Ku$$

$$\mathcal{L}^{-1}\{Js^2y + Bsy\} = Ku$$

$$J\ddot{y} + B\dot{y} = Ku$$

$$x_1 = y$$

$$x_2 = \dot{y} = \dot{x}_1$$

$$x_3 = \ddot{y} = \dot{x}_2$$

$$J\ddot{y} = Ku - B\dot{y}$$

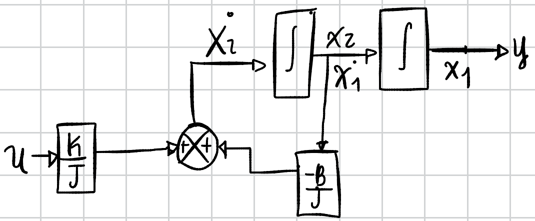
$$\ddot{y} = \frac{Ku}{J} - \frac{B\dot{y}}{J}$$

$$\dot{x}_2 = \frac{Ku}{J} - \frac{Bx_2}{J}$$

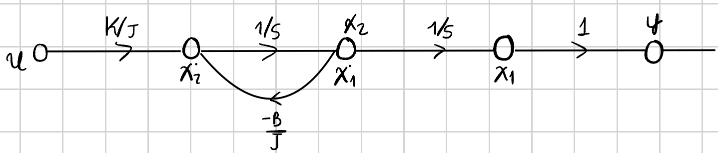
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K}{J} \end{bmatrix} u$$

$$y = [1 \; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

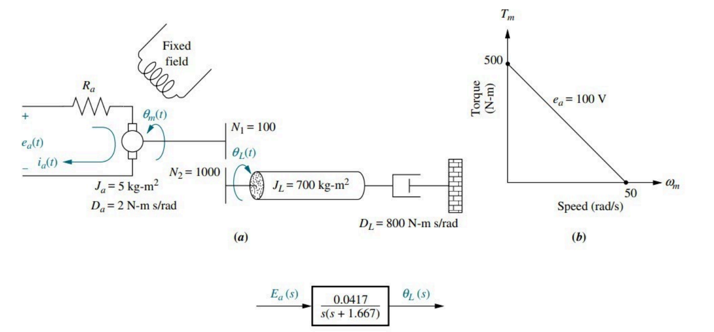
diagrama de bloques



Flujo



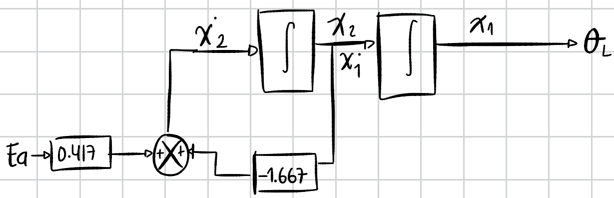
Ejemplo 3.



Con la función de transferencia expresion en el Espacio de estados

$$\frac{\theta_L(s)}{E_a(s)} = \frac{0.0417}{s(s+1.667)} = \frac{0.0417}{s^2+1.667s} \longrightarrow$$

Diagrama de bloques.



$$\theta_L(s)(s^2+1.667s) = 0.0417\,E_a(s)$$

$$\mathcal{L}^{-1}\{\theta_L s^2 + 1.667 s\theta_L\} = 0.0417 E_a$$

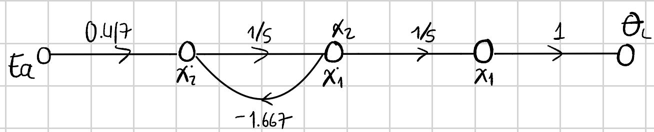
$$\ddot{\theta}_L + 1.667\,\dot{\theta}_L = 0.0417 E_a$$

$$x_1 = \theta_L$$

$$x_2 = \dot{\theta}_L = \dot{x}_1$$

$$\cancel{x_3} = \ddot{\theta}_L = \dot{x}_2$$

Diagrama de Flujo de señal



$$\ddot{\theta}_L = 0.0417 E_a - 1.667\,\dot{\theta}_L$$

$$\dot{x}_2 = 0.0417 E_a - 1.667 x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1.667 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.417 \end{bmatrix} E_a$$

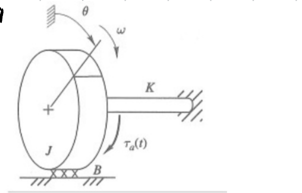
$$\theta_L = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

4 Conclusiones:

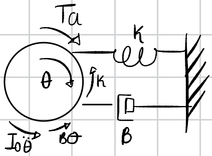
El modelo en ambos muestra una funcion de tranferencia del mismo formato, sin embargo solo uno es adaptable a cambios en el sistema ya que usa variables en lugar de valores puntuales. haciendo facil la emulación de diferentes escenarios. pero más alla del eso el planteamiento es igual. evidenciado en sus diagramas.
ambos son de segundo orden en su numerador. lo que da lugar a 2 variables de estado., ambos son la simplificación.

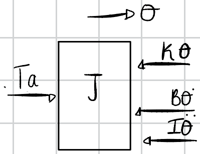
Julian Santiago Moreno Cucaita

20201005114.



① Punto





$$\mathcal{L}\{T_a = I_0\ddot{\theta} + B\dot{\theta} + K\theta\} \longrightarrow T_a(s) = I_0 s^2\theta + Bs\theta + K\theta$$

$$T_a(s) = \theta(s)\,(I_0 s^2 + Bs + K)$$

$$\frac{T_a(s)\,1}{(I_0 s^2 + Bs + K)} = \theta(s)$$

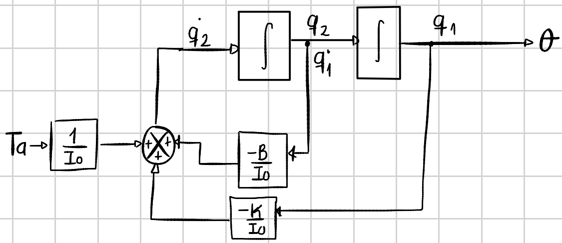
$$\frac{1}{(I_0 s^2 + Bs + K)} = \frac{\theta(s)}{T_a(s)}$$

$$\boxed{\frac{1}{(I_0 s^2 + Bs + K)} = \frac{\theta(s)}{T_a(s)}}$$

$$T_a - I_0\ddot{\theta} - B\dot{\theta} - K\theta = 0$$

$$T_a = +I_0\ddot{\theta} + B\dot{\theta} + K\theta$$

$$-I_0\ddot{\theta} = -T_a + B\dot{\theta} + K\theta$$



$$q_1 = \theta$$

$$q_2 = \dot{\theta} = \dot{q}_1$$

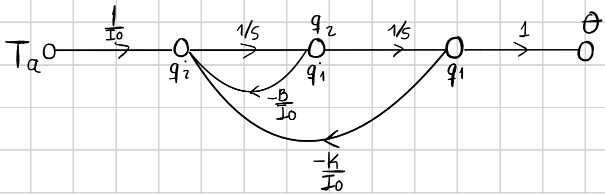
$$\cancel{q_3} = \ddot{\theta} = \dot{q}_2$$

$$-I_0\dot{q}_2 = -T_a + Bq_2 + Kq_1$$

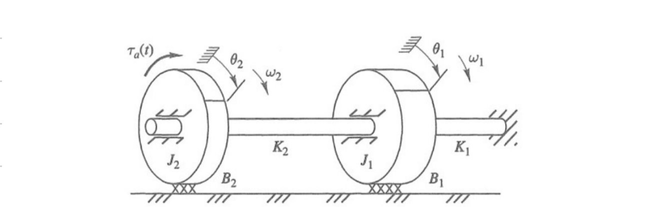
$$\dot{q}_2 = \frac{T_a}{I_0} - \frac{Bq_2}{I_0} - \frac{Kq_1}{I_0}$$

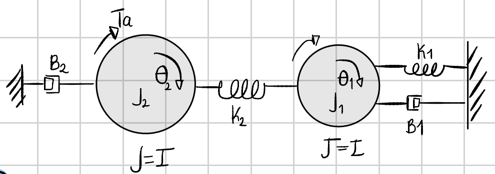
$$\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}1 & 0 \\ \frac{-K}{I_0} & \frac{-B}{I_0}\end{bmatrix}\begin{bmatrix}q_1 \\ q_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{I_0}\end{bmatrix}T_a$$

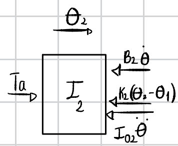
$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}q_1 \\ q_2\end{bmatrix}$$



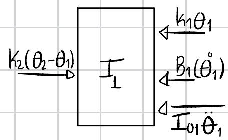
(2)







Para j2



(b)

$$T_a - I_{20}\ddot{\theta}_2 - B_2\dot{\theta}_2 - K_2(\theta_2-\theta_1) = \emptyset$$

$$T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2-\theta_1)$$

$$\theta_2 - \theta_1$$

$$-I_{02}\ddot{\theta} = -T_a + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1$$

$$\ddot{\theta} = \frac{T_a}{I_{02}} - \frac{B_2}{I_{02}}\dot{\theta}_2 - \frac{K_2}{I_{02}}\theta_2 + \frac{K_2\theta_1}{I_{02}}$$

$$x_1 = \theta_2 \qquad x_3 = \theta_1$$

$$x_2 = \dot{\theta}_2 = \dot{x}_1 \qquad x_4 = \dot{\theta}_1 = \dot{x}_3$$

$$\cancel{x_3} = \ddot{\theta}_2 = \dot{x}_2 \qquad x_5 = \ddot{\theta}_1 = \dot{x}_4$$

para j1

$$\dot{x}_2 = \frac{T_a}{I_{02}} - \frac{B_2 x_2}{I_{02}} - \frac{K_2 x_1}{I_{02}} + \frac{K_2 x_3}{I_{02}}$$

Para j2

$$K_2(\theta_2-\theta_1) - K_1(\theta_1) - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$K_2(\theta_2-\theta_1) - K_1\theta_1 - B_1\dot{\theta}_1 - I_{01}\ddot{\theta}_1$$

$$K_2\theta_2 - K_2\theta_1 - K_1\theta_1 - B_1\dot{\theta}_1 = I_{01}\ddot{\theta}_1$$

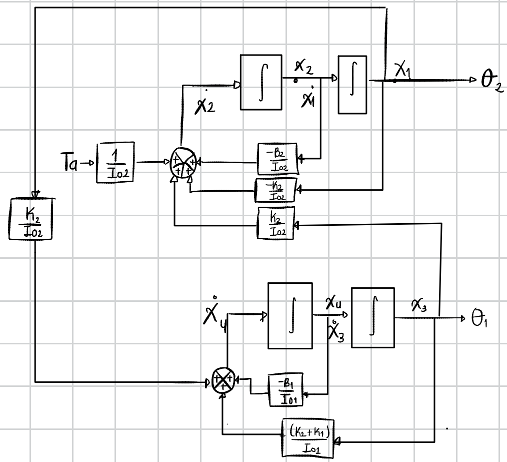
$$\frac{K_2\theta_2}{I_{01}} - \frac{K_2\theta_1}{I_{01}} - \frac{K_1\theta_1}{I_{01}} - \frac{B_1\dot{\theta}_1}{I_{01}}$$

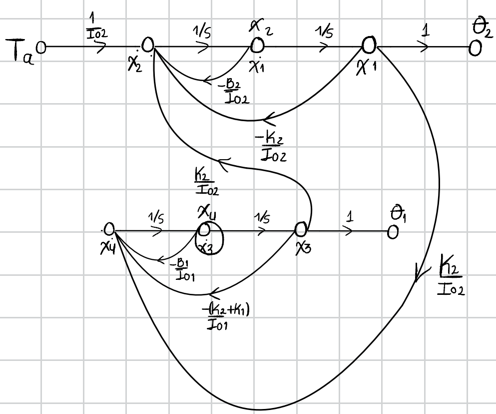
$$\frac{K_2\theta_2}{I_{01}} - \left(\frac{K_2+K_1}{I_{01}}\right)\theta_1 - \frac{B_1\dot{\theta}_1}{I_{01}} = \ddot{\theta}_1$$

$$\dot{x}_4 = \frac{K_2 x_1}{I_{01}} - \frac{(K_2+K_1)}{I_{01}}x_3 - \frac{B_1 x_4}{I_{01}}$$

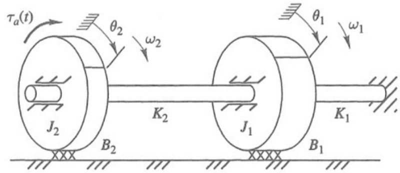
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_2}{I_{02}} & -\frac{B_2}{I_{02}} & \frac{K_2}{I_{02}} & 0\\ 0 & 0 & 0 & 1\\ \frac{K_2}{I_{01}} & 0 & -\frac{(K_2+K_1)}{I_{01}} & -\frac{B_1}{I_{01}}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{I_{02}}\\ 0\\ 0\end{bmatrix} T_a$$

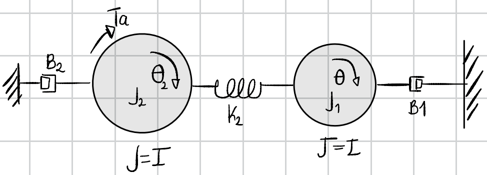
$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

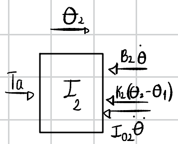




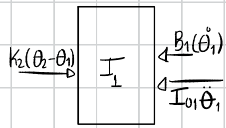
③







Para j2



$$T_a - I_{20}\ddot{\theta}_2 - B_2\dot{\theta}_2 - K_2(\theta_2-\theta_1) = \emptyset$$

$$T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2-\theta_1)$$

$$-I_{02}\ddot{\theta} = -T_a + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1$$

$$\ddot{\theta} = \frac{T_a}{I_{02}} - \frac{B_2}{I_{02}}\dot{\theta}_2 - \frac{K_2\theta_2}{I_{02}} + \frac{K_2\theta_1}{I_{02}}$$

$$x_1 = \theta_2 \qquad x_3 = \theta_1$$

$$x_2 = \dot{\theta}_2 = \dot{x}_1 \qquad x_4 = \dot{\theta}_1 = \dot{x}_3$$

$$\cancel{x_3} = \ddot{\theta}_2 = \dot{x}_2 \qquad x_5 = \ddot{\theta}_1 = \dot{x}_4$$

para j1

$$\dot{x}_2 = \frac{T_a}{I_{02}} - \frac{B_2 x_2}{I_{02}} - \frac{K_2 x_1}{I_{02}} + \frac{K_2 x_3}{I_{02}}$$

Para j2 con $K_1=0$

$$K_2(\theta_2-\theta_1) - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$K_2(\theta_2-\theta_1) - B_1\dot{\theta}_1 - I_{01}\ddot{\theta}_1$$

$$K_2\theta_2 - K_2\theta_1 - B_1\dot{\theta}_1 = I_{01}\ddot{\theta}_1$$

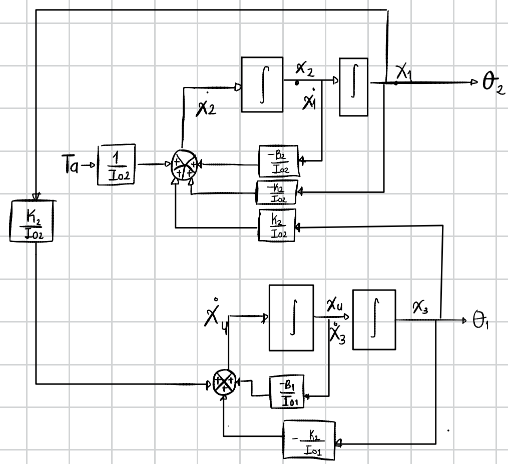
$$\frac{K_2\theta_2}{I_{01}} - \frac{K_2\theta_1}{I_{01}} - \frac{B_1\dot{\theta}_1}{I_{01}} = I_{01}\ddot{\theta}_1$$

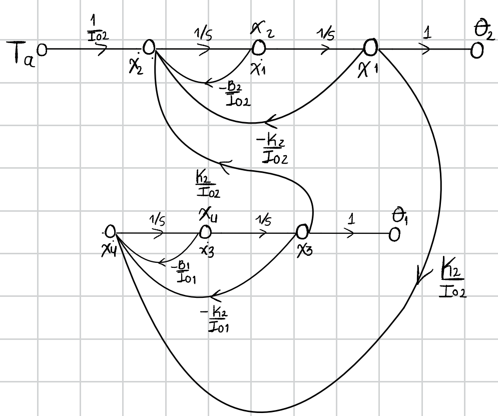
$$\frac{K_2\theta_2}{I_{01}} - \frac{K_2}{I_{01}}\theta \quad \frac{B_1\dot{\theta}_1}{I_{01}} = \ddot{\theta}_1$$

$$\dot{x}_4 = \frac{K_2 x_1}{I_{01}} - \frac{K_2 x_3}{I_{01}} - \frac{B_1 x_4}{I_{01}}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_2}{I_{02}} & -\frac{B_2}{I_{02}} & \frac{K_2}{I_{02}} & 0\\ 0 & 0 & 0 & 1\\ \frac{K_2}{I_{01}} & 0 & -\frac{K_2}{I_{01}} & -\frac{B_1}{I_{01}}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{I_{02}}\\ 0\\ 0\end{bmatrix} T_a$$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$





ⓐ Función de transferencia para punto ②

$$T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2 - \theta_1)$$

$$\mathcal{L}\{T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2) - K_2\theta_1$$

$$T_a(s) = I_{20}s^2\theta_2 + B_2 s\theta_2 + K_2\theta_2 - K_2\theta_1$$

$$T_a(s) = \theta_2(s)(I_{20}s^2 + B_2 s + K_2 - K_2\,\underline{\theta_1(s)})$$

Despejar θ_1 en terminos de θ_2

$$K_2(\theta_2 - \theta_1) - K_1(\theta_1) - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$K_2(\theta_2) - K_2\theta_1 - K_1\theta_1 - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$\mathcal{L}\{K_2\theta_2 = -K_2\theta_1 - K_1\theta_1 - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1)$$

$$-K_2\theta_2(s) = -K_2\theta_1 - K_1\theta_1 - B_1\theta_1 s - I_1\theta_1 s^2)$$

$$\theta_2(s) = \theta_1 - \frac{K_1}{K_2}\theta_1 - \frac{B_1}{K_2}\theta_1 s - \frac{I_1\theta_1 s^2}{K_2}$$

$$\theta_2(s) = \theta_1(s)\left[-\frac{K_1}{K_2} - \frac{B_1 s}{K_2} - \frac{I_{01}s^2}{K_2}\right]$$

$$\frac{1\quad \theta_2(s)}{\left[-\frac{K_1}{K_2} - \frac{B_1 s}{K_2} - \frac{I_{01}s^2}{K_2}\right]} = \theta_1(s) =$$

$$\to -\left(\frac{K_1 + B_1 s + I_1 s^2}{K_2}\right)$$

Reemplazando en Funcion de θ_2

$$T_a(s) = \theta_2(s)\left[I_{20}s^2 + B_2 s + K_2 + \frac{K_2}{\left[\frac{K_1 + B_1 s + I_1 s^2}{K_2}\right]}\right]$$

$$\frac{\theta_2(s)}{T_a(s)} = \frac{1}{\left[I_{20}s^2 + B_2 s + K_2 + \frac{K^2}{K_1 + B_1 s + I_{01}s^2}\right]} =$$

ⓐ Función de transferencia para punto ③

$$T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2 - \theta_1)$$

$$\mathcal{L}\{T_a = I_{20}\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2) - K_2\theta_1$$

$$T_a(s) = I_{20}s^2\theta_2 + B_2 s\theta_2 + K_2\theta_2 - K_2\theta_1$$

$$T_a(s) = \theta_2(s)(I_{20}s^2 + B_2 s + K_2 - K_2\,\underline{\theta_1(s)})$$

Despejar θ_1 en terminos de θ_2 considerando $K_1 = \emptyset$ se elimino.

$$K_2(\theta_2 - \theta_1) - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$K_2(\theta_2) - K_2\theta_1 - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1 = \emptyset$$

$$\mathcal{L}\{K_2\theta_2 = -K_2\theta_1 - B_1\dot{\theta}_1 - I_1\ddot{\theta}_1)$$

$$-K_2\theta_2(s) = -K_2\theta_1 - B_1\theta_1 s - I_1\theta_1 s^2)$$

$$\theta_2(s) = \theta_1 - \frac{B_1}{K_2}\theta_1 s - \frac{I_1\theta_1 s^2}{K_2}$$

$$\theta_2(s) = \theta_1(s)\left[-\frac{B_1 s}{K_2} - \frac{I_{01}s^2}{K_2}\right]$$

$$\frac{1\quad \theta_2(s)}{\left[-\frac{B_1 s}{K_2} - \frac{I_{01}s^2}{K_2}\right]} = \theta_1(s) =$$

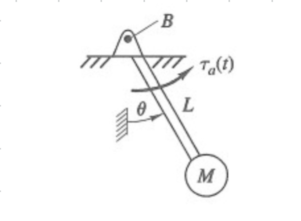
$$\to -\left(\frac{B_1 s + I_1 s^2}{K_2}\right)$$

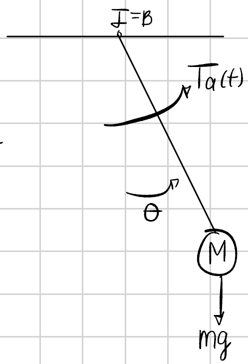
Reemplazando en Funcion de θ_2

$$T_a(s) = \theta_2(s)\left[I_{20}s^2 + B_2 s + K_2 + \frac{K_2}{\left[\frac{B_1 s + I_1 s^2}{K_2}\right]}\right]$$

$$\frac{\theta_2(s)}{T_a(s)} = \frac{1}{\left[I_{20}s^2 + B_2 s + K_2 + \frac{K_2^2}{B_1 s + I_{01}s^2}\right]} =$$

④





si $ml^2 = I$

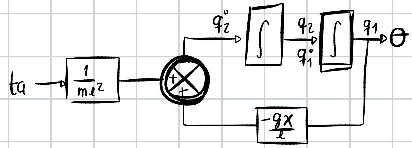
$T_a(t) - mgl\,sen = I\ddot{\theta}$

$q_1 = \theta$

$q_2 = \dot{q}_1 = \dot{\theta}$

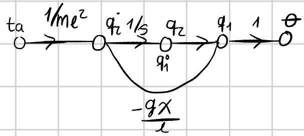
$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$

$$\mathcal{L}\left\{\ddot{\theta} + \frac{g}{l}sen(\theta) = \frac{ta(t)}{ml^2}\right\} \rightarrow \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}} = \frac{1}{s^2ml^2 + glm} = ta(s)$$



para $\ddot{\theta}$

$$\ddot{\theta} = \frac{ta(t)}{ml^2} - \frac{g\,sen\theta}{l} \qquad ta(t) = ml^2\ddot{\theta} + mlg\,sen(\theta)$$



$$\dot{q}_2 = \frac{ta}{ml^2} - \frac{g}{l}sen(q_1) \qquad \text{Con } sen(q_1) = x \qquad \dot{q}_2 = \frac{ta}{ml^2} - \frac{g}{l}x$$

$$\begin{bmatrix}\dot{q}_1 \\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{gx}{1} & 0\end{bmatrix}\begin{bmatrix}q_1 \\ q_2\end{bmatrix} + \begin{bmatrix}1 \\ \frac{1}{ml^2}\end{bmatrix} ta$$

$$\theta = [1 \quad 0]\begin{bmatrix}q_1 \\ q_2\end{bmatrix}$$

Made with Goodnotes