Appendix

A. Figures and tables

Table 1: Data description

Variables	Observations	Mean/ Percent	Minimum	Maximum
Exporter (ex_id)	12,445		1	43
Importer (im_id)	12,445		1	43
Year (year)	12,445		1995	2013
Trade share (x)	12,445	0.00056	$1.44e-11$	0.331
Regional trade agreements (RTA)	12,445	0.520	0	1
EU15 (since 1992)	12,445	0.127	0	1
EU25 (since 2004)	12,445	0.352	0	1
EU27 (since 2007)	12,445	0.410	0	1
EU28 (since 2013)	12,445	0.432	0	1
Number of international border (BORDER)	12,445	0.976	0	1

Notes: This table reports descriptive statistics of the key variables used for the estimation. Data comes from the WIOD database release 2013 merged with the release 2016 (Timmer et al., 2015) and information about RTA comes from Mario Larch's Regional Trade Agreement Database (Egger and Larch, 2008). Due to 3-year intervals suggested by Cheng and Wall (2005), the sample size is reduces from 35,735 to 12,445.

Table 2: Description of groups of trade flows

Trade flows	Country blocs	Number of observations							All years
		1995	1998	2001	2004	2007	2010	2013	
domestic	eastern EU				10	12	12	13	47
	western EU	15	15	15	15	15	15	15	105
	ROW	25	25	28	18	16	16	15	143
international	EU^{EAST}				90	132	132	156	510
	EU^{WEST}	210	210	210	210	210	210	210	1,470
	$EU^{EAST\text{-}WEST}$				300	360	360	390	1,410
	EU - ROW	750	750	840	900	864	864	840	5,808
	ROW - ROW	600	600	756	306	240	240	210	2,952
	All countries	1,600	1,600	1,849	1,849	1,849	1,849	1,849	12,445

Notes: This table shows the country blocs of bilateral trade flows and the corresponding number of observations per year. Trade flows are distinguished into two major groups of domestic and international trade flows, which capture trade flows within a country and between countries respectively. A further distinction is made by differing between trade flows concerning EU and non-EU members (ROW). Moreover, EU member countries are assigned to the western or the eastern part of the EU. While the western part comprises 15 members since 1995, the eastern part defines new joining members of the eastwards expansion in 2004, 2007 and 2013.

Table 3: First step coefficient estimates

First Step Dependent variable: $s_{ij,t}$	coefficients	lower CI	upper CI
$\text{RTA}_{ij,t}$	0.167	0.045	0.291
$\text{RTA}_{ij,t-3}$	0.093	0.028	0.162
$\text{RTA}_{ij,t-6}$	0.121	0.076	0.163
$\text{RTA}_{ij,t-9}$	0.048	−0.002	0.097
$\text{RTA}_{ij,t-12}$	0.059	0.010	0.108
$\text{RTA}_{ij,t-15}$	−0.046	−0.177	0.078
$\text{EU}^{EAST}_{ij,2004}$	0.445	0.271	0.618
$\text{EU}^{EAST}_{ij,2007}$	0.757	0.522	0.973
$\text{EU}^{EAST}_{ij,2010}$	0.838	0.593	1.093
$\text{EU}^{EAST}_{ij,2013}$	0.966	0.709	1.224
$\text{EU}^{WEST}_{ij,1998}$	0.164	0.127	0.202
$\text{EU}^{WEST}_{ij,2001}$	0.026	−0.031	0.085
$\text{EU}^{WEST}_{ij,2004}$	0.087	0.035	0.138
$\text{EU}^{WEST}_{ij,2007}$	0.193	0.133	0.253
$\text{EU}^{WEST}_{ij,2010}$	0.168	0.101	0.236
$\text{EU}^{WEST}_{ij,2013}$	0.185	0.098	0.280
$\text{EU}^{EAST}_{ij,2004} \times \text{EU}^{WEST}_{ij,2004}$	0.388	0.286	0.483
$\text{EU}^{EAST}_{ij,2007} \times \text{EU}^{WEST}_{ij,2007}$	0.653	0.536	0.771
$\text{EU}^{EAST}_{ij,2010} \times \text{EU}^{WEST}_{ij,2010}$	0.702	0.560	0.837
$\text{EU}^{EAST}_{ij,2013} \times \text{EU}^{WEST}_{ij,2013}$	0.769	0.625	0.904
$\text{EU}_{ij,1998} \times \text{ROW}_{ij,1998}$	0.244	0.214	0.274
$\text{EU}_{ij,2001} \times \text{ROW}_{ij,2001}$	0.158	0.115	0.196
$\text{EU}_{ij,2004} \times \text{ROW}_{ij,2004}$	0.217	0.165	0.270
$\text{EU}_{ij,2007} \times \text{ROW}_{ij,2007}$	0.304	0.239	0.369
$\text{EU}_{ij,2010} \times \text{ROW}_{ij,2010}$	0.317	0.236	0.399
$\text{EU}_{ij,2013} \times \text{ROW}_{ij,2013}$	0.338	0.264	0.414
$\text{ROW}_{ij,1998}$	0.108	0.073	0.144
$\text{ROW}_{ij,2001}$	0.039	−0.017	0.096
$\text{ROW}_{ij,2004}$	0.156	0.045	0.275
$\text{ROW}_{ij,2007}$	0.150	0.023	0.275
$\text{ROW}_{ij,2010}$	0.110	−0.019	0.233
$\text{ROW}_{ij,2013}$	0.121	0.005	0.235
Exporter-time fixed effect	YES		
Importer-time fixed effect	YES		
Pair fixed effect	YES		
N	12445		

Notes: This table presents the first step coefficient estimates and the corresponding 95 percent percentile bootstrapped confidence intervals.

Table 4: Second step coefficient estimates

Second step Dependent variable: $\bar{\mu}_{ij}$	coefficients	lower CI	upper CI
RTA_{ij}	0.167	0.045	0.291
lndistance_{ii}	-0.449	-0.469	-0.428
lndistance_{ij}	-0.537	-0.553	-0.522
contiguity_{ij}	-0.538	-0.548	-0.527
colony_{ij}	-0.285	-0.303	-0.267
$\text{commonlanguage}_{ij}$	-0.328	-0.345	-0.310
BORDER_{ij}	-1.733	-1.867	-1.580
EU_{ij}	0.491	0.372	0.610
Exporter fixed effect	YES		
Importer fixed effect	YES		
N	1764		

Notes: This table presents the second step coefficient estimates and the corresponding 95 percent percentile bootstrapped confidence intervals.

Table 5: GE effects by country: Counterfactual Experiment I (without level)

Country	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	GE trade changes						
	1995	**1998**	**2001**	**2004**	**2007**	**2010**	**2013**
Australia	1.000	0.990	0.999	0.965	0.930	0.934	0.923
Austria	1.000	1.063	1.010	1.090	1.194	1.209	1.252
Belgium	1.000	1.025	1.004	1.071	1.151	1.173	1.210
Bulgaria	1.000	0.984	0.998	0.959	1.837	1.917	2.065
Brazil	1.000	0.990	0.999	0.965	0.931	0.934	0.923
Canada	1.000	0.990	0.999	0.966	0.933	0.936	0.925
Switzerland	.	.	.	0.964	0.927	0.930	0.919
China	1.000	0.992	0.999	0.967	0.935	0.937	0.926
Cyprus	0.999	0.989	0.999	1.392	1.818	1.915	2.078
Czech Republic	1.000	0.983	1.001	1.303	1.594	1.660	1.771
Germany	1.000	1.013	0.998	1.058	1.122	1.144	1.178
Denmark	1.000	1.066	1.002	1.098	1.223	1.230	1.274
Spain	1.000	1.058	1.010	1.093	1.205	1.213	1.250
Estonia	1.001	0.984	1.009	1.361	1.781	1.815	1.993
Finland	1.000	1.057	0.998	1.092	1.202	1.218	1.261
France	1.000	1.021	1.009	1.072	1.152	1.174	1.211
Great Britain	1.000	1.029	1.004	1.075	1.159	1.181	1.221
Greece	1.000	1.092	1.005	1.115	1.260	1.263	1.299
Croatia	.	.	.	0.959	0.916	0.921	2.104
Hungary	1.000	0.984	1.014	1.303	1.584	1.645	1.773
India	1.000	0.990	0.998	0.965	0.931	0.934	0.923
Indonesia	1.000	0.990	0.998	0.965	0.930	0.933	0.922
Ireland	1.000	1.043	0.999	1.085	1.189	1.199	1.241
Italy	1.000	1.041	0.999	1.079	1.168	1.186	1.218
Japan	1.000	0.993	1.007	0.967	0.935	0.937	0.926
South Korea	1.000	0.991	1.006	0.967	0.934	0.937	0.926
Lithuania	1.000	0.982	1.000	1.349	1.787	1.793	1.922
Luxembourg	1.000	1.084	0.999	1.113	1.258	1.276	1.337
Latvia	1.000	0.983	0.998	1.418	1.943	1.944	2.158
Mexico	1.000	0.990	1.014	0.966	0.932	0.935	0.924
Malta	1.000	0.990	0.998	1.349	1.743	1.864	2.024
Netherlands	1.000	1.020	0.999	1.068	1.142	1.167	1.216
Norway	.	.	.	0.963	0.924	0.928	0.916
Poland	1.000	0.983	1.000	1.297	1.605	1.720	1.830
Portugal	1.000	1.091	1.003	1.116	1.260	1.269	1.304
Romania	1.000	0.983	0.999	0.960	1.851	1.923	2.078
Russia	1.000	0.987	0.998	0.964	0.925	0.929	0.917
Slovakia	1.000	0.982	1.015	1.346	1.668	1.738	1.890
Slovenia	1.000	0.983	0.998	1.359	1.730	1.811	1.953
Sweden	1.000	1.054	0.999	1.090	1.199	1.214	1.258
Turkey	1.000	0.988	0.998	0.964	0.929	0.932	0.921
Taiwan	1.000	0.991	0.998	0.966	0.933	0.936	0.925
USA	1.000	0.996	1.009	0.969	0.938	0.940	0.929
Average	1.000	1.011	1.002	1.097	1.249	1.274	1.354

Notes: This table presents GE trade integration effects of Counterfactual Experiment I by country and year. The numbers do not include the initial degree of integration, but only the change over time of EU integration effects.

Table 6: GE effects by country: Counterfactual Experiment I and II (with level)

Country	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	GE trade changes								
	1995	**1998**	**2001**	**2004**	**2007**	**2010**	**2013**	**Potential wrt BLN in 2013**	**Potential wrt CFL I in 2013**
Australia	0.971	0.965	0.976	0.941	0.906	0.910	0.899	0.819	0.736
Austria	1.243	1.332	1.227	1.320	1.439	1.452	1.504	1.626	2.445
Belgium	1.092	1.153	1.102	1.164	1.249	1.270	1.309	1.287	1.684
Bulgaria	0.956	0.945	0.956	0.916	1.742	1.825	1.972	2.268	4.471
Brazil	0.972	0.968	0.977	0.943	0.909	0.912	0.900	0.824	0.742
Canada	0.973	0.968	0.977	0.944	0.911	0.915	0.904	0.828	0.749
Switzerland	0.000	0.000	0.000	0.936	0.900	0.904	0.891	0.807	0.719
China	0.976	0.973	0.972	0.948	0.915	0.917	0.905	0.835	0.756
Cyprus	0.958	0.973	0.981	1.339	1.739	1.833	1.998	2.281	4.557
CzechRepublic	0.954	0.943	0.970	1.243	1.519	1.586	1.697	1.700	2.885
Germany	1.070	1.099	0.958	1.117	1.187	1.212	1.250	1.220	1.526
Denmark	1.243	1.337	1.065	1.352	1.519	1.498	1.544	1.777	2.744
Spain	1.210	1.300	1.240	1.321	1.455	1.432	1.457	1.593	2.322
Estonia	0.959	0.947	1.213	1.309	1.698	1.738	1.914	2.210	4.230
Finland	1.200	1.284	0.962	1.296	1.423	1.438	1.485	1.728	2.566
France	1.100	1.137	1.198	1.176	1.263	1.286	1.326	1.308	1.735
GreatBritain	1.107	1.166	1.105	1.184	1.275	1.301	1.349	1.376	1.855
Greece	1.337	1.453	1.113	1.470	1.659	1.635	1.637	2.099	3.436
Croatia	0.000	0.000	0.000	0.916	0.872	0.880	2.005	2.182	4.375
Hungary	0.957	0.946	1.320	1.247	1.513	1.576	1.702	1.727	2.940
India	0.972	0.966	0.956	0.942	0.907	0.911	0.900	0.822	0.740
Indonesia	0.971	0.966	0.961	0.941	0.906	0.910	0.899	0.820	0.737
Ireland	1.154	1.225	0.976	1.237	1.360	1.346	1.397	1.521	2.126
Italy	1.148	1.221	0.976	1.232	1.327	1.344	1.367	1.425	1.949
Japan	0.978	0.977	1.158	0.951	0.919	0.920	0.908	0.843	0.765
SouthKorea	0.975	0.971	1.152	0.947	0.914	0.917	0.905	0.833	0.754
Lithuania	0.954	0.942	0.984	1.290	1.697	1.713	1.842	2.083	3.836
Luxembourg	1.304	1.422	0.980	1.471	1.655	1.697	1.796	2.280	4.093
Latvia	0.953	0.945	0.960	1.346	1.837	1.850	2.058	2.426	4.992
Mexico	0.972	0.968	1.327	0.943	0.909	0.913	0.901	0.823	0.742
Malta	0.959	0.973	0.955	1.301	1.670	1.785	1.949	2.104	4.099
Netherlands	1.082	1.130	0.977	1.143	1.216	1.245	1.333	1.339	1.785
Norway	0.000	0.000	0.000	0.931	0.893	0.897	0.885	0.789	0.699
Poland	0.958	0.943	0.973	1.240	1.530	1.641	1.752	1.760	3.083
Portugal	1.320	1.460	1.087	1.496	1.681	1.685	1.676	2.033	3.408
Romania	0.955	0.942	0.969	0.918	1.753	1.829	1.982	2.200	4.360
Russia	0.963	0.956	0.960	0.931	0.891	0.896	0.884	0.791	0.699
Slovakia	0.955	0.939	1.354	1.280	1.587	1.659	1.808	1.933	3.496
Slovenia	0.955	0.944	0.958	1.296	1.646	1.728	1.867	1.964	3.666
Sweden	1.197	1.278	0.968	1.284	1.417	1.427	1.478	1.658	2.451
Turkey	0.965	0.959	0.954	0.935	0.898	0.902	0.890	0.796	0.709
Taiwan	0.974	0.970	0.959	0.946	0.912	0.915	0.904	0.832	0.752
USA	0.984	0.986	1.195	0.959	0.929	0.930	0.918	0.864	0.793
Average	1.048	1.074	1.051	1.140	1.292	1.316	1.394	1.475	2.057

Notes: This table presents GE effects of Counterfactual Experiment I and II by country and year. Trade integration effects include both the initial degree of integration and its change over time. Trade potential is shown by the last two columns. The first displays trade potential as calculated in equation 9 and the second column relates it to the scenario CFL I, as depicted in Figure 7.

Table 7: GE effects by type: Counterfactual Experiment I and II

Trade flows	Country groups	1995	1998	2001	2004	2007	2010	2013	Potential
EU integration (without level)									
domestic	eastern EU	.	.	.	0.937	0.910	0.906	0.896	
	western EU	1.000	0.960	0.994	0.977	0.948	0.953	0.947	
	ROW	1.000	1.002	1.000	1.001	1.002	1.002	1.001	
international	EU^{EAST}	.	.	.	1.451	1.874	2.046	2.274	
	EU^{WEST}	1.000	1.119	1.019	1.058	1.130	1.112	1.126	
	$EU^{EAST\text{-}WEST}$	.	.	.	1.307	1.543	1.640	1.728	
	EU - ROW	1.000	0.984	0.997	0.990	0.978	0.979	0.977	
	ROW - ROW	1.000	1.003	1.000	1.002	1.004	1.003	1.003	
EU integration (with level) and potential									
domestic	eastern EU	.	.	.	0.946	0.915	0.911	0.901	0.702
	western EU	0.901	0.871	0.903	0.887	0.862	0.867	0.862	0.570
	ROW	1.004	1.008	1.006	1.006	1.006	1.004	1.003	1.010
international	EU^{EAST}	.	.	.	1.463	1.886	2.056	2.286	4.992
	EU^{WEST}	1.433	1.608	1.464	1.514	1.620	1.603	1.630	3.012
	$EU^{EAST\text{-}WEST}$	.	.	.	1.243	1.470	1.562	1.648	2.952
	EU - ROW	0.960	0.946	0.958	0.953	0.942	0.942	0.941	0.794
	ROW - ROW	1.007	1.010	1.007	1.009	1.012	1.010	1.010	1.037

Notes: This table presents GE effects of Counterfactual Experiment I and II by type and year. These numbers are visually depicted by Figures 5 - 7.

B. Poisson fixed effects Conditional Maximum Likelihood (CML)

Formally, the Poisson fixed effects estimator solves the FOC of the concentrated log likelihood:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left[s_{ij,t}-\frac{\bar{s}_{ij}}{\bar{\varphi}_{ij}}\varphi_{ij,t}\right]=0 \tag{10}$$

where $\varphi_{ij,t} = z'_{ij,t}\hat{\delta}$ and $z'_{ij,t}$ contains K elements of all exponentiated explanatory variables, $\bar{\varphi}_{ij} = \frac{1}{T}\sum_t^T \varphi_{ij,t}$, $\bar{s}_{ij} = \frac{1}{T}\sum_t^T s_{ij,t}$. Subtraction of the rescaled average over all periods leads to the drop of $\frac{\varphi_{ij,t}}{\bar{\varphi}_{ij}}$ and hence, eliminates time-invariant fixed effects. The only requirements in this multiplicative model is the correct specification of the conditional mean $E\left[s_{ij,t}|d_{ij,1},...,d_{ij,T},\mu_{ij}\right] = \varphi_{ij,t}\mu_{ij}$ and strictly exogenous regressors (Cameron and Trivedi, 2013).

Consequently, without identification of the time-invariant fixed effects the time varying parameters of interest can be estimated consistently. In detail, all the parameters $\hat{\delta}$ can be estimated consistently including (N-1)(T-1) destination-time and N(T-1) origin-time fixed effects.

C. Normalization and reparameterization

The specification of the system of equations (2.1)-(2.3) imposes two sets of restrictions. The first set of T restrictions comes from the system of MR´s. The interrelated system is uniquely solved up to a constant because it represents only a ratio of remoteness.[24] In detail this means, origin and destination specific trade cost terms differ not only for i≠j, but also for $i = j$ ($\Pi_{i,t}^{1-\sigma} \neq P_{j,t}^{1-\sigma}$). To obtain a unique solution a normalization is required. Normalization solves the system in relative terms with respect to a reference country. The reference country should possibly unaffected by the counterfactual changes and should have reliable data. In this application the USA ($j = N$) is the chosen reference country. Thus, the solution of the system of multilateral resistances is normalized to one inward multilateral resistance $P_{N,t}^{1-\sigma}$ and their incorporated prices are interpreted relative to the reference country (Yotov et al., 2016).

The second set of $2N-1$ restrictions for each year is imposed by the dummy design that restricts identification of parameters. While there are numerous choices of which parameters to restrict and thus select as reference, the choice influences the interpretation of the results. Since the second step of the TSFE method requires a complete set of pair fixed effect, it is appealing leave them unrestricted. Restrictions are therefore imposed on one year $t = 0$.

In sum, normalizing to one year, using $j = N$ as numeraire and without a constant one obtains $N(T-1)$ origin-time $\hat{\beta}_{i,t}$ and $(N-1)(T-1)$ destination-time $\hat{\gamma}_{j,t}$ fixed effects. N^2 recovered pair fixed effects $\hat{\mu}_{ij}$ and

[24] The expression as ratio is due to asymmetric trade costs and unbalanced trade. For symmetric trade costs and balanced trade, the system of multilateral resistance terms collapses by implicitly normalizing $\Pi_{i,t}^{1-\sigma} = P_{j,t}^{1-\sigma}$.

the parameters $\hat{\alpha}$ are fully identified. Thus, the parametrization reads as:

$$
\begin{aligned}
\hat{\beta}_{i,t} &= \beta_{i,t} - \beta_{i,0} && (11)\\
\text{N(T-1)} &= \text{NT - N} \\
\hat{\gamma}_{j,t} &= \gamma_{j,t} - \gamma_{j,0} - \gamma_{N,t} + \gamma_{N,0} && (12)\\
\text{(N-1)(T-1)} &= \text{NT - N - T +1} \\
\hat{\mu}_{ij} &= \mu_{ij} && (13)\\
\text{N}^2 &= \text{N}^2
\end{aligned}
$$

with $\hat{\beta}_{i,0} = 0$,$\hat{\gamma}_{j,0} = 0$ and the numeraire $\hat{\gamma}_{N,t} = 0$.

However, this parametrization raises two problems. First, it omits the GE solution for one year and second, results are not interpretable with respect to internal trade costs μ_{jj}. A more intuitive interpretation of gravity estimates relates effects on international trade to domestic trade flows of each country rather than to a year. The naturally arising restrictions yield NT origin-time $\tilde{\beta}_{i,t}$, $(N-1)T$ destination-time $\tilde{\gamma}_{j,t}$ fixed effects and $(N-1)^2$ pair fixed effects $\tilde{\mu}_{ij}$. This intuitive normalization allows to interpret the estimated effects with reference to domestic trade flows and one importing country. For this purpose, the set of fixed effects is reparametrized by inserting the estimated values for the unidentified parameters $\hat{\beta}_{i,0} = 0$ and $\hat{\gamma}_{j,0} = 0$. Normalization to one importer is kept equal. The set of fixed effects of equations (11) - (13) is reparametrized to $\tilde{\beta}_{i,t}, \tilde{\gamma}_{j,t}$ and $\tilde{\mu}_{ij}$ by maintaining $2N-1$ restrictions for each year (Oberhofer and Pfaffermayr, 2017). The reparametrized set of fixed effects is expressed by:

$$
\begin{aligned}
\tilde{\beta}_{i,t} &= \hat{\beta}_{i,t} + \hat{\mu}_{iN} && (14)\\
\text{NT} &= \text{N(T-1) + N} \\
\tilde{\gamma}_{j,t} &= \hat{\gamma}_{j,t} + \hat{\mu}_{jj} - \hat{\mu}_{NN} && (15)\\
\text{(N-1)T} &= \text{(N-1)(T-1) + N - 1} \\
\tilde{\mu}_{ij} &= \hat{\mu}_{ij} - \hat{\mu}_{jj} - \tilde{\mu}_{iN} && (16)\\
\text{(N-1)}^2 &= \text{N}^2 \text{ - N - N + 1}
\end{aligned}
$$

with $\tilde{\beta}_{i,0} = \hat{\mu}_{iN}$ and $\tilde{\gamma}_{j,0} = -\hat{\mu}_{jj} + \hat{\mu}_{NN}$. And normalizations $\tilde{\gamma}_{N,t} = 0$, $\tilde{\mu}_{iN} = 0$, $\tilde{\mu}_{NN} = 0$, $\tilde{\mu}_{iN} = -\hat{\mu}_{iN} + \hat{\mu}_{NN} = 0$ and $\tilde{\mu}_{jj} = 0$. Expressed in words, one destination country and all domestic trade flows are used as reference group.

D. The structure of the error term ϵ_{ij}

Because the recovered term of equation (3.2) is an estimated one, the error term need special attention. This error term is three-fold. A first error component ϵ_η comes from the estimation of equation (3.1). Since pair fixed effects are recovered this heteroskedastic error component has to be taken into account. Second, recovering requires an estimation in the first step that yields estimates of time variant parameter δ (Pesaran and Zhou, 2018). The variance of this estimate ϵ_δ adds to the error component of the recovered pair fixed effects. Hence, by taking both error terms ϵ_η and ϵ_δ into account the recovered term (3.2) becomes an estimated one and is defined as:

$$
\begin{aligned}
e^{\hat{\hat{\mu}}_{ij}} &= \sum_t^T \frac{s_{ij,t}}{e^{(\hat{z}'_{ij,t}\hat{\delta})}} + \underbrace{\left(\sum_t^T \frac{s_{ij,t}}{e^{(z'_{ij,t}\hat{\delta})}} - \sum_t^T \frac{s_{ij,t}}{e^{(z'_{ij,t}\delta)}} \right)}_{\epsilon_\delta} \\
&= e^{\mu_{ij}} \underbrace{\sum_t^T \eta_{ij,t}}_{\epsilon_\eta} + \underbrace{\left(e^{\mu_{ij}} \sum_t^T \hat{\eta}_{ij,t} - e^{\mu_{ij}} \sum_t^T \eta_{ij,t} \right)}_{\epsilon_\delta} \\
&= e^{\mu_{ij}} \underbrace{\sum_t^T \eta_{ij,t}}_{\epsilon_\eta} + \underbrace{e^{\mu_{ij}} \left(\sum_t^T \hat{\eta}_{ij,t} - \sum_t^T \eta_{ij,t} \right)}_{\epsilon_\delta}.
\end{aligned}
$$

The third error component ϵ_a comes from the reformulated term $e^{\mu_{ij}} = e^{w'_{ij}\psi + a_{ij}}$. As $e^{a_{ij}}$ denotes the random error it adds to the error term. Moreover, the compound error term is compactly reformulated to be additively.

$$
\begin{aligned}
e^{\hat{\hat{\mu}}_{ij}} &= e^{w'_{ij}\psi} \underbrace{e^{a_{ij}}}_{\epsilon_a} \underbrace{\sum_t^T \eta_{ij,t}}_{\epsilon_\eta} + \underbrace{e^{w'_{ij}\psi} \underbrace{e^{a_{ij}}}_{\epsilon_a} \left(\sum_t^T \hat{\eta}_{ij,t} - \sum_t^T \eta_{ij,t} \right)}_{\epsilon_\delta} \\
&= e^{w'_{ij}\psi} \underbrace{e^{a_{ij}}}_{\epsilon_a} \underbrace{\sum_t^T \eta_{ij,t}}_{\epsilon_\eta} + \underbrace{e^{w'_{ij}\psi} \underbrace{e^{a_{ij}}}_{\epsilon_a} \left(\sum_t^T \hat{\eta}_{ij,t} - \sum_t^T \eta_{ij,t} \right)}_{\epsilon_\delta} + e^{w'_{ij}\psi} - e^{w'_{ij}\psi} \\
&= e^{w'_{ij}\psi} + \underbrace{e^{w'_{ij}\psi} \left[\underbrace{e^{a_{ij}}}_{\epsilon_a} \underbrace{\sum_t^T \eta_{ij,t}}_{\epsilon_\eta} + \underbrace{e^{a_{ij}}}_{\epsilon_a} \underbrace{\left(\sum_t^T \hat{\eta}_{ij,t} - \sum_t^T \eta_{ij,t} \right)}_{\epsilon_\delta} - 1 \right]}_{\epsilon} \\
&= e^{w'_{ij}\psi} + \epsilon
\end{aligned} \tag{4}
$$

The reformulation is observationally equivalent to the multiplicative version in a PPML framework and is implemented by simultaneously adding and subtracting $e^{w'_{ij}\psi}$(Windmeijer, 1997).