# A Modern Conceptual Framework for the Analysis of Factors in Retirement Decisions

Xiaojuan Zhu

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**Abstract**

This article focuses on analytical methodologies useful for analyzing retirement decision making in large organizations. We discuss a variety of biases that can occur in retirement databases along with modelling strategies and other factors. We also discuss approaches to using retirement models for both the analysis of prior policies and incentives and the prediction of future retirement behavior. Simulation is used to demonstrate the potential impact of sampling biases on predictions.

## 1 Introduction

Employee turnover is a topic that has drawn the attention of management researchers and practitioners for decades, because employee turnover is both costly and disruptive to the functioning of most organizations (Staw, 1980; Mueller and Price, 1989; Kacmar et al., 2006), and both private firms and governments spend billions of dollars every year managing the issue according to Leonard (2001). Therefore, the objectives of this study is 1) predict the probability that an individual will retire in a certain window and/or time until retirement (Mean residual life). 2) Predict aggregate numbers of retirements in a fixed time frame at the or division level to facilitate plannning. 3) To determine the impact of internal and external economic variables on retirement. 4) Quantifying the effect of a buyout policy. As a funded research project, a large organizational secondary dataset including 12-year employees demographic information and records is transformed, analyzed and modeled by Cox proportional hazard regression models with a time dependent covariate using competing risks analysis to examine the statistically significant factors and to predict employees' conditional retiring probabilities. This study also examines the forecasting capability of Cox proportional hazard model on the data with two kinds of bias (left truncation and right censor) by simulation.

# 2 Literature Review

## 2.1 impact of employee retirement

## 2.2 Methods for retirement forecasting

## 2.3 Survival analysis application

Survival analysis is widely used to analyze lifetime data in many areas, such as medical health, business,and reliability area. Claus et al. (1991) investigated the familial risk of breast cancer in a large population-based, case-control study using recurrent life time analysis. The effect of genotypon the risk of breast cancer is illustrated as a function of a woman's age. The carriers of the allele appear to be at greater risk at all ages compared with non-carriers. And the specific risks is greatest at young ages and declines steadily thereafter. De Angelis et al. (1999) applied a parametric mixture model to survival rates of colon cancer patients from the Finnish population-based cancer registry. The study divided survival into two different components to effectively perform the analysis and to interpret the role of prognostic factors on survival patterns. The studies found that age plays a different role in determining the probability of cure and life expectancy of fatal cases. Lu (2002) applied survival analysis techniques to predict customer churn by using data from a telecommunications company. The study helped the companies understand customer churn risk and hazard by predicting which customers will churn and when they will churn. Their study provided a tool for telecommunication companies to make retention plan to reduce the customer churn. Braun and Schweidel (2011) used a hierarchical competing risks analysis to model when and why customers terminate their service by using the data from a provider of land-based telecommunication services. Their study focused on three mainly causes: value (price), personal, non-pay or abuse. This study provided a tool for assistant market manager to target their customers and to determine retention strategies to prevent or delay the customer churn due to different causes. Carrión et al. (2010) estimates the time to failure of the pipes in water supply network dataset under left-truncation and right-censoring (2000-2005) by using the extend Nelson estimator (ESE) developed by Pan and Chappell (1998). The Cox regression model is applied into the data to identify the factors / covariates which affect the reliability for the water supply network. The result shows that the materials of the water pipe, length, and diameter of the pipe section and road traffic conditions are all significant.

# 3 Data Preparation

The turnover dataset is a large real world secondary dataset from a multipurpose research organization in the U.S. The dataset consists 4316 current active and 3782 terminated full-time employees' information including metrics such as payroll category, hired date, company start date, company credit service date, termination date, age at hired , years of service at hired (YCSH), gender, job classification (named as Cocs code), and Organization level (named as division). The company credit service date is the date that the organization starts to credit their retirement plan. Years of service (YCS) is the total years credit for employees' pension plan. The employees are eligible to get a full pension, when their age is at least 65 or their points is greater than 85, which is the sum of age and year of service. Employees have different YCS when they are hired because their YCS can be transferred from their previous job if their previous job also accounts for the pension plan. Common Occupational Classification System (COCS) code is a standardized code used to describe the job category by the organization for reporting to Common Occupational Classification System. In this study, COCS code is highly correlated with payroll category: managers, engineers, administrative, and scientists are monthly payroll, general administrative and technicians are weekly payroll, the other categories are hourly payroll. Organization level code is used to distinguish the departments. In this study, the division in the organization do not stabilize like COCS code for an employee, because the division can be renamed, reduced, or dismissed by the change of production plan or organization's budget. The division is considered as time independent variable for employees due to no historical record for divisions provided by HR department.

The window of time for the turnover dataset is from November 2000 to December 2012, i.e. the dataset consists the records only for the employees working in the organization from November 2000 to December 2012, indicating there is no records for employees leaving the organization before November 2000 and no termination date for 4316 current employees. These two kinds of unknown information cause two kinds of bias: right censor and left truncation. The right censor is due to the no termiation date for current employees, and the left truncation is due to no records for employee leaving before November 2000.

In the organization, all the employees hired before January 2012 are eligible for define benefit retirement plan. That organization started to use 401k retirement plan for part of new employees from January 2012. The two requirements for getting the full define benefit retirement plan are either a employee is at least 65 years old, or their points reach 85. The points is the summation of age and credit years of service.

The covariates identified from the turnover dataset and used to build the models are payroll, gender, division, cocs code (Job category), age at hired, and year of service at hired:

- Payroll (PR): hourly, weekly, or monthly payroll,
- Gender: male, female
- division (ORG): ten divisions in the organization.
- Cocs code: Crafts(C), Engineers (E), General Administrative (G), Laborers (L), General Managers (M), Administrative (P), Operators (O), Scientists (S), Technicians (T))
- Age at hired: most recent age when an employee is hired.
- Years of service at hired (YCSH): the years of service which accounts for pension plan when employee is most recently hired.

Several economic indices are being considered and tested their as a variable impact on employee turnover. These indices include unemployment index, housing price index (HPI), investment index, and marketing index. Seasonal adjusted unemployment rate is published by Bureau of Labor Statics from United department of Labor (U.S Bureau of Labor Statistics, 2015). U.S housing price index, U.S. and southeastern monthly purchase-only index are considered as another economic indicator variables in the study (Federal Housing Finance Agency, 2015). S&P 500 indices published from S&P Dow Jones Indices are also considered as investment index including S&P 500, Dividend, Earnings, Consumer index, Long Interest Rate, Real Price, Real Dividend, Real Earnings, P/E 10 ratio (S&P Dow Jones Indices, 2015). Wilshire 5000 total market full cap index published by Wilshire Associates is considered as market index in the forecasting model(Wilshire Associates, 2015). All these twelve indices are treated as variables using their twelve-month lag term in yearly data format. All these indices selected are indicators in various economic areas, such as job market, house market, and stock market, representing the fluctuation of these economic areas. The economic indices are originally in the dayly or monthly form. The average values by twelve month for each year are used into the model fitting.

# 4 Model Development and Evaluation

Several questions have to be addressed by this study: Can turnover in term of retirement or voluntary quit be predicted? When will a employees turnover? Who will retire or voluntary quit in term of job categories or divisions? what age groups are more likely to retire or voluntary quit? What economic conditions related to retirement or voluntary quit? What is the magnitude or impact of buyout program? How do the tenure and age impact the retirement or voluntary quit? Besides, how to deal with the data biases: right censor and left truncation existing in the dataset. All these questions and problems can be solved by lifetime analysis, also called survival analysis. The survival analysis is to analyze the time duration for the occurrence of an events or certain events. The events can be the death of the patients, the failure of the machine, and the leaving of the employees by any reasons for this study. There are two kinds of survival statistic models: parametric survival models and Cox proportional hazards (PH) models. In this study, Cox PH model is employed to build the forecasting model, to generate a employees' working life baseline (distribution), and to identify significant factors for turnover. The parametric models are not appropriate for this study, because it is hard to fit the employees' working life distribution to any parametric distributions, such as Weibull or log-normal distribution. Time dependent covariates are incorporated for fitting the 2008 intervention event due to the downsize policy in the organization and for examining the effects of economic indicators. Competing risks analysis is applied for modeling employee retirement and voluntary quit. Besides, A simulation study is performed to examine the forecasting capability of cox proportional hazard model on the left truncation and right censor dataset.

## 4.1 Two data bias: right censor and left truncation

Right censor and left truncation are common in survival analysis. The right censor is that the event of interest (failure) occurs after the study window. Let $T$ denotes the time of main event of interest to occur and let $C$ denotes the end time of study. An observation is right censored when $T > C$, indicating the study do not have the failure time of the right censored observation. In this study, the study window is from November 2000 to December 2012 as shown in the figure 1. Thus, the current active employees have unknown terminated

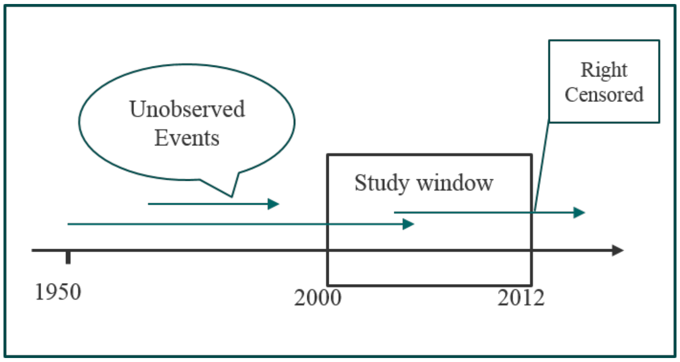



**Figure 1:** Right censor and left truncation

date. They are treated as right censor. These right censored observations require special treatment in survival analysis: a censor indicator variable is created:

$$\delta_i = \begin{cases} 1 & \text{if } t_i \leq c_i \text{ (uncensored)}, \\ 0 & \text{if } t_i > c_i \text{ (censored)}, \end{cases}$$

where, $i$ denotes the ith observation, and the failure time of event for ith observation is minimum time between $t_i$ and $c_i$, i.e., $min(t_i, c_i)$, that is when $c_i < t_i$, $c_i$ is taken as end time of the ith observation in order to do next analysis.

Left truncation is that the occurrence of an intermediate event prior to the event of interest appear in the sample dataset. Let $T$ denotes the time of event of interest to occur and let $X$ denotes the time an individual enters the study, that is time of truncation events occurs. Only the individuals with $T \geq X$ are observed in the study window. Left truncation in this study occurs due to no records for employees leaving the organization before November 2000 as unobserved events shown in the figure 1. The left truncation leads to another bias. As shown in the figure 1, the longest arrow represents a life span for an employee hired in 1950 and left in 2006. Those employees who remain in the study window increase the apparent lifetimes. The existence of truncation in the data must be taken into account in order to overcome this bias and to achieve accurate estimation of survival analysis (Carrión et al., 2010). Let $t_{i0}$ denotes the start time of the ith observation, i.e., hired time or age at hired of ith employee, $x_i$ denotes the entry time of the ith observation, i.e., the start time of study (November 1st, 2000) or age at November 1st, 2000. The start time of the observation is maximum value between $t_{i0}$ and $x_i$, that is when $t_{i0} < x_i$, $x_i$ is taken as start time of the ith observation in order to eliminate the left truncation bias (Allison, 1995). The number of failures in the $t_j$ is redefined for left truncation. When $x_i < t_j \leq t_i$, the observation is in the risk set. When $t_j < x_i \leq t_i$, the ith observation has not entered study yet at $t_j$ and it cannot be considered in the risk set. When $x_i \leq t_i < t_j$, it indicates the ith observation whose failure time before $t_j$, and it cannot be considered in the risk set at time $t_j$ neither (Carrión et al., 2010).

## 4.2 Cox PH regression model

Cox proportional hazards (PH) regression is a widely used method for estimating survival life events, introduced in a seminal paper by Cox (1972). The Cox PH model is usually taken the form of hazard model formula as shown in the equation 1:

$$h(t, x) = h_0(t)e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_k)$, $h_0(t)$ is the baseline hazard occurring when $x = 0$, $\beta$ is the coefficients of $x$. The model provides a hazard expression at time t for an individual with a given specification of a set of explanatory variables denoted by the $x$. The Cox PH formula is the product of quantities at hazard time $t$: $h_0(t)$ as the baseline hazard function and the exponential expression to the linear combination of $\beta_i x_i$, $x$ does not involve time $t$, so it is time-independent covariates. $x$ can also be time-dependent covariates, which named extended Cox PH regression as discussed in the section 4.3. The key assumption for Cox PH regression model is proportion hazards. However, Cox regression can handle non proportional hazards using time-dependent covariate or stratification. The Cox PH regression is "robust" and popular, because the baseline hazard function $h_0(t)$ is an unspecified function and its estimation can closely approximate correct parametric model (Kleinbaum, 1998). Taking the logarithm of both sides of the equation, the Cox PH model is rewritten in the equation 2:

$$\log h(t, x) = \alpha(t) + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

where $\alpha(t) = \log h_0(t)$. If $\alpha(t) = \alpha$, the baseline is exponential distribution. In the Cox PH regression, $\alpha(t)$ do not limited on specific parametric distributions and it can take any form. The partial likelihood method is used to estimated $\beta$ coefficients of the Cox model without having to specify the baseline (Allison, 1995). The Cox PH model is performed by SAS.

## 4.3 Time dependent covariate and counting process

A time dependent covariate is that a covariate is not constant through the whole study and its value changes over the course of the study. The extended Cox PH regression model incorporates both time-independent and time-dependent covariates as shown in the equation 3:

$$h(t, x) = h_0(t)e^{(\sum_{i=1}^{k_1} \beta_i x_i + \sum_{j=1}^{k_2} \gamma_j x_j(t))} \tag{3}$$

where $x = (x_1, x_2, \ldots, x_{k_1}, x_1(t), x_2(t), \ldots, x_{k_2}(t))$, $h_0(t)$ is the baseline hazard occurring when $x = 0$, $\beta$ and $\gamma$ are the coefficients of $x$. There are two time dependent covariates in this study: policy and economic indicators. Policy is to handle the downsizing policy issued in January 2008 with three months response time window to accommodate a voluntary reduction in force from the organization. Policy is a dummy variable across years:

$$Policy = \begin{cases} 1 & \text{if employee works in year 2008,} \\ 0 & \text{if employee does not work in year 2008.} \end{cases}$$

Counting process method in SAS programming statements is used to handle time dependent covariates, which is each employee have multiple records. Each record is related to a time interval and the covariates in this record remain constant. Therefore, Each employee has up to 3 records: before 2008, in-between 2008, and after 2008. Two variables, age, year of service, are used for repenting two time terminals of each interval or record. For age, one time point is age at beginning of the certain period, named "age at start"; and the other one is age at end of the curtain period, named "age at end". Two year of services points are also generated for each record: one is year of service at the beginning of the period, named "YCS at start"; the other one is the year of service at the end of the period, named "YCS at end".

Economic indicators is another time dependent covariates. Because economic indicators are fluctuated across the year, all the employees have up to 12 years records based on the calender year, which interval starts from hired date or January 1, and ends at terminated date or December 31 of certain year during the study window as shown in equation . The economic indicators are taken the average value for each year into the optimal model identified from the internal covariates to examine their impacts on turnover.

$$\begin{aligned}(\text{start point}, \text{end point}) = (&max(\text{hired date}, \text{January 1 of a certain year}), \\ &min(\text{terminated date}, \text{December 31 of a certain year}))\end{aligned} \tag{4}$$

## 4.4 Stratification model

An alternative for handling nonproportional hazards is stratification. A stratified model allows each subgroup of data as defined by a grouping variable to have its own baseline hazard while sharing parameters for other covariates across. If the proportional hazards assumption holds within these subgroups then this model allows us to get valid common estimates of covariate effects using all of the observations. Equation 5 below represents the hazard function for strata $z$;

$$h(t, x, z) = h_0^z(t)e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{5}$$

where $z$ represents the grouping variable, and $h^z\sigma_0(t)$ is a baseline hazard based for stratam z and $\beta_i$ are common effects of covariates ac. Note that the strata covariates cannot be the covariates in the Cox PH model.

The proportional hazard assumption can be tested using Schoenfeld residuals which works even if the model includes time-dependent covariates; see Allison (2010); Collett (2015). An alternative is to test the interaction between time-dependent and time-independent covariates in the Cox PH model. The assumption is valid if the interaction is not statistically significant ($P > 0.05$). Including a stratified covariate, when appropriate, can improve the Cox model's performance. The C-statistic is used to compare models with and without stratification with a higher C value indicating a better model (Lemke, 2012).

## 4.5 Competing risks

A competing risk is an event whose occurrence either precludes the occurrence of the event of interest or fundamentally alters the probability of occurrence of this event of interest

(Tableman and Kim, 2003). For example, turnover causes of an employee are exclusive and independent, i.e. an employee can experience only one event such as voluntary quit rather than retirement. This alters the probability of experiencing the event of interest, like retirement. Such events are known as competing risks events where one event of several different types of possible events can occur and hence the survival analysis for each event is calculated separately with the other events set as censored. Two mutually exclusive causes: retirement and voluntary quit are considered as the event of interests for each employees in this study, and the other events are treat as censored.

There are several reasons for selecting these two causes. One main reasean is because the organization are interested in forecasting the turnover of retirement and voluntary quit. There are 1/3 employees in that organization are over 50 years old who are eligible for retirement. The employee who voluntary quit usually is the one organization would like to keep (Allen et al., 2010). And also voluntary quit costs highly for the organizations and firms (Selden and Moynihan, 2000). Finally, the other reasons of turnover, such as layoff, transfer, death, or disability are caused by the factors which occurrence are random and hard to predict. The Cox PH regression for competing risks as shown in equation 6:

$$h_j(t,x) = h_{j0}(t)e^{(\sum_{i=1}^{k}\beta_{ij}x_i)} \tag{6}$$

where, $x_j$ is the covariate for a specific type of turnover. Note that the coefficient $\beta$ is the effects of the covariates may be different from different turnover types. If $\beta_{ij}$ is the same for all j, the model simplified to Cox PH model as shown in equation 1.

## 4.6 Variable selection

All the covariates are putting into Cox PH regression model and selected by manually backwards selection method based on $P < 0.05$. The variable selection procedure is as follow: first, all the covariates are used to build the model. Second, remove the non-significant variable ($P > 0.05$) with the largest P value, and rerun the model with the other variables. Then, repeat the second step until there is no significant variable remaining in the model.

## 4.7 Model evaluation and comparison

The Cox PH model is evaluated by four statistics criteria: Akaikes information (AIC), Schwartzs Bayesian criterion (SBC), mean absolute percentage value (MAPE) and $G^2$. The optimal model should have low AIC, SBC, MAPE, $G^2$ value for both training and holdout dataset. In this study, the model performance on holdout dataset is considered more important than that on the training dataset. AIC and SBC are both information criteria using likelihood value. Usually, the best model comes with lowest AIC or SBC values. AIC, SBC values are automatically generated by the models.

The predicted failure (retirement) probability is actually the conditional failure probability for an employee at time $t_j$, given that the employee is active at time $t_{j-1}$. It is calculated based on the baseline and coefficients from Cox PH models for both training and holdout dataset as shown in equation 7.

$$
\begin{aligned}
P\{t_{j-1} < T < t_j\} &= 1 - P\{T > t_j | T \geq t_j\} \\
&= 1 - \frac{S_{t_j}}{S_{t_{j-1}}} \\
&= 1 - \frac{S_0(t_j)^{(\sum_{i=1}^{k} \beta_i x_i)}}{S_0(t_{j-1})^{(\sum_{i=1}^{k} \beta_i x_i)}}
\end{aligned} \tag{7}
$$

where, $T$ is survival time, $t_j$ is a specific value for $T$, $S_0(t)$ is the baseline function generated by Cox PH model, $x$ is the covariates, and $\beta$ is the coefficient.

MAPE is another measure for comparing the accuracy of the model fitting between different forecast models since it measures relative performance (Chu, 1998) as shown in the equation 8.

$$
MAPE = \sum_{t=1}^{n} \left| \frac{y_t - \hat{y}_t}{y_t} \right| \frac{1}{n} \% \tag{8}
$$

MAPE is calculated by using the yearly actual and predicted retirement number as $y_t$ and $\hat{y}_t$, respectively. The predicted retirement or voluntary quit number is the expected retirement or voluntary number summarized by aggregating all the failure probabilities for the active employees in the risk set at $t_j$ as shown in 9.

$$
E(\text{turnover number at } t_j) = \sum_{i=1}^{k} P_i\{t_{j-1} < T < t_j\} \tag{9}
$$

where, $i$ denotes the ith employee.

$G^2$ is another criteria to evaluate the model prediction performance. The calculation takes the form as shown in the equation 10 (Simonoff, 2013),

$$
G^2 = 2 \sum_{t} [y_t \log (\frac{\bar{p}_t}{\hat{p}_t}) + (n_t - y_t) \log (\frac{1 - \bar{p}_t}{1 - \hat{p}_t})] \tag{10}
$$

where, $y_t$ is the number of employees retired in year $t$, $n_t$ is the workforce number in year $t$, $\bar{p}_t = y_t/n_t$, and $\hat{p}_t = \hat{y}_t/n_t$.

The logistic regression and time series moving average methods are also employed to compare with the performance of Cox PH regression model by MAPE value.

## 4.8 Simulation on right censor and left truncation

In order to understand the performance and efficiency of the Cox PH model in right censored and left truncated data we perform a simulation study.

Generated $n = 100, 200, 500, 1000, 2000,$ and $4000$ observations from a Weibull regression model with one covariate which we referred to as age.

Age is uniformly distributed from 22 to 70 years of age, which is chosen to mimic the actual distribution of workers ages in our sample.

In the regression model, the coefficients for $\beta_{age} = -.025$ (Why?) and the coefficient for $\beta_0 = 1.5$.

The survival times $T_i$ are randomly generated from a Weibull distribution with shape parameter $\alpha$ and scale parameter $\lambda$, where $\alpha = 1.5$ and $\lambda = exp(-0.025age + \beta_0)^{\frac{1}{\alpha}}$.

The simulation is performed on right censoring and left truncation separately, in order to observe the effects for different bias. For right censor simulation, two simulation procedure are conducted with different start points. First, the start point for all the observations are equal to 0, and stop point is equal to the survival time $t_i$ for ith observation where $T = (t_1, t_2, \ldots, t_n)$. After that, the censor time $C$ is equal to first quarter, median, third quarter, and maximum of the survival time, respectively, to get 75%, 50%, 25% and 0% censor proportions. When the survival time $t_i$ for ith observation is not greater than the censor time ($c_i$), the stop point is survival time $t_i$ and censor variable $\delta_i$ is 1. When survival time $t_i$ for ith observation is greater than censor time ($c_i$), the stop point is change to censor time ($c_i$) and censor variable $\delta_i$ is 0. The second right censor simulation procedure is to set the start points $S$ to follow uniform distribution from 0 to 10, representing the observations (employees) start at various time points within 10 years study window. The stop point is equal to the summation of start point and survival time: $S + T$. The censor time is a cutoff point ($C$) identified by R to get fix number of censor proportion (25%, 50%, and 75%). When the survival time $t_i$ for ith observation is not greater than the censor time ($c_i$), the stop point is survival time $t_i$ and censor variable $\delta_i$ is 1. The stop points are set to censor time $C$ for the observations with the stop points $S + T$ being great than censor time $C$, the survival time is reset to $C - S$, and censor variable $\delta_i$ is 0. Otherwise, the other observations with stop point being less than the censor time remain the same and the censor variable $\delta_i$ is 1. For the second simulation, there are 4000 observations are generated. Because some observations occur after the cutoff point (censor time) and the sample size are various for different censor proportion, only 400 are randomly selected with 75%, 50%, 25% and 0% censor proportions to keep the sample size same. The start point and the stop point are dependent variable in the cox regression model. $\delta$ is censor variable.

For left truncation simulation, the start point $U$ is generated as uniform distribution with $a = 0$ and $b = max(T)$ which indicates an observation start randomly from time 0 to time $max(T)$. The stop point $S$ is $U + T$. The histogram is generated for $S$. The truncation time $L$ is set as 0, first quarter, median, and third quarter of $S$, respectively, to get 0%, 25%, 50%, and 75% truncation proportions. When start point $u_i$ for ith observation is less than truncation time $l_i$, the start point is reset as truncation time $l_i$. When start point $u_i$ for ith observation is not less than truncation time $l_i$, the start point does not change ($u_i$). In left truncation, the censor variable $\delta$ for all the observations are equal 1. For right censor and left truncation simulation, the Cox regression models are modeled by "coxreg" and "phreg" function in eha package. The coxreg function is the regular cox regression modeling procedure to generate coefficient estimates and a non-parametric baseline. The phreg funcation is using parametric distribution like Weibull, Extrem value (EV) to estimate the baseline. The "phreg" function performs Cox PH model and also provides a parametric baseline hazards estimation (Broström, 2012). The phreg function is used to compare the non-parametric with parametric baseline and to demonstrate the prediction when the parametric baseline is not right. Total predicted failure number is calculated as shown in equation 7 and 9. The actual and predicted failure number are compared to show right censor and left truncation's impacts on the coefficient and baseline estimation .

# 5 Results

## 5.1 Right censor and left truncation simulation results

The right censoring simulation results are shown in the left part of Table 1 which all the observations start at the same time 0. The events in the second column of the table are the actual total failure events simulated without considering censoring, which is $n = 100, 200, 500, 1000, 2000, \text{and } 4000$. The number of events in the dataset applying to the Cox model are reduced due to the right censoring, which is equal to $events = \sum_{i=0}^{n} \delta_i$. The values in the other column are the average value of 100 replications for Cox PH model coefficient and baseline parameter estimates based on Weibull distribution using phreg. The simulation results show censoring proportion and the number of events are two influential factors for the coefficient estimation. The model overestimates the coefficients of age, $\lambda$, and $\alpha$, when the dataset has high proportion of censoring. For example, when 75% of the data are censored with only 25 events, the estimates for three parameters are 0.028, 4.043, and 1.564, respectively, which are the highest among all the estimates. As the event number increasing, the estimation is approaching to the actual value. For example, the estimation of age, $\lambda$, and $\alpha$ are close to 0.025, 2.7, and 1.5, respectively, after the number of event is at least 500. The predicted events in the sixth column is the total predicted failure number calculated by applying the coefficient estimates and the non-parametric baseline from Cox PH models into the dataset without considering censoring. The predicted events using censoring models are all lower than the actual total failure number, but close to the number of events after censoring. For example, the number of predicted events is 24.84 when using the estimates of Cox PH model with 75% censoring to calculate the dataset with 100 events, which is close to 25. The prediction is less than the actual dataset, because the baseline is lack of information for the events after censored time. The baseline does not include hazards or survival probability information for the long life time observation, because the baseline just captures the events before the censor time and the observation with long life time are censored in this simulation. As a result, there is no failure probability for long life time observation.

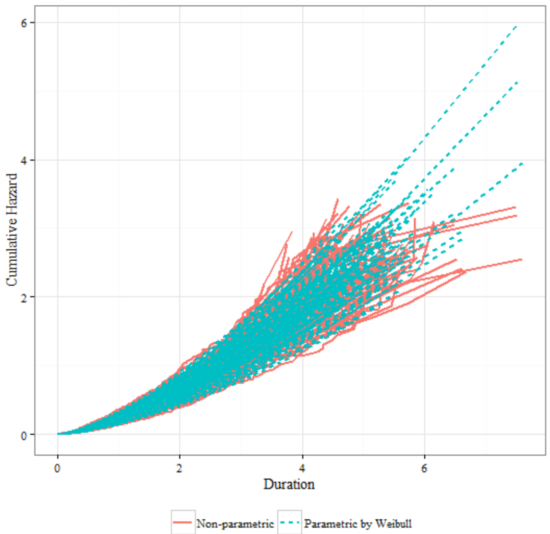
To simulate the realistic situation, the second right censor simulation is conducted with randomly occurrence of the observations followed by uniform distribution. The results are shown in the table 2. The events without considering censoring are all equal to 400 for easy comparison. The predicted events include the prediction by non-parametric baselines using "coxreg" and by parametric baseline using "phreg" function Weibull distribution as baseline estimation. The estimation of age and $\alpha$ are all close to the simulated value (0.025 and 1.5). And the estimation of $\lambda$ are around 2.8. These indicate the right censoring does not impact the coefficients estimation. However, right censoring does impact the baseline function estimation for Cox PH model as shown in the figure 2. The red lines in the figure 2 are the non-parametric baseline for 100 replications. And blue dash lines are the parametric baseline generated base on Weibull distribution for 100 replications, which represent the actual baseline. All these baselines are overlap together by various censoring, which indicates

**Table 1:** Right censoring and left truncation simulation statistics

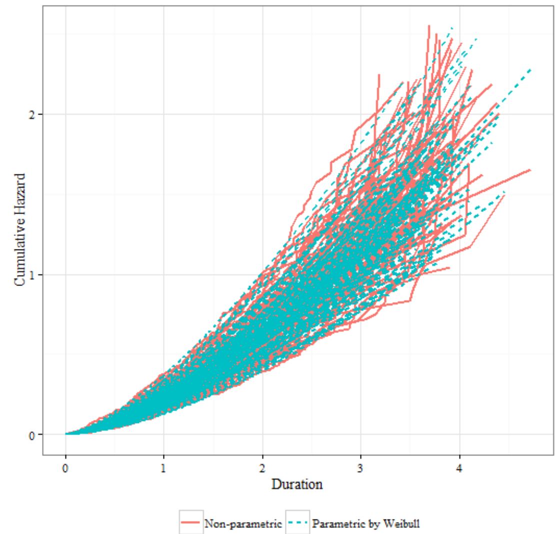
| Censor proportion | Events | Variable Estimates | | | Predicted Events | Truncation proportion | Events | Variable Estimates | | | Predicted Events |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Age | $\lambda$ | $\alpha$ | | | | Age | $\lambda$ | $\alpha$ | |
| 0% | 100 | 0.026 | 2.931 | 1.509 | 97.42 | 0% | 100 | 0.027 | 2.865 | 1.534 | 96.60 |
| 25% | 100 | 0.027 | 2.962 | 1.527 | 74.31 | 25% | 75 | 0.027 | 2.917 | 1.546 | 72.86 |
| 50% | 100 | 0.028 | 3.237 | 1.530 | 49.66 | 50% | 50 | 0.027 | 2.899 | 1.577 | 47.32 |
| 75% | 100 | 0.028 | 4.043 | 1.564 | 24.84 | 75% | 25 | 0.029 | 3.280 | 1.757 | 21.75 |
| 0% | 200 | 0.026 | 2.841 | 1.508 | 197.23 | 0% | 200 | 0.025 | 2.777 | 1.506 | 196.13 |
| 25% | 200 | 0.026 | 2.856 | 1.513 | 149.34 | 25% | 150 | 0.025 | 2.756 | 1.515 | 147.97 |
| 50% | 200 | 0.026 | 2.925 | 1.527 | 99.68 | 50% | 100 | 0.025 | 2.825 | 1.532 | 97.44 |
| 75% | 200 | 0.026 | 3.167 | 1.540 | 49.86 | 75% | 50 | 0.026 | 2.927 | 1.572 | 47.17 |
| 0% | 500 | 0.025 | 2.731 | 1.500 | 496.77 | 0% | 500 | 0.025 | 2.732 | 1.509 | 494.78 |
| 25% | 500 | 0.025 | 2.718 | 1.508 | 374.31 | 25% | 375 | 0.025 | 2.737 | 1.514 | 373.42 |
| 50% | 500 | 0.025 | 2.744 | 1.514 | 249.65 | 50% | 250 | 0.026 | 2.778 | 1.514 | 247.70 |
| 75% | 500 | 0.025 | 2.787 | 1.525 | 124.89 | 75% | 125 | 0.026 | 2.835 | 1.547 | 124.15 |
| 0% | 1000 | 0.025 | 2.748 | 1.509 | 996.41 | 0% | 1000 | 0.025 | 2.710 | 1.504 | 993.77 |
| 25% | 1000 | 0.025 | 2.747 | 1.512 | 749.23 | 25% | 750 | 0.025 | 2.709 | 1.504 | 748.94 |
| 50% | 1000 | 0.025 | 2.748 | 1.514 | 499.61 | 50% | 500 | 0.025 | 2.715 | 1.506 | 503.37 |
| 75% | 1000 | 0.026 | 2.844 | 1.509 | 249.80 | 75% | 250 | 0.025 | 2.694 | 1.524 | 250.93 |
| 0% | 2000 | 0.025 | 2.714 | 1.502 | 1996.19 | 0% | 2000 | 0.025 | 2.740 | 1.503 | 1993.65 |
| 25% | 2000 | 0.025 | 2.713 | 1.503 | 1499.29 | 25% | 1500 | 0.025 | 2.731 | 1.502 | 1507.94 |
| 50% | 2000 | 0.025 | 2.742 | 1.500 | 999.68 | 50% | 1000 | 0.025 | 2.724 | 1.503 | 1012.37 |
| 75% | 2000 | 0.025 | 2.733 | 1.502 | 499.89 | 75% | 500 | 0.025 | 2.718 | 1.508 | 512.30 |
| 0% | 4000 | 0.025 | 2.719 | 1.504 | 3995.80 | 0% | 4000 | 0.025 | 2.720 | 1.500 | 3988.90 |
| 25% | 4000 | 0.025 | 2.718 | 1.505 | 2998.86 | 25% | 3000 | 0.025 | 2.722 | 1.501 | 3014.31 |
| 50% | 4000 | 0.025 | 2.724 | 1.503 | 1999.52 | 50% | 2000 | 0.025 | 2.710 | 1.502 | 2032.03 |
| 75% | 4000 | 0.025 | 2.729 | 1.513 | 999.86 | 75% | 1000 | 0.025 | 2.703 | 1.503 | 1028.39 |

**Table 2:** Right censor simulation results by various start time

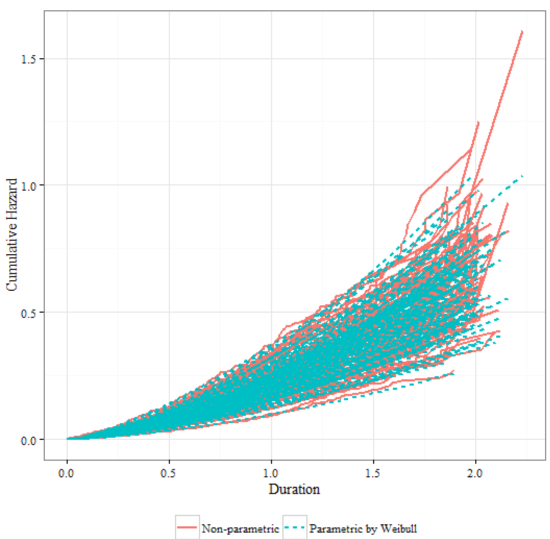
| Censor proportion | Events | Variable Estimaties | | | Predicted Events | |
|---|---|---|---|---|---|---|
| | | Age | $\lambda$ | $\alpha$ | "coxreg" | "phreg" |
| 0% | 400 | 0.025 | 2.694 | 1.508 | 398.52 | 400.44 |
| 25% | 400 | 0.026 | 2.802 | 1.518 | 394.24 | 401.72 |
| 50% | 400 | 0.026 | 2.828 | 1.514 | 340.73 | 398.51 |
| 75% | 400 | 0.025 | 2.821 | 1.518 | 215.92 | 400.80 |



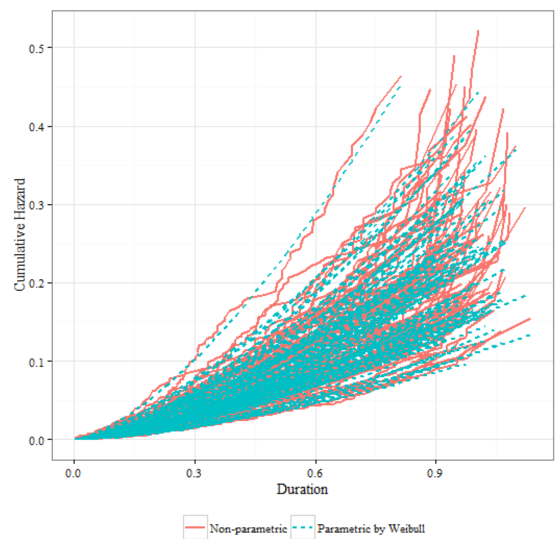

**(a)** No right censor




**(b)** 25% right censor



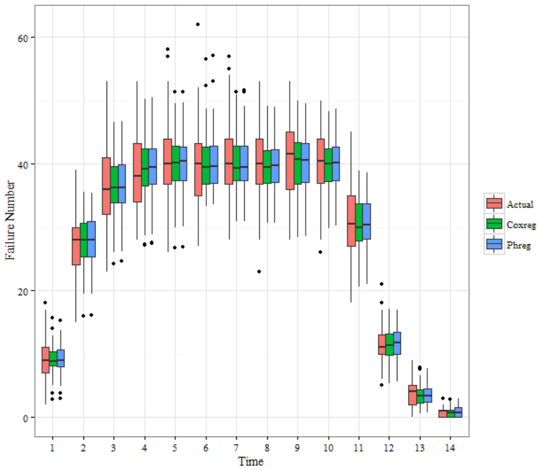

**(c)** 50% right censor



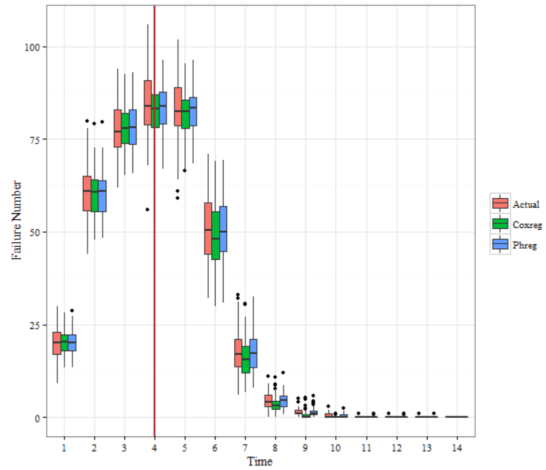

**(d)** 75% right censor

**Figure 2:** Baseline comparison by various censoring

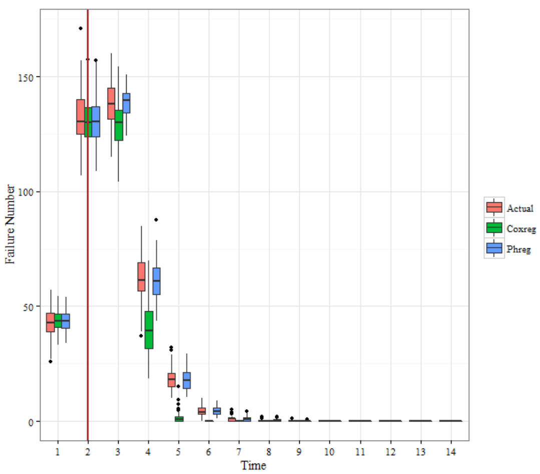
non-parametric baseline can capture the true parametric baseline. However, the duration of the non-parametric baseline is getting short by increasing the censoring proportion. As a result, the predicted number of events using non-parametric baseline is less than the actual failure number. However, it does not affect the prediction by using parametric baseline, which are all close to 400, because the parametric baseline is not limited by the baseline duration if the parameters are correctly estimated. The prediction by "coxreg" and "phreg" are plotted across time by box plot to compare to the actual failure values as shown in the figure 3. The vertical red line is the average value of censor time. The predicted number of events by "coxreg" are close to actual and the prediction by "phreg" before the censor time line. It



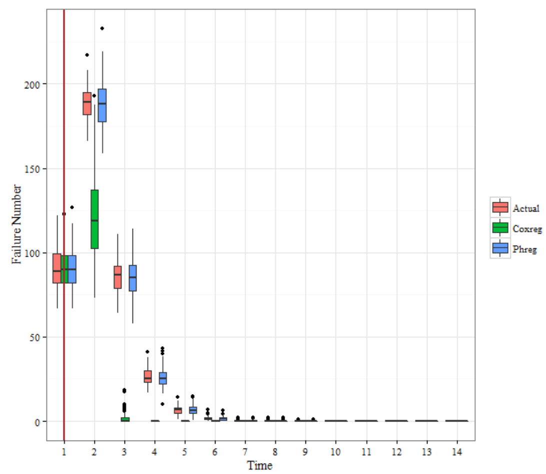

**(a)** No right censor




**(b)** 25% right censor




**(c)** 50% right censor

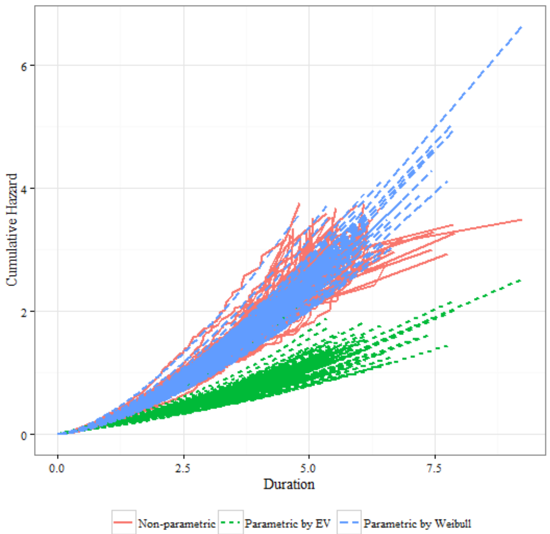



**(d)** 75% right censor

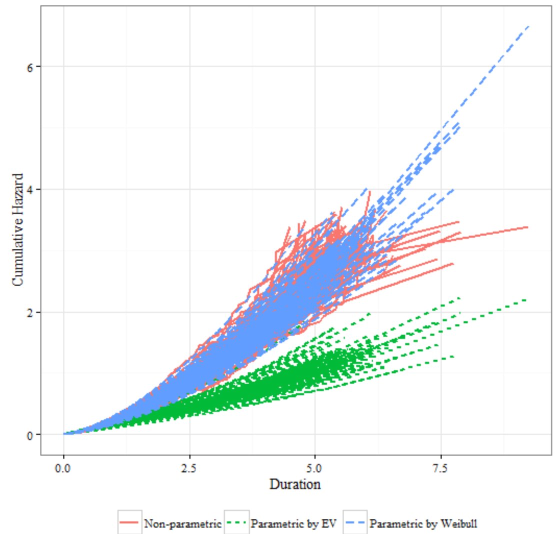
**Figure 3:** Right censor simulation results: actual vs. predicted failure number

is still close to actual after censor time line for the data with no censoring and with 25% of censoring as shown in the figure 3a and 3b, because of the long baseline duration. However, It drops down gradually after the censor time for the data with 50% and 75% of censoring as shown in figure 3c and 3d, because the baseline duration ends around censor time. This simulation clearly shows the limitation of Cox PH model with non-parametric baseline: it cannot accurate predict far beyond the duration of the longest events. Therefore, a baseline can be highly variable in the extremes leading to poor predictions. Although this simulates the real problem, the employee hiring time points are followed by uniform distribution in the real world.

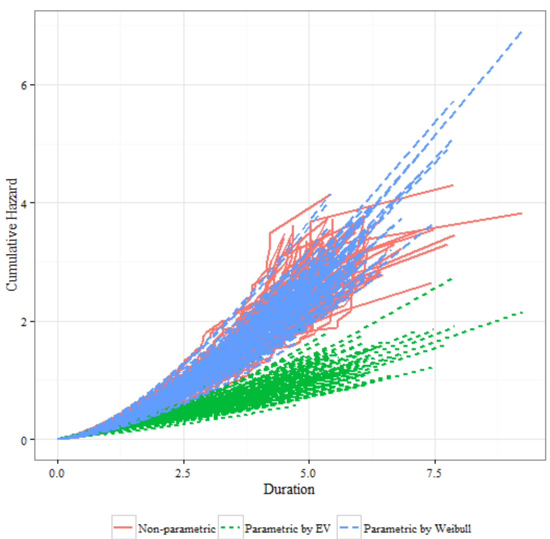
The results of the left truncation bias simulation statistics are shown in the right part of table 1. All the values shown are the average value for coefficient estimate of age, and baseline parameter estimates of $\lambda$ and $\alpha$ by "phreg" Weibull distribution. The last column is the total predicted number of events using non-parametric baseline by "coxreg" function. Similar as the right censor simulation result, left truncation proportion and the number of



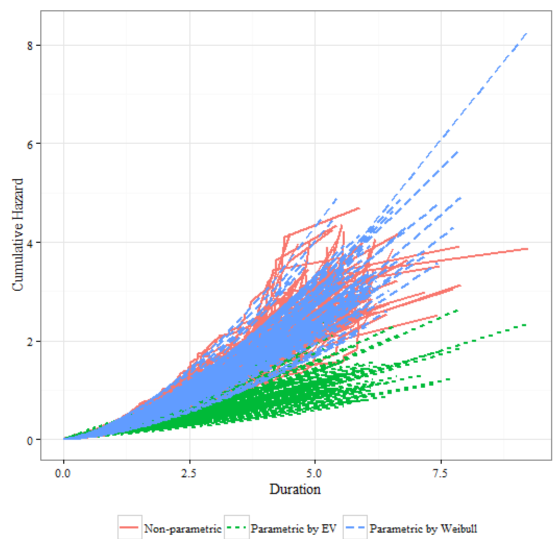

**(a)** No left truncation




**(b)** 25% left truncation



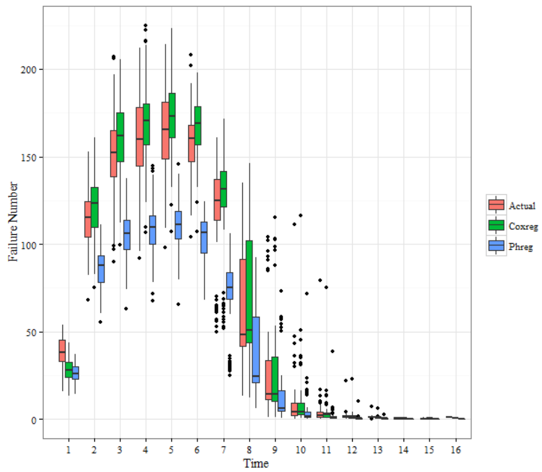

**(c)** 50% left truncation



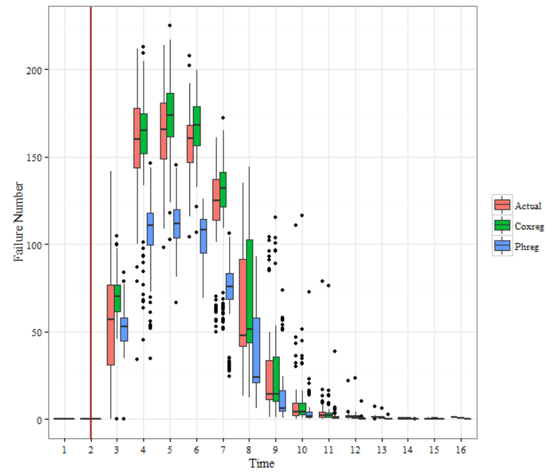

**(d)** 75% left truncation

**Figure 4:** Left truncation simulation results: Baseline Comparison

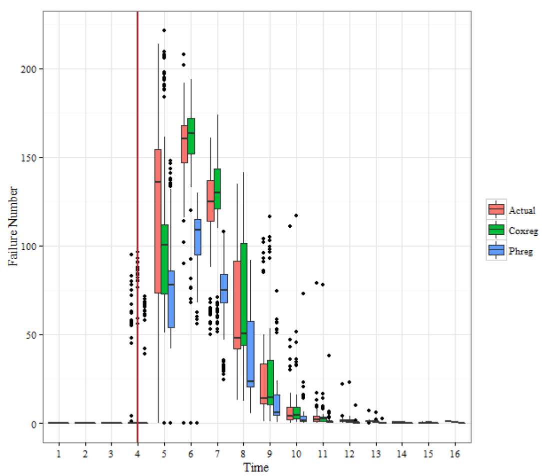
events are two key factors for coefficients estimation. As table 1 shown, the coefficients are overestimated when the left truncation proportion is 75% or the number of events are less than 200. However, increasing the number of events can offset (reduce) the left truncation effects. For example, the estimates for age and $\alpha$ are all close to simulated value, when the number of events is 1000 with 75% truncation proportion. The predicted number of events is close to the actual one as shown the green and red box in the figure 5, which is the box plot of total 2000 events simulated with 100 replication by various proportion of left truncation. The red vertical line is the average of left truncation time. It is slightly overestimated using "coxreg" non-parametric baseline by four left truncation proportions, because the baseline



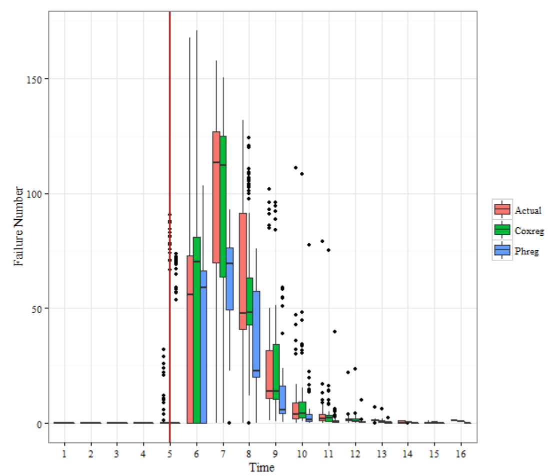

**(a)** No left truncation




**(b)** 25% left truncation




**(c)** 50% left truncation




**(d)** 75% left truncation

**Figure 5:** Left truncation simulation results: actual vs. predicted failure number

estimation is not affected by the left truncation. The figure 4 clear shows the duration of non-parametric baseline are the same for four left truncation proportion. It also shows the non-parametric baselines (red lines) are overlap with parametric baseline (blue dash lines).

To further test how baseline estimation impacts on the prediction, another simulation is conducted accompanied with left truncation simulation. In the previous left truncations study, the parametric baseline using Extreme Value (EV) distribution is generated by "phreg" and predicted the number of events based on it shape and scale parameter estimation as shown in figure 4. Because the data is generated by Weibull distribution, the parametric baselines (green dash line) by EV are much lower than the other two baselines by all four left truncation proportions. The predicted number of events based on the EV baseline are also much lower than the actual number of events across the time as the blue box shown in the figure 5. This study shows the inaccuracy estimation of the baseline lead to poor prediction.

Therefore, accurate estimation of coefficients and the baseline are two key factors impacts the perdition of the events of the Cox PH model. The simulation test shows that the Cox PH model can accurately estimates the coefficients when events number is at least 1000 even with high proportion of censor and left truncation. The baseline is another key factor to predict the number of events. The prediction will be accurate if the parametric distribution of the baseline is known or identify the correctly. Otherwise, a wrong baseline can also deteriorate the prediction. Compared to parametric baselines, a non-parametric baseline is more robust. However, it still needs enough number of events with curtain long period to get a smoothed and long baseline to predict accurately. As a result, it is hard to accurately predict the employee turnover for a company just formed recently, or when the employee population characteristic changed, due to high proportion of censor and lack of events. Although this study has more than 50% right censor, it still has more than 3000 events with long duration (around 50 years length).

## 5.2 Data pattern/Descriptive Analysis

The construction of a predictive model for retirement involves consideration of several factors. Among the variables available for analysis we must choose the set that offers the maximum predictive power, i.e. a model that includes variables that provide the best possible predictions on out of sample testing data sets and not simply on data used to train the model. We must also evaluate potential strengths and weaknesses of the baseline hazard estimate and its impact on prediction accuracy. Finally, it is useful to also understand the impact of various predictors on retirement age.

As described in Section 3, the current data provides five demographic and career history variables: payroll(hourly, weekly, or monthly payroll), gender(M,F), division(ten divisions), occupational codes(crafts, engineers, general administrative, laborers, managers, administrative, operators, scientists, technicians), age at hire, and years of service at hire. Table 3 provides marginal counts of the number of workers in the sample within each category. From that we see that among occupation codes there are four large categories (C,E,M,& P) with over one thousand employees observed throughout the data sample, four medium sized groups with 500-700 employees (G,L,R & T), and one small group, S, with 208 employees. Payroll data shows that the largest group is paid monthly, followed by hourly, and weekly. In terms of gender, approximately 72% of employees are male. Finally, divisions, while not fixed over the life of the employee, are distributed similarly to occupational codes with four larger groups and a number of smaller groups.

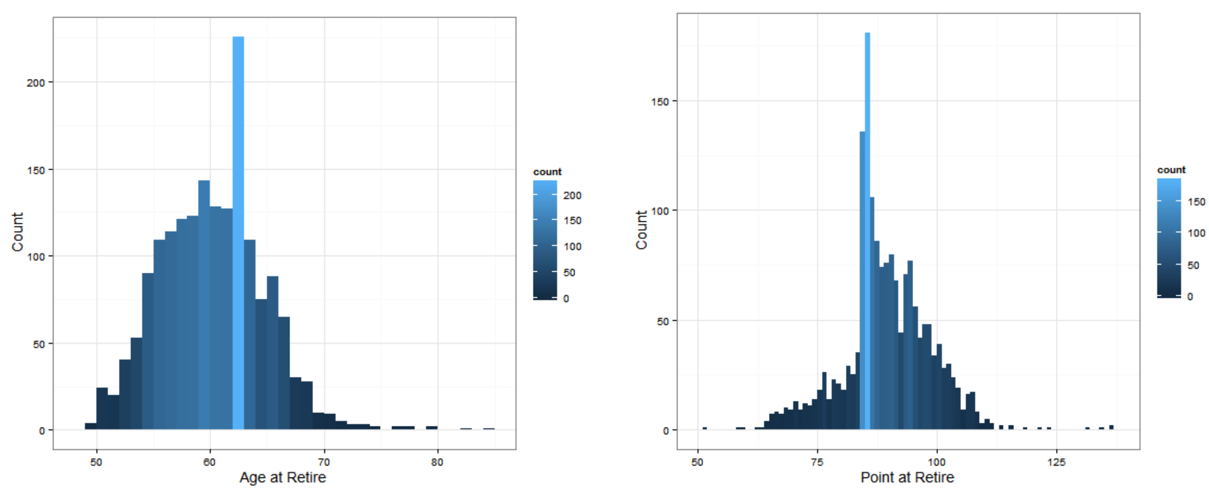
Beyond these demographic and career factors, our models also include behavioral variables derived from policy requirements for retirement and early retirement incentives that occur throughout the observation period. Histograms of retirement age and accumulated pension points are shown in Figure 6. The histogram of for age at retirement is right skewed and shows an anomalous spike at age equals 62 which is the mode, because age 62 is the earliest age for a person to receive social security retirement benefit. The average age is 59.72 demonstrating that many individuals retire before 62 and most before age 70. In terms of points accrued at time of retirement, recall that points are the sum of years of service plus current age, we see an irregular distribution with the vast majority retiring with point totals in the range of 85 to 100 and relatively few taking a reduced pension are retiring with diminished benefits with points below 85. Again we see that 85 points is the mode indicating that retiring immediately after become fully vested in the pension is a popular choice.

**Table 3:** Descriptive Statistics

| | Count | N % | | Count | N % |
|---|---|---|---|---|---|
| **Cocscode** | | | **Gender** | | |
| C | 1295 | 16.0% | F | 2296 | 28.4% |
| E | 1361 | 16.8% | M | 5802 | 71.6% |
| G | 574 | 7.1% | **Division** | | |
| L | 613 | 7.6% | divison1 | 1542 | 19.0% |
| M | 1178 | 14.5% | divison2 | 751 | 9.3% |
| P | 1621 | 20.0% | divison3 | 1042 | 12.9% |
| R | 595 | 7.3% | divison4 | 369 | 4.6% |
| S | 208 | 2.6% | divison5 | 398 | 4.9% |
| T | 652 | 8.1% | divison6 | 1199 | 14.8% |
| Missing | 1 | 0.0% | divison7 | 302 | 3.8% |
| **Payroll** | | | divison8 | 823 | 10.2% |
| Hourly | 2503 | 30.9% | divison9 | 404 | 5.0% |
| Monthly | 4369 | 54.0% | divison10 | 1268 | 15.7% |
| Weekly | 1226 | 15.1% | | | |

**Table 4:** Discriptive Statistics 2

| | Count | Mean | Median | Mode | Minimum | Maximum | Std. Deviation |
|---|---|---|---|---|---|---|---|
| Age at Retire | 1757 | 59.72 | 60.00 | 62.00 | 49.00 | 84.00 | 4.56 |
| Years of Service at Retire | 1757 | 29.72 | 30.90 | 30.06 | 0.05 | 55.68 | 7.74 |
| Point at Retire | 1757 | 89.44 | 88.67 | 85.47 | 51.05 | 136.66 | 9.15 |




(a) (b)

**Figure 6:** Histogram of Age and point at retire

## 5.3 Retirement model without external variables

Extensive model selection using a variety of metrics including Log likelihood, AIC, BIC,and out of sample predictive scoring (MAPE and $G^2$) was used to identify key predictive factors

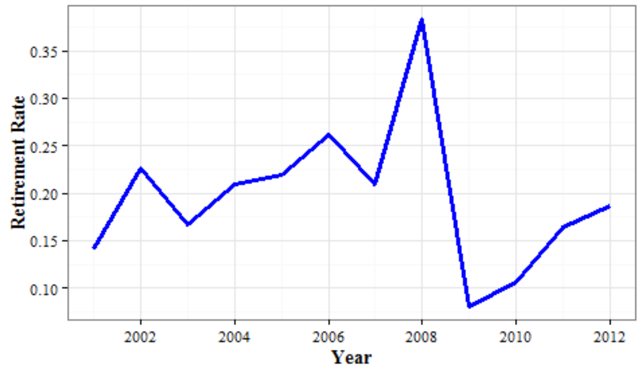



**Figure 7:** Retirement rate

in the model as shown in table 5.

**Table 5:** Models statistics

| Model | LR | AIC0 | AIC | SBC0 | SBC | Pred. MAPE | Hold-out MAPE | Pred.G2 | Holdout G2 |
|---|---|---|---|---|---|---|---|---|---|
| Division Gender Payroll Cocscode YCSH Ageh | 1194.3 | 20425.6 | 19271.3 | 20425.6 | 19377.3 | 39.44 | 56.78 | 381.77 | 85.19 |
| Division Cocscode YCSH Ageh | 1193.77 | 20425.6 | 19269.8 | 20425.6 | 19370.5 | 39.45 | 56.84 | 381.92 | 85.29 |
| Division Policy YCSH Ageh | 1451.62 | 20425.6 | 18998 | 20425.6 | 19061.6 | 25.91 | 15.51 | 128.75 | 2.64 |
| Division Policy YCSH Ageh Cocscode | 1469.92 | 20425.6 | 18995.7 | 20425.6 | 19101.7 | 25.91 | 15.24 | 129.47 | 2.56 |
| Division Policy YCSH Ageh and Strata on Cocscode | 1426.34 | 14602.1 | 13199.8 | 14602.1 | 13263.4 | 43.45 | 87.92 | 186.89 | 50.00 |
| 9 models by Cocscode | N/A | N/A | N/A | N/A | N/A | 24.62 | 3.40 | 51.19 | 0.17 |
| Division Policy YCSH Ageh P85 | 1826.95 | 20425.6 | 18624.6 | 20425.6 | 18693.6 | 25.59 | 19.04 | 109.17 | 3.40 |
| Division Policy YCSH Ageh P85 P85*A65 | 1873.69 | 20425.6 | 18579.9 | 20425.6 | 18654.1 | 25.38 | 7.97 | 111.55 | 0.79 |
| Division Policy YCSH Ageh P85 P85*A65P85*POLICY | 1881.02 | 20425.6 | 18574.6 | 20425.6 | 18654.1 | 25.42 | 4.20 | 112.27 | 0.81 |
| Logistic regression | 4103.91 | 13618.4 | 9556.53 | 13627.3 | 9752.1 | 28.20 | 31.73 | 232.84 | 117.38 |
| Time series | N/A | N/A | N/A | N/A | N/A | 11.17 | 34.38 | 32.10 | 8.54 |

Based on this analysis the optimal modelling variables, excluding aggregate economic factors, chosen for the prediction of age at retirement include *division*, *years of service at hire*, and *age at hire*. In addition, based on our understanding of the covenants and parameters of the retirement program we tested a number of additional variables and found several that increase the predictive power of the model. These include *policy*(an indicator for the 2008 ERI program), *P85*(An indicator that the individual has accrued 85 points and can retire with full benefits), *A65*P85* (An interaction term that moderates the impact of the *P85* effect after the individual has exceeded 65 years of age), and *Policy*P85* (An interaction term that moderates the impact of the *P85* effect while the ERI is in place).

Table 6 describes the fit parameters and hazard ratios. As noted above, we did not find that gender, occupational code and payroll category were not significant predictors in the presence of the other variables. This indicates that employees' gender, job types, and payroll status are not associated with choice of retirement age conditional on the other variables in the model.

**Table 6:** Parameter estimates for models

| | | Period Model | | Yearly model | |
|---|---|---|---|---|---|
| Parameter | Label | Parameter (Standard Error) | Hazard Ratio | Parameter (Standard Error) | Hazard Ratio |
| division | dir2 | -0.965(0.179)***[1] | 0.381 | -1.025(0.177)*** | 0.359 |
| division | dir3 | -0.241(0.112)* | 0.786 | -0.246(0.111)* | 0.782 |
| division | dir4 | 0.078(0.195) | 1.081 | -0.039(0.195) | 0.962 |
| division | dir5 | -0.131(0.190) | 0.877 | -0.246(0.190) | 0.782 |
| division | dir6 | 2.136(0.095)*** | 8.463 | 2.252(0.095)*** | 9.511 |
| division | dir7 | 2.435(0.129)*** | 11.418 | 2.437(0.129)*** | 11.437 |
| division | dir8 | 0.864(0.106)*** | 2.373 | 0.816(0.106)*** | 2.261 |
| division | dir9 | -3.023(0.581)*** | 0.049 | -2.774(0.504)*** | 0.062 |
| division | dir10 | 0.793(0.093)*** | 2.211 | 0.709(0.093)*** | 2.031 |
| YCSH | | 0.019(0.004)*** | 1.019 | 0.043(0.004)*** | 1.044 |
| Policy | 1 | 0.859(0.169)*** | . | 0.942(0.109)*** | . |
| Age_Start | | -0.172(0.013)*** | 0.842 | -0.187(0.013)*** | 0.829 |
| P85 | 1 | 1.435(0.091)*** | . | 0.682(0.072)*** | . |
| A65*P85 | 1 | -1.610(0.206)*** | . | -0.662(0.177)*** | . |
| Policy*P85 | 1 | 0.469(0.179)** | . | 0.600(0.130)*** | . |

[1] * denotes $P < 0.05$, ** denotes $P < 0.01$, and *** denotes $P < 0.001$.

*P85* is an indicator that a person is eligible for maximum retirement benefits and naturally this has a strong impact on the probability that a person will retire. From a quantitative point of view the hazard ratio is $e^{1.44} = 4.22$. So the hazard of retirement becomes 4.22 times more likely after the person exceeds 85 points. While not surprising, this quantification is important in predicting individual and aggregate retirement time and reflects the modal spike observed in the histogram in Figure 6. The survival function after 85 points is achieved can be su (Julia, why is the hazard ratio not published in the data??? because it has interaction term)

An alternative eligibility criteria for retirement occurs when individuals exceed an age of 65 years and so we would anticipate the hazard increasing at this point in an employees career. Because the response variable in our model is age, we cannot estimate the effect of this within the proportional hazards setting because the impact is included in the baseline hazard which should increase after this point; see figure 8 (explain). However, by including an interaction between the indicators of Age greater than 65 and points greater than 85, *A65*P85*, we can estimate how the impact of reaching 85 points diminishes when a person exceeds regular retirement age. In this case, the interaction term is estimated at -1.61 indicating a diminishing effect on the *P85* criteria to $e^{1.44-1.61} = 0.84$. This suggests that people that exceed both criteria actually have a reduced hazard of retiring over those have only met the Age 65 criteria. In other words, the fact that the individual remains on the job after hitting either criteria indicates that the other criteria has less impact (or that they are intending to work longer?). (Discuss that the baseline may increase so that the overall probability is actually higher.)

According to our model, retirement can also be influenced by an employee's age at hire and their years of service at the time of hiring, *YCSH.* The coefficient estimate for age is -0.17. As the reference age is 45.49, this means that the hazard ratio for retirement of an employee that started working at age 46.49 is $exp(-.017) = .84$ indicating a 16% drop in hazard for each additional year later that an employee started. The employee's survival probability at any time, $t$, can be computed as $S(t)^{1.19} = (S(t)^{e^{0.17}})$ when age at hire is one year below 45.49, where $S(t)$ is the baseline survival probability for a reference employee of average age at the time of hiring. Moving in the other direction, employee's survival probability is $S(t)^{0.84} = (S(t)^{e^{-0.17}})$ for a one year increase beyond 45.49 in the employee's starting age. Together, this implies that at any given retirement age, the employee who starts earlier than 45.49 years old is more likely to retire because they have more years of service and closer to vesting full benefits (85 points) than an equivalent employee who starts working at an older age. Similarly, the employee's years of service at hire show a positive estimate (0.019) with a hazard ratio (1.019) indicating that each year of service at hire beyond the baseline of 2.75 is associated with an approximately 2% increase in the hazard of retirement. This leads to a survival probability $S(t)^{0.98}$ for a one year decrease in the reference years of service at time of hiring. On the other hand, the survival probability is $S(t)^{1.019}$ for an employee with one year of service above average at the time of hire. Together, both age at hire and YCSH effects reflect the intuitive fact that, all else being equal, an employee who has more years of service and therefore is closer to full vesting is more likely to retire. What is non-intuitive about this finding is that while one might suspect that the effects should be of similar magnitude, we actually see that the effect of one year difference in age seems to have about 8 times the impact that one year of previous service does on the hazard of retirement.

In the fiscal year 2008, the employer in this study created a temporary early retirement buyout option. The response window for this option was 3 months although the specific details beyond this are unknown. In order to deal with the increased level of retirement during this period we including a time dependent indicator variable. The coefficient for this indicator was 0.86 leading to a hazard ratio of $e^{.86} = 2.21$ which indicates that, on average, an individuals hazard of retirement increased by almost 2.2 times during this period. If more information were known about the requirements or targets of this policy, a more case specific estimate may be possible. However, this would not effect the overall aggregate retirement estimates. It is important to include this one-time effect in order to improve the estimates of other factors(work on this last part - why do we need to include).

As a second step, we further test the policy effect on the employees who are eligible for getting a pension. The test results show that the policy had a significant effect for an employee who is eligible for getting the full pension benefit rather than the employee who is only eligible for getting partial retirement as the interaction term of between policy and indicator variables that employees achieve points 75 or points 65 are not statistically significant. The hazard ratio for the policy effect on those employees increases substantially to 15.85 from 2.21, which is more than seven times of the basic policy effect, after the model adding a interaction term of policy and the indicator that a person exceeds 85 points.

The *division* variable was a significant predictor. For analysis, the baseline level was chosen arbitrarily as division 1 so that it's hazard rate is determined by the baseline. Relative to this baseline, divisions 6 and 7 have very high hazard ratios, 8.363 and 11.405 respectively, which indicates, other factors being equal, that the employees in division 6 and division 7 are much more likely to retire at any age than those from division 1. Conversely, division 9 has a hazard ratio of exp(-3.023) = .049 indicating that individuals within this group have 1/20 the hazard of group 1. This may indicate that the division is new and contains younger employees(how to deal with this?? Time varying covariate?). In general, differences in retirement rates could be caused by differences in age demographics, leadership, departmental and job function, or departmental leadership.

The baseline survival function and log hazard function are shown in figure 8. The survival probability is 1 before age 49 as shown in figure 8a, which indicates that no employees retire before this age. The survival probability starts to slowly decease from age 50 to age 62. By age 62 the survival probability has decreased to close to 0.75, which indicates that 75% of employees retire at an age greater than 62 years. The slope of survival function decreases sharply at this point indicating the increased retirement rates for workers between age 62 and 65. After 65 the probability drops off even further as most of the remaining population retires by age 68 or 69. Accompanying the survival function is the log of the cumulative hazard ratio. Again, the steep rise in the cumulative hazard between age 62 and 65 indicates the increased retirement activity during this period. After this the cumulative hazard levels off indicating a drop in the hazard rate at these future points.

Summarize external

Summarize the predictive capabilities at the individual level. Strengths and weaknesses.

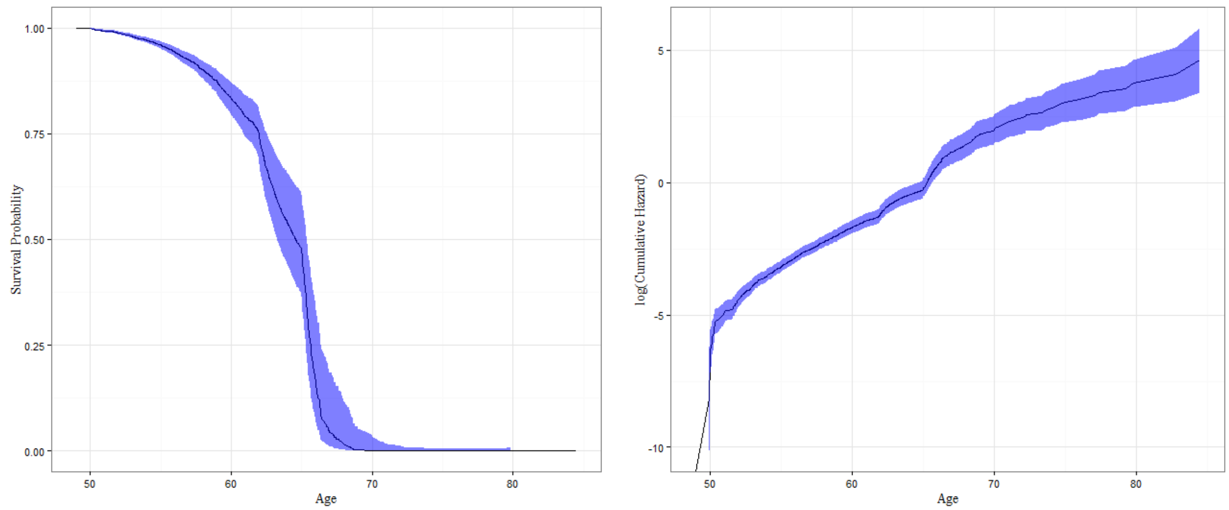
Summarize the predictive capabilities for aggregates. Strengths and weaknesses.

**Julia: We need to describe the prediction process results.**

The prediction for the employee retirement is shown in the figure 9, table 7 and 8. The model predictions capture the fluctuations in actual retirement, and also capture the peak of the special year 2008 when an early retirement incentive(policy) was introduced as shown in Figure 9. The out of sample predictions for holdout years (2011 and 2012) are very close to the actual number, indicating that the model performs well on both the training and holdout samples. Besides predicting the overall retirement, the model can also provide predictions by category. The table 7 and 8 shows the prediction by occupation code and division by year, which is the summation of the retirement probabilities of individuals by job classification and division, separately. As the tables shown, the predicted values are also close to the actual values in these two categories.

## 5.4 Retirement model with external variables

Because the previous model cannot account for the time varying features of the economic indicators, we use a new counting process model with yearly interval based on calendar year to test their effects on retirement. This model has the similar parameter estimation




**(a)** Survival function **(b)** Cumulative hazard

**Figure 8:** Baselines with 95% confident intervals

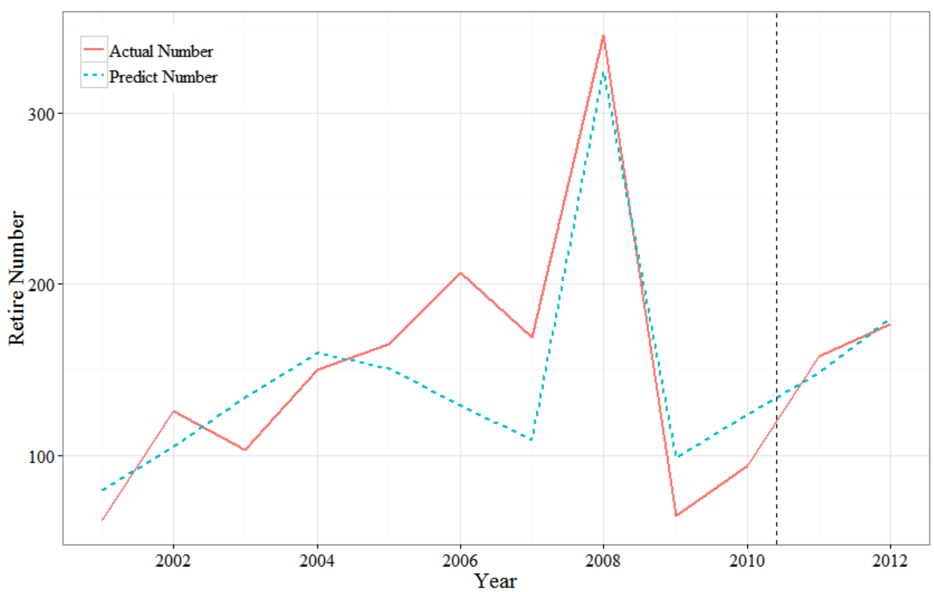



**Figure 9:** Retirement Forecasting

as the selected model as shown in the right part of the table 6. We found three indicators are statistically significant and also improve the model forecasting due to lower MAPE and $G^2$ than the values of selected model without economic indicator, which are S&P500, Real Earnings and Wilshire 5000 as shown in table 9. Although Real price is also statistically significant, its coefficient estimates is 0.0004 leading hazard ratio is 1, which indicate it does not impact the employee's retirement behaviors. As shown in table 9, Real earnings is the most important factor among all the indicators as it has lowest $G^2$. The test results show that it has strong impact on the retire behaviors. As shown in figure **??**, the fluctuation of

**Table 7:** Predictions by job classification

| Year | Crafts | Engineers | General Admin. | Laborers | Managers | Prof. Admin. | Operators | Scientists | Technicians | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| 2001 | 23[1](16)[2] | 12 (10) | 4 (1) | 6 (5) | 11 (14) | 11 (9) | 8 (2) | 2 (1) | 4 (4) | 81 (62) |
| 2002 | 31 (29) | 15 (16) | 5 (15) | 7 (11) | 15 (26) | 13 (18) | 11 (6) | 2 (1) | 6 (4) | 105 (126) |
| 2003 | 37 (26) | 18 (13) | 6 (5) | 9 (6) | 17 (21) | 18 (13) | 16 (15) | 4 (2) | 8 (2) | 133 (103) |
| 2004 | 43 (32) | 23 (23) | 8 (9) | 11 (7) | 21 (30) | 23 (25) | 15 (15) | 4 (3) | 10 (6) | 158 (150) |
| 2005 | 40 (39) | 24 (17) | 8 (13) | 9 (7) | 20 (27) | 24 (31) | 12 (15) | 3 (4) | 11 (12) | 151 (165) |
| 2006 | 31 (58) | 20 (29) | 6 (10) | 8 (9) | 19 (32) | 25 (37) | 9 (13) | 2 (4) | 9 (15) | 129 (207) |
| 2007 | 19 (44) | 14 (25) | 7 (9) | 6 (9) | 18 (26) | 27 (40) | 8 (6) | 3 (4) | 7 (6) | 109 (169) |
| 2008 | 55 (71) | 30 (33) | 19 (20) | 22 (12) | 64 (63) | 79 (84) | 27 (32) | 6 (7) | 21 (23) | 323 (345) |
| 2009 | 16 (14) | 9 (6) | 7 (3) | 7 (7) | 20 (8) | 25 (10) | 8 (11) | 1 (1) | 5 (5) | 98 (65) |
| 2010 | 18 (19) | 11 (17) | 9 (1) | 7 (8) | 28 (23) | 34 (16) | 8 (4) | 1 (3) | 7 (3) | 123 (94) |
| 2011 | 22 (36) | 13 (25) | 11 (8) | 9 (9) | 34 (27) | 40 (34) | 9 (5) | 2 (1) | 8 (13) | 148 (158) |
| 2012 | 24 (29) | 16 (23) | 14 (11) | 12 (10) | 41 (46) | 49 (36) | 12 (4) | 3 (2) | 9 (16) | 180 (177) |

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

**Table 8:** Prediction by division

| Year | Division1 | Division2 | Division3 | Division4 | Division5 | Division6 | Division7 | Division8 | Division9 | Division10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2001 | 3[1](0)[2] | 0 (0) | 1 (0) | 0 (0) | 0 (0) | 57 (43) | 11 (2) | 5 (10) | 0 (0) | 5 (7) |
| 2002 | 4 (0) | 0 (0) | 2 (0) | 0 (0) | 0 (0) | 72 (54) | 14 (2) | 6 (38) | 0 (0) | 9 (32) |
| 2003 | 6 (0) | 1 (0) | 2 (0) | 1 (0) | 0 (0) | 89 (44) | 19 (7) | 6 (18) | 0 (0) | 9 (34) |
| 2004 | 9 (0) | 1 (0) | 4 (0) | 1 (0) | 1 (0) | 101 (96) | 26 (29) | 7 (13) | 0 (0) | 10 (12) |
| 2005 | 13 (0) | 1 (0) | 5 (0) | 1 (0) | 1 (0) | 87 (114) | 20 (26) | 8 (18) | 0 (0) | 14 (7) |
| 2006 | 18 (34) | 2 (0) | 8 (0) | 2 (5) | 2 (3) | 58 (105) | 12 (32) | 9 (12) | 0 (0) | 17 (16) |
| 2007 | 23 (59) | 3 (0) | 12 (5) | 3 (7) | 3 (9) | 26 (53) | 3 (10) | 12 (6) | 0 (0) | 24 (20) |
| 2008 | 87 (97) | 14 (23) | 52 (79) | 11 (11) | 12 (13) | 16 (16) | 0 (0) | 45 (24) | 1 (0) | 85 (82) |
| 2009 | 26 (17) | 5 (4) | 15 (21) | 4 (3) | 5 (3) | 0 (0) | 0 (0) | 16 (4) | 0 (0) | 27 (13) |
| 2010 | 32 (25) | 7 (10) | 18 (20) | 6 (4) | 6 (4) | 0 (0) | 0 (0) | 23 (6) | 1 (4) | 32 (21) |
| 2011 | 38 (51) | 9 (15) | 23 (25) | 7 (8) | 7 (9) | 0 (0) | 0 (0) | 28 (12) | 1 (15) | 35 (23) |
| 2012 | 42 (44) | 13 (16) | 30 (33) | 9 (3) | 10 (7) | 0 (0) | 0 (0) | 32 (21) | 1 (15) | 42 (38) |

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

retirement plot is corresponding with 1 year lag of the trend of Real earnings. Unadjusted Monthly Housing Price (MHP) is another influential index with the lowest MAPE value. It has significantly impacts on the retire number as it increases as shown in figure 10a. However, the retire number does not decrease coinciding with the decreasing of MHP. Two market indicators(S&P500 and Wilshire 5000) are both significantly impact the employee retirement behavior with $> 1$ hazard ratio, which indicating the employee are more likely to retire when the economics is under good condition.

# 6 Conclusions and Managerial Implications

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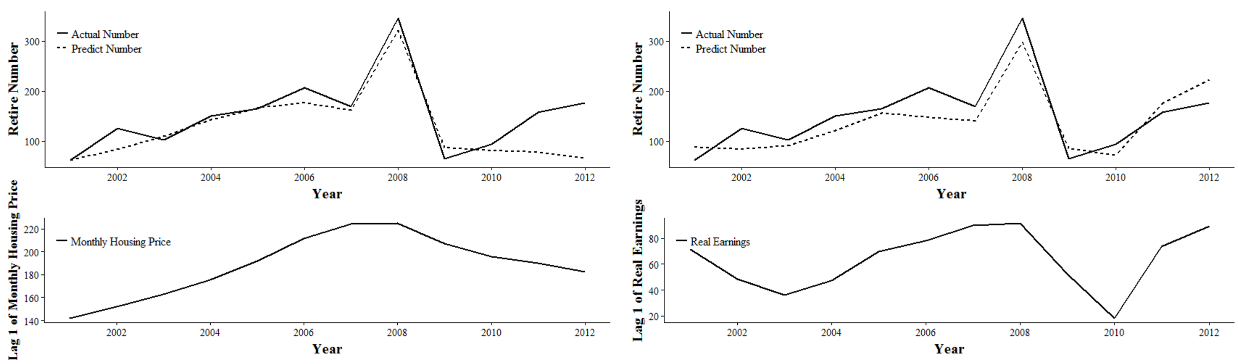
**Table 9:** Economic index test statistics

| Economic Inidcator | Chi-square | P-value | Hazard ratio | MAPE | $G^2$ |
|---|---|---|---|---|---|
| Without Econmic indicator[1] | | | | 21.31 | 147.43 |
| MHP NSA | 129.614 | $< .001$ | 1.020 | 17.84 | 220.65 |
| MHP SA | 129.516 | $< .001$ | 1.020 | 17.86 | 221.66 |
| Southeast MHP NSA | 68.055 | $< .001$ | 1.030 | 23.37 | 178.38 |
| Southeast MHP SA | 67.871 | $< .001$ | 1.030 | 23.40 | 179.87 |
| S&P500 | 13.319 | $< .001$ | 1.001 | 20.79 | 129.83 |
| Dividend | 1.045 | 0.307 | 1.015 | 22.63 | 150.03 |
| Earnings | 84.895 | $< .001$ | 1.016 | 21.40 | 105.06 |
| Consumer Price Index | 5.404 | 0.020 | 1.013 | 21.36 | 133.57 |
| Real Price | 5.522 | 0.019 | 1.000 | 21.22 | 138.72 |
| Real Dividend | 1.925 | 0.165 | 1.022 | 23.39 | 154.84 |
| Real Earnings | 80.358 | $< .001$ | 1.013 | 20.33 | 95.02 |
| Long Interest Rate | 1.539 | 0.215 | 1.082 | 22.03 | 149.01 |
| Unemployment Rate | 32.212 | $< .001$ | 0.849 | 25.08 | 179.49 |
| P E10 | 0.041 | 0.839 | 0.998 | 21.31 | 147.97 |
| Wilshire5000 | 22.392 | $< .001$ | 1.028 | 20.83 | 121.58 |

[1] it is the selected model without economic indicator.




**(a)** Monthly Housing Price

**(b)** Real Earnings

**Figure 10:** Economic indicators and retirement prediciting plot

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