A Modern Conceptual Framework for the Analysis of Factors in Retirement Decisions

Xiaojuan Zhu

September 24, 2015

Abstract

This article focuses on analytical methodologies useful for analyzing retirement decision making in large organizations. We discuss a variety of biases that can occur in retirement databases along with modelling strategies and other factors. We also discuss approaches to using retirement models for both the analysis of prior policies and incentives and the prediction of future retirement behavior. Simulation is used to demonstrate the potential impact of sampling biases on predictions.

1 Introduction

Employee turnover is a topic that has drawn the attention of management researchers and practitioners for decades, because employee turnover is both costly and disruptive to the functioning of most organizations (**???**), and both private firms and governments spend billions of dollars every year managing the issue according to **?**. Therefore, the objectives of this study is 1) predict the probability that an individual will retire in a certain window and/or time until retirement (Mean residual life). 2) Predict aggregate numbers of retirements in a fixed time frame at the or division level to facilitate plannning. 3) To determine the impact of internal and external economic variables on retirement. 4) Quantifying the effect of a buyout policy. As a funded research project, a large organizational secondary dataset including 12-year employees demographic information and records is transformed, analyzed and modeled by Cox proportional hazard regression models with a time dependent covariate using competing risks analysis to examine the statistically significant factors and to predict employees' conditional retiring probabilities. This study also examines the forecasting capability of Cox proportional hazard model on the data with two kinds of bias (left truncation and right censor) by simulation.

2 Literature Review

3 Data Preparation

The turnover dataset is a large real world secondary dataset from a multipurpose research organization in the U.S. The dataset consists 4316 current active and 3782 terminated full-

time employees' information including metrics such as payroll category, hired date, company start date, company credit service date, termination date, age at hired , years of service at hired (YCSH), gender, job classification (named as Cocs code), and Organization level (named as division). The company credit service date is the date that the organization starts to credit their retirement plan. Years of service (YCS) is the total years credit for employees' pension plan. The employees are eligible to get a full pension, when their age is at least 65 or their points is greater than 85, which is the sum of age and year of service. Employees have different YCS when they are hired because their YCS can be transferred from their previous job if their previous job also accounts for the pension plan. Common Occupational Classification System (COCS) code is a standardized code used to describe the job category by the organization for reporting to Common Occupational Classification System. In this study, COCS code is highly correlated with payroll category: managers, engineers, administrative, and scientists are monthly payroll, general administrative and technicians are weekly payroll, the other categories are hourly payroll. Organization level code is used to distinguish the departments. In this study, the division in the organization do not stabilize like COCS code for an employee, because the division can be renamed, reduced, or dismissed by the change of production plan or organization's budget. The division is considered as time independent variable for employees due to no historical record for divisions provided by HR department. The window of time for the turnover dataset is from November 2000 to December 2012, i.e. the dataset consists the records only for the employees working in the organization from November 2000 to December 2012, indicating there is no records for employees leaving the organization before November 2000 and no termination date for 4316 current employees. These two kinds of unknown information cause two kinds of bias: right censor and left truncation. The right censor is due to the no termiation date for current employees, and the left truncation is due to no records for employee leaving before November 2000.

The turnover dataset is split into two datasets: training and holdout dataset. The training part is used to build the model and the holdout part is to validate the model performance. Two methods are used to slipt the dataset in order to validate the model performance: One is split data by a time point November 1 2010: training (November 1, 2000 - October 31, 2010) and holdout (November 1st, 2010 - December 31, 2012). The other on is to random split the turnover dataset into 2/3 of the dataset as training and 1/3 of the dataset as holdout. The covariates identified from the turnover dataset and used to build the models are payroll, gender, division, cocs code (Job category), age at hired, and year of service at hired:

- Payroll (PR): hourly, weekly, or monthly payroll,
- Gender: male, female
- division (ORG): ten divisions in the organization.
- Cocs code: Crafts(C), Engineers (E), General Administrative (G), Laborers (L), General Managers (M), Administrative (P), Operators (O), Scientists (S), Technicians (T))
- Age at hired: most recent age when an employee is hired.

- Years of service at hired (YCSH): the years of service which accounts for pension plan when employee is most recently hired.

Several economic indices are being considered and tested their as a variable impact on employee turnover. These indices include unemployment index, housing price index (HPI), investment index, and marketing index. Seasonal adjusted unemployment rate is published by Bureau of Labor Statics from United department of Labor (**?**). U.S housing price index, U.S. and southeastern monthly purchase-only index are considered as another economic indicator variables in the study (**?**). S&P 500 indices published from S&P Dow Jones Indices are also considered as investment index including S&P 500, Dividend, Earnings, Consumer index, Long Interest Rate, Real Price, Real Dividend, Real Earnings, P/E 10 ratio (**?**). Wilshire 5000 total market full cap index published by Wilshire Associates is considered as market index in the forecasting model(**?**). All these twelve indices are treated as variables using their twelve-month lag term in yearly data format. All these indices selected are indicators in various economic areas, such as job market, house market, and stock market, representing the fluctuation of these economic areas. The economic indices are originally in the dayly or monthly form. The average values by twelve month for each year are used into the model fitting.

4 Model Development and Evaluation

Several questions have to be addressed by this study: Can turnover in term of retirement or voluntary quit be predicted? When will a employees turnover? Who will retire or voluntary quit in term of job categories or divisions? what age groups are more likely to retire or voluntary quit? What economic conditions related to retirement or voluntary quit? What is the magnitude or impact of buyout program? How do the tenure and age impact the retirement or voluntary quit? Besides, how to deal with the data biases: right censor and left truncation existing in the dataset. All these questions and problems can be solved by lifetime analysis, also called survival analysis. The survival analysis is to analyze the time duration for the occurrence of an events or certain events. The events can be the death of the patients, the failure of the machine, and the leaving of the employees by any reasons for this study. There are two kinds of survival statistic models: parametric survival models and Cox proportional hazards (PH) models. In this study, Cox PH model is employed to build the forecasting model, to generate a employees' working life baseline (distribution), and to identify significant factors for turnover. The parametric models are not appropriate for this study, because it is hard to fit the employees' working life distribution to any parametric distributions, such as Weibull or log-normal distribution. Time dependent covariates are incorporated for fitting the 2008 intervention event due to the downsize policy in the organization and for examining the effects of economic indicators. Competing risks analysis is applied for modeling employee retirement and voluntary quit. Besides, A simulation study is performed to examine the forecasting capability of cox proportional hazard model on the left truncation and right censor dataset.

4.1 Two data bias: right censor and left truncation

Right censor and left truncation are common in survival analysis. The right censor is that the event of interest (failure) occurs after the study window. Let T denotes the time of

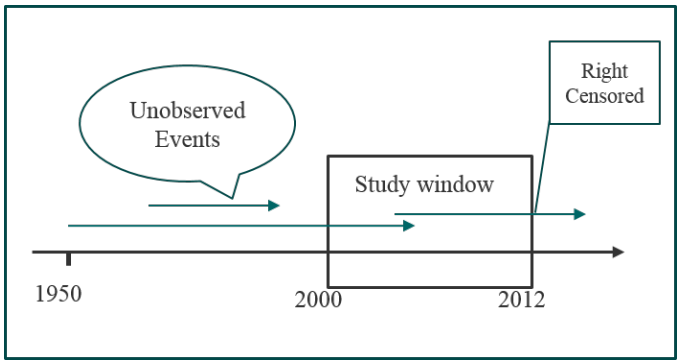


Figure 1: Right censor and left truncation

main event of interest to occur and let C denotes the end time of study. An observation is right censored when $T > C$, indicating the study do not have the failure time of the right censored observation. In this study, the study window is from November 2000 to December 2012 as shown in the figure 1. Thus, the current active employees have unknown terminated date. They are treated as right censor. These right censored observations require special treatment in survival analysis: a censor indicator variable is created:

$$\delta_i = \begin{cases} 1 & \text{if } t_i \leq c_i \text{ (uncensored)}, \\ 0 & \text{if } t_i > c_i \text{ (censored)}, \end{cases}$$

where, i denotes the ith observation, and the failure time of event for ith observation is minimum time between t_i and c_i, i.e., $min(t_i, c_i)$, that is when $c_i < t_i$, c_i is taken as end time of the ith observation in order to do next analysis.

Left truncation is that the occurrence of an intermediate event prior to the event of interest appear in the sample dataset. Let T denotes the time of event of interest to occur and let X denotes the time an individual enters the study, that is time of truncation events occurs. Only the individuals with $T \geq X$ are observed in the study window. Left truncation in this study occurs due to no records for employees leaving the organization before November 2000 as unobserved events shown in the figure 1. The left truncation leads to another bias. As shown in the figure 1, the longest arrow represents a life span for an employee hired in 1950 and left in 2006. Those employees who remain in the study window increase the apparent lifetimes. The existence of truncation in the data must be taken into account in order to overcome this bias and to achieve accurate estimation of survival analysis (**?**). Let t_{i0} denotes the start time of the ith observation, i.e., hired time or age at hired of ith employee, x_i denotes the entry time of the ith observation, i.e., the start time of study (November 1st, 2000) or age at November 1st, 2000. The start time of the observation is maximum value between t_{i0} and x_i, that is when $t_{i0} < x_i$, x_i is taken as start time of the ith observation in order to eliminate the left truncation bias (**?**). The number of failures in the t_j is redefined for left truncation. When $x_i < t_j \leq t_i$, the observation is in the risk set. When $t_j < x_i \leq t_i$,

the ith observation has not entered study yet at t_j and it cannot be considered in the risk set. When $x_i \leq t_i < t_j$, it indicates the ith observation whose failure time before t_j, and it cannot be considered in the risk set at time t_j neither (**?**).

4.2 Cox PH regression model

Cox proportional hazards (PH) regression is a widely used method for estimating survival life events, introduced in a seminal paper by **?**. The Cox PH model is usually taken the form of hazard model formula as shown in the equation 1:

$$h(t, x) = h_0(t)e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_k)$, $h_0(t)$ is the baseline hazard occurring when $x = 0$, β is the coefficients of x. The model provides a hazard expression at time t for an individual with a given specification of a set of explanatory variables denoted by the x. The Cox PH formula is the product of quantities at hazard time t: $h_0(t)$ as the baseline hazard function and the exponential expression to the linear combination of $\beta_i x_i$, x does not involve time t, so it is time-independent covariates. x can also be time-dependent covariates, which named extended Cox PH regression as discussed in the section 4.3. The key assumption for Cox PH regression model is proportion hazards. However, Cox regression can handle non proportional hazards using time-dependent covariate or stratification. The Cox PH regression is "robust" and popular, because the baseline hazard function $h_0(t)$ is an unspecified function and its estimation can closely approximate correct parametric model (**?**). Taking the logarithm of both sides of the equation, the Cox PH model is rewritten in the equation 2:

$$\log h(t, x) = \alpha(t) + \sum_{i=1}^{k} \beta_i x_i \tag{2}$$

where $\alpha(t) = \log h_0(t)$. If $\alpha(t) = \alpha$, the baseline is exponential distribution. In the Cox PH regression, $\alpha(t)$ do not limited on specific parametric distributions and it can take any form. The partial likelihood method is used to estimated β coefficients of the Cox model without having to specify the baseline (**?**). The Cox PH model is performed by SAS.

4.3 Time dependent covariate and counting process

A time dependent covariate is that a covariate is not constant through the whole study and its value changes over the course of the study. The extended Cox PH regression model incorporates both time-independent and time-dependent covariates as shown in the equation 3:

$$h(t, x) = h_0(t)e^{(\sum_{i=1}^{k_1} \beta_i x_i + \sum_{j=1}^{k_2} \gamma_j x_j(t))} \tag{3}$$

where $x = (x_1, x_2, \ldots, x_{k_1}, x_1(t), x_2(t), \ldots, x_{k_2}(t))$, $h_0(t)$ is the baseline hazard occurring when $x = 0$, β and γ are the coefficients of x. There are two time dependent covariates in this study: policy and economic indicators. Policy is to handle the downsizing policy issued

in January 2008 with three months response time window to accommodate a voluntary reduction in force from the organization. Policy is a dummy variable across years:

$$Policy = \begin{cases} 1 & \text{if employee works in year 2008,} \\ 0 & \text{if employee does not work in year 2008.} \end{cases}$$

Counting process method in SAS programming statements is used to handle time dependent covariates, which is each employee have multiple records. Each record is related to a time interval and the covariates in this record remain constant. Therefore, Each employee has up to 3 records: before 2008, in-between 2008, and after 2008. Two variables, age, year of service, are used for repenting two time terminals of each interval or record. For age, one time point is age at beginning of the certain period, named "age at start"; and the other one is age at end of the curtain period, named "age at end". Two year of services points are also generated for each record: one is year of service at the beginning of the period, named "YCS at start"; the other one is the year of service at the end of the period, named "YCS at end".

Economic indicators is another time dependent covariates. Because economic indicators are fluctuated across the year, all the employees have up to 12 years records based on the calender year, which interval starts from hired date or January 1, and ends at terminated date or December 31 of certain year during the study window as shown in equation . The economic indicators are taken the average value for each year into the optimal model identified from the internal covariates to examine their impacts on turnover.

$$\begin{aligned} (\text{start point}, \text{end point}) = (&max(\text{hired date}, \text{January 1 of a certain year}), \\ &min(\text{terminated date}, \text{December 31 of a certain year})) \end{aligned} \tag{4}$$

4.4 Stratification model

An alternative for handling nonproportional hazards is stratification. A stratified model allows each subgroup of data as defined by a grouping variable to have its own baseline hazard while sharing parameters for other covariates across. If the proportional hazards assumption holds within these subgroups then this model allows us to get valid common estimates of covariate effects using all of the observations. Equation 5 below represents the hazard function for strata z;

$$h(t, x, z) = h_0^z(t) e^{(\sum_{i=1}^{k} \beta_i x_i)} \tag{5}$$

where z represents the grouping variable, and $h^z\sigma_0(t)$ is a baseline hazard based for stratam z and β_i are common effects of covariates ac. Note that the strata covariates cannot be the covariates in the Cox PH model.

The proportional hazard assumption can be tested using Schoenfeld residuals which works even if the model includes time-dependent covariates; see **??**. An alternative is to test the interaction between time-dependent and time-independent covariates in the Cox PH model. The assumption is valid if the interaction is not statistically significant ($P > 0.05$). Including a stratified covariate, when appropriate, can improve the Cox model's performance. The C-statistic is used to compare models with and without stratification with a higher C value indicating a better model (**?**).

4.5 Competing risks

A competing risk is an event whose occurrence either precludes the occurrence of the event of interest or fundamentally alters the probability of occurrence of this event of interest (**?**). For example, turnover causes of an employee are exclusive and independent, i.e. an employee can experience only one event such as voluntary quit rather than retirement. This alters the probability of experiencing the event of interest, like retirement. Such events are known as competing risks events where one event of several different types of possible events can occur and hence the survival analysis for each event is calculated separately with the other events set as censored. Two mutually exclusive causes: retirement and voluntary quit are considered as the event of interests for each employees in this study, and the other events are treat as censored.

There are several reasons for selecting these two causes. One main reasean is because the organization are interested in forecasting the turnover of retirement and voluntary quit. There are 1/3 employees in that organization are over 50 years old who are eligible for retirement. The employee who voluntary quit usually is the one organization would like to keep (**?**). And also voluntary quit costs highly for the organizations and firms (**?**). Finally, the other reasons of turnover, such as layoff, transfer, death, or disability are caused by the factors which occurrence are random and hard to predict. The Cox PH regression for competing risks as shown in equation 6:

$$h_j(t,x) = h_{j0}(t)e^{(\sum_{i=1}^{k}\beta_{ij}x_i)} \tag{6}$$

where, x_j is the covariate for a specific type of turnover. Note that the coefficient β is the effects of the covariates may be different from different turnover types. If β_{ij} is the same for all j, the model simplified to Cox PH model as shown in equation 1.

4.6 Variable selection

All the covariates are putting into Cox PH regression model and selected by manually backwards selection method based on $P < 0.05$. The variable selection procedure is as follow: first, all the covariates are used to build the model. Second, remove the non-significant variable ($P > 0.05$) with the largest P value, and rerun the model with the other variables. Then, repeat the second step until there is no significant variable remaining in the model.

4.7 Model evaluation and comparison

The Cox PH model is evaluated by four statistics criteria: Akaikes information (AIC), Schwartzs Bayesian criterion (SBC), C-statistics, and mean absolute percentage value (MAPE). The optimal model should have low AIC, SBC, and MAPE value, and high C-statistics for both training and holdout dataset. In this study, the model performance on holdout dataset is considered more important than that on the training dataset. AIC and SBC are both information criteria using likelihood value. Usually, the best model comes with lowest AIC or SBC values. AIC, SBC values are automatically generated by the models.

C-statistics or the area under the receiver operating characteristic (ROC) curve is to test whether the probability of predicting the outcome is better than chance. It ranges

from 0.5 to 1. Models are considered acceptable when the C-statistic is higher than 0.7 (**?**). C-statistics are calculated by using the predicted failure probability compared with the actual outcomes by SAS proc logistic. The predicted failure (retirement or voluntary quit) probability is actually the conditional failure probability for an employee at time t_j, given that the employee is active at time t_{j-1}. It is calculated based on the baseline and coefficients from Cox PH models for both training and holdout dataset as shown in equation 7.

$$\begin{aligned} P\{t_{j-1} < T < t_j\} &= 1 - P\{T > t_j | T \geq t_j\} \\ &= 1 - \frac{S_{t_j}}{S_{t_{j-1}}} \\ &= 1 - \frac{S_0(t_j)^{(\sum_{i=1}^{k} \beta_i x_i)}}{S_0(t_{j-1})^{(\sum_{i=1}^{k} \beta_i x_i)}} \end{aligned} \tag{7}$$

where, T is survival time, t_j is a specific value for T, $S_0(t)$ is the baseline function generated by Cox PH model, x is the covariates, and β is the coefficient.

MAPE is another measure for comparing the accuracy of the model fitting between different forecast models since it measures relative performance (**?**) as shown in the equation 8.

$$MAPE = \sum_{t=1}^{n} \left| \frac{y_t - \hat{y_t}}{y_t} \right| \frac{1}{n}\% \tag{8}$$

MAPE is calculated by using the yearly actual and predicted retirement or voluntary quit number as y_t and $\hat{y_t}$, respectively. The predicted retirement or voluntary quit number is the expected retirement or voluntary number summarized by aggregating all the failure probabilities for the active employees in the risk set at t_j as shown in 9.

$$E(\text{turnover number at } t_j) = \sum_{i=1}^{k} P_i\{t_{j-1} < T < t_j\} \tag{9}$$

where, i denotes the ith employee. The logistic regression and time series moving average methods are also employed to compare with the performance of Cox PH regression model by MAPE value.

4.8 Simulation on right censor and left truncation

In order to understand the performance and efficiency of the Cox PH model in right censored and left truncated data we perform a simulation study.

Generated $n = 100, 200, 500, 1000, 2000,$ and 4000 observations from a Weibull regression model with one covariate which we referred to as age.

Age is uniformly distributed from 22 to 70 years of age, which is chosen to mimic the actual distribution of workers ages in our sample.

In the regression model, the coefficients for $\beta_{age} = -.025$ (Why?) and the coefficient for $\beta_0 = 1.5$.

The survival times T_i are randomly generated from a Weibull distribution with shape parameter α and scale parameter λ, where $\alpha = 1.5$ and $\lambda = exp(-0.025age + \beta_0)^{\frac{1}{\alpha}}$.

The simulation is performed on right censoring and left truncation separately, in order to observe the effects for different bias. For right censor simulation, two simulation procedure are conducted with different start points. First, the start point for all the observations are equal to 0, and stop point is equal to the survival time t_i for ith observation where $T = (t_1, t_2, \ldots, t_n)$. After that, the censor time C is equal to first quarter, median, third quarter, and maximum of the survival time, respectively, to get 75%, 50%, 25% and 0% censor proportions. When the survival time t_i for ith observation is not greater than the censor time (c_i), the stop point is survival time t_i and censor variable δ_i is 1. When survival time t_i for ith observation is greater than censor time (c_i), the stop point is change to censor time (c_i) and censor variable δ_i is 0. The second right censor simulation procedure is to set the start points S to follow uniform distribution from 0 to 10, representing the observations (employees) start at various time points within 10 years study window. The stop point is equal to the summation of start point and survival time: $S + T$. The censor time is a cutoff point (C) identified by R to get fix number of censor proportion (25%, 50%, and 75%). When the survival time t_i for ith observation is not greater than the censor time (c_i), the stop point is survival time t_i and censor variable δ_i is 1. The stop points are set to censor time C for the observations with the stop points $S + T$ being great than censor time C, the survival time is reset to $C - S$, and censor variable δ_i is 0. Otherwise, the other observations with stop point being less than the censor time remain the same and the censor variable δ_i is 1. For the second simulation, there are 4000 observations are generated. Because some observations occur after the cutoff point (censor time) and the sample size are various for different censor proportion, only 400 are randomly selected with 75%, 50%, 25% and 0% censor proportions to keep the sample size same. The start point and the stop point are dependent variable in the cox regression model. δ is censor variable.

For left truncation simulation, the start point U is generated as uniform distribution with $a = 0$ and $b = max(T)$ which indicates an observation start randomly from time 0 to time $max(T)$. The stop point S is $U + T$. The histogram is generated for S. The truncation time L is set as 0, first quarter, median, and third quarter of S, respectively, to get 0%, 25%, 50%, and 75% truncation proportions. When start point u_i for ith observation is less than truncation time l_i, the start point is reset as truncation time l_i. When start point u_i for ith observation is not less than truncation time l_i, the start point does not change (u_i). In left truncation, the censor variable δ for all the observations are equal 1. For right censor and left truncation simulation, the Cox regression models are modeled by "coxreg" and "phreg" function in eha package. The coxreg function is the regular cox regression modeling procedure to generate coefficient estimates and a non-parametric baseline. The phreg funcation is using parametric distribution like Weibull, Extrem value (EV) to estimate the baseline. The "phreg" function performs Cox PH model and also provides a parametric baseline hazards estimation (**?**). The phreg function is used to compare the non-parametric with parametric baseline and to demonstrate the prediction when the parametric baseline is not right. Total predicted failure number is calculated as shown in equation 7 and 9. The actual and predicted failure number are compared to show right censor and left truncation's impacts on the coefficient and baseline estimation .

5 Results

5.1 Right censor and left truncation simulation results

The right censoring simulation results are shown in the left part of Table 1 which all the observations start at the same time 0. The events in the second column of the table are the actual total failure events simulated without considering censoring, which is $n = 100, 200, 500, 1000, 2000,$ and 4000. The number of events in the dataset applying to the Cox model are reduced due to the right censoring, which is equal to $events = \sum_{i=0}^{n} \delta_i$. The values in the other column are the average value of 100 replications for Cox PH model coefficient and baseline parameter estimates based on Weibull distribution using phreg. The simulation results show censoring proportion and the number of events are two influential factors for the coefficient estimation. The model overestimates the coefficients of age, λ, and α, when the dataset has high proportion of censoring. For example, when 75% of the data are censored with only 25 events, the estimates for three parameters are 0.028, 4.043, and 1.564, respectively, which are the highest among all the estimates. As the event number increasing, the estimation is approaching to the actual value. For example, the estimation of age, λ, and α are close to 0.025, 2.7, and 1.5, respectively, after the number of event is at least 500. The predicted events in the sixth column is the total predicted failure number calculated by applying the coefficient estimates and the non-parametric baseline from Cox PH models into the dataset without considering censoring. The predicted events using censoring models are all lower than the actual total failure number, but close to the number of events after censoring. For example, the number of predicted events is 24.84 when using the estimates of Cox PH model with 75% censoring to calculate the dataset with 100 events, which is close to 25. The prediction is less than the actual dataset, because the baseline is lack of information for the events after censored time. The baseline does not include hazards or survival probability information for the long life time observation, because the baseline just captures the events before the censor time and the observation with long life time are censored in this simulation. As a result, there is no failure probability for long life time observation.

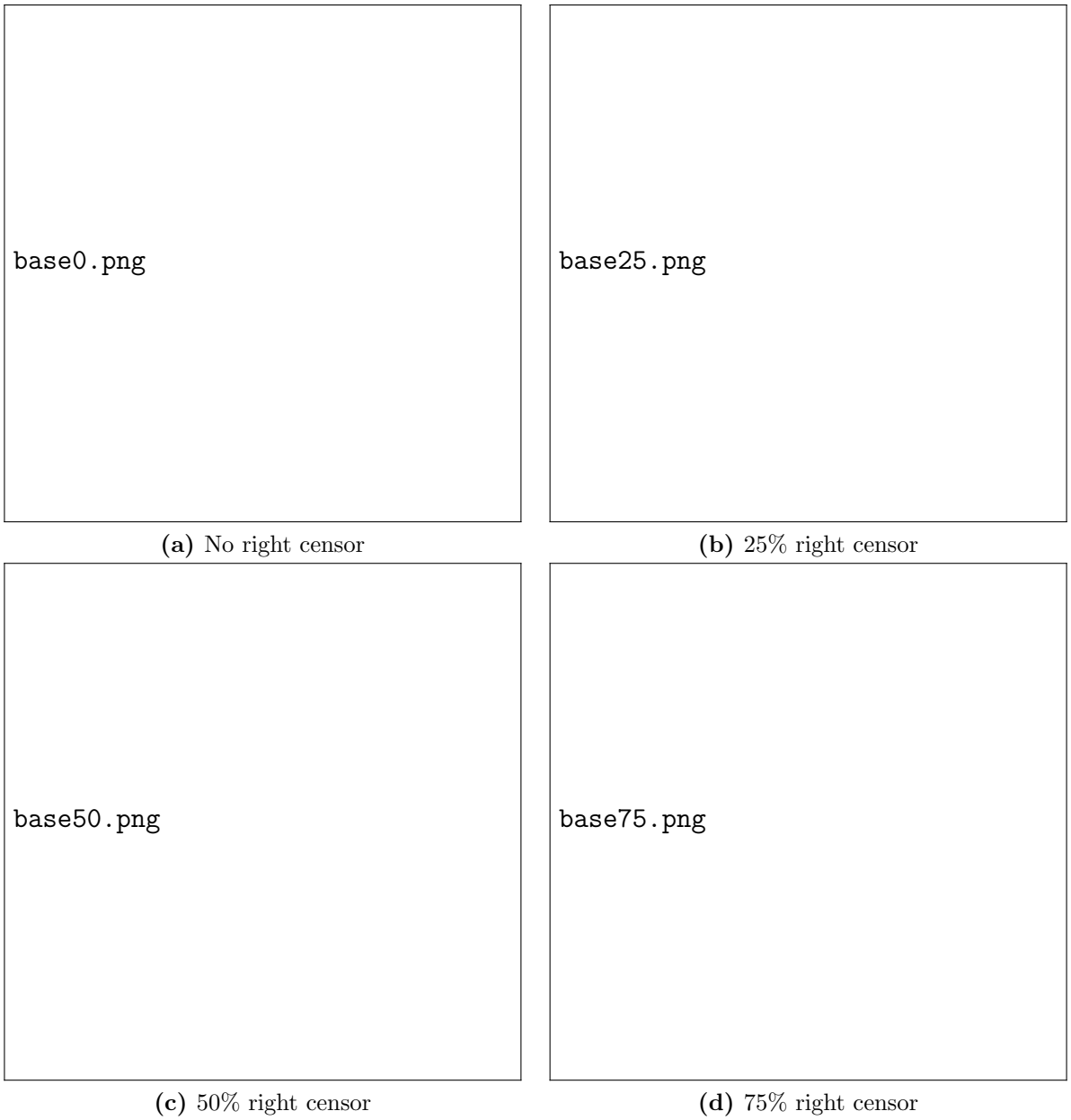
To simulate the realistic situation, the second right censor simulation is conducted with randomly occurrence of the observations followed by uniform distribution. The results are shown in the table 2. The events without considering censoring are all equal to 400 for easy comparison. The predicted events include the prediction by non-parametric baselines using "coxreg" and by parametric baseline using "phreg" function Weibull distribution as baseline estimation. The estimation of age and α are all close to the simulated value (0.025 and 1.5). And the estimation of λ are around 2.8. These indicate the right censoring does not impact the coefficients estimation. However, right censoring does impact the baseline function estimation for Cox PH model as shown in the figure 2. The red lines in the figure 2 are the non-parametric baseline for 100 replications. And blue dash lines are the parametric baseline generated base on Weibull distribution for 100 replications, which represent the actual baseline. All these baselines are overlap together by various censoring, which indicates non-parametric baseline can capture the true parametric baseline. However, the duration of the non-parametric baseline is getting short by increasing the censoring proportion. As a result, the predicted number of events using non-parametric baseline is less than the actual

Table 1: Right censoring and left truncation simulation statistics

Censor proportion	Events	Variable Estimates			Predicted Events	Truncation proportion	Events	Variable Estimates			Predicted Events
		Age	λ	α				Age	λ	α	
0%	100	0.026	2.931	1.509	97.42	0%	100	0.027	2.865	1.534	96.60
25%	100	0.027	2.962	1.527	74.31	25%	75	0.027	2.917	1.546	72.86
50%	100	0.028	3.237	1.530	49.66	50%	50	0.027	2.899	1.577	47.32
75%	100	0.028	4.043	1.564	24.84	75%	25	0.029	3.280	1.757	21.75
0%	200	0.026	2.841	1.508	197.23	0%	200	0.025	2.777	1.506	196.13
25%	200	0.026	2.856	1.513	149.34	25%	150	0.025	2.756	1.515	147.97
50%	200	0.026	2.925	1.527	99.68	50%	100	0.025	2.825	1.532	97.44
75%	200	0.026	3.167	1.540	49.86	75%	50	0.026	2.927	1.572	47.17
0%	500	0.025	2.731	1.500	496.77	0%	500	0.025	2.732	1.509	494.78
25%	500	0.025	2.718	1.508	374.31	25%	375	0.025	2.737	1.514	373.42
50%	500	0.025	2.744	1.514	249.65	50%	250	0.026	2.778	1.514	247.70
75%	500	0.025	2.787	1.525	124.89	75%	125	0.026	2.835	1.547	124.15
0%	1000	0.025	2.748	1.509	996.41	0%	1000	0.025	2.710	1.504	993.77
25%	1000	0.025	2.747	1.512	749.23	25%	750	0.025	2.709	1.504	748.94
50%	1000	0.025	2.748	1.514	499.61	50%	500	0.025	2.715	1.506	503.37
75%	1000	0.026	2.844	1.509	249.80	75%	250	0.025	2.694	1.524	250.93
0%	2000	0.025	2.714	1.502	1996.19	0%	2000	0.025	2.740	1.503	1993.65
25%	2000	0.025	2.713	1.503	1499.29	25%	1500	0.025	2.731	1.502	1507.94
50%	2000	0.025	2.742	1.500	999.68	50%	1000	0.025	2.724	1.503	1012.37
75%	2000	0.025	2.733	1.502	499.89	75%	500	0.025	2.718	1.508	512.30
0%	4000	0.025	2.719	1.504	3995.80	0%	4000	0.025	2.720	1.500	3988.90
25%	4000	0.025	2.718	1.505	2998.86	25%	3000	0.025	2.722	1.501	3014.31
50%	4000	0.025	2.724	1.503	1999.52	50%	2000	0.025	2.710	1.502	2032.03
75%	4000	0.025	2.729	1.513	999.86	75%	1000	0.025	2.703	1.503	1028.39

Table 2: Right censor simulation results by various start time

Censor proportion	Events	Variable Estimaties			Predicted Events	
		Age	λ	α	"coxreg"	"phreg"
0%	400	0.025	2.694	1.508	398.52	400.44
25%	400	0.026	2.802	1.518	394.24	401.72
50%	400	0.026	2.828	1.514	340.73	398.51
75%	400	0.025	2.821	1.518	215.92	400.80

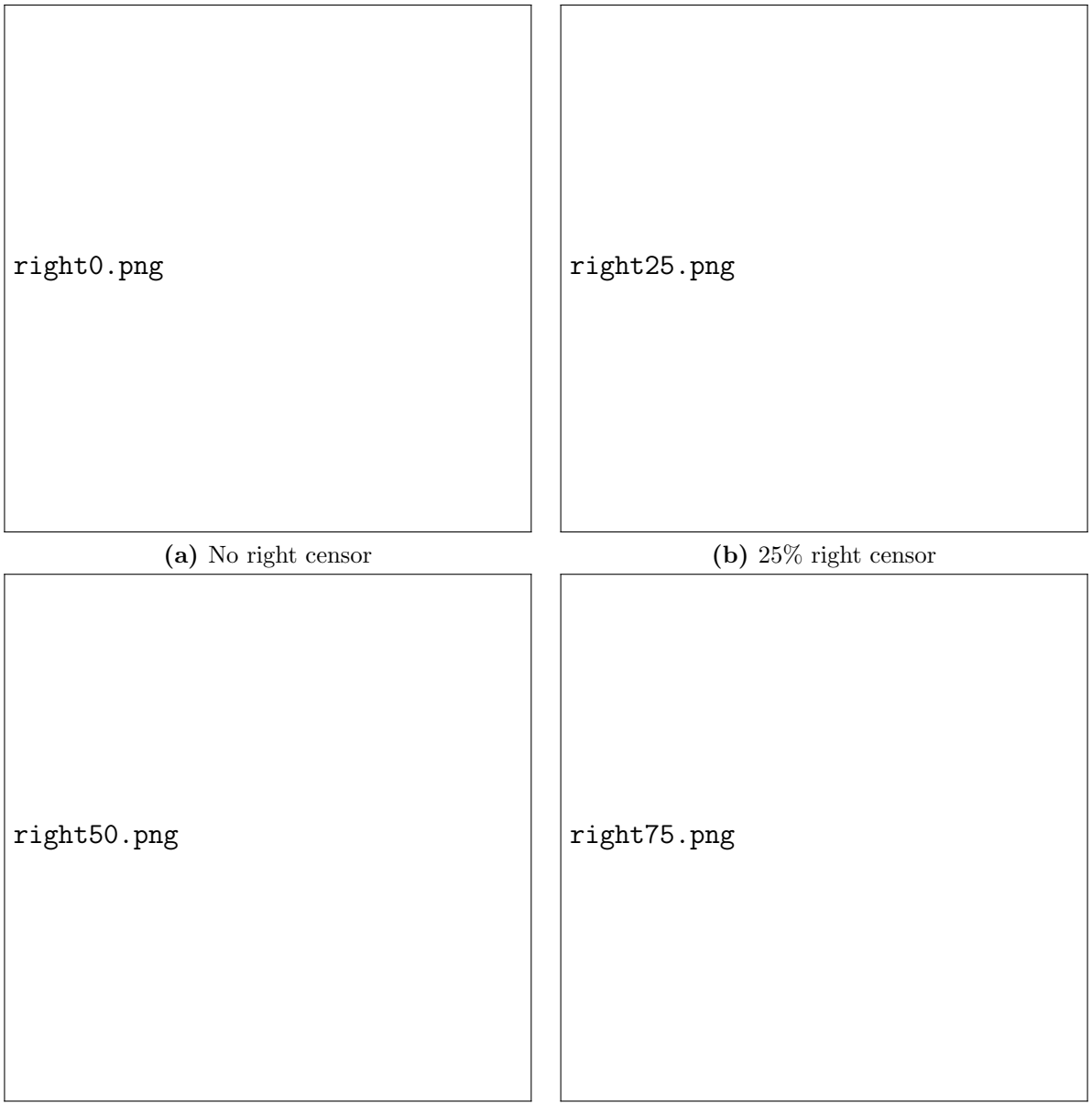


(a) No right censor **(b)** 25% right censor

(c) 50% right censor **(d)** 75% right censor

Figure 2: Baseline comparison by various censoring

failure number. However, it does not affect the prediction by using parametric baseline, which are all close to 400, because the parametric baseline is not limited by the baseline duration if the parameters are correctly estimated. The prediction by "coxreg" and "phreg" are plotted across time by box plot to compare to the actual failure values as shown in the figure 3. The vertical red line is the average value of censor time. The predicted number of events by "coxreg" are close to actual and the prediction by "phreg" before the censor time line. It is still close to actual after censor time line for the data with no censoring and with 25% of censoring as shown in the figure 3a and 3b, because of the long baseline duration. However, It drops down gradually after the censor time for the data with 50% and 75% of censoring



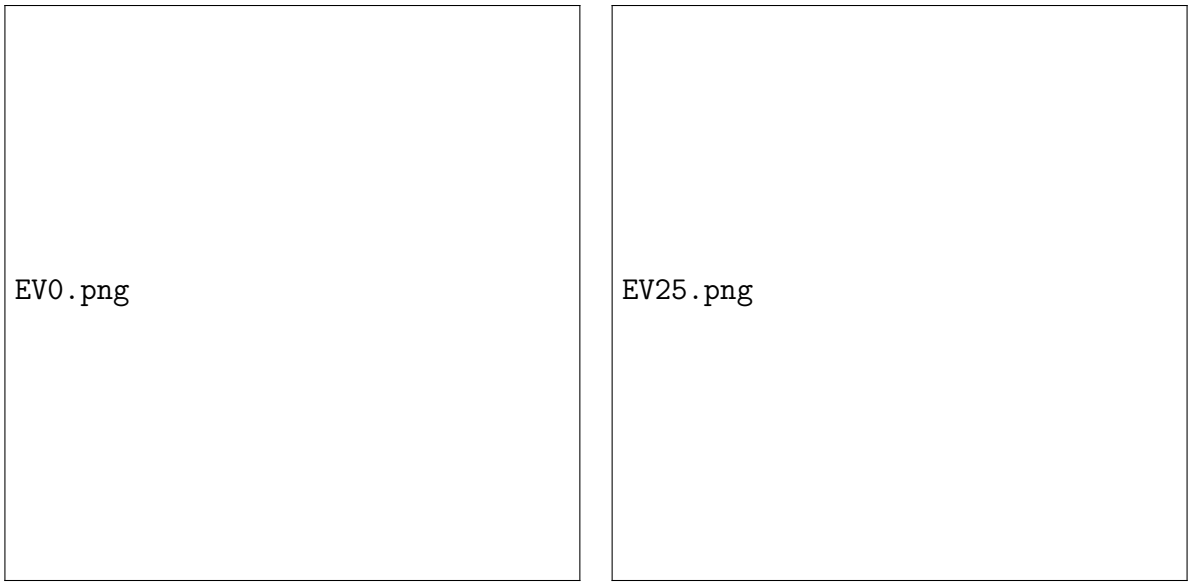
(a) No right censor **(b)** 25% right censor

(c) 50% right censor **(d)** 75% right censor

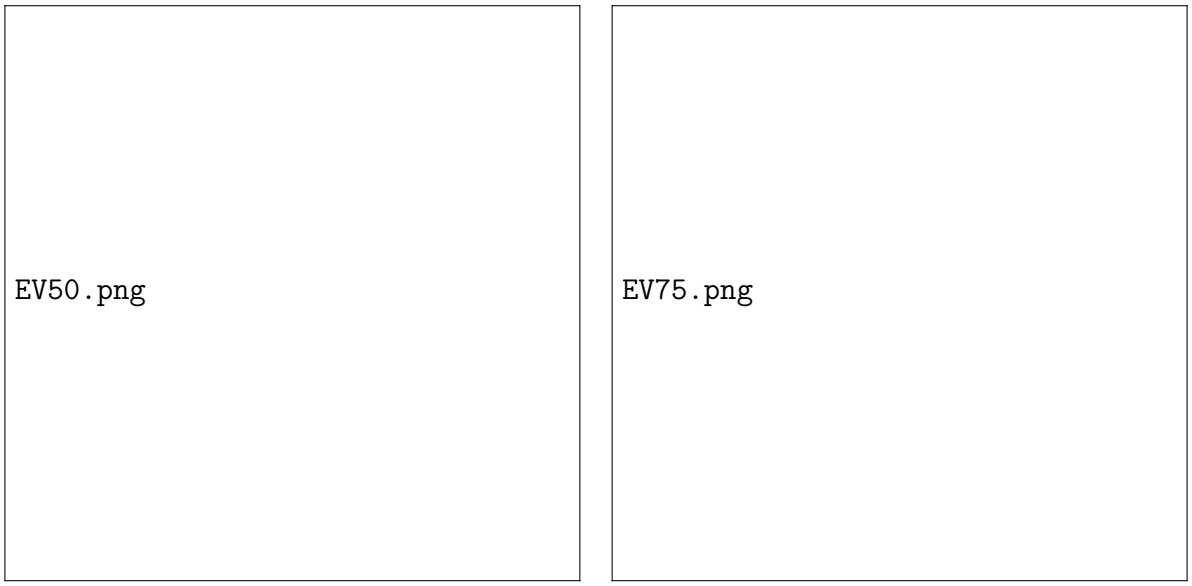
Figure 3: Right censor simulation results: actual vs. predicted failure number

as shown in figure 3c and 3d, because the baseline duration ends around censor time. This simulation clearly shows the limitation of Cox PH model with non-parametric baseline: it cannot accurate predict far beyond the duration of the longest events. Therefore, a baseline can be highly variable in the extremes leading to poor predictions. Although this simulates the real problem, the employee hiring time points are followed by uniform distribution in the real world.

The results of the left truncation bias simulation statistics are shown in the right part of table 1. All the values shown are the average value for coefficient estimate of age, and baseline parameter estimates of λ and α by "phreg" Weibull distribution. The last column



(a) No left truncation **(b)** 25% left truncation



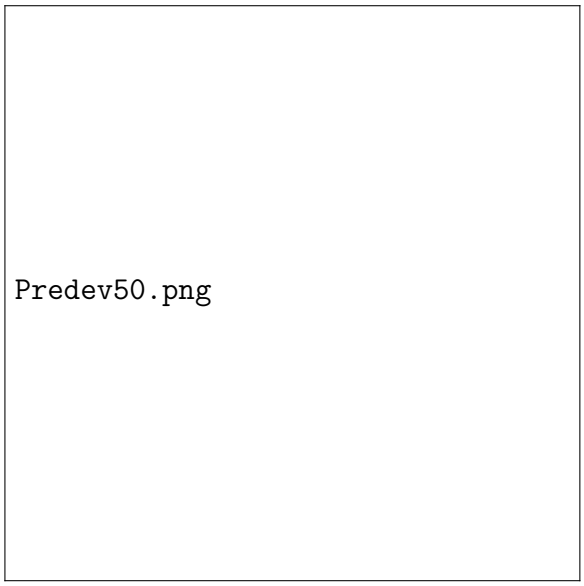
(c) 50% left truncation **(d)** 75% left truncation

Figure 4: Left truncation simulation results: Baseline Comparison

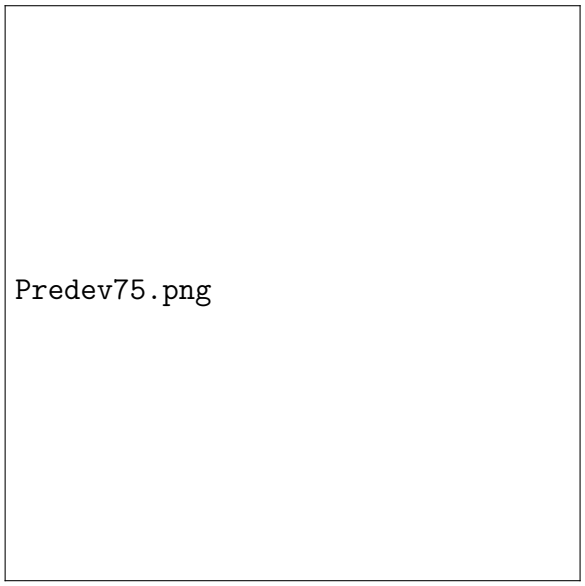
is the total predicted number of events using non-parametric baseline by "coxreg" function. Similar as the right censor simulation result, left truncation proportion and the number of events are two key factors for coefficients estimation. As table 1 shown, the coefficients are overestimated when the left truncation proportion is 75% or the number of events are less than 200. However, increasing the number of events can offset (reduce) the left truncation effects. For example, the estimates for age and α are all close to simulated value, when the number of events is 1000 with 75% truncation proportion. The predicted number of events is close to the actual one as shown the green and red box in the figure 5, which is the box plot of total 2000 events simulated with 100 replication by various proportion of left truncation.

(a) No left truncation

(b) 25% left truncation



(c) 50% left truncation



(d) 75% left truncation

Figure 5: Left truncation simulation results: actual vs. predicted failure number

The red vertical line is the average of left truncation time. It is slightly overestimated using "coxreg" non-parametric baseline by four left truncation proportions, because the baseline estimation is not affected by the left truncation. The figure 4 clear shows the duration of non-parametric baseline are the same for four left truncation proportion. It also shows the non-parametric baselines (red lines) are overlap with parametric baseline (blue dash lines).

To further test how baseline estimation impacts on the prediction, another simulation is conducted accompanied with left truncation simulation. In the previous left truncations study, the parametric baseline using Extreme Value (EV) distribution is generated by "phreg" and predicted the number of events based on it shape and scale parameter es-

timation as shown in figure 4. Because the data is generated by Weibull distribution, the parametric baselines (green dash line) by EV are much lower than the other two baselines by all four left truncation proportions. The predicted number of events based on the EV baseline are also much lower than the actual number of events across the time as the blue box shown in the figure 5. This study shows the inaccuracy estimation of the baseline lead to poor prediction.

Therefore, accurate estimation of coefficients and the baseline are two key factors impacts the perdition of the events of the Cox PH model. The simulation test shows that the Cox PH model can accurately estimates the coefficients when events number is at least 1000 even with high proportion of censor and left truncation. The baseline is another key factor to predict the number of events. The prediction will be accurate if the parametric distribution of the baseline is known or identify the correctly. Otherwise, a wrong baseline can also deteriorate the prediction. Compared to parametric baselines, a non-parametric baseline is more robust. However, it still needs enough number of events with curtain long period to get a smoothed and long baseline to predict accurately. As a result, it is hard to accurately predict the employee turnover for a company just formed recently, or when the employee population characteristic changed, due to high proportion of censor and lack of events. Although this study has more than 50% right censor, it still has more than 3000 events with long duration (around 50 years length).

5.2 Retirement model without external variables

1. Descriptive Analysis - number of females, histograms of age ranges by division, min and max, means. summary() and hist(age division) in R.

2. Set up - what are the reference groups for the categorical variables. Division 1 is the reference group with hazard ratio as 1. Two divisions (6 and 7) have very high hazard ratio: 8.363 and 11.405, which indicates that the employee in division 6 and division 7 are much more likely to retire than the division 1.

3. Practical Significance & Interpretations. Provide Hazard ratios and change in survival functions related to each variable/parameter. Explain what the parameters mean.

   The variables tested in the model are payroll, cocscode, division, gender, years of service at hire, age at start, policy, P85 (binary indicator of 85 retirement points) and the interaction term of P85 and A65. We find that gender, cocs code and payroll were not significant predictors in the presence of the other variables. These indicates employees' gender, job types, or working full time or part time does not impact on retirement. The statistically significant factors are division, years of service at hire, age at start, policy, P85 and the interaction term of P85 and A65 in the Cox PH model as shown in the table 3. The reference levels of these variables are division 1, policy at 0, P85 at 0, P85*A65 at 0, 2.75 years of service at hire and 45.59 years of age, which is equal to the average age of the whole population. P85 is an indicator that a person is eligible for maximum retirement benefits and naturally this has a strong impact on the probability that a person will retire. From a quantitative point of view, the hazard ratio is $e^{1.53} = 4.62$. So the hazard of retirement becomes 4.62 times more likely after

the person exceeds 85 points. While not surprising, this quantification is important in predicting individual and aggregate retirement time.

Another alternative eligibility criteria occurs when individuals exceed an age of 65 years and so we would anticipate the hazard increasing at this point in an employees career. Because the response variable in our model is age, we cannot estimate the effect of this within the proportional hazards setting because the impact is included in the baseline hazard which should increase after this point; see figure 6 (explain). However, by including an interaction between the Age 65 and points 85 indicator, we can estimate how the impact of reaching 85 points diminishes beyond regular retirement age. In this case, the interaction term is estimated at -1.59 leading to a decreasing effect of the 85 points criteria to $e^{1.53-1.59} = 0.943$ indicating that people that exceed both criteria actually have a reduced hazard of retiring over those have only met the Age 65 criteria. In other words, the fact that the individual remains on the job after hitting either criteria indicates that the other criteria has less impact (or that they are intending to work longer).

Starting age(age at beginning of period). Naturally

According to our model, retirement can also be influenced by employee's age and their years of service at the time of hiring (YCSH). The estimates for age are -0.171. As the reference age is 45.49, it means the employee's survival probability is $S(t)^{1.186}(S(t)^{e^{0.171}})$ when the employee started age is one year younger than 45.49, where $S(t)$ is the baseline survival probability at time t for a reference employee of average age, in the case 45.49 years old at the start of the current interval. The employee's survival probability is $S(t)^{0.843}(S(t)^{e^{-0.171}})$ for every one year increase beyond 45.49 in the employee's starting age. This indicates that at any given age, the employee who starts earlier than 45.49 years old is more likely to retire , because they have more years of service and are much likely to reach retirement requirement (85 points) earlier than the employee who starts at older age. Similar as the years of service, the employee with less than 2.75 years of service when they are hired has negative estimates (-0.019) and lower hazard ratio (0.981). The survival probability is $S(t)^{0.9811}$ for the employee is one year of service less than 2.75. On the other hand, the employee who is hired with more than 2.75 years of service has positive estimates (0.019) and higher hazard ratio (1.019) than those have 2.75 years of service. The survival probability is $S(t)^{1.019}$ for the employee is one year of service more than 2.75. This indicates that the employee who has more years of service are more likely to retire than the employee who has less years of service.

Years of service at hire.

? review

In the fiscal year 2008, the employer in the study created a temporary early retirement buyout option. The response window for this option was 3 months although the specific details of this are unknown. In order to deal with the increased level of retirement during this period we including a time dependent indicator variable. The coefficient for this indicator was 1.252 leading to a hazard ratio of 3.496 which indicates that, on average, an individuals hazard of retirement increased by almost 3.5 times during this period. If more information were known about the requirements or targets of this

policy, a more case specific estimate would be possible. However, this would not effect the overall aggregate retirement estimates. It is important to include this one-time effect in order to improve the estimates of other factors(work on this last part - why do we need to include). We further test the policy effect on the employee who are eligible for getting pension. The test results shows the policy had a significant effect for the employee who are eligible for getting the full pension benefit rather than the employee who are eligible for getting partial retirement plan as the interaction term of between policy and points 75 or points 65 are not statistically significant. The hazard ratio for the policy effect on those employee substantially increase to 15.85 from 2.36, which is about seven times of the basic policy effect, after the model adding a interaction term of policy and points 85.

Then, we further test the policy effects on divisions. The test results shows policy have different impact on divisions. Three divisions (2, 4, and 8) are significantly impacted by policy as their p value of interaction term are less than 0.05. The employee in division 2 and 3 are more likely to accept the early retirement incentive. On the other hand, the employee in division 8 are not likely to accept the retirement incentive, since their interaction term estimation is negative, leading the hazard ratio from 3.2 to 2.7.

Summarize external

Summarize the predictive capabilities at the individual level. Strengths and weaknesses.

Summarize the predictive capabilities for aggregates. Strengths and weaknesses.

Three divisions (2, 3, and 9) have negative estimates (-0.969, -0.239, and -3.026) and the hazard ratios lower than 1 (0.380, 0.788, and 0.049). The survival probabilities for the employees in these division are $S(t)^{0.380}, S(t)^{0.788}$, and $S(t)^{0.049}$ adjusting for the other variables, where $S(t)$ is the baseline function or the survival probability for the employee of division 1 at time t. The division 9 have the lowest hazard ratio, which indicates the employee in division 9 are much less likely to retire than division 1. Three divisions (6, 7 and 8) have positive estimates (2.124, 2.434, and 2.360). Two divisions (division 6 and 7) have very high hazard ratio: 8.363 and 11.405. The survival probability for the employee in these two division are $S(t)^{8.363}$ and $S(t)^{11.405}$ adjusting for the other variables, which indicates that the employee in division 6 and division 7 are much more likely to retire than the employee in division 1. The other divisions (4 and 5) are not statistically significant, which indicates that the employee in division 4 and 5 has the similar chance to retire with the employee in the division 1.

4. Baseline - describe the features of the baseline maybe use a plot.

   The survival function, hazard function, and cumulative hazard function are shown in figure 6 as the baseline function. The survival probability is 1 before age 49 as shown in figure 6a, which indicates that no employee retire before age at 49. Survival probability starts to slowly decease from age 50 to age 62. The survival probability is around 0.75 for the employee who less than 62 years old, which indicates the employee who is less than 62 years has a probability of 25% to retire. The slope of survival function decrease sharply and causes the survival probability steeply decreasing from 0.75 to almost 0

from age 62 to age 70, which indicates the employee are much more likely to retire from age 62 to 70. Accompany with the survival function, the cumulative hazard starts to rise up from age 62 as shown in figure 6c. The trend for hazard function tend to increase by age increasing as shown in figure 6b. The hazard ratio causes the employee retire probability increasing sharply.

5. prediction results

   The prediction for the employee retirement is shown in the figure 7, table 4 and 5. The prediction of the model captures the fluctuation of actual retirement, and also captures the peak of the special year 2008 which is the buyout policy of workforce due to budget reduction in the organization as shown in the figure 7. The prediction number for holdout years (2011 and 2012) are very close to the actual number, indicates the model performs well. The table 4 and 5 shows the prediction by cocscode (job classification) and division for years, which is the summation of the retire probability by job classification and division, separately. The Cox PH model can also provides the prediction by gender, payroll, and other categories.

Table 3: Parameter estimates

Parameter	Label	DF	Parameter Estimates	Std. Error	Chi-Square	P-value	Hazard Ratio	95% Hazard Ratio Confidence Limits	
division	2	1	-0.969	0.179	29.161	<.001	0.380	0.267	0.540
division	3	1	-0.239	0.112	4.566	0.033	0.788	0.633	0.980
division	4	1	0.067	0.195	0.118	0.731	1.069	0.730	1.566
division	5	1	-0.146	0.190	0.593	0.441	0.864	0.596	1.253
division	6	1	2.124	0.094	505.416	<.001	8.363	6.950	10.065
division	7	1	2.434	0.128	358.840	<.001	11.405	8.866	14.671
division	8	1	0.859	0.106	65.611	<.001	2.360	1.917	2.906
division	9	1	-3.026	0.581	27.106	<.001	0.049	0.016	0.152
division	10	1	0.785	0.093	71.249	<.001	2.193	1.827	2.631
Policy	Policy 1	1	1.252	0.071	313.263	<.001	3.496	3.043	4.015
YCSH		1	0.019	0.004	27.842	<.001	1.019	1.012	1.026
AGE_START		1	-0.171	0.013	162.192	<.001	0.843	0.821	0.865
P85	P85 1	1	1.531	0.085	326.389	<.001	.	.	.
P85*A65	P85 1 * A65 1	1	-1.590	0.207	59.314	<.001	.	.	.

5.3 Retirement model with external variables

best model and tested which variable does significantly impact on retirement.

6 Conclusions and Managerial Implications

Table 4: Predictions by job classification

Year	Crafts	Engineers	General Admin.	Laborers	Managers	Prof. Admin.	Operators	Scientists	Technicians	Total
2001	23[1](16)[2]	12 (10)	4 (1)	6 (5)	11 (14)	10 (9)	8 (2)	2 (1)	4 (4)	80 (62)
2002	31 (29)	15 (16)	5 (15)	7 (11)	15 (26)	13 (18)	11 (6)	2 (1)	6 (4)	105 (126)
2003	37 (26)	19 (13)	6 (5)	9 (6)	17 (21)	18 (13)	16 (15)	4 (2)	8 (2)	134 (103)
2004	44 (32)	23 (23)	8 (9)	11 (7)	21 (30)	23 (25)	16 (15)	4 (3)	10 (6)	160 (150)
2005	40 (39)	24 (17)	8 (13)	9 (7)	20 (27)	24 (31)	12 (15)	3 (4)	11 (12)	151 (165)
2006	31 (58)	20 (29)	6 (10)	8 (9)	19 (32)	25 (37)	9 (13)	2 (4)	9 (15)	129 (207)
2007	19 (44)	14 (25)	7 (9)	6 (9)	18 (26)	27 (40)	8 (6)	3 (4)	7 (6)	109 (169)
2008	54 (71)	30 (33)	20 (20)	22 (12)	64 (63)	80 (84)	26 (32)	6 (7)	22 (23)	324 (345)
2009	16 (14)	9 (6)	7 (3)	7 (7)	21 (8)	25 (10)	8 (11)	1 (1)	5 (5)	99 (65)
2010	19 (19)	11 (17)	9 (1)	7 (8)	28 (23)	34 (16)	8 (4)	1 (3)	7 (3)	124 (94)
2011	22 (36)	13 (25)	11 (8)	9 (9)	34 (27)	40 (34)	9 (5)	2 (1)	8 (13)	148 (158)
2012	24 (29)	16 (23)	14 (11)	12 (10)	41 (46)	49 (36)	12 (4)	3 (2)	9 (16)	180 (177)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

Table 5: Prediction by division

Year	Division1	Division2	Division3	Division4	Division5	Division6	Division7	Division8	Division9	Division10
2001	3[1](0)[2]	0 (0)	1 (0)	0 (0)	0 (0)	56 (43)	11 (2)	5 (10)	0 (0)	5 (7)
2002	4 (0)	0 (0)	2 (0)	0 (0)	0 (0)	72 (54)	14 (2)	6 (38)	0 (0)	8 (32)
2003	6 (0)	1 (0)	2 (0)	0 (0)	0 (0)	89 (44)	19 (7)	6 (18)	0 (0)	9 (34)
2004	9 (0)	1 (0)	4 (0)	1 (0)	1 (0)	101 (96)	26 (29)	7 (13)	0 (0)	10 (12)
2005	13 (0)	1 (0)	5 (0)	1 (0)	1 (0)	88 (114)	20 (26)	8 (18)	0 (0)	14 (7)
2006	19 (34)	2 (0)	8 (0)	2 (5)	2 (3)	58 (105)	12 (32)	9 (12)	0 (0)	17 (16)
2007	23 (59)	3 (0)	12 (5)	3 (7)	3 (9)	26 (53)	3 (10)	12 (6)	0 (0)	24 (20)
2008	85 (97)	14 (23)	51 (79)	11 (11)	12 (13)	16 (16)	0 (0)	45 (24)	1 (0)	86 (82)
2009	27 (17)	5 (4)	15 (21)	4 (3)	4 (3)	0 (0)	0 (0)	16 (4)	0 (0)	27 (13)
2010	32 (25)	7 (10)	19 (20)	6 (4)	6 (4)	0 (0)	0 (0)	23 (6)	1 (4)	32 (21)
2011	38 (51)	9 (15)	23 (25)	7 (8)	7 (9)	0 (0)	0 (0)	28 (12)	1 (15)	34 (23)
2012	42 (36)	13 (15)	30 (15)	9 (3)	10 (3)	0 (0)	0 (0)	32 (14)	1 (8)	41 (15)

[1] the number before the parentheses is predicted retirement number.
[2] the number inside the parentheses is actual retirement number.

(a) Survival function **(b)** Hazard function **(c)** Cumulative hazard

Figure 6: Baselines with 95% confident intervals

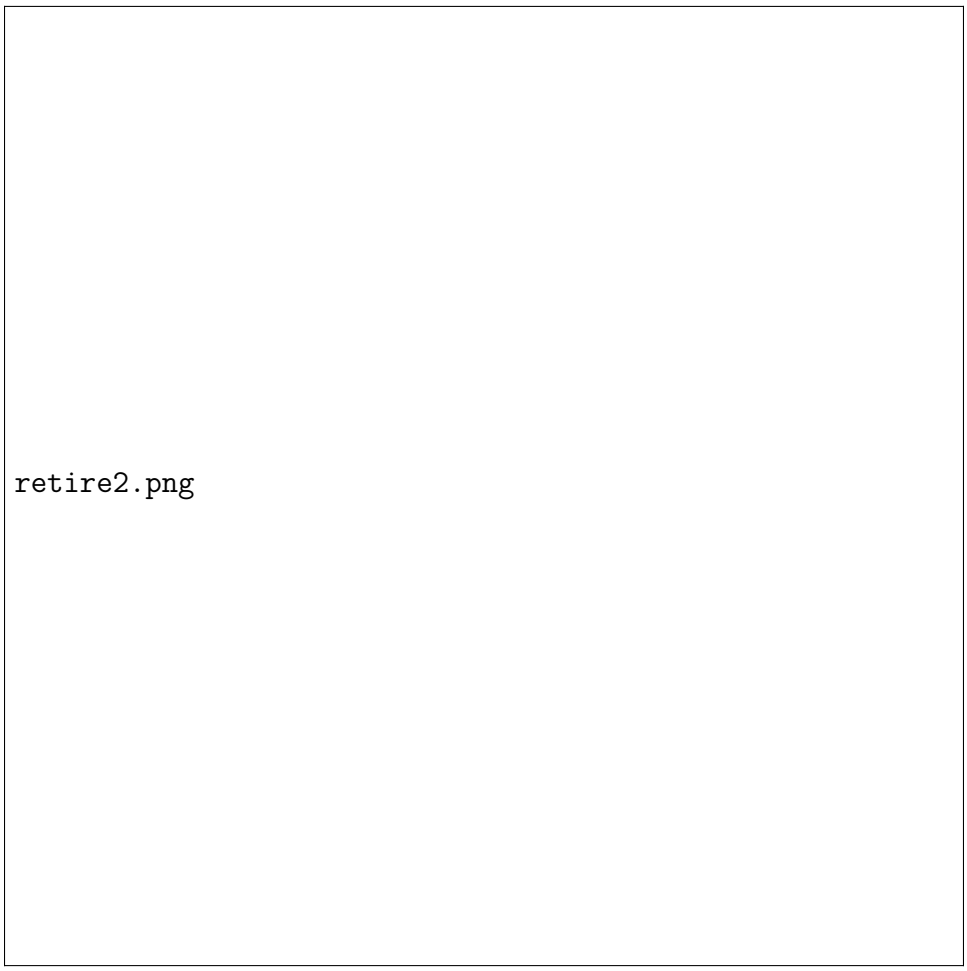


Figure 7: Retirement Forecasting